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The intrinsic relation between envelope solitons and plasma instabilities

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Abstract. A general form of the one-dimensional nonlinear Schrödinger equation is derived; the coefficients depend mainly on the linear dispersion relation, as does the nonlinear term. Envelope solitons may only exist together with some plasma instabilities. An example of solar radio bursts is selected as evidence for envelope solitons in the electromagnetic waveband.

1. Introduction

Plasma envelope solitons at the lower-hybrid frequency and the ion gyrofrequency have been detected by a series of spacecraft in the Earth's magnetosphere, and play important roles in plasma energization (heating or acceleration) by the interaction between the solar wind and magnetosphere (Chang 1993; Shapiro et al. 1993; Seyler 1994). Unfortunately, such direct measurements are not possible for most astrophysical objects, including the Sun. So far, the photosphere, chromosphere, and lower corona are characterized by electromagnetic radiation in different wavebands. The question arises whether envelope solitons at electromagnetic wavebands can exist? In particular, solar flares show a variety of pulses on different time scales, which are usually considered as evidence for plasma energization or energy release. Moreover, these activities may be associated with plasma instabilities (Huang 1998). Might envelope solitons provide another possible explanation for these activities – at least in some special cases?

2. A general form of the 1D NLSE

The dispersion relation of a weakly magnetized plasma was derived by Huang (1997) as

$$\varepsilon[(N^2-\varepsilon)^2-g^2\cos^2\theta]=0, \tag{1}$$

$$N_{\pm}^2=\varepsilon\pm g\cos\theta, \tag{2}$$

where

$$\varepsilon=1-\frac{\omega_{pe}^2}{\omega^2}, \qquad g=\frac{\omega_{pe}^2\omega_{ce}}{\omega^3},$$

$$N_{\pm}=\frac{k_{\pm}c}{\omega}$$

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are the refractive indices of ordinary (+) and extraordinary (−) modes, ω_{pe} and ω_{ce} are the electron plasma and cyclotron frequencies respectively, and θ is the angle between the wave vector $k_{\pm}$ and the ambient magnetic field. When $\theta = 90°$, the one-dimensional form of the cubic nonlinear wave equation can be obtained from the dispersion relation using the transformation $\omega \sim \partial/\partial t$ and $k \sim \partial/\partial x$:

$$\frac{\partial^2 E}{\partial t^2} - c^2 \frac{\partial^2 E}{\partial x^2} + \omega_{pe}^2 E = \gamma \frac{\partial^2 E^3}{\partial z^2}. \tag{3}$$

The nonlinear coefficient is derived as usual (Huang et al. 1997):

$$\gamma = \sum_{\alpha} \frac{\omega_{p\alpha}^2}{(\omega^2 - \omega_{c\alpha}^2)^2} \frac{q_\alpha^2}{m_\alpha^2}, \tag{4}$$

where α indicates the species of the charged particles, and $\omega_{p\alpha}$, $\omega_{c\alpha}$, q_α and m_α are respectively the plasma frequency, gyrofrequency, charge and mass of particle α. Let

$$E = \rho(\xi, t) \exp(ikz - i\omega t) + \text{c.c.}, \qquad \xi = x - ut,$$

where u is the group velocity of the envelope. For $\rho = O(\epsilon)$, with ϵ a small parameter,

$$\frac{\partial \rho}{\partial x} = O(\epsilon^2), \qquad \left(\frac{\partial \rho}{\partial t}\right)_{z-ut} = O(\epsilon^3).$$

If all terms of order ϵ^3 are kept, the nonlinear Schrödinger equation (NLSE) can be derived from (3):

$$i \frac{\partial \Lambda}{\partial \omega} \left(\frac{\partial \rho}{\partial t}\right)_{\xi} + \Lambda(k, \omega)\rho + \frac{1}{2} \frac{\partial \Lambda}{\partial \omega} \frac{d^2\omega}{dk^2} \frac{\partial^2 \rho}{\partial \xi^2} + \frac{9}{2} \gamma k^2 \rho^3 = 0. \tag{5}$$

The coefficients of the NLSE depend simply on the linear dispersion relation Λ_0, $\partial\Lambda_0/\partial_\omega$, and $d^2\omega/dk^2$, as does the nonlinear term. It is easy to find that the form of the NLSE is the same in the direction parallel to the magnetic field ($\theta = 0°$), even at lower frequencies, such as the lower-hybrid frequency and the ion gyrofrequency, which will be studied in forthcoming papers.

3. Relation between solitons and instabilities

The stable solution for envelope solitons in (5) is given by

$$\rho = \chi \operatorname{sech} \nu\xi, \tag{6}$$

$$\chi^2 = -\frac{4\Lambda_0}{9\gamma k^2}, \tag{7}$$

$$\nu^2 = -\frac{2\Lambda_0}{\dfrac{\partial \Lambda_0}{\partial \omega} \dfrac{d^2\omega}{dk^2}}, \tag{8}$$

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Note that ω and k for envelope solitons should satisfy the linear dispersion relation, at least in the initial phase, so that the solution does not exist for waves with small amplitudes ($\Lambda_0(k,\omega)=0$).

When plasma instabilities occur as a result of non-thermal components ($|\delta\Lambda| \ll \Lambda_0$), $\omega \approx \omega_r + \delta\omega (|\delta\omega| \ll \omega_r)$, the dispersion relation can be expanded as ($\Lambda_0(k,\omega_r) \equiv 0$)

$$\Lambda_0(k,\omega) \approx \Lambda_0(k,\omega_r) + \frac{\partial \Lambda_0}{\partial \omega_r}\delta\omega. \tag{9}$$

The coefficients of the envelope solitons become

$$\chi^2 = -\frac{4\dfrac{\partial \Lambda_0}{\partial \omega_r}\delta\omega}{9\gamma k^2}, \tag{10}$$

$$\nu^2 = -\frac{2\delta\omega}{\dfrac{d^2\omega}{dk^2}}. \tag{11}$$

The value of $\delta\omega$ is usually complex, with the imaginary part representing the growth rate of the instability; the real (or imaginary) part of the solution in (6) is still an envelope soliton ($\partial\Lambda_0/\partial\omega_r < 0$, $d^2\omega/dk^2 > 0$). Therefore an envelope soliton exists under the same conditions as the plasma instabilities (such as frequency, propagation angle, and ambient plasma parameters), and envelope solitons may be amplified simultaneously by plasma instabilities.

4. Observational evidence

A solar radio burst event observed by the 1–2 GHz spectrometer of Beijing Astronomical Observatory may provide evidence for envelope solitons at electromagnetic wavebands. There are 50 channels with 50 ms time resolution; the details of the spectrometer are given by Ji et al. (1997). Figure 1 shows a group of type III bursts with reversed frequency drifts on 15 April 1998, which may be explained by non-thermal electrons propagating upwards and downwards from an acceleration region (Huang et al. 1998). Solar type III bursts are usually associated with the beam–plasma instability amplification of Langmuir waves or electromagnetic waves (Huang 1998). For typical coronal parameters (density 10^{15} m^{-3} and magnetic field 10^{-2} T), the magnitude of γ in (4) is 10^4 MKS units, the amplitude of the electric field in (10) is 10^{-4} V m^{-1}, with a radio flux of 10^{-20} W m^{-2} Hz^{-1} or 100 s.f.u., which is comparable to the observations in Fig. 1. The growth rate, amplitude, and rise time of each pulse in Fig. 1 can be measured. Statistical linear correlations between growth rate and amplitude (Fig. 2) and between growth rate and rise time (Fig. 3) give correlation coefficients of 0.55 and -0.34 at 0.01 significance, which are consistent with the theoretical predictions for the envelope solitons in (10) and (11). Therefore the bursts in Fig. 1 may be explained by amplification of seed envelope solitons by beam–plasma instability at different frequencies (or heights) and times.

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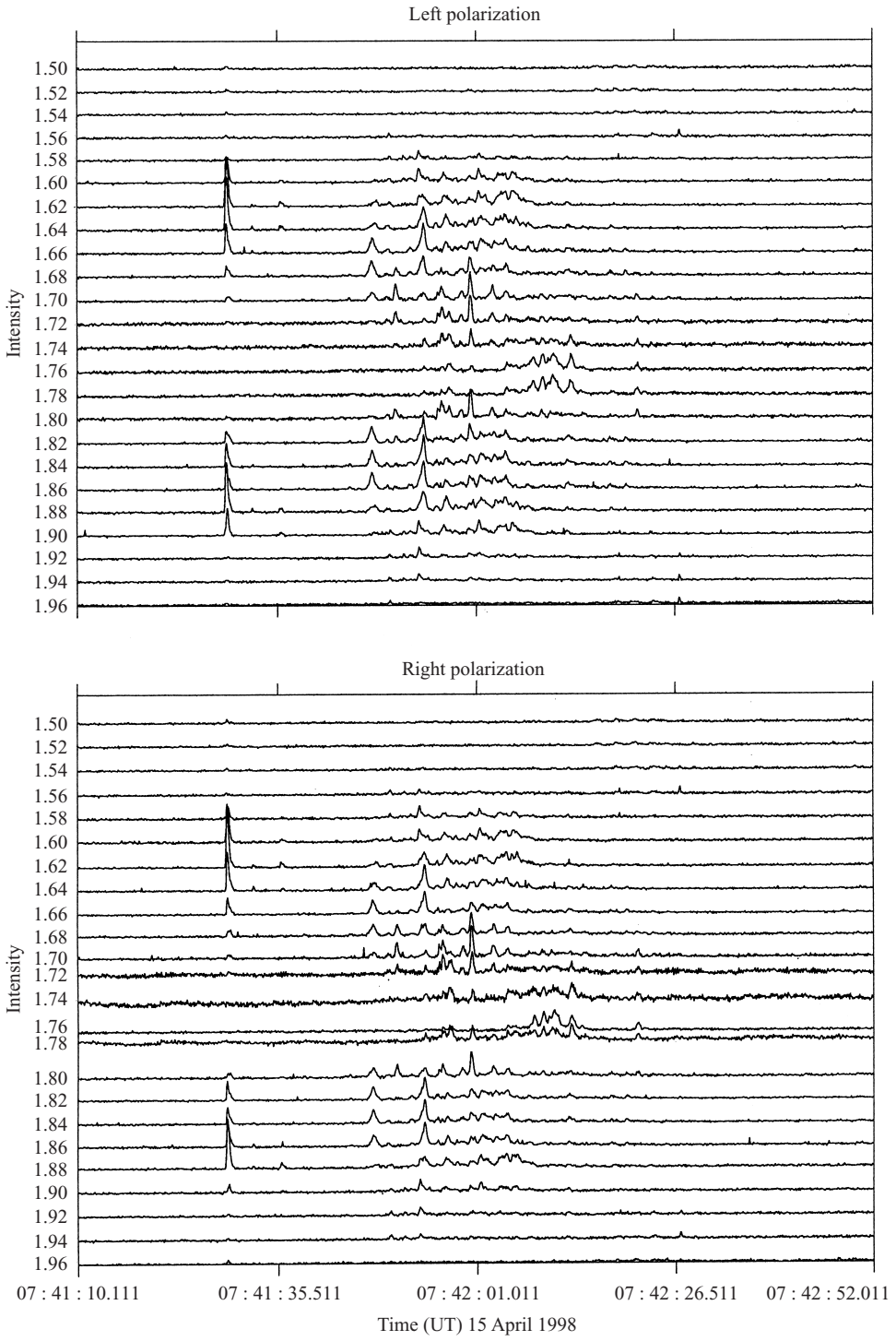


Figure 1. Observations using the Beijing Astronomical Observatory spectrometer at 1–2 GHz from 07:41:44.611 UT to 07:42:17.211 UT on 15 April 1998 (top: LC; bottom: RC).

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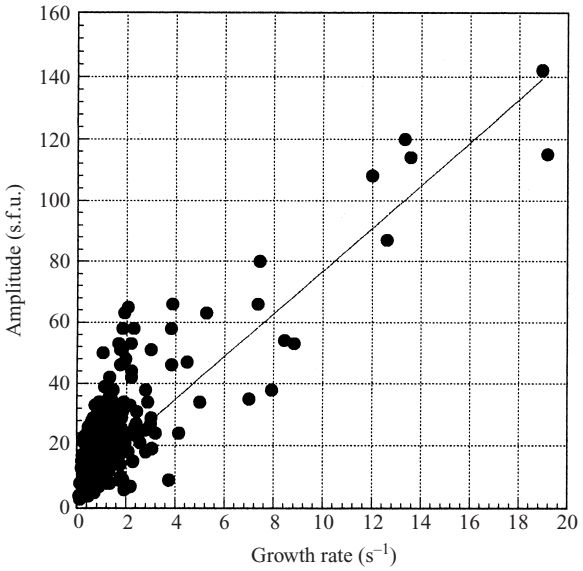


Figure 2. Linear correlation between the growth rate and the amplitude of the pulses in Fig. 1.

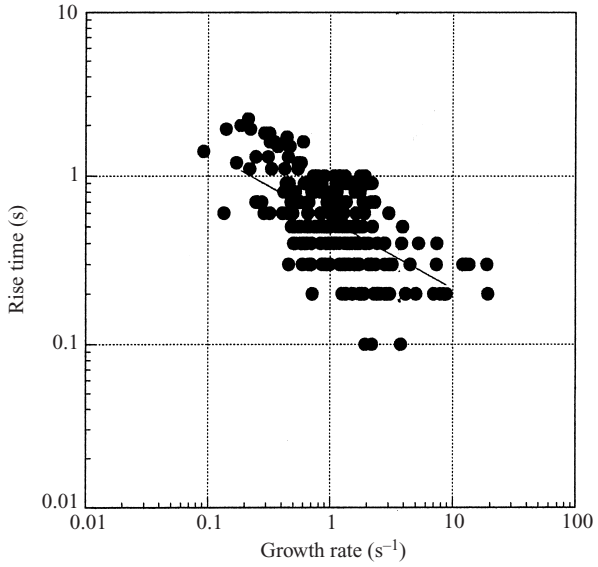


Figure 3. Linear correlation between the growth rate and the rise time of the pulse in Fig. 1.

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