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Adaptive control schemes for autonomous underwater vehicle

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SUMMARY

In this paper, two adaptive proportional-derivative (PD) control laws are proposed for autonomous underwater vehicle (AUV). The proposed adaptive controllers require only model of gravity and buoyancy regressor matrix and do not need any knowledge of inertia matrix, Coriolis and centripetal force, hydrodynamic damping, and the parameters of the gravity and buoyancy force. Hence, the proposed controllers have the advantages of simplicity and ease of implementation. We shall also prove that the setpoint controller is a special case of the region reaching controller when a desired region is reduced to a point. Furthermore, the inverse Jacobian matrix is not required in the control laws. Lyapunov-like functions are proposed for the stability analysis. Simulations are presented to demonstrate the effectiveness of the proposed controllers.

KEYWORDS: underwater vehicle; adaptive control; region reaching; stability.

1. Introduction

The ocean covers about 70% of the earth surface and has great effect on the future existence of all human beings, beside the land and aerospace. In order to explore the full depths of the ocean and its abundant resources, underwater research and development has been carried out rapidly during recent decades. Underwater robotics is no doubt an important scientific area due to its great applications that vary from scientific research of ocean, surveillance, inspection of commercial undersea facilities and installations to various military operations. Remotely operated vehicles (ROVs) and autonomous underwater vehicles (AUVs) are the main types of the underwater robots that received great attention from both the industrial and underwater research community. Due to advantages of operational efficiency, mobility, and low operational cost, AUV has received wider attention than ROV.

Several control schemes have been proposed for the AUV.[1–13] The proportional-derivative (PD) control plus gravitational and buoyancy compensation[1,5] is the simplest global setpoint control for the underwater vehicles. However, a drawback of this control strategy is that the exact knowledge of the gravitational and buoyancy force of the underwater vehicle, which depends on some parameters like water density and vehicle mass, is required. To overcome parametric uncertainties on the underwater vehicles dynamics, adaptive controllers have been introduced in refs. [6–10]. However, these controllers require a regressor matrix of the dynamic model which includes the inertia matrix, Coriolis and centripetal force, hydrodynamic damping, and gravitational and buoyancy force. Hence, the number of dynamic parameters to be updated by the adaptive law is very large. On the other hand, the common practice of using the proportional-integral-derivative (PID) control[1,13] in the control of underwater vehicles, which does not require any information of the underwater vehicles dynamics, is only valid in a local sense. Besides the conventional setpoint control problems of AUV, where the desired region is specified as a point, it is interesting to note that in some applications, the desired target could be specified as a region, for example, AUV traveling inside the pipeline for specific task, maintaining the AUV within a minimum and maximum depth in water, station keeping within a specified region for observation and data collection, and avoiding an obstacle located at a specified region.

Motivated by the results developed for robot manipulators,[14–19] we present an adaptive setpoint controller and an adaptive region reaching controller for AUV. This paper shows that the proposed controllers require only the gravity regressor matrix as compared with the previous adaptive control laws[6–10] for underwater vehicles which require the regressor matrix of inertia matrix, Coriolis and centripetal matrix, and hydrodynamic damping matrix. Since the dynamics of the underwater vehicles is very complex, the number of physical parameters to be updated by adaptive law is very large. Therefore, the proposed control laws for AUV which used only the gravity regressor are very simple and hence easy to implement. Furthermore, we also show that the inverse Jacobian matrix which results in large control actions at singular points is not required in the adaptive laws as compared with the previous adaptive control laws in the literature. We also prove formally that the region reaching controller is a generalization of setpoint control problem of AUV. By defining different desired regions and then reducing them to zero, the region controllers are reduced to setpoint controllers and new setpoint control laws can also be derived. Lyapunov-like functions are proposed for the stability analysis. Simulations are presented to demonstrate the effectiveness of the proposed controllers.

The organization of this paper is as follows. Section 2 summarizes an underwater vehicle's kinematic and dynamics model and its main properties. Section 3 proposes an adaptive setpoint controller and provides conditions on the controller gains to ensure stability. Section 4 proposes an adaptive

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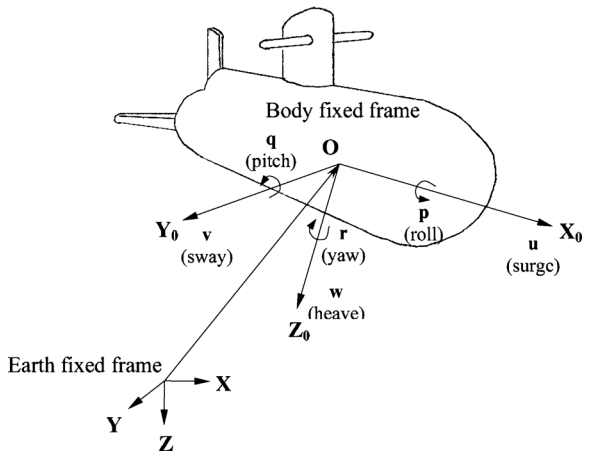


Fig. 1. Body fixed and earth fixed reference frames.

region reaching control and the stability analysis. Section 5 shows the simulations of the proposed control laws. Finally, Section 6 concludes this paper.

2. Dynamics

In this section, the structure and properties of an underwater vehicle's kinematics and dynamics are briefly reviewed. It is convenient to define the underwater vehicle state vectors according to the Society of Naval Architects and Marine Engineers (SNAME) notation.[1] Two common vectors that are being used in defining the underwater vehicle state vector are η and v. The vector η is defined as $\eta = [\eta_1^T \ \eta_2^T]^T$, where $\eta_1 = [x \ y \ z]^T$ is the vehicle position vector in the earth fixed frame and $\eta_2 = [\phi \ \theta \ \psi]^T$ is the vehicle Euler angle in the earth fixed frame. And the vector v is defined as $v = [v_1^T \ v_2^T]^T$, where $v_1 = [u \ v \ w]^T$ is the body fixed linear velocity vector and $v_2 = [p \ q \ r]^T$ is the body fixed angular velocity vector. In Fig. 1, the defined coordinate frames are illustrated. Besides the Euler angle representation, Euler parameters or unit quaternions[1,12] can also be used.

The vehicle's motion path relative to the earth fixed frame coordinate system is given by the kinematics equation[1] as follows:

$$\dot{\eta} = J(\eta)v = \begin{bmatrix} J_1(\eta) & 0 \\ 0 & J_2(\eta) \end{bmatrix} v, \tag{1}$$

where $J(\eta)$ is a 6×6 kinematics transformation matrix or a Jacobian matrix.

The dynamics behavior of an underwater vehicle is described through Newton's laws of linear and angular momentum. The equations of motion of underwater vehicles are highly nonlinear and coupled due to hydrodynamic added mass, lift, and drag forces, which act on the vehicle. The equations of motion of the underwater vehicle is given as [1]:

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau, \tag{2}$$

where $v \in R^6$ is the vector of linear and angular velocity of the AUV with respect to the vehicle's body fixed frame. The vector $\tau \in R^6$ is the vector of generalized forces on the AUV which is supplied by the thrusters. The matrix M is the inertia matrix of the AUV which includes both the rigid body and the added mass term. The matrix $C(v)$ denotes the matrix of Coriolis and centripetal forces which includes both the rigid body and the added mass terms. The vector $D(v)$ represents the hydrodynamic damping and lift force and the vector $g(\eta)$ denotes the gravitational and buoyancy force.

The underwater vehicle dynamics described in Eq. (2) has the following properties:[1]

Property 1. The inertia matrix M including the added mass is symmetric and positive definite such that $M = M^T > 0$.

Property 2. The Coriolis and centripetal matrix $C(v)$ is skew-symmetric matrix such that $C(v) = -C^T(v) \ \ \forall \ \ v \in R^6$.

Property 3. The hydrodynamic damping matrix $D(v)$ is strictly positive such that $D(v) > 0 \ \ \forall \ \ v \in R^6$.

Property 4. The gravitational and buoyancy force $g(\eta)$ can be completely characterized by a set of parameters $\theta = (\theta_1, \ldots, \theta_m)^T$ [16] as

$$g(\eta) = Z(\eta)\theta, \tag{3}$$

where $Z(\theta) \in R^{n \times m}$ is the gravity regressor which represents the known part of $g(\eta)$ and $\theta \in R^{m \times 1}$ is the unknown part of $g(\eta)$.

3. Adaptive Setpoint Control of Autonomous Underwater Vehicle

In this paper, we propose an adaptive saturated proportional-derivative (SP-D) control law with uncertain gravitational and buoyancy force compensation using gravity regressor.

The adaptive SP-D control law is proposed as

$$\tau = -K_p J^T(\eta)s(e) - K_v v + Z(\eta)\hat{\theta}, \tag{4}$$

$$\dot{\hat{\theta}} = -L^{-1}Z^T(\eta)\left(v + \alpha J^T(\eta)s(e)\right), \tag{5}$$

where $e = \eta - \eta_d = (e_1, \ldots, e_n)^T \in R^n$ is the positional deviation from a desired position; $s(e) = (s_1(e_1), \ldots, s_n(e_n))^T$ $[s_i(\cdot), i = 1, \ldots, n]$ are saturated functions to be defined; $K_p = k_p I$ and $K_v = k_v I$ are positive feedback gains for the position and velocity, respectively; L is a positive definite diagonal matrix; α is a positive constant.

Note that the above controller does not require the knowledge of the inertia matrix, Coriolis and centripetal matrix, hydrodynamic damping matrix, and the inverse of the Jacobian matrix, $J(\eta)$.

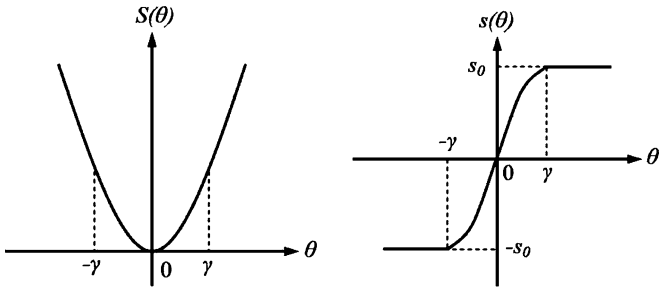
Let us define a scalar function $S_i(\theta)$ and its derivative $s_i(\theta)$ as shown in Fig. 2 with the following properties:[16]

1. $S_i(\theta) > 0$ for $\theta \neq 0$ and $S_i(0) = 0$.
2. $S_i(\theta)$ is twice continuously differentiable, and the derivative $s_i(\theta) = \frac{dS_i(\theta)}{d\theta}$ is strictly increasing in θ for $|\theta| < \gamma_i$ with some γ_i and saturated for $|\theta| \geq \gamma_i$, i.e. $S_i(\theta) = \pm s_i$ for $\theta \geq +\gamma_i$ and $\theta \leq -\gamma_i$, respectively.
3. There is constant $\bar{c}_i > 0$ such that

$$S_i(\theta) \geq \bar{c}_i {s_i}^2(\theta) \tag{6}$$

for $\theta \neq 0$.

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(a) (b)

Fig. 2. (a) Quasi-natural potential: $S(\theta)$; (b) Derivative of $S(\theta)$: $s(\theta)$.

Some examples of the saturation function can be found in refs. [16] and [21].

Substituting (4) into (2) and using (3), we obtain the closed-loop equation as follows:

$$M\dot{v} + C(v)v + D(v)v + K_p J^T(\eta)s(e) + K_v v + Z(\eta)\Delta\theta = 0, \tag{7}$$

where $\Delta\theta = \theta - \hat{\theta}$. We define a Lyapunov function candidate V as

$$V = \frac{1}{2}v^T Mv + \alpha s(e)^T J(\eta)Mv + \frac{1}{2}\Delta\theta^T L\Delta\theta + \sum_{i=1}^{n}(k_p + \alpha k_v)S_i(e_i). \tag{8}$$

To show the positive definiteness of the Lyapunov function candidate V in (8), we note that

$$\begin{aligned}
&\frac{1}{4}v^T Mv + \alpha s(e)^T J(\eta)Mv + \sum_{i=1}^{n}(k_p + \alpha k_v)S_i(e_i) \\
&= \frac{1}{4}(v + 2\alpha J^T(\eta)s(e))^T M(v + 2\alpha J^T(\eta)s(e)) \\
&- \alpha^2 s(e)^T J(\eta)MJ^T(\eta)s(e) + \sum_{i=1}^{n}(k_p + \alpha k_v)S_i(e_i) \\
&\geq \sum_{i=1}^{n}(k_p + \alpha k_v)\bar{c}_i s_i^2(e_i) - \alpha^2 s(e)^T J(\eta)MJ^T(\eta)s(e) \\
&\geq \sum_{i=1}^{n}(k_p + \alpha(k_v\bar{c}_i - \alpha\lambda_m))s_i^2(e_i),
\end{aligned} \tag{9}$$

where $\lambda_m \triangleq \lambda_{\max}[J(\eta)MJ^T(\eta)]$. Substituting the above inequality into (8), we have

$$V \geq \frac{1}{4}v^T Mv + \frac{1}{2}\Delta\theta^T L\Delta\theta + \sum_{i=1}^{n}\left(k_p + \alpha(k_v\bar{c}_i - \alpha\lambda_m)\right)s_i^2(e_i) > 0, \tag{10}$$

where k_v can be chosen sufficiently large and α can be chosen sufficiently small so that

$$k_v\bar{c}_i - \alpha\lambda_m > 0. \tag{11}$$

Hence, V is positive definite in v, $s(e)$, and $\Delta\theta$.

Next, differentiating V with respect to time and using **Property 1**, we have

$$\begin{aligned}
\dot{V} &= v^T M\dot{v} + \alpha\dot{s}(e)^T J(\eta)Mv + \alpha s(e)^T \dot{J}(\eta)Mv \\
&+ \alpha s(e)^T J(\eta)M\dot{v} - \Delta\theta^T L\dot{\hat{\theta}} \\
&+ \sum_{i=1}^{n}(k_p + \alpha k_v)s(e)^T J(\eta)v.
\end{aligned} \tag{12}$$

From (7), we have

$$\begin{aligned}
M\dot{v} = &- \{C(v)v + D(v)v + K_p J^T(\eta)s(e) \\
&+ K_v v + Z(\eta)\Delta\theta\}.
\end{aligned} \tag{13}$$

Substituting (13) into (12) and using (5) and **Property 2**, we have

$$\dot{V} = -W, \tag{14}$$

where

$$\begin{aligned}
W = &v^T K_v v + v^T Dv + \alpha s(e)^T J(\eta)K_p J^T(\eta)s(e) \\
&+ \alpha\{s(e)^T J(\eta)C(v)v + s(e)^T J(\eta)D(v)v \\
&- \dot{s}(e)^T J(\eta)Mv - s(e)^T \dot{J}(\eta)Mv\}.
\end{aligned} \tag{15}$$

From the last term on the right-hand side of (15), since $s(e)$ is bounded, there exist constants $c_0 > 0$ and $c_1 > 0$ so that[16]

$$\begin{aligned}
&\alpha \mid s(e)^T J(\eta)C(v)v + s(e)^T J(\eta)D(v)v \\
&\quad - \dot{s}(e)^T J(\eta)Mv - s(e)^T \dot{J}(\eta)Mv \mid \\
&\geq -\alpha c_0\|v\|^2 - \alpha c_1\|s(e)\|^2.
\end{aligned} \tag{16}$$

Substituting inequality (16) into (15) yields

$$\begin{aligned}
W &\geq v^T(k_v I + D(v) - \alpha c_0 I)v \\
&+ s(e)^T(\alpha J(\eta)k_p J^T(\eta) - \alpha c_1 I)s(e), \\
&\geq (k_v + \lambda_{\min}[D(v)] - \alpha c_0)\|v\|^2 \\
&+ \alpha(k_p\lambda_{\min}[J(\eta)J^T(\eta)] - c_1)\|s(e)\|^2,
\end{aligned} \tag{17}$$

where k_p and k_v can be chosen sufficiently large and α can be chosen sufficiently small so that

$$\begin{aligned}
k_v + \lambda_{\min}[D(v)] - \alpha c_0 > 0, \\
k_p\lambda_{\min}[J(\eta)J^T(\eta)] - c_1 > 0,
\end{aligned} \tag{18}$$

and hence W is positive definite in v and $s(e)$.

The asymptotic stability of the adaptive setpoint control for AUV with uncertain gravitational and buoyancy force compensation is specified by the following theorem.

Theorem 1. *The closed-loop system described by (7) gives rise to the convergence of η to η_d and v to 0 as $t \to \infty$ if the feedback gains k_p, k_v, and α are chosen to satisfy conditions (11) and (18).*

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Proof: Since V is positive definite in v, $s(e)$, and $\Delta\theta$ and W are positive definite in v and $s(e)$, from (14), we have

$$\dot{V} = -W \leq 0. \tag{19}$$

Therefore, LaSalle's invariance theorem implies that $\eta_d - \eta \to 0$, $v \to 0$ as $t \to \infty$ for any initial position $\eta(0)$ and $v(0)$.

4. Region Reaching Control of Autonomous Underwater Vehicle

In some applications of AUV, the desired target is a region instead of a point, for example, maintaining the AUV within a minimum and maximum depth in water, AUV traveling inside the pipeline for specific task, station keeping within a specified region for observation and data collection. In this section, we present an adaptive region reaching control law where the desired objective can be specified as a closed or open region, instead of a point. The region reaching controller is also proposed for macro/mini structures such as an underwater vehicle manipulator system.[20] However, the inverse Jacobian matrix is required in the update law and hence it results in large control actions near singular points. In this paper, the region controller does not require the use of inverse Jacobian matrix. In addition, we shall also prove formally that the region reaching controller is a generalization of setpoint controller in the sense that when a desired region is reduced to zero, the region controller is reduced to the setpoint controller.

In the region reaching control, the desired region can be specified as multiple objective functions as follows:

$$f(\delta\eta_o) = \begin{bmatrix} f_1(\delta\eta_{o_1}) \\ f_2(\delta\eta_{o_2}) \\ \vdots \\ f_N(\delta\eta_{o_N}) \end{bmatrix} \leq 0, \tag{20}$$

where $\delta\eta_{o_i} = \eta_i - \eta_{o_i}$, η_{o_i} is the reference point of the ith desired region ($i = 1, 2, \ldots, N$), N is the total number of objective functions, and $f_i(\delta\eta_{o_i}) \in R$ are scalar functions with continuous partial derivatives. Note that the desired region is a single region specified as the intersections of all the objective functions.

For example, the desired region can be specified as spheres using the following objective function:

$$f_1(\delta\eta_{o_1}) = (x - x_{o_1})^2 + (y - y_{o_1})^2 + (z - z_{o_1})^2 - r_1^2 \leq 0, \tag{21}$$

They can also be specified as an intersection of two or multiple spherical regions, by adding additional objective functions as follows:

$$\begin{aligned} &f_2(\delta\eta_{o_2}) = \\ &\quad (x - x_{o_2})^2 + (y - y_{o_2})^2 + (z - z_{o_2})^2 - r_2^2 \leq 0, \\ &\qquad\qquad \vdots \\ &f_N(\delta\eta_{o_N}) = \\ &\quad (x - x_{o_N})^2 + (y - y_{o_N})^2 + (z - z_{o_N})^2 - r_N^2 \leq 0. \end{aligned} \tag{22}$$

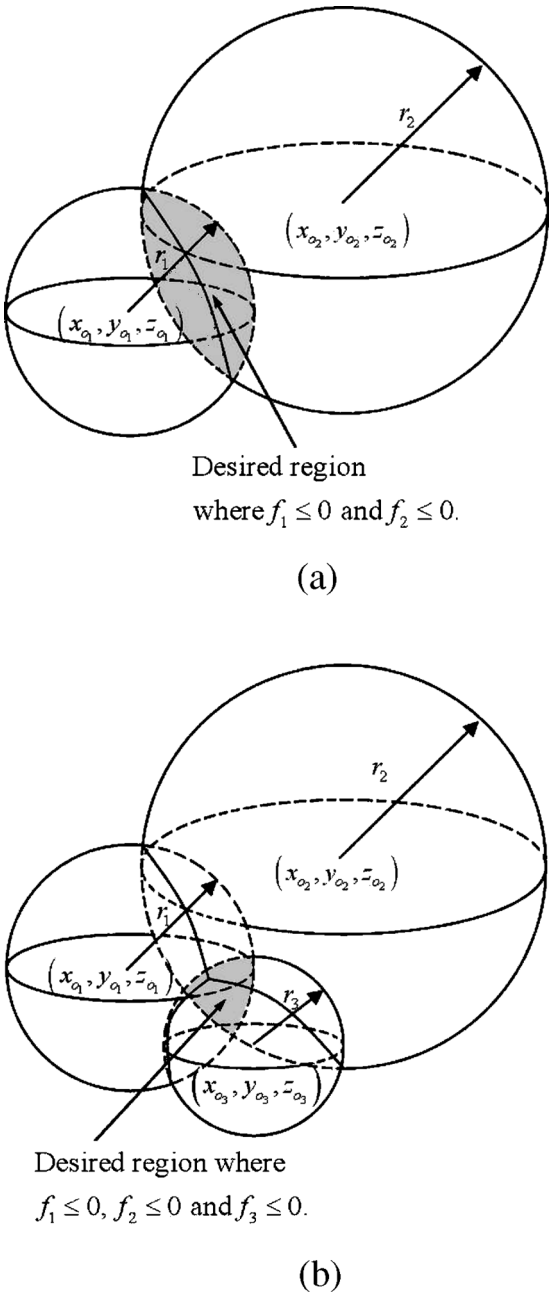


Fig. 3. Desired region as the intersection of multiple spherical regions. (a) Intersection of two spherical regions; (b) Intersection of three spherical regions.

That is, when both $f_1(\delta\eta_{o_1}) \leq 0$ and $f_2(\delta\eta_{o_2}) \leq 0$, the desired region is represented by the intersection of the two spherical regions as shown in Fig. 3(a). Figure 3(b) shows the desired region as an intersection of three spherical regions.

Another example of the desired region is shown in Fig. 4. The inequality functions that describe this region are specified as

$$\begin{aligned} f_1(\delta\eta_{o_1}) &= (x - x_o)^2 - \epsilon_x^2 \leq 0, \\ f_2(\delta\eta_{o_2}) &= (y - y_o)^2 - \epsilon_y^2 \leq 0, \\ f_3(\delta\eta_{o_3}) &= (z - z_o)^2 - \epsilon_z^2 \leq 0, \end{aligned} \tag{23}$$

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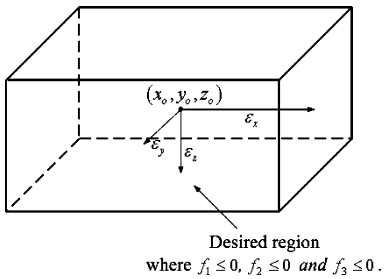


Fig. 4. Desired region specified as a rectangular block.

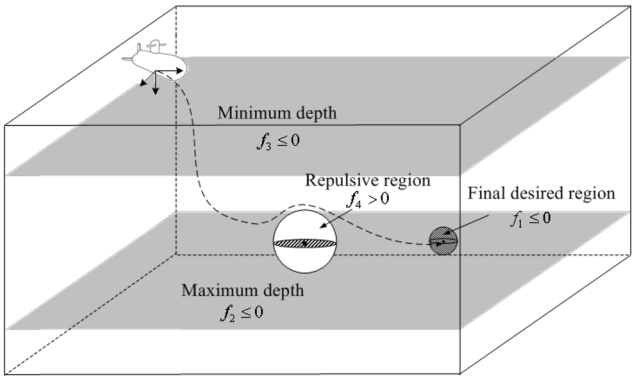


Fig. 5. Illustration of desired region and repulsive region.

where ϵ_x, ϵ_y, and ϵ_z are the individual regional bounds for each axis.

Besides the desired regions, repulsive regions can also be specified for obstacle avoidance purpose. For example, an objective function for repulsive region can be added as follows:

$$f_4(\delta\eta_{o4}) = r_o^2 - (x - x_{o4})^2 - (y - y_{o4})^2 - (z - z_{o4})^2 < 0, \tag{24}$$

where r_o is the reference point of the repulsive region. Figure 5 shows the graphical illustration of the desired region and the repulsive region.

The potential energy function for the desired regions described in inequality (20) can be specified respectively as

$$P(\delta\eta_o) = \sum_{i=1}^{N} P_i(\delta\eta_{o_i}), \tag{25}$$

where

$$P_i(\delta\eta_{o_i}) = \frac{k_{p_i}}{2}\left[\max(0,\, f_i(\delta\eta_{o_i}))\right]^2. \tag{26}$$

That is,

$$P_i(\delta\eta_{o_i}) = \begin{cases} 0, & f_i(\delta\eta_{o_i}) \le 0, \\ \frac{k_{p_i}}{2} f_i^2(\delta\eta_{o_i}), & f_i(\delta\eta_{o_i}) > 0, \end{cases} \tag{27}$$

where k_{p_i} are positive constants. Note that $P(\delta\eta_o) = 0$ only if all the inequality functions (25) are satisfied.

Partial differentiating the potential energy function described by (25) and (27) with respect to η yields

$$\left(\frac{\partial P(\delta\eta_o)}{\partial\eta}\right)^T = \sum_{i=1}^{N} \frac{\partial P_i(\delta\eta_{o_i})^T}{\partial\eta} = \sum_{i=1}^{N} k_{p_i} \max(0,\, f_i(\delta\eta_{o_i})) \left(\frac{\partial f_i(\delta\eta_{o_i})}{\partial\eta}\right)^T. \tag{28}$$

The region reaching controller with uncertain gravitational and buoyancy force compensation is proposed as

$$\tau = -J^T(\eta) \sum_{i=1}^{N} k_{p_i} \max(0,\, f_i(\delta\eta_{o_i})) \left(\frac{\partial f_i(\delta\eta_{o_i})}{\partial\eta}\right)^T - J^T(\eta) K_v J(\eta) v + Z(\eta)\hat{\varphi}, \tag{29}$$

$$\dot{\hat{\varphi}} = -L^{-1} Z^T(\eta) * \left(v + \alpha J^T(\eta)\left[\sum_{i=1}^{N} a_i \max(0,\, f_i(\delta\eta_{o_i})) \left(\frac{\partial f_i(\delta\eta_{o_i})}{\partial\eta}\right)^T\right]\right), \tag{30}$$

where $K_v = k_v I \in R^{n\times n}$ is a positive diagonal matrix, L is a positive definite matrix, α is a positive constant, $a_i = k_{p_i}/k_p$ is defined as the weight for each objective function, k_p is a positive constant, $\max(0,\, f_i(\delta\eta_{o_i}))(\frac{\partial f_i(\delta\eta_{o_i})}{\partial\eta})^T$ is the vehicle region error, and $*$ denotes multiplication.

For clarity of presentation, let

$$\overline{\delta\xi_o} = \sum_{i=1}^{N} \overline{\delta\eta_{o_i}}, \tag{31}$$

where

$$\overline{\delta\eta_{o_i}} = a_i \max(0,\, f_i(\delta\eta_{o_i})) \left(\frac{\partial f_i(\delta\eta_{o_i})}{\partial\eta}\right)^T. \tag{32}$$

Substituting (29) into (2) and using (3) and (31), the closed-loop equation is obtained as

$$M\dot{v} + C(\eta)v + D(\eta)v + J^T(\eta)K_p\overline{\delta\xi_o} + J^T(\eta)K_v J(\eta)v + Z(\eta)\Delta\varphi = 0, \tag{33}$$

where $\Delta\varphi = \varphi - \hat{\varphi}$.

Next, a Lyapunov-like function is defined as

$$V_1 = \frac{1}{2}v^T M v + \alpha\overline{\delta\xi_o}^T J(\eta) M v + \frac{1}{2}(k_p + \alpha k_v) \sum_{i=1}^{N} a_i \left[\max(0,\, f_i(\delta\eta_{o_i}))\right]^2 + \frac{1}{2}\Delta\varphi^T L \Delta\varphi. \tag{34}$$

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From (32) note that

$$\left[\max(0,\ f_i(\delta\eta_{o_i}))\right]^2 = \frac{1}{a_i^2}\frac{\overline{\delta\eta_{o_i}}^T\overline{\delta\eta_{o_i}}}{\left\|\frac{\partial f_i(\delta\eta_{o_i})}{\partial\eta}\right\|^2}. \tag{35}$$

Taking norm on (31) gives

$$\|\overline{\delta\xi_o}\| \le \|\overline{\delta\eta_{o_1}}\| + \cdots + \|\overline{\delta\eta_{o_N}}\|, \tag{36}$$

and hence

$$\left\|\overline{\delta\xi_o}\right\|^2 \le N\sum_{i=1}^{N}\left\|\overline{\delta\eta_{o_i}}\right\|^2. \tag{37}$$

By using (35) and (37), note that a part of the terms in the proposed Lyapunov-like function (34) can be written as

$$\begin{aligned}
&\frac{1}{4}v^TMv + \alpha\overline{\delta\xi_o}^TJ(\eta)Mv + \frac{1}{2}(k_p+\alpha k_v)\sum_{i=1}^{N}\frac{1}{a_i}\frac{\overline{\delta\eta_{o_i}}^T\overline{\delta\eta_{o_i}}}{\left\|\frac{\partial f_i(\delta\eta_{o_i})}{\partial\eta}\right\|^2}\\
&= \frac{1}{4}\left(v+2\alpha J^T(\eta)\overline{\delta\xi_o}\right)^TM\left(v+2\alpha J^T(\eta)\overline{\delta\xi_o}\right)\\
&-\alpha^2\overline{\delta\xi_o}^TJ(\eta)MJ^T(\eta)\overline{\delta\xi_o}\\
&+\frac{1}{2}(k_p+\alpha k_v)\sum_{i=1}^{N}\frac{1}{a_i}\frac{\overline{\delta\eta_{o_i}}^T\overline{\delta\eta_{o_i}}}{\left\|\frac{\partial f_i(\delta\eta_{o_i})}{\partial\eta}\right\|^2}\\
&\ge \frac{1}{2}(k_p+\alpha k_v)a_{\min}b_{\min}\sum_{i=1}^{N}\left\|\overline{\delta\eta_{o_i}}\right\|^2 - \alpha^2\lambda_1N\sum_{i=1}^{N}\left\|\overline{\delta\eta_{o_i}}\right\|^2,
\end{aligned} \tag{38}$$

where $a_{\min} \triangleq \min_i(\frac{1}{a_i})$, $b_{\min} \triangleq \min_i\left(\frac{1}{\|\frac{\partial f_i(\delta\eta_{o_i})}{\partial\eta}\|^2}\right)(i=1,2,\ldots,N)$, and $\lambda_1 \triangleq \lambda_{\max}[J(\eta)MJ^T(\eta)]$.

Substituting the above inequality into the Lyapunov-like function (34) yields

$$\begin{aligned}
V_1 &\ge \frac{1}{4}v^TMv + \frac{1}{2}\Delta\varphi^TL\Delta\varphi\\
&+\left(\frac{1}{2}(k_p+\alpha k_v)a_{\min}b_{\min} - \alpha^2\lambda_1N\right)\sum_{i=1}^{N}\left\|\overline{\delta\eta_{o_i}}\right\|^2.
\end{aligned} \tag{39}$$

Therefore, α can be chosen sufficiently small and k_p and k_v can be chosen sufficiently large so that

$$\frac{1}{2}(k_p+\alpha k_v)a_{\min}b_{\min} - \alpha^2\lambda_1N > 0, \tag{40}$$

and hence V_1 is locally positive definite in v, $\overline{\delta\eta_{o_i}}$, and $\Delta\varphi$.

Differentiating (34) with respect to time, substituting (30) and (33) into it, and using **Property 2** gives

$$\dot{V}_1 = -W_1, \tag{41}$$

where

$$\begin{aligned}
W_1 &= v^TJ^T(\eta)K_vJ(\eta)v + v^TD(\eta) + \alpha\overline{\delta\xi_o}^TK_p\overline{\delta\xi_o}\\
&+v^TJ^T(\eta)K_p\overline{\delta\xi_o} + \alpha\overline{\delta\xi_o}^TK_vJ(\eta)\zeta\\
&-(k_p+\alpha k_v)\sum_{i=1}^{N}\overline{\delta\eta_{o_i}}\,\dot{\eta} + \alpha\left\{\overline{\delta\xi_o}^TJ(\eta)C(\eta)v\right.\\
&+\overline{\delta\xi_o}^TJ(\eta)D(\eta)v - \dot{\overline{\delta\xi_o}}^TJ(\eta)Mv - \overline{\delta\xi_o}^T\dot{J}(\eta)Mv\\
&\left.-\overline{\delta\xi_o}^TJ(\eta)\dot{M}v\right\}.
\end{aligned} \tag{42}$$

Note that from the last term on the right-hand side of (42), there exists a constant $c_0 > 0$ locally so that[16]

$$\begin{aligned}
\alpha\Big|&\overline{\delta\xi_o}^TJ(\eta)C(\eta)v + \overline{\delta\xi_o}^TJ(\eta)D(\eta)v - \dot{\overline{\delta\xi_o}}^TJ(\eta)Mv\\
&-\overline{\delta\xi_o}^T\dot{J}(\eta)Mv - \overline{\delta\xi_o}^TJ(\eta)\dot{M}v\Big| \ge -\alpha c_0\|v\|^2,
\end{aligned} \tag{43}$$

where it is noted that $\dot{\overline{\delta\xi_o}}$ is a function of η and $\dot{\eta} = J(\eta,q)v$. Substituting (43) into (42) yields

$$\begin{aligned}
W_1 &\ge v^T(J^T(\eta)K_vJ(\eta) + D(\eta) - \alpha c_0I)v + \alpha\overline{\delta\xi_o}^TK_p\overline{\delta\xi_o}\\
&\ge \left(\lambda_{\min}\left[J^T(\eta)K_vJ(\eta)\right] + \lambda_{\min}[D(\eta)] - \alpha c_0\right)\|v\|^2\\
&+\alpha(\lambda_{\min}[K_p])\|\overline{\delta\xi_o}\|^2,
\end{aligned} \tag{44}$$

where $K_p = k_pI$ and $K_v = k_vI$ can be chosen sufficiently large and α can be chosen sufficiently small so that

$$\begin{aligned}
\lambda_{\min}\left[J^T(\eta)K_vJ(\eta)\right] + \lambda_{\min}[D(\eta)] - \alpha c_0 > 0,\\
\alpha\lambda_{\min}[K_p] > 0,
\end{aligned} \tag{45}$$

and hence W_1 is positive definite in v and $\overline{\delta\xi_o}$.

A compact set Ω is considered in the state space as

$$\Omega = \{(v,\eta) : V_1 \le \gamma\} \tag{46}$$

where $P(\delta\eta_o)$ has isolated minimums at the desired regions with a positive γ. If $v(0)$, $\eta(0) \in \Omega$, then $(v,\eta) \in \Omega$ since $\dot{V}_1 \le 0$. The stability of the region control for AUV with uncertain gravitational and buoyancy force is specified by the following theorem.

Theorem 2. *The closed-loop system described by (33) gives rise to the convergence of η to the desired region $f(\delta\eta_o) = \left[f_1(\delta\eta_{o_1})\ \ f_2(\delta\eta_{o_2})\ \cdots\ f_N(\delta\eta_{o_N})\right]^T \le 0$ and v to 0 as $t \to \infty$ if the feedback gains K_p and K_v are chosen to satisfy conditions (40) and (45).*

Proof: By using invariant set theorem, since $\dot{V}_1 = 0$ implies that $v = 0$ and $\overline{\delta\xi_o} = 0$, hence $v \to 0$, $\overline{\delta\xi_o} \to 0$ as $t \to \infty$. This implies finally that $f(\delta\eta_o) = \left[f_1(\delta\eta_{o_1})\ \ f_2(\delta\eta_{o_2})\ \cdots\ f_N(\delta\eta_{o_N})\right]^T \le 0$ in Ω as $t \to \infty$.

Remark: The stability conditions (11), (18), (40), and (45) presented in the theorem are sufficient conditions to

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guarantee the stability of the underwater vehicle system in the presence of uncertain gravitational and buoyancy force. Conditions (11), (18), (40), and (45) simply mean that the feedback gains k_v and k_p should be chosen sufficiently large as compared to α. A larger k_p typically implies faster response and a larger k_v decreases overshoot, but slows down transient response.

4.1. Setpoint control as a special case

When the desired region is specified arbitrarily small, it reduces to a point. Hence, the region control concept is a generalization of the setpoint control problem. In fact, it is interesting to note that *any closed region with arbitrary shape reduces to a point as the region reduces to zero*. In this section, we revisit the setpoint control problem from region control point of view to gain new insights into the setpoint control problem. We show that the setpoint controller is a special case of the proposed region reaching controller.

First, consider an AUV with six degrees of freedom and let $\eta_1 = [x \;\; y \;\; z]^T$ represent the position and $\eta_2 = [\phi \;\; \theta \;\; \psi]^T$ represent the orientation of the AUV. Let the desired regions of the position and orientation be specified as follows:

$$\begin{aligned} &f_1(\delta\eta_{o_1}) = \\ &\quad \sqrt{(x-x_o)^2 + (y-y_o)^2 + (z-z_o)^2} - \epsilon_{xyz} \le 0, \\ &f_2(\delta\eta_{o_2}) = \\ &\quad \sqrt{(\phi-\phi_o)^2 + (\theta-\theta_o)^2 + (\psi-\psi_o)^2} - \epsilon_{\phi\theta\psi} \le 0, \end{aligned} \tag{47}$$

where ϵ_{xyz} and $\epsilon_{\phi\theta\psi}$ are nonnegative scalars. Hence

$$\begin{aligned} &\left(\frac{\partial f_1(\delta\eta_{o_1})}{\partial \eta}\right)^T = \\ &\quad \frac{1}{\sqrt{(x-x_o)^2 + (y-y_o)^2 + (z-z_o)^2}} \begin{bmatrix} \delta\eta_{o_1} \\ 0 \end{bmatrix}, \\ &\left(\frac{\partial f_2(\delta\eta_{o_2})}{\partial \eta}\right)^T = \\ &\quad \frac{1}{\sqrt{(\phi-\phi_o)^2 + (\theta-\theta_o)^2 + (\psi-\psi_o)^2}} \begin{bmatrix} 0 \\ \delta\eta_{o_2} \end{bmatrix}, \end{aligned} \tag{48}$$

where $\delta\eta_{o_1} = [x - x_o \;\; y - y_o \;\; z - z_o]^T$ and $\delta\eta_{o_2} = [\phi - \phi_o \;\; \theta - \theta_o \;\; \psi - \psi_o]^T$. If ϵ_{xyz} and $\epsilon_{\phi\theta\psi}$ reduce to zero, then $f_1(\delta\eta_{o_1})$ and $f_2(\delta\eta_{o_2})$ are nonnegative and hence

$$\begin{aligned} &\max\left(0,\, f_1(\delta\eta_{o_1})\right)\left(\frac{\partial f_1(\delta\eta_{o_1})}{\partial \eta}\right)^T \\ &\quad = f_1(\delta\eta_{o_1})\left(\frac{\partial f_1(\delta\eta_{o_1})}{\partial \eta}\right)^T = \begin{bmatrix} \delta\eta_{o_1} \\ 0 \end{bmatrix}, \\ &\max\left(0,\, f_2(\delta\eta_{o_2})\right)\left(\frac{\partial f_2(\delta\eta_{o_2})}{\partial \eta}\right)^T \\ &\quad = f_2(\delta\eta_{o_2})\left(\frac{\partial f_2(\delta\eta_{o_2})}{\partial \eta}\right)^T = \begin{bmatrix} 0 \\ \delta\eta_{o_2} \end{bmatrix}. \end{aligned} \tag{49}$$

Based on Eq. (29), the region reaching controller becomes

$$\tau = -J^T(\eta)K_p\delta\eta - J^T(\eta)K_vJ(\eta)v + Z(\eta)\hat{\varphi}, \tag{50}$$

$$\dot{\hat{\varphi}} = -L^{-1}Z^T(\eta)\left(v + \alpha J^T(\eta)\delta\eta\right), \tag{51}$$

which is the setpoint controller where $\delta\eta = [\delta\eta_{o_1}^T \;\; \delta\eta_{o_2}^T]^T$.

Next, we consider the case where the desired regions are specified as rectangular regions as follows:

$$\begin{aligned} f_1(\delta\eta_{o_1}) &= (\eta_{x_1} - \eta_{o_1})^2 - \epsilon_{\eta_{x_1}}^2 \le 0, \\ f_2(\delta\eta_{o_2}) &= (\eta_{x_2} - \eta_{o_2})^2 - \epsilon_{\eta_{x_2}}^2 \le 0, \\ &\vdots \\ f_n(\delta\eta_{o_n}) &= (\eta_{x_n} - \eta_{o_n})^2 - \epsilon_{\eta_{xn}}^2 \le 0, \end{aligned} \tag{52}$$

and hence

$$\begin{aligned} \left(\frac{\partial f_1(\delta\eta_{o_1})}{\partial \eta}\right)^T &= [2(\eta_{x_1} - \eta_{o_1}), 0, \ldots, 0]^T, \\ \left(\frac{\partial f_2(\delta\eta_{o_2})}{\partial \eta}\right)^T &= [0, 2(\eta_{x_2} - \eta_{o_2}), \ldots, 0]^T, \\ &\vdots \\ \left(\frac{\partial f_n(\delta\eta_{o_n})}{\partial \eta}\right)^T &= [0, 0, \ldots, 2(\eta_{x_n} - \eta_{o_n})]^T. \end{aligned} \tag{53}$$

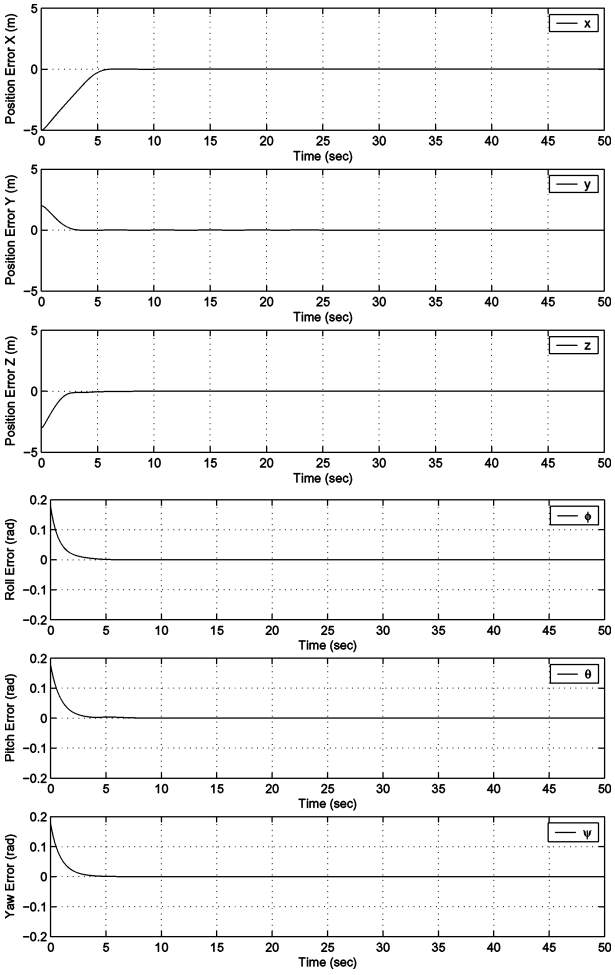


Fig. 6. Adaptive setpoint control: AUV position and orientation errors.

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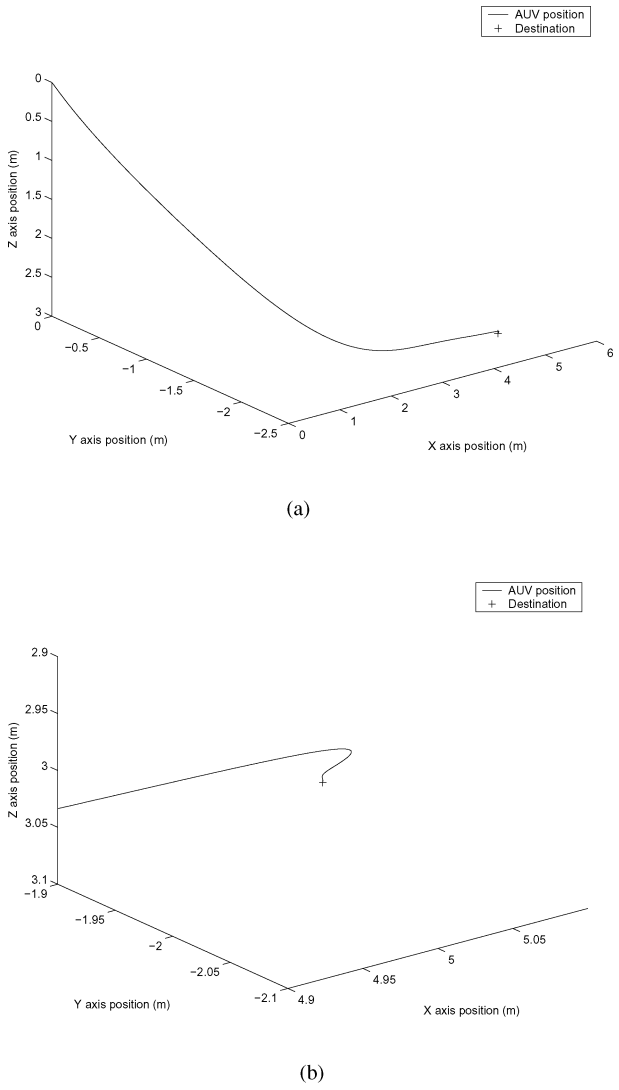


Fig. 7. Adaptivesetpoint control: 3D illustration of AUV position. (a) AUV position plot, (b) A close-up view of Fig. 7(a).

If $\epsilon_{\eta_{x_i}}$ reduces to zero for $i = 1, 2, \ldots, n$, then Eq. (29) becomes a new setpoint controller as follows:

$$\tau = -J^T(\eta)K_p e^3 - J^T(\eta)K_v J(\eta)v + Z(\eta)\hat{\varphi}, \quad (54)$$

$$\dot{\hat{\varphi}} = -L^{-1}Z^T(\eta)\left(v + \alpha J^T(\eta)e^3\right), \quad (55)$$

where $K_p = \text{diag}\{2k_{p1}, 2k_{p2}, \ldots, 2k_{pn}\}$ and $e^3 = [(\eta_{x_1} - \eta_{o_1})^3, (\eta_{x_2} - \eta_{o_2})^3, \ldots, (\eta_{x_n} - \eta_{o_n})^3]^T$.

5. Simulations

In this section, simulations are presented to illustrate the effectiveness of the proposed controllers. The simulations are performed on the omni directional intelligent navigator (ODIN). ODIN is a spherical underwater vehicle designed in the University of Hawaii. The parameters of the dynamic model of ODIN can be found in ref. [22]. In the simulations, the underwater vehicle is required to move from an initial position to a desired destination.

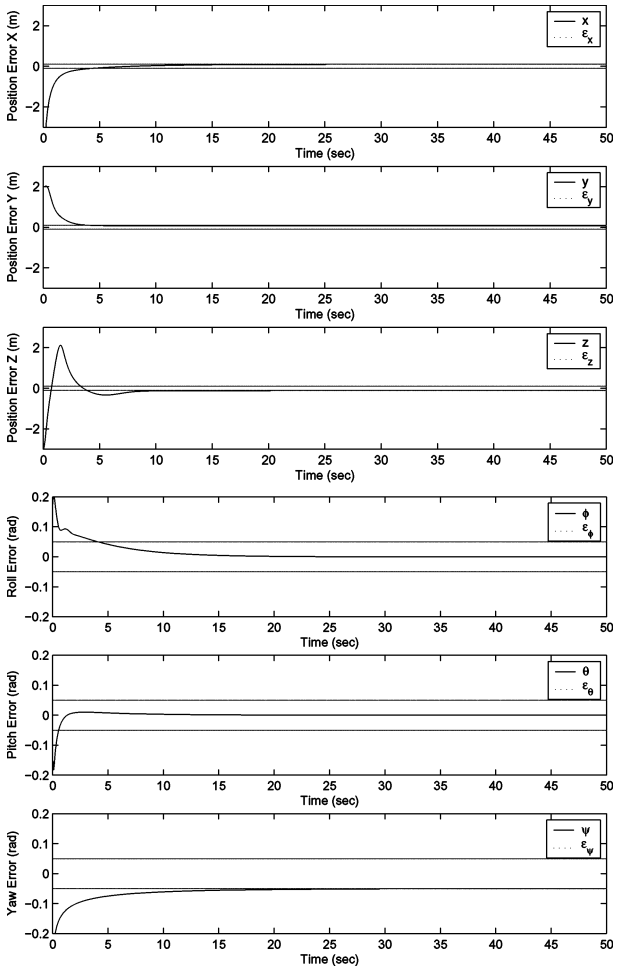


Fig. 8. Region reaching control: AUV position and orientation errors.

The gravitational and buoyancy force vector $g(\eta)$ can be expressed as[22]

$$g(\eta) = \begin{bmatrix} (mg - \frac{4}{3}\pi r^3 \rho g)\sin(\theta) \\ -(mg - \frac{4}{3}\pi r^3 \rho g)\cos(\theta)\sin(\phi) \\ -(mg - \frac{4}{3}\pi r^3 \rho g)\cos(\theta)\cos(\phi) \\ z_G mg\cos(\theta)\sin(\phi) \\ z_G mg\sin(\theta) \\ 0 \end{bmatrix}, \quad (56)$$

where $r = 0.3$ m is the radius of ODIN, $m = 125$ kg is the mass of ODIN, $z_G = 0.05$ m is the distance of the center of gravity from the geometrical center, $\rho = 1000\,\text{kg/m}^3$ is the density of water, and $g = 9.81\,\text{m/s}^2$.

From the gravitational and buoyancy force, the gravity regressor can be obtained as $g(\eta) = Z(\eta)\theta$, where

$$Z(\eta) = \begin{bmatrix} \sin(\theta) & 0 \\ -\cos(\theta)\sin(\phi) & 0 \\ -\cos(\theta)\cos(\phi) & 0 \\ 0 & \cos(\theta)\cos(\phi) \\ 0 & \sin(\theta) \\ 0 & 0 \end{bmatrix},$$

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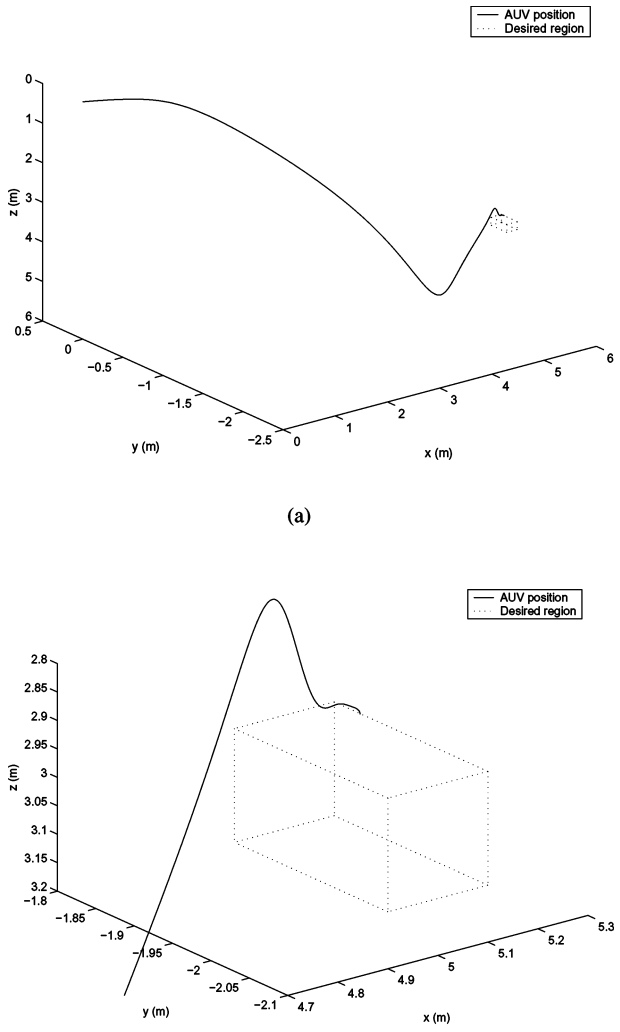


Fig. 9. Region reaching control: 3D illustration of AUV position. (a) AUV position plot, (b) A close-up view of Fig. 9(a).

$$\theta = \begin{bmatrix} mg - \frac{4}{3}\pi r^3 \rho g \\ z_G mg \end{bmatrix}. \tag{57}$$

Note that in the proposed control law described by (4), only the structure of the gravity regressor, $Z(\eta)$, is required. In addition, only two parameters need to be updated for the ODIN vehicle. Hence, it is simpler and easy to implement. The uncertain parameters in the gravitational and buoyancy force are assumed to be unknown.

5.1. Adaptive setpoint control

In this simulation, the position and velocity gain are chosen as $K_p = 750I$ and $K_v = 500I$, respectively, the gain in the adaptive update law is chosen as $L = 0.001$, and $\alpha = 0.85$. The vehicle is required to move from the initial location $[0, 0, 0, 0.17, 0.17, 0.17]^T$ to $[5, -2, 3, 0, 0, 0]^T$. As seen from Fig. 6, the position and orientation errors of the AUV converge to zero within 5 s. The position error in X-axis took a slightly longer time to converge because of larger initial error. Figure 7(a) shows the path of the AUV from the initial position to the desired end position, and a close-up view of the path when the vehicle is near the end position is shown in Fig. 7(b).

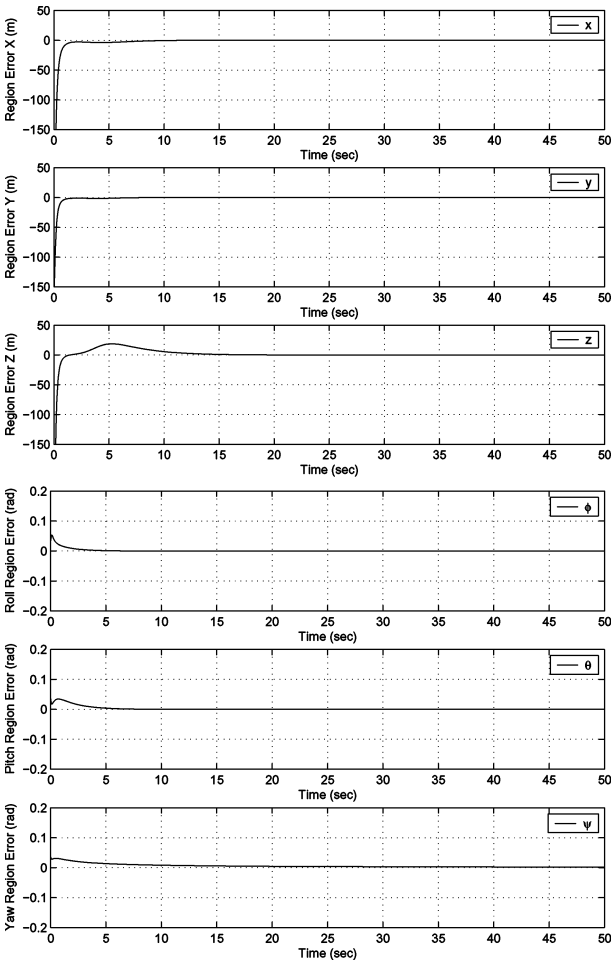


Fig. 10. Region reaching control with spherical region: AUV region errors.

5.2. Region reaching control

In this simulation, the AUV is required to move from an initial position
$[0, 0, 0, 0.17, -0.17, -0.26]^T$ to a desired region specified as

$$\begin{aligned} &(x - x_o)^2 \le \epsilon_x, \quad (y - y_o)^2 \le \epsilon_y, \quad (z - z_o)^2 \le \epsilon_z, \\ &(\phi - \phi_o)^2 \le \epsilon_\phi, \quad (\theta - \theta_o)^2 \le \epsilon_\theta, \quad (\psi - \psi_o)^2 \le \epsilon_\psi, \end{aligned} \tag{58}$$

where $[x_o, y_o, z_o, \phi_o, \theta_o, \psi_o]^T = [5, -2, 3, 0, 0, 0]^T$ is the reference point within the desired regions, and $[\epsilon_x, \epsilon_y, \epsilon_z, \epsilon_\phi, \epsilon_\theta, \epsilon_\psi]^T$ = [0.1 m, 0.1 m, 0.1 m, 0.05 rad, 0.05 rad, 0.05 rad]^T is the individual regional bound for each axis. Figure 8 shows the simulation results for region reaching controller with $K_p = \text{diag}[200I_{3\times3}, 3000I_{3\times3}]$, $K_v = \text{diag}[250I_{3\times3}, 50I_{3\times3}]$, $L = \text{diag}[0.005, 0.5]$, and $\alpha = 1.6$. From Fig. 8, it can be seen that all positions and orientation errors converge to desired regions in about 10 s. The yaw error took a slightly longer time to converge due to larger initial error. Figure 9(a) shows the path of the AUV from the initial position to the desired end position, and a close-up view of the path when the vehicle is near the end position is shown in Fig. 9(b).

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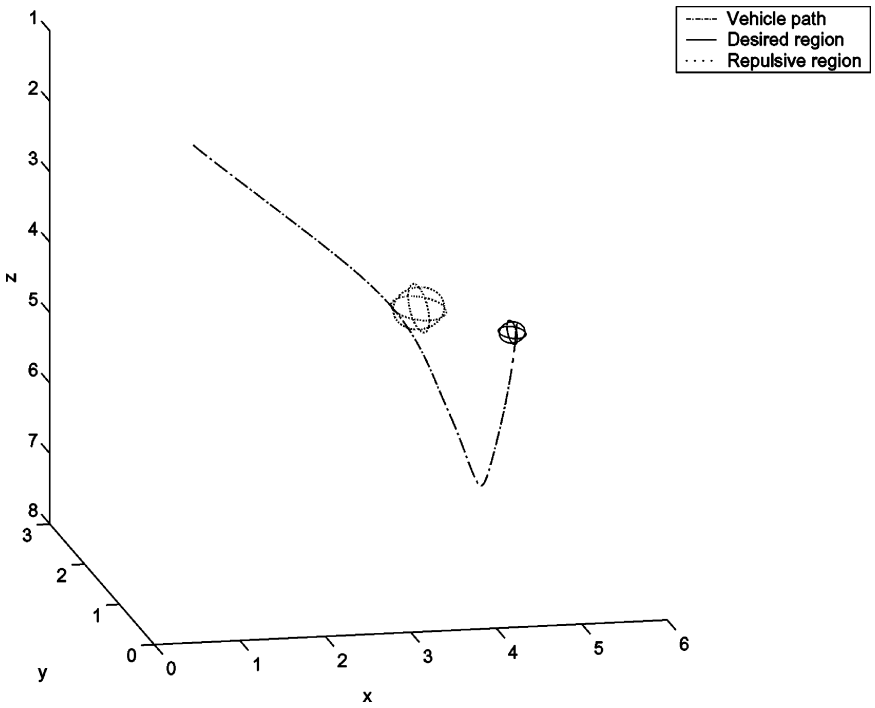


Fig. 11. Region reaching control with spherical region (both desired and repulsive regions): 3D illustration of AUV position.

Various desired regions could be specified based on different applications. In the practical implementation, region control can also be used for obstacle avoidance purposes. For example, when there is an obstacle within the operating area, repulsive regions can be specified for the region occupied by the obstacle, and therefore the system can be prevented from collision with the obstacle. In this simulation, the objective functions for the desired spherical region is specified as f_{V_1}, the repulsive spherical region is specified as f_{V_2}, and f_{V_3}, f_{V_4}, f_{V_5} are the objective functions for AUV orientation, respectively. The AUV is required to move from an initial position $[0.5, 0.1, 1.0, 0.26, 0.26, 0.26]^T$ to a desired region specified by the objective functions as follows:

$$f_{V_1} = (x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2 - \epsilon_{xyz}^2 \leq 0,$$
$$f_{V_2} = \epsilon_{r_{xyz}}^2 - (x - x_{r_o})^2 - (y - y_{r_o})^2 - (z - z_{r_o})^2 < 0,$$
$$f_{V_3} = (\phi - \phi_o)^2 - \epsilon_\phi^2 \leq 0,$$
$$f_{V_4} = (\theta - \theta_o)^2 - \epsilon_\theta^2 \leq 0,$$
$$f_{V_5} = (\psi - \psi_o)^2 - \epsilon_\psi^2 \leq 0,$$

where $[x_o, y_o, z_o, \phi_o, \theta_o, \psi_o]^T = [5, 2, 5, 0, 0, 0]^T$ is the reference point within the desired region, $[x_{r_o}, y_{r_o}, z_{r_o}]^T = [3.5, 1.0, 4.0]^T$ is the reference point within the repulsive region, and the tolerance for each axis is specified as $[\epsilon_{xyz}, \epsilon_{r_{xyz}}, \epsilon_\phi, \epsilon_\theta, \epsilon_\psi]^T = [0.15\text{ m}, 0.30\text{ m}, 0.05\text{ rad}, 0.05\text{ rad}, 0.05\text{ rad}]^T$. The vehicle position converges to the desired region, with $K_p = \text{diag}[80, 80, 105, 50, 50, 150]$, $K_v = \text{diag}[1300, 1200, 1100, 100, 100, 100]$, $L = \text{diag}[0.005, 0.05]$, and $\alpha = 0.1$, and the control gain for repulsive region is chosen as $K_{p_r} = 70$. Figure 10 shows the plots of region errors versus time. The region errors in X- and Z-axis took a slightly longer time to converge because of larger initial error. As seen from Fig. 11, the vehicle moved toward the repulsive region but the controller managed to pull the vehicle out from the repulsive region and finally stopped at the desired position.

From the simulation results, we observed that the proposed control schemes performed effectively in moving the AUV to the desired destination.

6. Conclusion

In this paper, an adaptive setpoint controller and an adaptive region reaching controller are proposed for autonomous underwater vehicle. Lyapunov-like functions have been presented for the stability analysis of the proposed controllers. Simulations illustrated the performance of the proposed controllers. The proposed region reaching control concept is also a generalization of setpoint control problem because when the desired region is specified arbitrarily small, the control objective is reduced to setpoint control.

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