LIMIT DEPENDENCES IN STABILITY CALCULATIONS WITH ACCOUNT FOR PHYSICAL NONLINEARITY

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ABSTRACT

Stability of bars, plates, shells, and other thin-walled structures in conditions of small physical nonlinearity is considered, when stresses exceed the proportionality limit, the amount of deformations being limited. Shanley's concept is used. The critical state is determined by means of some limit dependences. In a large number of cases, when creating efficient highly-stressed constructions, limited plastic deformations are allowed in them. When analysing stability in the critical state, the calculated stresses turn out to exceed the proportionality limit and the Young's modulus of elasticity turns out to be greater than the tangent modulus corresponding to the calculated stress on the diagram "deformation-stress". The objective of this work is to show that stability calculation beyond the proportionality limit is reduced to the analysis of some limit dependences as well as to develop a general solution algorithm for similar problems.

Keywords: Physical nonlinearity, Critical state, Limit dependence.

1. INTRODUCTION

Recently much attention is given to methods of calculation of building constructions in nonlinear statement [1-3]. It is explained by the fact that in a large number of cases, when creating efficient highly-stressed constructions, limited plastic deformations are allowed in them. When analysing stability in the critical state, the calculated stresses turn out to exceed the proportionality limit and the Young's modulus of elasticity turns out to be greater than the tangent modulus corresponding to the calculated stress on the diagram "deformation-stress". The objective of this work is to show that stability calculation beyond the proportionality limit is reduced to the analysis of some limit dependences as well as to develop a general solution algorithm for similar problems.

2. METHODS. PROBLEM FORMULATION

Stability of bars, plates and shells is considered in conditions of small physical nonlinearity when stresses exceed the proportionality limit, but the amount of deformations and plasticity zones is limited. It satisfies operating conditions of the majority of building construction elements. The material properties are defined by some random diagram, $\sigma_c = \sigma_c(\lambda)$, connecting deformations and stresses. The iteration algorithm of moduli correction [3] is used for determination of the critical load when deforming the material beyond the proportionality limit. Shanley's concept, according to which appearance and further development of the contiguous form of equilibrium takes place at monotonically increasing load, forms the basis of the whole theory. According to Shanley's concept, critical stress is expressed by means of tangent modulus [4].

3. EULER PROBLEM IN PHYSICALLY NONLINEAR STATEMENT

The amount of critical load for a bar in compression is determined by Euler formula. It is offered to substitute the calculated value of modulus EY, which depends on the level of stresses, in Euler formula when solving such problems in conditions of physical nonlinearity [3]:

$$\sigma_c = \frac{\pi^2 \cdot E_{\mathrm{p}}(\sigma_c)}{\lambda^2} \qquad (1)$$

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Thus, Euler formula turns into the equation allowing to obtain the dependence of critical stresses on flexibility of the bar:

$$\sigma_c = \sigma_c(\lambda) \tag{2}$$

If we divide both parts of the Eq. (1) by the value of the proportionality limit and enter dimensionless parameters

$$\bar{\sigma} = \frac{\sigma}{\sigma_{пц}}, \quad \lambda^* = \pi\sqrt{\frac{E}{\sigma_{пц}}}, \quad \bar{\lambda} = \frac{\lambda}{\lambda^*}, \quad \bar{\mathrm{E}} = \mathrm{m} = \frac{\mathrm{E_p}(\sigma)}{\mathrm{E}}. \tag{3}$$

then the Eq. (1) can be presented in the following form:

$$\bar{\lambda}^2\bar{\sigma}_c = \bar{E}(\bar{\sigma}_c). \tag{4}$$

In coordinates $\bar{\sigma}$, $\bar{E}$ the solution represents a point of intersection of the line, equation of which is in the left part of equality, with some curve, equation of which is in the right part. If we consider a loading process as motion along a straight line, then the right part in expression Eq. (4) can be considered as limit dependence at the achievement of which a critical condition occurs in some point of R (Fig. 1).

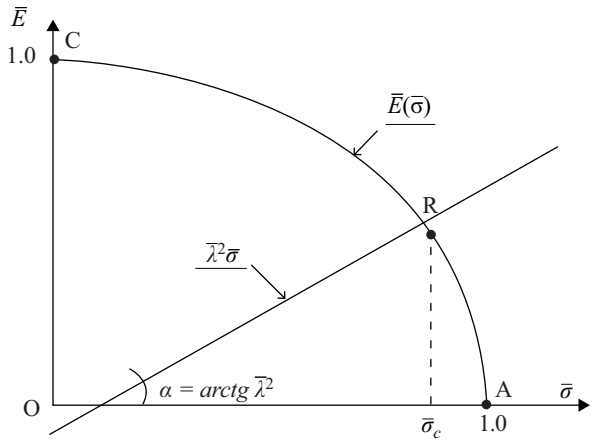


Fig. 1 Limit curve, solution of the Euler problem.

The work Eq. (3) shows that such approach is applicable to shells, plates and other thin-walled structures in which stresses are constant over the section thickness in the subcritical state. In all cases the critical stress is proportional to the modulus of elasticity and determined by the following formula:

$$\bar{\sigma}_c = \bar{E}(\bar{\sigma}_c)U(c_k). \tag{5}$$

In this expression function U depends on parameters c_k which define properties of the structure. This expression is reduced to Eq. (4) if we assume that

$$U(c_k) = 1/\bar{\lambda}^2. \tag{6}$$

Thus, the problem is reduced to solution of Euler Eq. (4) within accepted assumptions for any systems, and the limit curve depends on material properties only.

4. MODULUS CORRECTION ALGORITHM

It can be seen from the Euler formula Eq. (1) that the value of critical stress depends on the modulus of elasticity. Consequently, it is enough to substitute the corresponding value of the tangent modulus in Eq. (1) for determination of the critical stress beyond the proportionality limit. Such an approach to solution of the problem was for the first time investigated in the work of [2] in details. The essence of the moduli correction algorithm (MCA) is that the calculated value of the tangent modulus is corrected depending on the stress level in the critical state. We assume that there is some program of rather high level (for example, FEM program) which provides construction design for static loading and stability calculation.

A construction in whole or its separate parts can take various forms when there is a violation of stability. In general, it is possible to distinguish parts of the construction, which do not change their form, and parts of the construction, which considerably change their form. Naturally, when creating a design model for MCA execution, it is necessary to consider only changeable parts (fragments) of the construction. It is necessary to determine the stress level in these parts and distinguish those elements in which stresses exceed the proportionality limit in the critical state. Correction of the secant modulus value is carried out for each such fragment of the construction at each stage of the calculation.

$$\bar{E}_{k+1} = \frac{\bar{\sigma}_{c,k}\bar{E}_k}{\bar{\sigma}_{c,k-1}}. \tag{7}$$

Such process continues until values of pcrit parameter on the subsequent step significantly differs from the previous one.

5. COMBINATION OF LOADS IN STABILITY CALCULATIONS. PAPKOVICH THEOREM

When calculating building constructions, a necessary stage is a number of calculation options for simultaneous action of various loads. The purpose of such calculations is determination of a load combination at which the absolute stability margin is minimal. As the stability analysis and static analysis are repeated several times during calculation taking into account physical nonlinearity in iteration process, a total number of single calculations can be rather large. It is possible to reduce a number of single calculations significantly by means of the theorem of [5].

Let us assume that each of the loads at unlimited increase can lead to loss of stability. We characterize the value of each load by P_i parameter.

In the critical state at action of this load its parameter

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assumes value P_i^* Critical parameter

$$p_{kp,i} = \frac{P_i^*}{P_i}. \tag{8}$$

Let us consider all these load parameters as elements of dimensional space. We introduce an orthogonal coordinates system against axes of which a parameter of one of the loads, changing from zero to is plotted. Then a point with a set of coordinates corresponds to each state, and a set of such points or a continuous line in this space corresponds to each loading, that is to increase of the total load before the critical state. The critical state occurs in some point at the end of each loading.

A set of these points forms a boundary surface $\Re$. According to P.F. Papkovich's theorem this surface is convex. The border of this area can be approximated by a plane in a first approximation with some small margin. The equation of this plane in sections on axes is the following:

$$\sum \frac{P_{r,i}}{P_i^*} = \sum \bar{P}_{r,i} = 1, \tag{9}$$

where $P_{r,i}$ is the value of the load parameters in the critical state. This expression is usually called Papkovich formula.

If we consider proportional loading, then from Eq. (8) we obtain the following

$$\bar{P}_{r,i} = p_{kp}\,\bar{P}_i, \quad p_{kp} = \frac{1}{\sum \bar{P}_i}. \tag{10}$$

A boundary surface is a straight line in a two-dimensional case (Fig. 2).

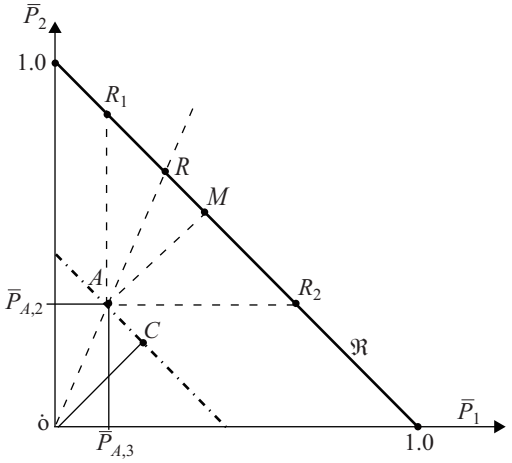


Fig. 2 Normed space of loadings: proportional loading along line OAR, loading at $\bar{P}_1 = const$ along line AR_1, loading at $\bar{P}_2 = const$ along line AR_2, loading on the shortest way – along line AM.

The loading from a point inside the area to the end plane on the shortest way, that is along the normal to the end plane, can be the most interesting among nonproportional loadings. In such case a stability coefficient will turn out minimum. If we accept the ratio of the whole path of loading to its initial section as a factor of safety, the minimum stability coefficient will be:

$$n_{y,\min} = 1 + \frac{1 - \sum \bar{P}_{A,i}}{\sqrt{m \sum \bar{P}_{A,i}^2}}. \tag{11}$$

Thus, the theorem of P.F. Papkovich allows to estimate the results of any loading subsequent to the solution results for separate fixed loadings. In such case all possible special cases of loading shall be taken into account (the sum in a denominator shall be full, that is it shall contain m summands).

6. CONCLUSIONS

Thus, stability calculation in conditions of small physical nonlinearity is reduced to the definition of the intersection point of the loading line with some limit curve or surface. The algorithms of critical state determination have the following properties:

1. When solving any special problem within the accepted assumptions, the limit curve is defined only by material properties.
2. When calculating a complex multielement structure, the use of the modulus correction algorithm allows to determine the critical load parameter by method of successive approximation.
3. Use of Papkovich theorem at multiple calculations allows to reduce a total number of steps and to determine the smallest stability coefficient.

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