*Bull. Aust. Math. Soc.* **114** (2026), 386–387
doi:10.1017/S0004972726101671

# SPLITTING ALGORITHMS WITH APPLICATIONS IN POWER SYSTEMS[‡]

**TAN NHAT PHAM**

(Received 30 May 2026 UTC; first published online 29 July 2026)

2020 *Mathematics subject classification*: primary 90C26; secondary 49M27, 65K05.

*Keywords and phrases*: splitting methods, decomposition methods, nonconvex, nonsmooth, numerical algorithm, ADMM, Douglas–Rachford, difference of convex functions, Kurdyka–Łojasiewicz inequality.

Modern engineering systems generate models that are inherently high-dimensional, nonconvex and nonsmooth, due to physical and other constraints. The emergence of new optimisation models beyond the scope of existing convex methods calls for the development of novel and efficient numerical algorithms, capable of addressing nonconvexity and nonsmoothness. A novel approach, initiated in the 1980s, explored the class of difference-of-convex (DC) functions with the pioneering work of [6]. The class of DC functions is broad enough to address many optimisation applications, while remaining close enough to convexity to use existing convex optimisation tools.

This doctoral thesis is dedicated to addressing practical challenges in nonconvex optimisation. Specifically, the possibility of developing algorithms for structural problems, where there is little or no convexity, is investigated, with a focus on generalising the DC problems. Novel proximal and splitting algorithms with guaranteed convergence, moving beyond traditional convex analysis by leveraging modern variational analysis, are developed. These methods are successfully applied to problems in power systems, demonstrating both theoretical novelty and practical utility. In particular, the following contributions have been published.

In [4], a forward–backward (proximal subgradient) type algorithm for a generalised DC problem is developed. This work shows the possibility of solving the DC problem without requiring the convexity of the components. It also provides a proof-of-concept for solving nonconvex optimal power flow problems without convex relaxation.

In [3], we extend the famous Alternating Direction Method of Multipliers (ADMM) for solving the DC problem. This work is the first to extend the DC program into

Thesis submitted to Federation University Australia in September 2024; degree approved on 2 December 2024; supervisors Minh Dao, Rakibuzzaman Shah, Nargiz Sultanova and Syed Islam.

[‡]This article has been updated since original publication. A notice detailing the change has been published.

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https://doi.org/10.1017/S0004972726101671 Published online by Cambridge University Press

a multi-block structure in a nonconvex setting. The theoretical convergence analysis also requires weaker assumptions than the pioneering work in [7].

In [5], we propose a lightweight deep feature selection framework that uses the DC structure in training neural networks. By adding this, the framework can select useful features in the training process itself.

In [1, 2], we explore the possibility of extending Doughlas–Rachford splitting to efficiently solve the DC problem with a weak convexity assumption, from two different perspectives. In [1], the algorithm still uses the subgradient of one component, while in [2], a new full proximal splitting strategy is proposed. Both algorithms in [1, 2] are the first in the literature to have guaranteed full sequential convergence without the common smooth assumption on the concave part in the DC structure. Both have shown superior performance on real datasets. The work in [2] also provides a lightweight, fast and explainable signal reconstruction method for power system operators, and it has the potential to be developed further into plug-and-play algorithms.

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TAN NHAT PHAM, School of Industrial Engineering and Management, International University, VNU-HCM, Ho Chi Minh City, Vietnam

Viet Nam National University Ho Chi Minh City, Vietnam
e-mail: pntan@hcmiu.edu.vn

https://doi.org/10.1017/S0004972726101671 Published online by Cambridge University Press