$$a \to a - 3b$$

$$b \to a + b$$

$$c \to a - 5b.$$

References

1. Adam McBride, A tale of two sixes, *Math. Gaz.* **105** (July 2021) pp, 285-290.
2. Chris Starr, Notes on Listener Crossword 4595 by Elap, *Math. Gaz.* **105** (July 2021) pp. 291-298.

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On 105.19: Nick Lord writes: I greatly enjoyed this short note linking Fermat's last theorem with a mensuration problem about cones which provides a lovely glimpse of the frontiers of mathematics for young students. Here, I just add two further thoughts.

First, the same situation considered in the note, but filling hypercones (or simplexes) in $\mathbb{R}^n$ ($n \geqslant 3$) rather than cones, leads to the replacement of equation (1) of the note with $(h - h_1)^n + h_2^n = h^n$ which also has no solutions in rationals, by the general case of Fermat's last theorem.

Second, the idea of the note leads to interesting problems in number theory for other shapes. For a closed, hollow hemisphere of radius r filled with liquid from its base to a depth x and then inverted to fill a cap of height $y > x$, we have the following analogue of equation (1) of the note:

$$C(y) + C(r - x) = \tfrac{2}{3}\pi r^3, \qquad (*)$$

where $C(t) = \frac{1}{3}\pi\left(3rt^2 - t^3\right)$ is the volume of a spherical cap of height t. Substituting this formula into $(*)$ and simplifying leads to

$$3r^2x - 3ry^2 - x^3 + y^3 = 0.$$

This equation *does* have integer solutions, for example $x = 9$, $y = 39$, $r = 155$. Substituting $X = r - x$ produces the symmetrical equivalent equation $2r^3 - 3r\left(X^2 + y^2\right) + X^3 + y^3 = 0$, and it would be good to know more about its integer solutions.

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On 105.23: Henry Ricardo writes: Toward the end of his interesting article, Clement Falbo writes that [proving the Goldbach conjecture] "would make some other prime number theorems easier to prove." In my brief note in the June-July 2005 issue of *The American Mathematical Monthly* (p. 492), I proved that Goldbach's Conjecture implies Bertrand's Postulate. This result was submitted independently by Yoshihiro Tanaka at about the same time. Later, in a Letter to the Editor of *Mathematical Spectrum* (January 2014: Vol. 46, No. 2, p. 91), Guido Lasters also stated and proved the result.

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