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Time-Varying Margin Requirements and Optimal Portfolio Choice

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Abstract

This paper studies the optimal consumption and portfolio problem of an investor with recursive preferences who is subject to time-varying margin requirements. The level of the requirements at each moment is determined by contemporaneous volatility of returns, which is stochastic and may have jumps. I show that the nonstandard hedging demand produced by margin requirements increases with their persistence and volatility. However, for realistic values of parameters, the hedging demand is small even in the presence of jumps, and contemporaneous jumps in prices have a much stronger effect on optimal portfolio than jumps in constraints.

I. Introduction

Many investors who want to magnify their returns trade on margin, that is, finance long positions in various securities with short-term debt. Although such debt can take various forms, those forms share several common features. First, lenders almost always require borrowers to put their portfolios as collateral. Second, to protect themselves against losses resulting from adverse price movements, lenders require margin debt to be overcollateralized so that the value of collateral exceeds the amount of debt. Effectively, the investor's portfolio is financed with both debt and the investor's own capital, and the overcollateralization imposes restrictions on portfolio leverage. Such restrictions are known as margin requirements.

There is abundant evidence that tightness of margin requirements for various asset classes varies over time (e.g., Gorton and Metrick (2010), Dudley and Nimalendran (2011), and Hedegaard (2011)). Prime brokers, which provide margin credit to hedge funds, can change margin requirements at their own discretion and even without advance notice, and they actively resorted to that right during

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the financial crisis of 2008–2009. The adjustment of margin requirements can be almost continuous when borrowers and lenders renegotiate the terms of margin credit on a daily basis, or discrete, when margin requirements are abruptly and unexpectedly changed by lenders. To minimize their possible loss in the case of adverse price changes, the providers of margin credit often link the tightness of margin requirements to the portfolio's value-at-risk (VaR). The latter is predominantly determined by contemporaneous volatility of returns, so changes in margin requirements mostly result from changes in asset volatility.

The main objective of this study is to quantitatively analyze the impact of time-varying margin requirements on optimal portfolio policies. I consider the consumption and portfolio choice problem of a long-term investor who has recursive preferences. The investor can trade a bond with a constant risk-free rate and a risky asset, but the investor's position in the risky asset is subject to margin constraints. To make the modeling of the variation in margin requirements as realistic as possible, I assume that their tightness is inversely proportional to the contemporaneous volatility of returns. This functional form follows from the VaR-based approach to setting margin requirements. The volatility is stochastic and can have jumps, and this determines the dynamics of the constraints.

Unfortunately, the optimal portfolio problem does not have a closed-form solution in the presence of portfolio constraints. Nevertheless, it can be reduced to an ordinary differential equation by using the standard dynamic programming approach. To solve this equation, I use a projection method, which approximates the solution by a linear combination of orthogonal polynomials (Judd (1998)). For numerical analysis, I calibrate the model parameters using the estimates of stochastic volatility processes available in the literature.

The paper contains several findings. First, I demonstrate that continuous variation in margin requirements produces a nonstandard hedging demand.[1] In general, when the coefficient of risk aversion is greater than 1 and changes in prices are negatively correlated with changes in tightness of margin requirements (i.e., when the price decreases, the volatility increases and margin requirements become more restrictive), the presence of constraints makes the optimal portfolio more conservative even when the constraints do not bind. The intuition for this result is as follows: When volatility goes up and margins become tighter, the probability that they will bind in the future and make the unconstrained optimal portfolio infeasible increases. Moreover, the constraints on average become more binding when they already bind. As a result, investment opportunities deteriorate. Due to a negative correlation between prices and tightness of margin requirements, an investor with relative risk aversion greater than 1 hedges the variation in the constraints and holds a smaller amount of the risky asset than in the unconstrained case.

Second, I find that for a realistic parametrization of price and volatility processes as well as investor preferences, the size of the hedging demand produced by time-varying margin requirements is small. Existing studies show that hedging

[1]The existing portfolio-choice literature mostly focuses on hedging demand produced by variations in the market price of risk and in the risk-free rate (the former can be caused by changes in expected returns and volatilities).

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demand can be large when it results from variation in expected returns (e.g., Barberis (2000), Lynch (2001), and Campbell, Chan, and Viceira (2003)) or variation in volatility in complete markets (e.g., Liu and Pan (2003), Zhou and Zhu (2012)), but tends to be smaller when generated by variation in the variance-covariance matrix of returns in incomplete markets (e.g., Ang and Bekaert (2002), Chacko and Viceira (2005), and Buraschi, Porchia, and Trojani (2010)). Effectively, I extend this line of research to the case of time-varying portfolio constraints and show that variation in margin requirements only weakly affects the optimal portfolio policy. Also, I estimate the certainty equivalent loss (CEL) produced by ignoring margin requirements when they do not bind and find it to be very small.[2]

To check robustness of my conclusions, I perform comparative statics with respect to various parameters. This analysis confirms that the small size of the hedging demand is not specific to the baseline calibration of the model. Also, it shows that the hedging demand tends to increase with the volatility of portfolio constraints and their persistence.

Finally, I study the effect of jumps in prices and in margin constraints (the latter are produced by jumps in the volatility of returns). Indeed, the small size of the hedging demand associated with time-varying margin requirements may result from the assumed ability of the investor to trade continuously without price impact, whereas jumps in margin constraints occurring simultaneously with jumps in prices and volatility may have a stronger impact on the optimal portfolio. Surprisingly, this is not the case. My analysis shows that the effects of constraints in the economies with and without jumps are very similar and tend to be small. For example, in an economy with 10% downward jumps in prices and 5% upward jumps in return variance that happen on average every other year, the hedging demand produced by constraints is around 0.02, which is approximately 1% of the demand in an economy with the same jumps but without constraints.

The hedging demand produced by constraints with jumps is small not only in absolute terms but also compared to the effect of large jumps in prices, which is measured by a difference in the optimal portfolio weights in unconstrained economies with and without jumps. For example, a 30% downward jump in the price accompanied by a 15% upward jump in variance that occurs on average once in 25 years reduces the optimal portfolio weight of the risky asset in the unconstrained economy from 2.06 to 1.63, that is, by almost 21%. In contrast, the effect of additionally imposing margin requirements that can jump simultaneously with the price and volatility reduces the optimal weight of the risky asset by less than 0.02, which is less than 2% of the unconstrained demand.

The last finding admits an interesting interpretation. In practice, an abrupt tightening of margin requirements is often accompanied by an illiquidity event,

[2]To compute the CEL, I compare indirect utility functions of two investors in the constrained economy: The first follows the optimal portfolio policy, whereas the second implements the best unconstrained portfolio policy in the states of the economy in which such a policy is feasible and maintains the highest possible leverage otherwise. The maximum fraction of wealth that the first investor wants to give up for having an ability to adjust portfolio weights in response to changing margin requirements (i.e., for not being in the position of the second investor) is the CEL of the second investor.

so their individual effects on the optimal portfolio are hard to disentangle.[3] Because jumps in prices can be interpreted as illiquidity risk (Liu, Longstaff, and Pan (2003)), my results imply that the illiquidity risk is much more important for a rational leveraged investor than the risk of tightening margin requirements. In other words, investors tend to have low leverage when they fear the possibility that liquidity dries up but not that margin requirements become more restrictive.

Although time variation in margin requirements is a ubiquitous phenomenon, there are only few papers that analyze its effect on optimal consumption and portfolio choice. Grossman and Vila (1992) and Vila and Zariphopoulou (1997) find the optimal portfolio of a constant relative risk aversion (CRRA) investor whose dollar amount invested in a risky asset is bounded from above and the upper limit is specified as a nonhomogeneous function of the investor's wealth. Detemple and Rindisbacher (2005) solve the portfolio choice problem of a CRRA investor whose position in one of the assets is constrained and identify the hedging demand produced by fluctuations in the shadow price of the constraint. Liu and Longstaff (2004) study an optimal dynamic portfolio choice of a logarithmic investor who can exploit an arbitrage opportunity modeled as a Brownian bridge process and whose investment is limited by a collateral constraint imposed on the number of "units of arbitrage." None of these papers focuses on the hedging demand produced by stochastic margin requirements, and none of them considers jumps in the constraints. Moreover, most of the papers do not provide any realistic quantitative estimates of how time-varying constraints affect optimal portfolios.

By allowing time variation in margin requirements, the paper extends the studies of the optimal portfolio choice in the presence of constant portfolio constraints. The most popular technique of solving the portfolio optimization problem with convex constraints is the duality approach (e.g., Cvitanić and Karatzas (1992), Cuoco (1997), Cuoco and Liu (2000), Tepla (2000), Schroder and Skiadas (2003), (2005), and Detemple and Rindisbacher (2005)). The vast majority of such studies assume that the investor has a time-separable utility function. The exceptions are the works by Schroder and Skiadas (2003), (2005), in which the duality approach is extended to models with recursive utility functions.[4]

The paper also fits into a vast literature on the optimal consumption and portfolio choice with time-varying investment opportunities.[5] Following Merton (1971), who applied stochastic dynamic programming to this problem and identified hedging demand, the literature examined hedging demands produced by variations in expected returns (e.g., Kim and Omberg (1996), Campbell and Viceira (1999), Barberis (2000), Lynch (2001), Wachter (2002), and Campbell et al. (2003)), the risk-free rate (e.g., Brennan, Schwartz, and Lagnado (1997),

[3]Recent studies of the optimal portfolio problem with illiquid assets include those by Gârleanu (2009), Longstaff (2009), and Ang, Papanikolaou, and Westerfield (2014), among others.

[4]Theoretically, the duality approach can be generalized to time-varying portfolio constraints, but I am unaware of any paper conducting such an analysis.

[5]This literature is reviewed by Campbell and Viceira (2002), Skiadas (2008), and Brandt (2010), among others.

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Campbell and Viceira (2001), and Liu (2007)), the volatility of returns (e.g., Chacko and Viceira (2005), Liu (2007)), the correlation of returns (e.g., Ang and Bekaert (2002), Buraschi et al. (2010)), the bond risk premia (e.g., Sangvinatsos and Wachter (2005)), and the simultaneous variations in the interest rate and the market price of risk (Detemple, Garcia, and Rindisbacher (2003)). Many of these studies assume that investors have recursive preferences.

II. The Model

I consider a consumption and portfolio choice problem of a risk-averse investor who is subject to time-varying margin requirements. I assume that the investor derives utility from an infinite consumption stream C_t and has recursive preferences (Epstein and Zin (1989), Weil (1989)). Following Duffie and Epstein (1992), the utility function is parametrized as

$$J_t = E_t\left[\int_t^\infty f(C_s, J_s)\, ds\right], \tag{1}$$

where

$$f(C_s, J_s) = \begin{cases} \dfrac{\beta}{1-1/\psi}(1-\gamma)J_s\left(\left(\dfrac{C_s}{((1-\gamma)J_s)^{1/(1-\gamma)}}\right)^{1-1/\psi} - 1\right), & \text{when } \psi \neq 1, \\ \beta(1-\gamma)J_s\left(\log(C_s) - \dfrac{1}{1-\gamma}\log((1-\gamma)J_s)\right), & \text{when } \psi = 1. \end{cases} \tag{2}$$

The constants $\beta > 0$, $\gamma > 0$, and $\psi > 0$ are the time preference parameter, the coefficient of relative risk aversion, and the elasticity of intertemporal substitution (EIS), respectively. Recursive utility is a generalization of standard CRRA utility; it allows for a separation of the investor's attitudes toward risk and consumption timing. When $\psi = 1/\gamma$, the recursive preferences reduce to the CRRA preferences.

There are two assets in the economy: a risk-free asset with the constant rate of return $r > 0$ and a risky asset with the price S_t. I assume that the dynamics of the risky asset are described by a stochastic volatility model with jumps in prices and volatility. Specifically, the price of the risky asset S_t follows the process

$$dS_t = \left(r + \mu_S \frac{v_t}{\bar{v}}\right) S_t\, dt + \sqrt{v_t} S_t\, dB_t^S + Q_S S_{t-}\left(dN_t - \frac{v_t}{\bar{v}}\lambda\, dt\right), \tag{3}$$

where the stochastic variance of returns v_t has the unconditional expectation $\bar{v}$ and evolves as

$$dv_t = \phi_v(\bar{v} - v_t)\, dt + \bar{\sigma}_v\sqrt{v_t}\, dB_t^v + Q_v\left(dN_t - \frac{v_t}{\bar{v}}\lambda\, dt\right). \tag{4}$$

The diffusion parts of the processes are driven by the standard Brownian motions B_t^S and B_t^v defined on the underlying probability space $(\Omega, \mathcal{F}, P)$. The shocks dB_t^S and dB_t^v are correlated with the correlation coefficient ρ_{vS}. The processes also contain jump components; jumps arrive according to a Poisson process N_t with the intensity $\lambda v_t/\bar{v}$, where λ is a constant. Using standard notation, S_{t-} is the price of the asset just before a jump. Note that the instantaneous risk premium $\mu_S v_t/\bar{v}$

as well as the jump intensity $\lambda v_t/\bar{v}$ are proportional to v_t, and the Sharpe ratio of the asset is proportional to $\sqrt{v_t}$. This specification of stochastic volatility largely follows Liu et al. (2003), who use it to assess the impact of jumps in prices and volatilities on the optimal portfolio choice. Because $\mathrm{E}[v_t]=\bar{v}$, the constants μ_S and λ are naturally interpreted as the unconditional risk premium and average jump intensity, respectively.

Equation (4) describes a Cox, Ingersoll, and Ross (1985) process with jumps. The square root in the diffusion term (together with the assumptions on the jump size) guarantees that v_t is always nonnegative. The boundary $v=0$ is inaccessible when the standard Feller condition $2\phi_v\bar{v} \geq \bar{\sigma}_v^2$ is satisfied (Karlin and Taylor (1981)). For simplicity, the sizes of jumps Q_S and Q_v are assumed to be constant.[6] Empirically, returns decrease more abruptly than they increase and volatility tends to jump upward (e.g., Eraker (2004), Broadie, Chernov, and Johannes (2007)), so I assume that $-1 < Q_S \leq 0$ and $Q_v \geq 0$.

The investor is subject to margin requirements that restrict portfolio leverage and imply that the proportion of the investor's wealth ω_t allocated to the risky asset cannot exceed a prespecified threshold m. The key element of the model is time variation in margin requirements. In practice, it is produced by variation in asset volatility: When volatility increases, lenders make the requirements tighter to protect themselves against possible losses, which are more likely when returns are more volatile. In particular, lenders often set margin requirements based on the portfolio's VaR. As shown by Danielsson, Shin, and Zigrand (2004), this rule implies that the tightness of margin constraints is inversely proportional to the contemporaneous volatility of returns. Motivated by this observation, I use the following functional form for the constraints:

$$\omega_t \leq m(v_t), \qquad m(v_t) = \max\left(\bar{m}\sqrt{\frac{\bar{v}}{v_t}}, 1\right). \tag{5}$$

The parameter $\bar{m}$ characterizes the average tightness of the constraints and is equal to the level of margin requirements when $v_t = \bar{v}$.[7] Equation (5) also implies that there are no restrictions on unleveraged portfolios.

Because margin requirements are inversely proportional to volatility, their dynamics also contain diffusive and jump components. Jumps in margin requirements are driven by the same Poisson process N_t that is responsible for jumps in the price level. Hence, jumps can be interpreted as rare events that simultaneously shift the price, volatility, and margin requirements.[8]

[6] Although this assumption is unrealistic, it is innocuous for my results. In the numerical analysis, I consider a wide range of constant jump sizes and find that my conclusions hold for all of them. Therefore, they are likely to hold for jumps with a random size whose realizations are predominantly in the considered range.

[7] Note that the total instantaneous variance of returns is $(\bar{v} + \lambda Q_S^2)(v_t/\bar{v})$, and its unconditional expectation is $\bar{v} + \lambda Q_S^2$. Their ratio is $v_t/\bar{v}$, and it determines the level of margin requirements in equation (5).

[8] Typically margin requirements are adjusted once a day, and the adjustments tend to be small (or even absent) in normal times, so modeling margin requirements as a continuous process provides a good approximation to reality. In times of market distress, margin requirements as well as prices may change abruptly during one day, and I use jumps to model such events. A good example is the

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Note that in my model the market is incomplete, and in general there are three sources of incompleteness. First, the instantaneous changes in volatility and stock returns may not be perfectly correlated, and a part of the variation in volatility remains unspanned by the available securities. Second, there are two sources of variation in volatility (diffusion and jump components), but only one asset that can be used for hedging them. Third, the presence of portfolio constraints makes some combinations of the assets unattainable.

The investor solves a standard optimization problem: At each moment, the investor has to choose the consumption policy C_t and the weight of the risky asset in the portfolio ω_t that maximize the recursive utility function subject to the budget constraint

$$(6)\quad dW_t = \omega_t W_t \left(\frac{\mu_S - \lambda Q_S}{\bar{\nu}} v_t\, dt + \sqrt{v_t}\, dB_t^S\right) + (rW_t - C_t)\, dt + W_{t-}\omega_{t-} Q_S\, dN_t$$

and margin requirements from equation (5). The solution to this problem is given by Proposition 1:

Proposition 1. The solution to the optimal consumption and portfolio problem of an investor with the recursive utility function from equations (1) and (2) with $\psi \neq 1$, $\gamma \neq 1$ subject to the budget constraint from equation (6) and margin requirements from equation (5) is

$$(7)\quad \begin{aligned} C^*(W, v) &= W \exp\left(\frac{1-\psi}{1-\gamma} H(v) + \psi \log \beta\right), \\ \omega^*(v) &= \min\left(\max\left(\bar{m}\sqrt{\frac{\bar{\nu}}{v}}, 1\right), \bar{\omega}(v)\right), \end{aligned}$$

where $\bar{\omega}(v)$ solves a nonlinear algebraic equation

$$(8)\quad \begin{aligned} \bar{\omega}(v) = {} & \frac{\mu_S}{\gamma \bar{\nu}} + \frac{\bar{\sigma}_v \rho_{vS}}{\gamma} H'(v) \\ & + \frac{\lambda Q_S}{\gamma \bar{\nu}} \left[(1 + \bar{\omega}(v) Q_S)^{-\gamma} \exp(H(v + Q_v) - H(v)) - 1\right]. \end{aligned}$$

The twice continuously differentiable function $H(v)$ is a solution to the following differential equation:

$$(9)\quad \begin{aligned} & \frac{1}{2}\bar{\sigma}_v^2 v (H'' + (H')^2) + H' \left(\phi_v(\bar{\nu} - v) - Q_v \lambda \frac{v}{\bar{\nu}}\right) \\ & \quad + (1-\gamma) v \left(\frac{\mu_S - \lambda Q_S}{\bar{\nu}} \omega^* - \frac{\gamma}{2} (\omega^*)^2 + H' \omega^* \bar{\sigma}_v \rho_{vS}\right) \end{aligned}$$

stock market crash of Oct. 19, 1987. On that day Standard & Poor's (S&P) 500 index lost about 20%. Simultaneously, all investors who traded on margin experienced a substantial increase in margin requirements. For example, margins for S&P futures jumped from 4% to more than 12%. A similar jump, although of a smaller size, occurred during the Long-Term Capital Management (LTCM) crisis in 1998.

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$$+ r(1-\gamma) - \beta\frac{1-\gamma}{1-1/\psi} - \frac{1-\gamma}{1-\psi}\exp\left(\frac{1-\psi}{1-\gamma}H + \psi\log\beta\right)$$
$$+ \lambda\frac{v}{\bar{v}}\left[(1+\omega^* Q_S)^{1-\gamma}\exp(H(v+Q_v) - H(v)) - 1\right] \quad = \quad 0$$

and the indirect utility function is

$$J(W,v) \quad = \quad \frac{1}{1-\gamma}W^{1-\gamma}\exp(H(v)). \tag{10}$$

In the special case with $\psi = 1$, the solution to the optimal consumption problem is

$$C^*(W,v) \quad = \quad \beta W \tag{11}$$

and the optimal portfolio is given by equations (7) and (8), where a twice continuously differentiable function $H(v)$ solves the following differential equation:

$$\begin{aligned}\frac{1}{2}\bar{\sigma}_v^2 v(H'' + (H')^2) + H'\left(\phi_v(\bar{v}-v) - Q_v\lambda\frac{v}{\bar{v}}\right) \\ + (1-\gamma)v\left(\frac{\mu_S - \lambda Q_S}{\bar{v}}\omega^* - \frac{\gamma}{2}(\omega^*)^2 + H'\omega^*\bar{\sigma}_v\rho_{vS}\right) \\ + (1-\gamma)(r - \beta + \beta\log(\beta)) - \beta H \\ + \lambda\frac{v}{\bar{v}}\left[(1+\omega^* Q_S)^{1-\gamma}\exp(H(v+Q_v) - H(v)) - 1\right] \quad = \quad 0.\end{aligned} \tag{12}$$

Proof. See the Appendix.

Proposition 1 shows that the investor's optimal portfolio policy has a 2-region structure. The boundary between the regions v^* is endogenously determined by the investor's decision at which state margin constraints become binding and solves the equation $m(v) = \bar{\omega}(v)$. When margin constraints are sufficiently tight (i.e., $v \geq v^*$), it is optimal for the investor to borrow as much as possible, and $\omega^*(v) = m(v)$. However, when $v < v^*$, the optimal portfolio is $\omega^*(v) = \bar{\omega}(v)$, where $\bar{\omega}(v)$ solves equation (8).

In the states with nonbinding margin constraints, the demand $\bar{\omega}$ has an intuitive structure. The first term $\mu_S/(\gamma\bar{v})$ is the standard myopic demand, which is the solution to the portfolio problem in the absence of portfolio constraints, time variation in volatility, and jumps in the price level. As demonstrated by Svensson (1989) and Obstfeld (1994), the solution to the portfolio problem of an Epstein–Zin investor with constant investment opportunities is identical to the solution of an investor with the CRRA preferences derived by Merton (1969).

The second term in equation (8) represents hedging demand produced by continuous changes in volatility and margin requirements. It disappears when $\gamma = 1$ (e.g., Campbell and Viceira (2002)), and its sign is determined by the correlation between diffusive innovations in the price and return variance ρ_{vS} as well as by the relation between the marginal utility and the state variable v_t, which is captured by $H'(v)$. The sign of the latter for recursive preferences is determined by the coefficient of risk aversion and the sign of the correlation between the

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state variable and investment opportunities: For a state variable that is positively (negatively) related to investment opportunities, $H'(v)$ is negative (positive) when $\gamma > 1$ and positive (negative) when $\gamma < 1$ (e.g., Bhamra and Uppal (2006)). In my model, the state variable v_t is i) positively related to investment opportunities because the Sharpe ratio increases with volatility and ii) negatively related to investment opportunities because margin requirements become tighter as volatility increases. Thus, when $\rho_{vS} < 0$ and $\gamma > 1$ ($\gamma < 1$), the hedging demand produced by variation in the Sharpe ratio is positive (negative), and the hedging demand produced by variation in margin requirements is negative (positive). The sign of the total hedging demand depends on the relative sizes of these demands. To separate the effect of time variation in margin requirements in the following numerical analysis, I compare portfolio policies with and without margin requirements in the presence of stochastic volatility and attribute the difference in optimal portfolios to the impact of the constraints.

The third term in equation (8) appears only when the price of the risky asset follows a jump-diffusion process (i.e., $\lambda > 0$, $Q_S \neq 0$) and corresponds to the event-risk hedging demand studied by Liu et al. (2003) and Wu (2003). It complicates the computation of the optimal portfolio by making the equation for $\bar{\omega}(v)$ nonlinear. Note that because the price, volatility, and tightness of the constraints jump simultaneously, the jumps in volatility (and constraints) may have an amplification effect on the event-risk hedging demand. Also, the equation for $H(v)$ is modified in the presence of jumps, so the hedging demand produced by continuous changes in volatility and margins is affected as well.

As a benchmark for assessing the impact of margin requirements, it is natural to consider the optimal portfolio policy without constraints. When investors have a unit EIS, a linear function $H(v) = A + Bv$ solves equation (9), and the unconstrained optimal portfolio problem admits a closed-form solution.[9] The exact result is stated in Corollary 1:

Corollary 1. Assume that $\psi = 1$ and there are no margin requirements. Then, the optimal portfolio policy is constant: $\omega^*(v) = \omega^*$, and ω^* together with the constant B solves the following system of nonlinear algebraic equations:

$$\omega^* = \frac{\mu_S}{\gamma\bar{\nu}} + \frac{\bar{\sigma}_v\rho_{vS}B}{\gamma} + \frac{\lambda Q_S}{\gamma\bar{\nu}}\left[(1+\omega^* Q_S)^{-\gamma}\exp(BQ_v) - 1\right], \tag{13}$$

$$\begin{aligned} &\frac{1}{2}\bar{\sigma}_v^2 B^2 - B\left(\phi_v + \frac{Q_v\lambda}{\bar{\nu}}\right) \\ &\quad + (1-\gamma)\left(\omega^*\left(\frac{\mu_S - \lambda Q_S}{\bar{\nu}} + B\bar{\sigma}_v\rho_{vS}\right) - \frac{\gamma}{2}(\omega^*)^2\right) - \beta B \\ &\quad + \frac{\lambda}{\bar{\nu}}\left[(1+\omega^* Q_S)^{1-\gamma}\exp(BQ_v) - 1\right] = 0. \end{aligned} \tag{14}$$

[9] A similar result is demonstrated by Schroder and Skiadas (1999), Chacko and Viceira (2005), and Zhou and Zhu (2012) for models without jumps.

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The indirect utility function is given by equation (10) with $H(v) = A + Bv$, where

$$A = \frac{1}{\beta}(B\phi_v\bar{v} + (1-\gamma)(r - \beta + \beta\log(\beta))).$$

In the absence of jumps (i.e., when $Q_S = 0$, $Q_v = 0$, and $\lambda = 0$), equations (13) and (14) provide a quadratic equation for B, which can be solved analytically.

Proof. See the Appendix.

Corollary 1 has two main implications. First, it shows that models with a unit EIS remain analytically tractable even with jumps in prices and state variables. Second, it indicates that $\psi = 1$ is a convenient choice for numerical analysis because in this case the unconstrained benchmark portfolio policy can be computed analytically, and numerical methods are required only for solving the constrained optimization problem.

III. Analysis and Numerical Results

A. Calibration of the Model Parameters

In this section, I illustrate the implications of Proposition 1 and study the optimal consumption and portfolio policy for various combinations of model parameters. In particular, I am interested in the effect of time-varying margin requirements on the optimal portfolio. I identify the risky asset with the aggregate stock market and assume that the investor is sufficiently risk-tolerant to hold a leveraged portfolio. I choose the parameters of the model to be as realistic as possible and set them based on empirical estimates of stochastic volatility models available in the literature. Many papers that study the size of hedging demand follow a similar approach and calibrate parameters using historical data on asset returns (e.g., Campbell and Viceira (1999), Lynch (2001), Detemple et al. (2003), Chacko and Viceira (2005), and Buraschi et al. (2010)).

To separate the impact of continuous variation and jumps in margin requirements, I consider two versions of the model: without jumps (i.e., with $\lambda = 0$, $Q_S = 0$, and $Q_v = 0$) and with jumps in the price and volatility. In all specifications, I set the time preference parameter β to 0.03 and assume that the risk-free rate is $r = 0.02$. The coefficient of risk aversion is $\gamma = 2$, and this choice implies that at least in some states the investor wants to hold a leveraged portfolio (otherwise margin requirements would be irrelevant).

The calibration of the elasticity of intertemporal substitution ψ is debated in the literature: Many empirical studies indicate that it is less than 1 (e.g., Hall (1988), Ogaki and Reinhart (1998), and Yogo (2004)), although some papers challenge this conclusion (e.g., Attanasio and Weber (1989), Bansal and Yaron (2004), and Chen, Favilukis, and Ludvigson (2013)). This controversy can be possibly explained by imprecision of the obtained estimates and heterogeneity in the EIS across agents (e.g., Guvenen (2006)). Motivated by analytical tractability of the unconstrained model with a unit EIS, I set $\psi = 1$ in the baseline calibration and explore the sensitivity of conclusions to this choice as a robustness test. Note that in the baseline case $\gamma > 1/\psi$, so the investor prefers early resolution of uncertainty.

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The model without jumps belongs to a standard class of continuous time models with stochastic volatility that were estimated and used for description of aggregate stock returns in numerous papers.[10] However, the obtained parameter estimates vary across studies because the authors use different model specifications (e.g., affine and nonaffine processes), different indexes as proxies for market returns (e.g., S&P 100 and S&P 500), different estimation techniques (e.g., generalized method of moments (GMM), efficient method of moments (EMM), maximum likelihood, and Bayesian Markov chain Monte Carlo (MCMC)), and different time periods. Many studies use not only index returns but also option prices (e.g., Chernov and Ghysels (2000), Pan (2002), and Eraker (2004)), futures option prices (e.g., Broadie et al. (2007)), volatility index (VIX) (Jones (2003)), and variance swap rates (Egloff et al. (2010)). Table 1 collects some of the estimates available in the literature.

TABLE 1

Estimates of Stochastic Volatility Models

Table 1 presents the estimates of stochastic volatility models available in the literature. The models SV and SVJ correspond to specifications without and with jumps in prices, respectively. The parameter μ_S is omitted for model specifications in which the risk premium does not depend on volatility. All parameters have been annualized.

Publication	Model	$\bar{v}$	ϕ_v	$\bar{\sigma}_v$	ρ_{vS}	μ_S
Andersen et al. (2002)	SV	0.013	3.93	0.199	−0.59	0.09
	SVJ	0.013	3.70	0.187	−0.62	0.13
Pan (2002)	SV	0.024	5.30	0.380	−0.57	0.11
	SVJ	0.013	7.10	0.280	−0.52	0.04
Eraker et al. (2003)	SV	0.023	5.80	0.357	−0.40	—
	SVJ	0.020	3.28	0.255	−0.47	—
Chernov et al. (2003)	SV	0.015	3.28	0.153	−0.28	—
	SVJ	0.016	2.77	0.102	−0.43	—
Eraker (2004)	SV	0.022	4.28	0.275	−0.37	—
	SVJ	0.021	3.02	0.201	−0.47	—
Egloff et al. (2010)	SV	0.004	4.49	0.250	−0.73	0.01

To calibrate the model, I choose one combination of the parameters that is in the ballpark of the empirical estimates (later I demonstrate that my results are not sensitive to this particular choice). In the benchmark specification without jumps, I set the mean of v_t at $\bar{v} = 0.02$, the persistence of volatility at $\phi_v = 5$, and the volatility of stochastic variance at $\bar{\sigma}_v = 0.3$. The correlation between innovations in the price and return variance is $\rho_{vS} = -0.5$. Also, I assume that the unconditional risk premium is 8% per year, so $\mu_S = 0.08$.

To calibrate the parameters in the model with jumps, I follow the approach of Liu et al. (2003). In particular, I start with the parameters of the benchmark model without jumps, specify ex ante the intensity of jumps λ and the sizes of price and variance jumps Q_S and Q_v, and adjust the parameters $\bar{v}$ and $\bar{\sigma}_v$ so that

[10] An incomplete list includes Chernov and Ghysels (2000), Andersen, Benzoni, and Lund (2002), Pan (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), Eraker, Johannes, and Polson (2003), Jones (2003), Eraker (2004), Aït-Sahalia and Kimmel (2007), Broadie et al. (2007), and Egloff, Leippold, and Wu (2010).

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they produce the same expected variance of returns $\mathrm{E}[\mathrm{Var}_t(dS_t/S_t)]/dt$ and the same expected variance of innovations in the state variable $\mathrm{E}[\mathrm{Var}_t(dv_t)]/dt$ as in the model without jumps.[11] I separately consider two cases: one with high intensity of jumps ($\lambda = 0.5$) and the other with low intensity of jumps ($\lambda = 0.04$). Because the average frequency of jumps is approximately $1/\lambda$, the chosen values imply on average 1 jump in 2 years in the first case and 1 jump in 25 years in the second case. The range is wide because there are diverse empirical estimates for intensity of jumps in the literature. In their baseline model, Liu et al. (2003) set $\lambda/\bar{\nu} = 1.84156$; together with their other parameters, this choice corresponds to 1 jump in 25 years. The empirical estimates of stochastic volatility models with jumps suggest a much higher frequency: 1 jump in 1.3 years in Eraker (2004) and approximately 1 jump per year in Eraker et al. (2003) and Chernov et al. (2003).

There is also a wide dispersion in the estimates of Q_S and Q_v available in the literature. For example, Liu et al. (2003) use $Q_S = -0.25$ and $Q_v = 0.22578$ in their benchmark specification. Similarly, Liu and Pan (2003) consider Q_S equal to -0.1, -0.25, and -0.5. However, the estimates for Q_S in Pan (2002) are of the order of -0.01. Similarly, the results of Eraker et al. (2003) and Eraker (2004) suggest that the size of price jumps is several percentage points. A comparable discrepancy is observed for Q_v. To ensure that my conclusions are not sensitive to the calibration of jump sizes, I consider Q_S and Q_v in the ranges from 0 to -0.3 and from 0 to 0.15, respectively, when $\lambda = 0.04$. In the case of more frequent jumps (when $\lambda = 0.5$), the ranges are from 0 to -0.1 and from 0 to 0.05, respectively. Note that for all considered combinations of λ, Q_S, and Q_v, the matching of the parameters just described is well defined and the Feller condition is satisfied.

Because margin requirements are set by creditors (brokerage firms) and may vary across borrowers, it is not easy to calibrate the parameter $\bar{m}$ that determines the average tightness of margins in the model. However, Regulation T of the Federal Reserve Board states that the initial margin requirement for equity cannot be below 50%, so in the baseline specification I set $\bar{m} = 2$.[12] This choice implies that the margin loan cannot exceed 50% of the current market value of the risky security when $v_t = \bar{\nu}$. As in the case of other parameters, I also examine the sensitivity of my results by considering alternative values of $\bar{m}$.

B. Time-Varying Margin Requirements without Jumps

To separate the effect of continuous changes in margin requirements from the effect of jumps, I first consider a setting without jumps and set $Q_S = 0$,

[11] These statistics are equal to $\bar{\nu}$ and $\bar{\sigma}_v^2\bar{\nu}$ in the model without jumps and to $\bar{\nu} + \lambda Q_S^2$ and $\bar{\sigma}_v^2\bar{\nu} + \lambda Q_v^2$ in the model with jumps. Thus, in terms of the parameters of the model without jumps, $\bar{\nu}$ and $\bar{\sigma}_v^2$ in the model with jumps are set to $\bar{\nu} - \lambda Q_S^2$ and $(\bar{\sigma}_v^2\bar{\nu} - \lambda Q_v^2)/(\bar{\nu} - \lambda Q_S^2)$, respectively. Note that the matching of $\mathrm{E}[\mathrm{Var}_t(dS_t/S_t)]/dt$ immediately implies that $\bar{\nu}$ differs across the models so that the additional volatility of returns brought by jumps is offset by reduction in the volatility associated with continuous price changes.

[12] Maintenance margin requirements are typically less restrictive. For example, the Financial Industry Regulation Authority (FINRA) mandates that the maintenance margin must be at least 25%. Individual brokerage firms usually use higher levels.

$Q_v = 0$, and $\lambda = 0$. In this case, equation (8) simplifies, and the optimal portfolio weight is

$$\omega^*(v) = \min\left(\max\left(\bar{m}\sqrt{\frac{\bar{v}}{v}}, 1\right), \frac{\mu_S}{\gamma\bar{v}} + \frac{\bar{\sigma}_v\rho_{vS}}{\gamma}H'(v)\right). \tag{15}$$

The function $H(v)$ is determined by equation (9), which even without the last term is a complicated nonlinear differential equation. Unfortunately, it does not have a closed-form solution and can be solved only numerically. For this purpose, I use a projection method, which approximates $H(v)$ by a linear combination of orthogonal polynomials that are chosen to be Chebyshev polynomials (Judd (1998)). This approach ensures that $H(v)$ is twice continuously differentiable for all $v \in (0, \infty)$. A detailed description of the employed numerical procedure is presented in Internet Appendix (available at www.jfqa.org).

To illustrate the impact of margin constraints on the optimal portfolio policy, I plot the optimal portfolio weights $\omega^*(v)$ in unconstrained and constrained models with identical parameters in the same graph. Also I indicate the upper bound on the portfolio weight implied by margin requirements in each state of the economy.

The center graph of Figure 1 corresponds to the baseline set of the parameters. As follows from Corollary 1, the unconstrained optimal portfolio weight is constant and equal to 2.062. The weight is a sum of the myopic component $\mu_S/(\gamma\bar{v})$ (equal to 2) and the hedging component $\bar{\sigma}_v\rho_{vS}B/\gamma$ (equal to 0.062) produced by stochastic volatility. Because the conditional Sharpe ratio increases with v_t and innovations in volatility and returns are negatively correlated ($\rho_{vS} = -0.5$), a negative shock to the Sharpe ratio, which implies a deterioration in investment opportunities, is often accompanied by a positive shock to the price. Therefore, the stock itself is a hedge against adverse changes in investment opportunities, the marginal utility of an investor with $\gamma > 1$ decreases with v ($B = -0.83$), and the hedging demand produced by stochastic volatility is positive. This result is consistent with the general discussion of hedging demand in Section II.

In the presence of margin requirements, the highest permissible weight of the risky asset (plotted as a dotted curve in Figure 1) is inversely proportional to the volatility of returns. Thus, when volatility is high, the unconstrained optimal portfolio is infeasible. In the vast majority of such states, it is optimal for the investor to exhaust borrowing capacity and have the highest possible leverage (the solid line coincides with the dotted line). In contrast, margin constraints are loose when volatility is low and the unconstrained solution is attainable. Nevertheless, the constraints make the investor more conservative even when they do not bind, and the optimal portfolio weight is smaller than in the unconstrained economy. The difference between the constrained and unconstrained solutions in the region where the constraint does not bind is equal to the hedging demand produced by time-varying margin requirements. Consistent with the discussion in Section II, it has the negative sign. Note that in general the presence of margin constraints changes both the level and the slope of the demand function. Thus, when volatility increases and margins become tighter, the investor preemptively sells the risky asset even when constraints do not bind.

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FIGURE 1

Optimal Portfolio Weights $\omega^*(v)$ for Alternative Values of ϕ_v and $\bar{\sigma}_v$

Figure 1 presents the optimal portfolio policy $\omega^*(v)$ in the absence of jumps in prices and volatility. Different graphs correspond to different combinations of the parameters ϕ_v and $\bar{\sigma}_v$, which characterize the stochastic variance process. The solid line shows the optimal portfolio weight in an economy with margin requirements; the dashed line shows the optimal portfolio weight in an identical economy without margin requirements; the dashed-dotted line shows the myopic portfolio weight; the dotted line indicates the upper bound on the portfolio weight implied by margin requirements. The combination of the parameters in the top-right graph violates the Feller condition $2\phi_v\bar{v} \geq \bar{\sigma}_v^2$.

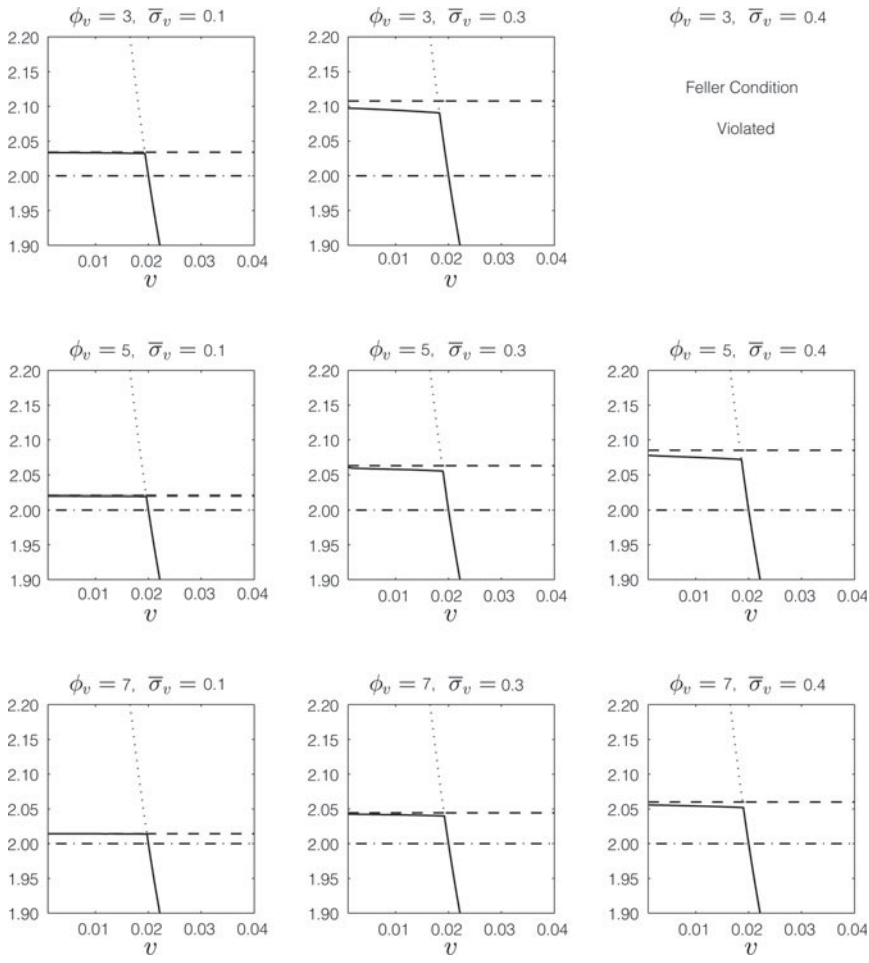


Figure 1 shows that the hedging demand is the largest when v_t is close to the point where the maximum leverage permitted by the constraint coincides with the optimal unconstrained leverage. However, even in this range the size of the hedging demand is not impressive: It does not exceed 0.01 and constitutes less than 0.5% of the unconstrained demand. When the probability of hitting the constraint is lower (v_t is lower), the hedging demand is even smaller. This is one of the main findings of the paper: Even though the variation in margin constraints produces additional hedging demand, its practical importance is limited, and it would not be a big mistake to ignore it completely. This message becomes even

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more compelling if the uncertainty in the parameter estimation is taken into account. However, the conclusion may rely on the ability of the investor to trade continuously without price impact. In reality, changes in margin requirements often coincide with illiquidity events or jumps in the price level. I explore the effect of these additional factors in Section III.D.

Along with the graph for the benchmark set of the parameters, Figure 1 presents graphs for alternative values of ϕ_v and $\bar{\sigma}_v$, which deliver several additional observations.[13] To save space, I consider only 1 point above and 1 point below the baseline values of ϕ_v and $\bar{\sigma}_v$.

First, the optimal portfolio policies for different combinations of the parameters are qualitatively similar and resemble the policy in the baseline model. Moreover, the hedging demand produced by margin requirements is small for all parameters, including those for which the constraints are relatively persistent and volatile. The largest hedging demand is observed for $\phi_v = 3$, $\bar{\sigma}_v = 0.3$, and even in this case it is less than 0.02, which is less than 1% of the unconstrained portfolio weight.

Second, Figure 1 demonstrates that the size of the hedging demand grows with the volatility of margin constraints and their persistence, which are determined by the volatility and persistence of v_t (the latter is higher for lower ϕ_v). This result has an intuitive explanation. Because v_t follows a Cox et al. (1985) process, its unconditional variance is $\bar{v}\bar{\sigma}_v^2/(2\phi_v)$, which grows with $\bar{\sigma}_v$ and $1/\phi_v$. Given that v_t is bounded from below and the tightness of the constraints is inversely related to $\sqrt{v_t}$, an increase in the unconditional volatility has two effects. First, the probability that margin requirements bind increases. Second, extreme realizations of v_t become more likely, so the constraints on average are more binding when they bind. Due to both effects, the incentives to hedge the variation in margins are stronger and the hedging demand is more pronounced for high $\bar{\sigma}_v$ and low ϕ_v.

Next, I explore the effect of margin requirements for alternative mean return variance $\bar{v}$ and tightness of margin requirements $\bar{m}$. The results are presented in Figure 2. Again, along with the optimal portfolio weight in the presence of margin requirements, I plot the unconstrained optimal portfolio and the upper bound on the portfolio weight implied by margin requirements. Obviously, the parameter $\bar{m}$ does not affect the unconstrained optimal portfolio, which is the same across the columns of Figure 2. However, an increase in $\bar{v}$ reduces the optimal unconstrained portfolio weight, and the effect is largely due to a decrease in the myopic demand (see equations (13) and (14)).

Figure 2 shows that the shape of the optimal portfolio policy in the presence of constraints is the same for all combinations of the parameters; again, the agent underinvests in the risky asset compared to the unconstrained case even when the constraints do not bind. As in the baseline model, the hedging demand produced by constraints is consistently small: Even the largest hedging demand observed for $\bar{v} = 0.015$ and $\bar{m} = 1.5$ does not exceed 0.04 and constitutes less than 1.5% of the unconstrained demand.

[13] The blank graph in Figure 1 appears for the combination of the parameters that violates the Feller condition $2\phi_v\bar{v} \geq \bar{\sigma}_v^2$; in this case the process for v_t is nonstationary and can reach the boundary $v = 0$ with a positive probability.

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FIGURE 2

Optimal Portfolio Weights $\omega^*(v)$ for Alternative Values of $\bar{v}$ and $\bar{m}$

Figure 2 presents the optimal portfolio policy $\omega^*(v)$ in the absence of jumps in prices and volatility. Different graphs correspond to different combinations of the average variance of returns $\bar{v}$ and the tightness of margin requirements $\bar{m}$. The solid line shows the optimal portfolio weight in an economy with margin requirements; the dashed line shows the optimal portfolio weight in an identical economy without margin requirements; the dashed-dotted line shows the myopic portfolio weight; the dotted line indicates the upper bound on the portfolio weight implied by margin requirements.

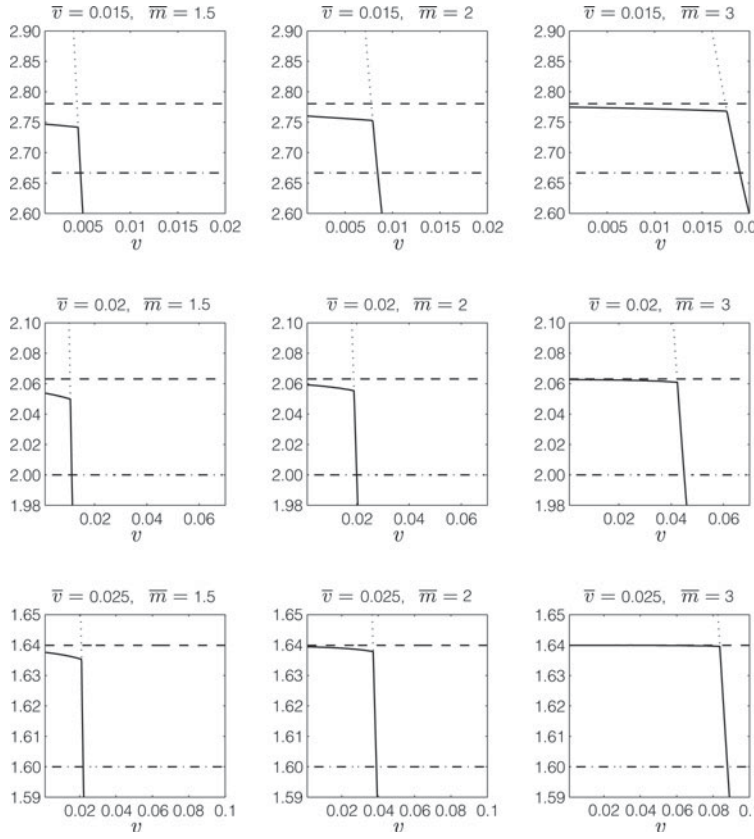


Figure 2 also demonstrates that the effect of the constraints becomes weaker as $\bar{m}$ increases. When $\bar{m}$ is high, the optimal unconstrained portfolio is infeasible only in the high-volatility states. Because the conditional variance v_t follows a mean-reverting process and its unconditional distribution has the highest density around $\bar{v}$, the probability that the economy reaches the states with high v is low. As a result, margin requirements have only a weak effect on the optimal portfolio strategy when they do not bind, and the hedging demand produced by them is small. In the other extreme with low $\bar{m}$, the constraints bind in the majority of the states and their presence affects the optimal portfolio more strongly even in the unconstrained states.

The relation between the size of the hedging demand and the average variance of returns $\bar{v}$ for a fixed $\bar{m}$ is more complicated and depends on which of the following two effects dominates. On the one hand, a higher $\bar{v}$ implies a higher

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probability to reach high-volatility states in which margin requirements bind, and this makes them more relevant. On the other hand, the optimal unconstrained portfolio becomes less leveraged as $\bar{v}$ increases, but the level of the constraints in the states around $\bar{v}$ (where the economy is most likely to be) stays the same. As a result, the relevance of the constraints decreases with $\bar{v}$. The latter effect is clearly illustrated by the middle column of graphs in Figure 2, which corresponds to $\bar{m} = 2$. When $\bar{v} = 0.015$, the unconstrained optimal portfolio is infeasible when the variance v_t is close to $\bar{v}$ and the investor is constrained most of the time. In contrast, when $\bar{v} = 0.025$, the optimal unconstrained portfolio is feasible in the most probable states around $\bar{v} = 0.025$, and the constraints bind only in the high volatility states with $v > 0.037$. Figure 2 unambiguously shows that the hedging demand produced by the constraints tends to decrease with $\bar{v}$. This result implies that the second effect dominates and margin constraints are less important for the investor when $\bar{v}$ is high.

In general, the size of hedging demand may depend on the preference parameters, such as the coefficient of risk aversion γ and the elasticity of intertemporal substitution ψ, so I find optimal portfolios for alternative values of these parameters. In particular, besides the baseline specification with $\gamma = 2$ and $\psi = 1$, I consider models with $\gamma = 1.5$ and $\gamma = 3$ as well as with $\psi = 0.7$ and $\psi = 1.25$. This particular choice of the parameters ensures that $\gamma > 1/\psi$ in all calibrations, so the investor prefers early resolution of uncertainty.

Figure 3 reports the optimal portfolio policy for various combinations of γ and ψ. Consistent with the previous analysis, the hedging demand produced by margin constraints is uniformly small: Its largest size is around 0.01 (observed for $\gamma = 1.5$), which is less than 0.5% of the unconstrained demand. In the considered range for γ, the hedging demand decreases with risk aversion. When γ is high, the investor has low optimal leverage and hits the constraint only in the high-volatility states. Because the probability of reaching these states is low, the size of the hedging demand is small.

Figure 3 also presents optimal portfolios for alternative values of ψ and demonstrates that the elasticity of intertemporal substitution has almost no impact on the optimal portfolio strategy in both unconstrained and constrained economies. This result echoes the conclusion of Campbell and Viceira (1999), Campbell et al. (2003), Chacko and Viceira (2005), and Zhou and Zhu (2012), who find little variation in hedging demand across investors with different EIS values in a 1-factor stochastic volatility model.

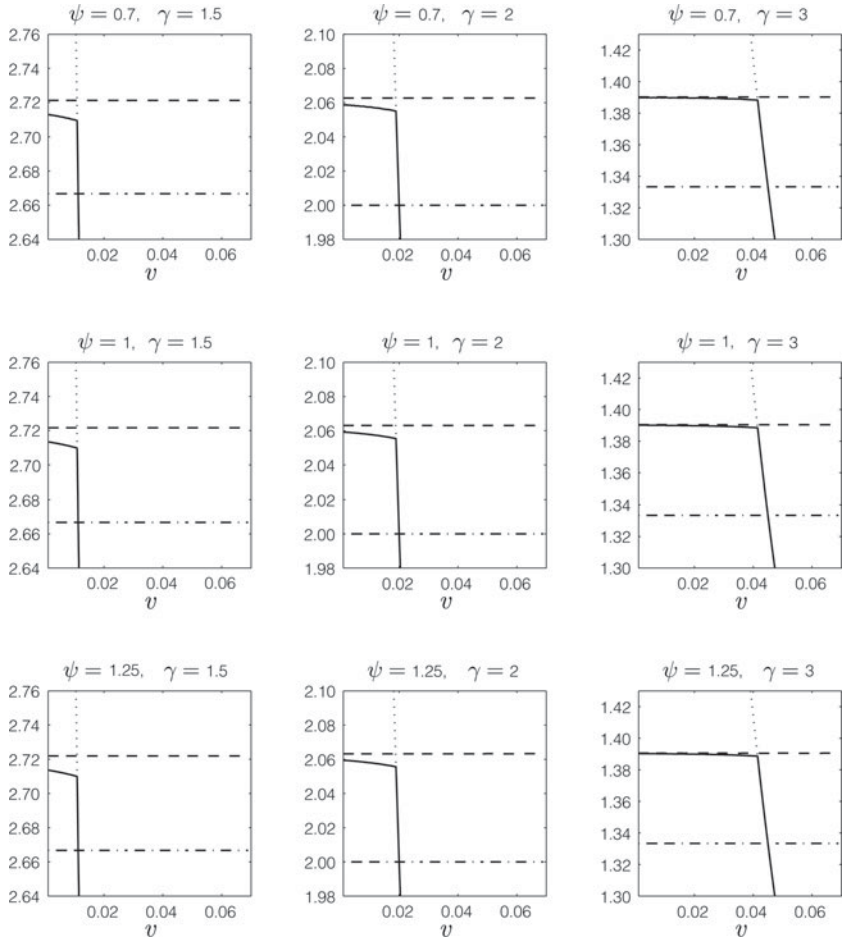
Although neither the EIS nor the constraints noticeably affect the optimal portfolio in the unconstrained region, they have a pronounced effect on the optimal consumption policy. Figure 4 plots C^*/W as a function of v for the unconstrained and constrained economies with various values of ψ and γ. It shows that the EIS has a strong impact on consumption in the unconstrained economy: When $\gamma = 1.5$, the consumption–wealth ratio is around 0.06 for $\psi = 0.7$ and only 0.005 for $\psi = 1.25$. This happens because the investor prefers to postpone consumption when the elasticity of intertemporal substitution is high and, therefore, reduces current consumption. Obviously, C^*/W is constant and equal to $\beta = 0.03$ when $\psi = 1$ (see equation (11)). It increases with v when $\psi < 1$ (the Sharpe ratio is an increasing function of v, and the income effect dominates the substitution effect

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FIGURE 3

Optimal Portfolio Weights $\omega^*(\nu)$ for Alternative Values of ψ and γ

Figure 3 presents the optimal portfolio policy $\omega^*(\nu)$ in the absence of jumps in prices and volatility. Different graphs correspond to different combinations of the elasticity of intertemporal substitution ψ and the coefficient of risk aversion γ. The solid line shows the optimal portfolio weight in an economy with margin requirements; the dashed line shows the optimal portfolio weight in an identical economy without margin requirements; the dashed-dotted line shows the myopic portfolio weight; the dotted line indicates the upper bound on the portfolio weight implied by margin requirements.



for agents with $\psi < 1$) and decreases with ν when $\psi > 1$ (this effect is weak and almost unnoticeable).

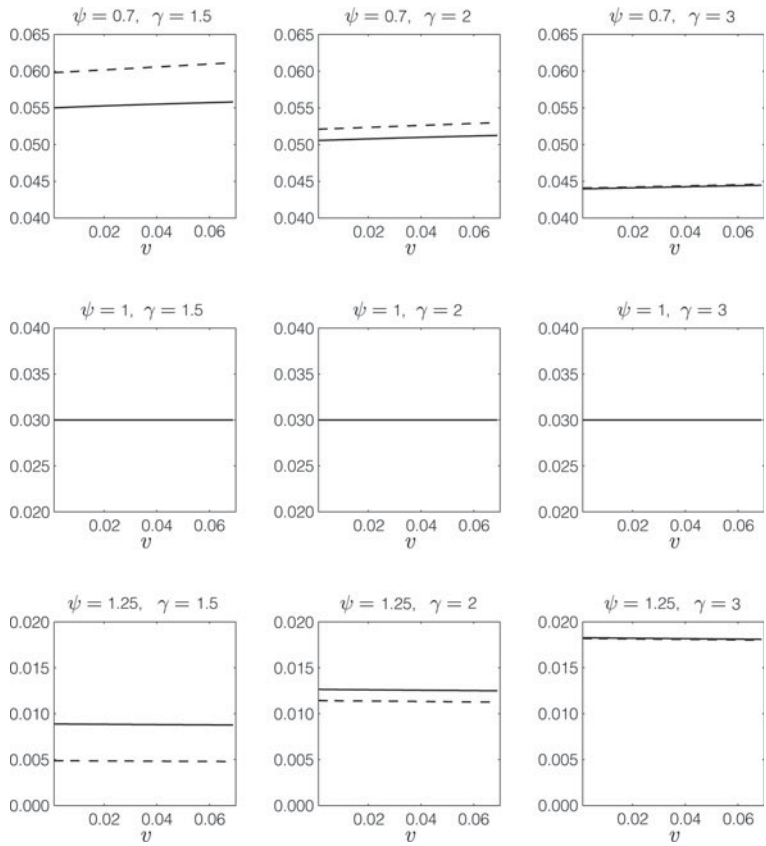
More interestingly, as opposed to the portfolio weights, consumption is strongly affected by margin requirements even in the states where they do not bind. For example, in the case with $\gamma = 1.5$, consumption falls by more than 8% when $\psi = 0.7$, but it increases by almost 80% when $\psi = 1.25$ due to the presence of the constraints. The direction of the effect is intuitive: The constraints worsen the investment opportunities, so an investor with $\psi > 1$ consumes more (the substitution effect dominates), but an investor with $\psi < 1$ consumes less (the income effect dominates). The impact of the constraints is particularly pronounced when

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FIGURE 4

Optimal Consumption–Wealth Ratio C^*/W for Alternative Values of ψ and γ

Figure 4 presents the optimal consumption–wealth ratio C^*/W as a function of v in the absence of jumps in prices and volatility. Different graphs correspond to different combinations of the elasticity of intertemporal substitution ψ and the coefficient of risk aversion γ. The solid and dashed lines show the optimal consumption–wealth ratios in the economies with and without margin requirements, respectively.



the risk aversion is low and the desired high leverage is inaccessible in many states of the economy (for $v > 0.01$ when $\gamma = 1.5$, see Figure 3), but becomes much weaker for high γ when the unconstrained optimal leverage is low and satisfies margin requirements in the most probable states.

C. Certainty Equivalent Loss

The CEL provides a natural way to quantify a reduction in investment opportunities caused by portfolio constraints as well as to measure inefficiencies produced by suboptimal portfolio strategies. Accordingly, I compute two CELs. The first CEL results from the presence of margin constraints when the investor uses optimal consumption and portfolio policies. The second CEL is caused by ignoring the hedging demand produced by margin constraints, that is, by holding the unconstrained optimal portfolio instead of the actual optimal portfolio in the

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presence of margin constraints when the unconstrained portfolio satisfies margin requirements. In all scenarios, I use the baseline specification of the model parameters.

The CEL produced by the presence of margin requirements is defined as a solution to the following equation:

$$J(W(1-\mathrm{CEL}(v)), v\,; \bar{m}=\infty) \quad = \quad J(W, v\,; \bar{m}), \tag{16}$$

where $J(W, v)$ is the investor's indirect utility function. Note that $J(W, v\,; \bar{m}=\infty)$ is the utility of an unconstrained investor whose optimal portfolio is given by Corollary 1. Effectively, $\mathrm{CEL}(v)$ is equal to the fraction of wealth that an unconstrained investor would give up for not being constrained by time-varying margin requirements when the variance of returns is v. $\mathrm{CEL}(v)$ is directly related to the function $H(v)$, which solves equation (9) when $\psi \neq 1$ or equation (12) when $\psi = 1$. Together with equation (10), equation (16) yields

$$\mathrm{CEL}(v) \quad = \quad 1 - \exp\left(\frac{H(v) - H_{\mathrm{unc}}(v)}{1-\gamma}\right),$$

where $H_{\mathrm{unc}}(v)$ solves equation (9) (or equation (12)) in the absence of the constraints. Because portfolio constraints can only reduce investment opportunities, the CEL is always nonnegative.

Graph A of Figure 5 plots $\mathrm{CEL}(v)$ produced by the presence of margin requirements. It shows that the constraints substantially change the investor's utility. For example, the investor would give up almost 16% of his or her wealth for not being constrained when $v = 0.06$. The impact of the constraints is stronger in the high-volatility states, where the constraints are tight. When v is low, the optimal portfolio policy is close to its unconstrained counterpart, and although the constraints can become binding in the future, the loss caused by them is reduced by time discounting.

I also use the CEL for measuring the impact of hedging the variation in margin requirements on the investor's utility. In particular, I compare the indirect utility functions of two investors in the economy with constraints. The first follows the optimal consumption and portfolio policies described in Proposition 1. The second holds the optimal unconstrained portfolio when such a portfolio is feasible and chooses to be bound by the constraints otherwise. The investor's consumption policy is still optimal and maximizes utility, although it is different from the consumption policy of the first investor because it takes the suboptimal portfolio policy into account. The proportion of wealth that the first investor wants to pay to be able to adjust portfolio weights in response to changes in margin requirements is the CEL of the second investor. Formally, the CEL solves the following equation:

$$J(W(1-\mathrm{CEL}(v)), v\,; \bar{m}) \quad = \quad J_0(W, v\,; \bar{m}), \tag{17}$$

where $J(W, v\,;\, \bar{m})$ is the indirect utility function from Proposition 1, and $J_0(W, v\,; \bar{m})$ is the utility of the investor who follows the optimal unconstrained

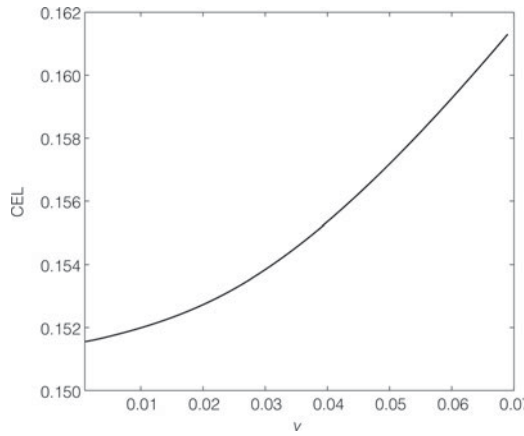
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FIGURE 5

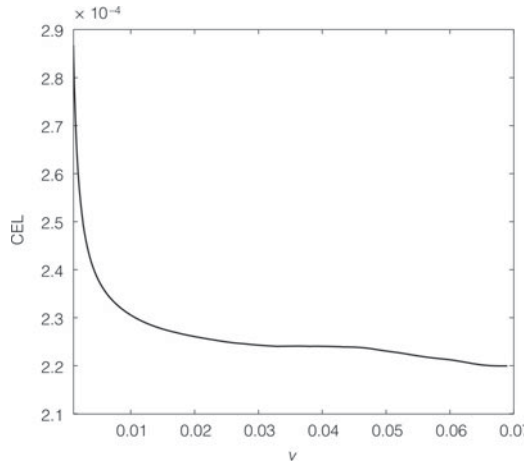
Certainty Equivalent Loss (CEL)

Figure 5 presents the CELs as functions of the state variable v. The first CEL (Graph A) results from the presence of margin requirements when the investor implements the optimal consumption and portfolio policies. The second CEL (Graph B) is produced by ignoring the hedging demand and implementing the optimal unconstrained portfolio strategy in the states of the constrained economy in which such a strategy is feasible.

Graph A. The CEL Produced by Margin Requirements



Graph B. The CEL Produced by Ignoring the Hedging Demand



portfolio strategy when the constraints do not bind. Using equation (10), the equation for $\mathrm{CEL}(v)$ can be rewritten as

$$\mathrm{CEL}(v) \quad = \quad 1 - \exp\left(\frac{H_0(v) - H(v)}{1-\gamma}\right),$$

where $H(v)$ solves equation (9) (or equation (12) when $\psi = 1$) with the optimal weight given by equation (7) and with $\bar{\omega}(v)$ from equation (8), whereas $H_0(v)$ solves equation (9) (or equation (12) when $\psi = 1$) with the weight from

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equation (7), where $\bar{\omega}(v)$ is equal to the optimal weight in the unconstrained economy (given by equation (13) in the case $\psi = 1$). The CEL is a function of the state variable v, and it is plotted in Graph B of Figure 5.

As known from the previous analysis, the hedging demand produced by margin constraints is relatively small. Thus, it would be natural to expect that the CEL associated with it is also small. This intuition is fully confirmed by Figure 5, which shows that the CEL does not exceed 0.03%. It is also not surprising that the CEL decreases with v. When the constraints are tight, both investors hit the constraints, and the disregard of the hedging demand in the unconstrained states by one of the investors only weakly affects the investor's utility.

D. Margin Requirements with Jumps

A relatively small size of the hedging demand produced by continuous variation in margin constraints could possibly be explained by the ability of the investor to constantly adjust his or her portfolio in response to even small revisions in margin requirements. Because in practice the requirements are tightened abruptly and unexpectedly when volatility jumps upward, the conclusions based on modeling the dynamics of constraints as a diffusion process may be misleading. To make the analysis more realistic, now I assume that the process for conditional variance v_t, which determines the tightness of margin requirements, also contains jumps. The model is calibrated as described in Section III.A: Following Liu et al. (2003), I consider various combinations of jump intensity λ and jump sizes Q_S and Q_v and adjust $\bar{\nu}$ and $\bar{\sigma}_v$ to keep the average return variance and the average variance of innovations in the state variable v_t as in the economy without jumps.[14]

It is known that even without margin requirements, jumps in prices and volatility affect optimal portfolios (Liu et al. (2003), Wu (2003)). To disentangle the impacts of jumps and portfolio constraints, I first compute optimal portfolio weights in the unconstrained economy for various values of Q_S, Q_v, and λ. As shown in Corollary 1, the optimal unconstrained portfolio in the presence of jumps does not depend on v_t and is given by equation (13). This equation is nonlinear and does not have an analytical solution, although it can be solved numerically.

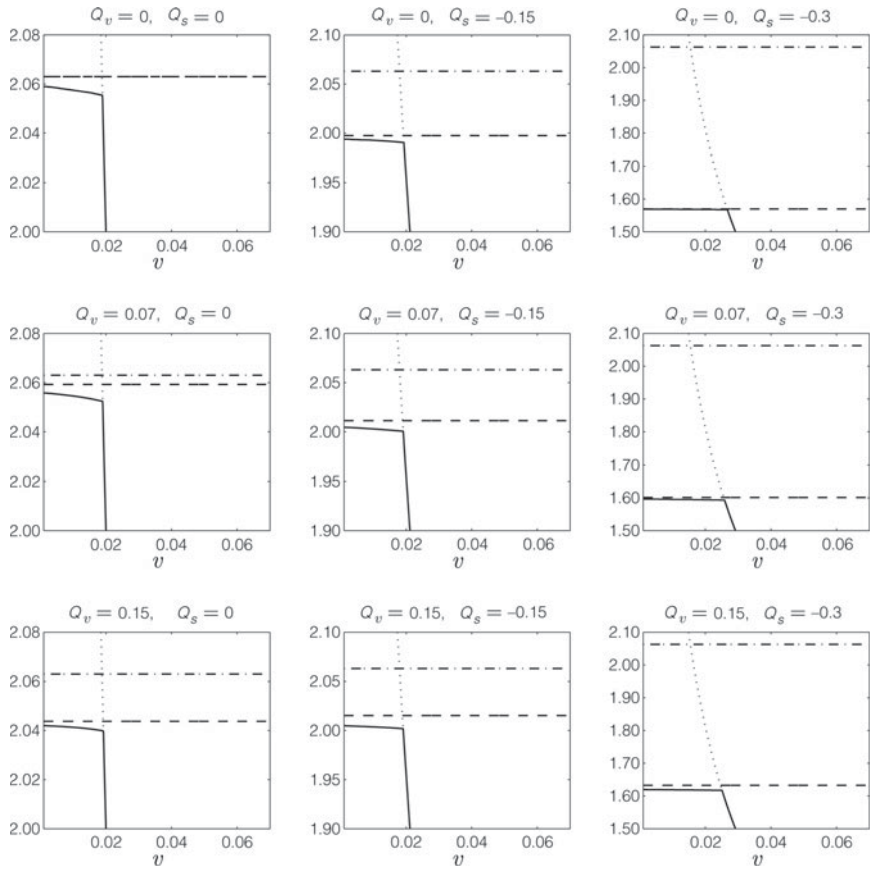
Figures 6 and 7 show optimal unconstrained weights (plotted by dashed lines) for different combinations of Q_S and Q_v when $\lambda = 0.04$ and $\lambda = 0.5$, respectively. Also, in all graphs, I indicate optimal unconstrained portfolios in economies without jumps (they are plotted by dashed-dotted lines). The figures demonstrate that the optimal investment in the risky asset is smaller in the presence of jumps. For example, a 15% jump in prices occurring once in 25 years (the case with $\lambda = 0.04$) changes the weight by approximately 0.05, which is

[14]Note that the adjustment of $\bar{\nu}$ affects the optimal portfolio by itself; as follows from equation (8), lower $\bar{\nu}$ corresponds to higher myopic demand, although the increase is largely offset by the reduction in the optimal portfolio weight caused by the presence of jumps. Moreover, without an increase in portfolio leverage, lower $\bar{\nu}$ corresponds to the smaller hedging demand produced by continuous variation in margin requirements because the probability that the constraint binds is also lower (this effect is discussed in Section III.B).

FIGURE 6

Optimal Portfolio Weights $\omega^*(v)$ for Alternative Values of Q_S and Q_v ($\lambda = 0.04$)

Figure 6 shows the optimal portfolio policy $\omega^*(v)$ in the presence of jumps. Different graphs correspond to different combinations of the size of price jumps Q_S and the size of volatility jumps Q_v. The solid line shows the optimal portfolio weight in an economy with margin requirements; the dashed line shows the optimal portfolio weight in an identical economy without margin requirements; the dashed-dotted line shows the optimal portfolio weight in an unconstrained economy without jumps; the dotted line indicates the upper bound on the portfolio weight implied by margin requirements.



approximately 2.5% of the demand without jumps. The effect is notably stronger for a 30% jump: Now the decrease in the portfolio weight is approximately 0.4, which is almost 20% of the weight without jumps. A similar pattern is observed for frequent jumps ($\lambda = 0.5$), although the magnitude of portfolio adjustments is smaller due to the smaller size of the jumps. Figures 6 and 7 also show that the effect of jumps in volatility is weaker than that of price jumps, and the optimal weight of the risky asset may increase or decrease with Q_v depending on the size of price jumps Q_S. These results are consistent with Liu et al. (2003).

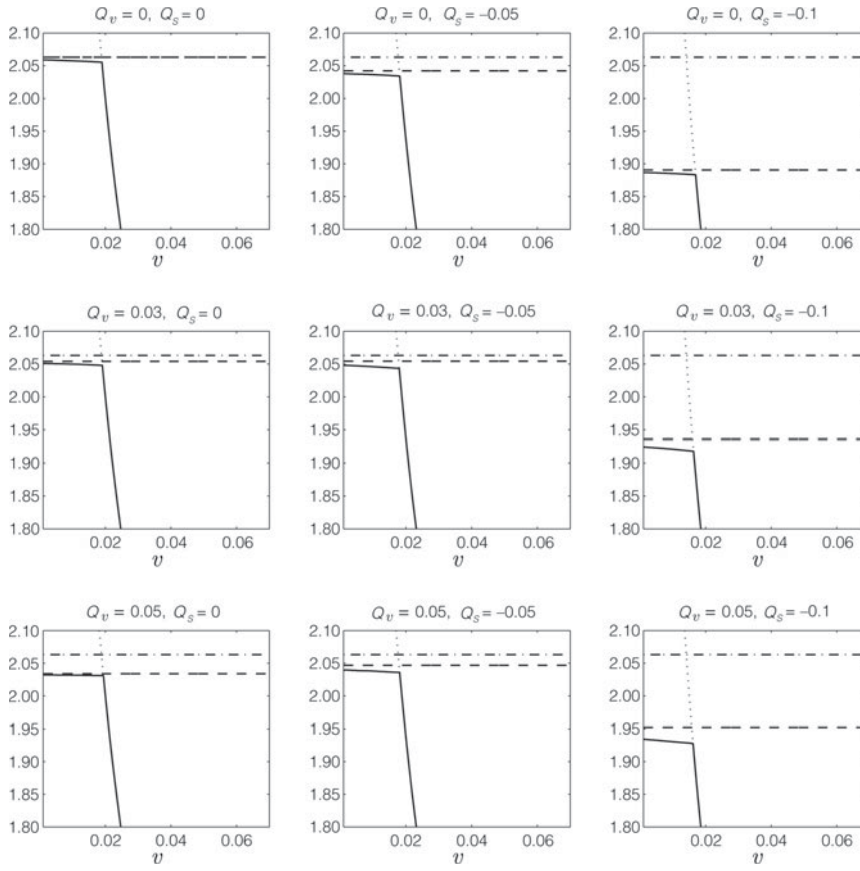
Next, I consider the effect of imposing margin requirements that can jump with volatility. The optimal portfolios in this case are plotted by solid lines in Figures 6 and 7. As in the model without jumps, margin requirements produce hedging demand, so it is optimal to underinvest in the risky asset compared to the

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FIGURE 7

Optimal Portfolio Weights $\omega^*(v)$ for Alternative Values of Q_S and Q_v ($\lambda = 0.5$)

Figure 7 shows the optimal portfolio policy $\omega^*(v)$ in the presence of jumps. Different graphs correspond to different combinations of the size of price jumps Q_S and the size of volatility jumps Q_v. The solid line shows the optimal portfolio weight in an economy with margin requirements; the dashed line shows the optimal portfolio weight in an identical economy without margin requirements; the dashed-dotted line shows the optimal portfolio weight in an unconstrained economy without jumps; the dotted line indicates the upper bound on the portfolio weight implied by margin requirements.



identical unconstrained economy even if the unconstrained portfolio is feasible. However, the hedging demand is small: For the considered parameters, it never exceeds 0.03 and typically constitutes less than 1.5% of the unconstrained demand. Comparison of the top-left graph, which corresponds to the economy without jumps, and other graphs reveals that the hedging demand in economies with jumps is very similar to that in the economy without jumps, so continuous variation in the constraints appears to be the major source of the hedging demand.

Figures 6 and 7 also demonstrate that price jumps typically have a much stronger effect on the optimal portfolio in the unconstrained region than the presence of margin requirements. This result has an interesting practical implication. Because price jumps and volatility jumps in the model are driven by the same

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Poisson process and jumps in prices can be interpreted as a form of illiquidity risk (Liu et al. (2003)), the model captures the fact that an abrupt tightening of margin requirements (as a result of a jump in volatility) is often accompanied by an illiquidity event. My results imply that the risk of tightening margin requirements is much less important for rational leveraged investors than the illiquidity risk, so they have low leverage when they anticipate an illiquidity event, not changes in margin requirements.

Although the hedging demand is uniformly small, its size still depends on the size of jumps. In particular, it tends to decrease with Q_v when $Q_S = 0$ and increase with Q_v in other considered cases. Also, the hedging demand tends to increase with the size of price jumps when volatility also jumps, although this effect is obscured in Figure 6 by the different scales used for different graphs. These effects are produced by nonlinearities in the equation for the optimal portfolio weight brought by the presence of jumps and to some extent mirror the effects of price jumps on the unconstrained optimal portfolio.

IV. Conclusion

This paper studies the optimal consumption and portfolio choice in the presence of time-varying margin requirements in a continuous time framework. It shows that the constraints produce hedging demand, which has not been quantitatively examined in the literature. The main finding of the paper is that the size of the hedging demand is relatively small: Even when the dynamics of margin requirements include jumps, the hedging demand does not exceed 1% of the unconstrained demand. Moreover, the impact of jumps in the constraints is much smaller than the impact of contemporaneous price jumps. Practically, this observation means that investors do not make a big mistake by ignoring the possibility of changes in margin requirements unless these changes coincide with deterioration in liquidity.

The paper assumes that investment opportunities of investors are exogenously specified and conducts a partial equilibrium analysis. As a result, it ignores that portfolio constraints may change expected returns and other characteristics of investment opportunities through general equilibrium effects (e.g., Detemple and Murthy (1997), Basak and Cuoco (1998), Basak and Croitoru (2000), Gârleanu and Pedersen (2011), Chabakauri (2013), and Rytchkov (2014)). However, this limitation is outweighed by advantages of focusing on the portfolio problem because the partial equilibrium approach simplifies the calibration of the model, makes it possible to rely on empirical estimates of the processes for volatility and returns, and delivers realistic quantitative estimates of how margin constraints affect optimal portfolios.

The model in the paper has only two assets. However, it may be interesting to consider a more general setup with multiple assets and explore how different margin requirements imposed on different assets affect the composition of the optimal portfolio. Also, the solution to the portfolio problem may be sensitive to whether margin requirements are imposed on individual assets or on the portfolio as a whole. A comprehensive analysis of these issues is a fruitful direction for future research.

Appendix. Proofs

Proof of Proposition 1. The stochastic optimization problem of an investor with a recursive utility function reduces to the following standard Hamilton–Jacobi–Bellman (HJB) equation:

$$\max_{\{C,\ \omega \le m(v)\}} [f(C,J) + DJ] \quad = \quad 0, \tag{A-1}$$

where $m(v)$ is the tightness of margin requirements from equation (5), the aggregator $f(C,J)$ is given by equation (2), and

$$\begin{aligned} DJ \quad = \quad & J_W \left(\frac{\mu_S - \lambda Q_S}{\bar{v}} v\omega W + rW - C \right) + \frac{1}{2} J_{WW} W^2 \omega^2 v + J_v \phi_v (\bar{v} - v) - J_v Q_v \lambda \frac{v}{\bar{v}} \\ & + \frac{1}{2} J_{vv} \bar{\sigma}_v^2 v + J_{Wv} W \omega \bar{\sigma}_v \rho_{vS} v + \frac{v}{\bar{v}} \lambda \left(J(W(1+\omega Q_S), v + Q_v) - J(W,v) \right). \end{aligned}$$

Because of homotheticity of preferences, I look for the value function in the standard form:

$$J(W,v) \quad = \quad \frac{1}{1-\gamma} W^{1-\gamma} \exp(H(v)), \tag{A-2}$$

where the function $H(v)$ is assumed to be twice continuously differentiable. The maximization in equation (A-1) with respect to C yields

$$J_W \quad = \quad \beta ((1-\gamma)J)^{\frac{1/\psi - \gamma}{1-\gamma}} C^{-\frac{1}{\psi}},$$

which together with equation (A-2) gives the optimal consumption policy stated in equation (7). Note that this consumption policy is optimal in the cases of both $\psi \neq 1$ and $\psi = 1$ (in the latter case it obviously reduces to $C^*(W,v) = \beta W$). Using the optimal consumption policy, the HJB equation can be written as

$$\max_{\{\omega \le m(v)\}} [G(\omega, v)] \quad = \quad 0, \tag{A-3}$$

where

$$\begin{aligned} G(\omega, v) \quad = \quad & \frac{1}{2(1-\gamma)} \bar{\sigma}_v^2 v (H'' + (H')^2) + \frac{1}{1-\gamma} H' \left(\phi_v (\bar{v} - v) - Q_v \lambda \frac{v}{\bar{v}} \right) \\ & + H' \omega \bar{\sigma}_v \rho_{vS} v + \frac{\mu_S - \lambda Q_S}{\bar{v}} v\omega - \frac{\gamma}{2} \omega^2 v \\ & + \frac{\lambda v}{(1-\gamma)\bar{v}} \left[(1+\omega Q_S)^{1-\gamma} \exp(H(v+Q_v) - H(v)) - 1 \right] + r \\ & + \begin{cases} \dfrac{1}{\psi - 1} \exp\left(\dfrac{1-\psi}{1-\gamma} H + \psi \log \beta \right) - \dfrac{\beta}{1 - 1/\psi}, & \text{when } \psi \neq 1, \\ \beta \log(\beta) - \beta - \dfrac{\beta}{1-\gamma} H, & \text{when } \psi = 1. \end{cases} \end{aligned}$$

Because wealth cannot become negative (otherwise the investor's utility is infinitely negative), it is sufficient to consider only those portfolio weights for which $1 + \omega Q_S > 0$ (see Liu et al. (2003) for a similar argument). It is easy to see that for $\omega < A < 0$, where A is a sufficiently negative constant, $G(\omega, v)$ increases with ω. Given that $G(\omega, v)$ is twice continuously differentiable, the solution ω^* to the maximization problem in equation (A-3) is either an internal point that satisfies the first-order condition $G_\omega(\omega^*, v) = 0$ or a point on the boundary $\omega^* = m(v)$ if $G_\omega(\omega, v) > 0$ for all $\omega \le m(v)$. The first-order condition gives equation (8), and its solution is denoted by $\bar{\omega}$. Note that

$$G_{\omega\omega} \quad = \quad -\gamma v - \frac{\gamma \lambda Q_S^2 v}{\bar{v}} (1+\omega Q_S)^{-1-\gamma} \exp(H(v+Q_v) - H(v)) \quad < \quad 0, \tag{A-4}$$

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for all admissible values of ω and v. Hence, $G(\omega, v)$ is concave in ω and has at most one local maximum in $\{\omega : \omega < m(v)\}$, and the optimal portfolio weight is $\omega^* = \min(m(v), \bar{\omega})$. The verification of $G_{\omega\omega} < 0$ is equivalent to checking the "verification theorem" (e.g., Fleming and Soner (2006)). The substitution of the optimal portfolio into equation (A-3) gives equation (9) when $\psi \neq 1$ and equation (12) when $\psi = 1$. □

Proof of Corollary 1. In the absence of margin requirements, the optimal portfolio problem from Proposition 1 admits a closed-form solution. Assuming that $H(v) = A + Bv$, where A and B are constants, and plugging this linear function into equation (8), one can see that the optimal portfolio is state independent and solves the following algebraic equation:

$$\text{(A-5)} \qquad \omega^* = \frac{\mu_S}{\gamma\bar{\nu}} + \frac{B\bar{\sigma}_v\rho_{vS}}{\gamma} + \frac{\lambda Q_S}{\gamma\bar{\nu}}\left[(1+\omega^* Q_S)^{-\gamma}\exp(BQ_v) - 1\right].$$

The constants A and B are determined by equation (12), which yields two additional algebraic equations:

$$\begin{aligned} &\frac{1}{2}\bar{\sigma}_v^2 B^2 - B\left(\phi_v + \frac{Q_v\lambda}{\bar{\nu}}\right) \\ &\quad + (1-\gamma)\left(\omega^*\left(\frac{\mu_S - \lambda Q_S}{\bar{\nu}} + B\bar{\sigma}_v\rho_{vS}\right) - \frac{\gamma}{2}(\omega^*)^2\right) \\ &\quad - \beta B + \frac{\lambda}{\bar{\nu}}\left[(1+\omega^* Q_S)^{1-\gamma}\exp(BQ_v) - 1\right] = 0, \\ &\quad B\phi_v\bar{\nu} + (1-\gamma)(r - \beta + \beta\log(\beta)) - \beta A = 0. \end{aligned}$$

In the absence of jumps (when $\lambda = 0$), the first equation reduces to a quadratic equation for B, which can be solved analytically. □

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