How should we conceive of Time?

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A (would-be) sophisticated answer to the question of the title might be, 'The question is senseless. We should not *conceive* of time at all. We should just get on with our ordinary lives, asking and answering the usual questions, such as "What Time is it?", "How long will it take?", and so on, which we understand perfectly well. St. Augustine understood such questions, phrased in Latin, as well as we do. He should have been content with that, instead of bothering his head with the misbegotten metaphysical question, "What is time?"'.

What this appears to say is, 'Don't ask philosophical questions, or, if you do, don't try to answer them: just disdain them. Don't do philosophy at all. Just carry on as do those to whom philosophical problems have never occurred'. Let us leave aside this philosophically philistine, or nihilist, response for the time being, and address ourselves as best we can to the question in the title.

In his 'Is "What is Time?" a Good Question to Ask?'[1], Dr. Rupert Read criticizes me (p. 195) for arguing[2] that the classical model of time implies the conceptual, though not the physical, possibility of conceptually impossible states of affairs. His ground of criticism is that I appear to understand the classical model, which I think to involve conceptual impossibilities, well enough to know what consequences it has. Well, I do. Read himself speaks of 'latent nonsense' (pp. 194, 197), presumably as contrasted with patent nonsense: latent nonsense is that which is not nonsensical on its face, but under examination proves to be such. Latent nonsense contains enough sense for it to be possible to derive consequences from it: it is often from the absurdity of such consequences that we come to recognize that it *was* nonsense from the start. That is why it is possible to understand latent nonsense to a sufficient degree to operate with it. L. E. J. Brouwer believed classical mathematics to be ultimately unintelligible. Yet he understood it well enough to prove several major theorems in classical topology before settling down to expound and campaign for intuitionistic mathematics.

The classical model of time is a *model:* it represents time on the

[1] *Philosophy,* 77, 2002, pp. 193–209.

[2] In 'Is time a continuum of instants?', *Philosophy,* 75, 2000, pp. 497–515.

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model of the classical continuum of real numbers, with the changing magnitudes of physical quantities corresponding to functions on the real numbers. I regard intuitionistic mathematics as being clearer and more intelligible than the classical variety, which I think to be conceptually confused in its foundations. But in my paper I was not attempting to argue this general point: I was trying to show, on grounds acceptable to a classical mathematician, that the classical continuum is an inadequate model of physical time. I understand the classical conception of the continuum well enough to be able to draw the consequences of using this model. Dr. Read appears not to understand it that much. He says, 'it is absurd to surmise that a collectivity, however large, of dimensionless points could actually result in something with dimension' (p. 193), and asks how you can 'develop' continuity, time and change out of changeless points (p. 196).

Classically the real line or continuum is composed of real numbers: it is the set of all real numbers. It was a *tour de force* of classical mathematics to start with the notion of an individual real number, and to characterize the continuity of the real line in terms of the order relation on the real numbers of which it is taken to be composed. Brouwer and Weyl objected to this approach. They wanted to take the continuum as fundamental, and then to explain the determination of a point on it—a particular real number—by means of the notion of an infinite sequence as they understood it, a sequence generated by a process that can always be carried further but cannot be completed and must not be represented as having been completed; this culminated in Brouwer's conception of choice sequences. On the classical conception, any one real number is dimensionless in the sense that it does not form a segment of the real line. But the real line itself has dimension: if you take real numbers as subject to addition, any segment of the real line, determined by two distinct real numbers as end-points, has a length, itself given by a real number. That real number gives, in the relevant units, the difference or distance between two magnitudes of some quantity, when those magnitudes are given by those real numbers in the same units (seconds, centimetres, grams, etc.).

The continuity of the real line depends classically only upon their order (given by the relation $<$). This order possesses the following properties:

a) it is linear;
b) it is dense (there is a third real number strictly between any two distinct real numbers);

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c) it is complete (any set of real numbers that has an upper bound has a least upper bound).

That is how classical mathematics 'develops' the continuity of the one-dimensional real line out of dimensionless real numbers.

As for change, it is modelled, classically, upon functions of real numbers; the continuity of a function is a different notion from that of the continuity of the real line itself, and classical functions do not need to be everywhere—nor, indeed, anywhere—continuous. It is easy to describe a function exhibiting a removable discontinuity:

$$f(x) = \begin{cases} 2 \text{ if } x \neq 3 \\ 4 \text{ if } x = 3 \end{cases}$$

It is equally easy to describe the two functions associated with a jump discontinuity:

$$f(x) = \begin{cases} 3 \text{ if } x < 2 \\ 5 \text{ if } x \geq 2 \end{cases} \qquad g(x) = \begin{cases} 3 \text{ if } x \leq 2 \\ 5 \text{ if } x > 2 \end{cases}$$

Of course, these definitions are intuitionistically illegitimate, because we cannot always decide whether one real number is less than or equal to another; there are no such functions in intuitionistic analysis. But they are legitimate in classical analysis; and what I called the classical model of time conceives of time by analogy with the classical continuum, which we can all understand in the sense in which, or to the degree to which, Brouwer understood it. Dr. Read repeatedly criticizes me for stigmatizing a type of change as conceptually abhorrent or impossible, and saying that it ought not to be describable, having previously described it (see, for example, his footnote 9 on p. 198). He quite misunderstands my argumentative strategy. This is to take a mathematical function that can readily be described in classical mathematics, and then to try to describe a change in physical magnitude that would be represented by such a function, in accordance with the classical representation of time and change on the model of the classical continuum; when the result proves to be nonsense, I take that as a good ground for saying that the classical model does not yield an intelligible conception of physical time. That is not describing what cannot be described. It is showing that no change in physical reality could correspond to a describable mathematical function, as the classical model of time requires that it could be conceived to occur, even if it is ruled out by the laws of physics.

Dr. Read quite fails to distinguish between the continuity of time

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and the continuity of motion and change. In terms of the classical model, the distinction is between the continuity of the real line, which is secured by its having a dense, complete linear order, and the continuity of (some) functions defined on it; these are two quite different things. For this reason, Dr. Read fails to perceive how central is discontinuity of motion and change to even the formulation of what is objectionable in the conception of time on the classical model. Thus he says (p. 196), 'It is not clear that anything at all has genuinely been pictured, if we are asked to imagine something being somewhere only for an instant that has no *duration*'. Well, if you throw a ball vertically and it comes straight down, on the standard account it is at its highest point (and at rest there) only for a (durationless) instant. Since the motion of the ball is continuous, this is *not* one of the consequences of the classical model that is conceptually repugnant to us: it is contrary neither to intuition nor to observation. What are needed are examples, logically possible on the classical model, in which there is *dis*continuous motion or change. Dr. Read does not see this: he merely persists in his crude rhetoric, saying that we must 'ask ourselves how we could ever have imagined in the first place that you can "develop" continuity, time, change, out of changeless points alone' (also p. 196). He is right, in my view, to reject the classical model; but he has completely failed to appreciate its power.

Around the middle of his article, Dr. Read appears to go berserk, lashing out with wild haymakers, unaware of where his opponent is standing. He rebukes me for casting no light on the question how change is possible (p. 204), oblivious of my never having raised that question (and in fact doubting whether it has any sense). He is convinced that I am trying to answer the general question, 'What is time?', and strongly repudiates the question. I was not endeavouring to answer the general question. To say that time is a continuum of instants, or that it is not, is not to attempt to say what time is, any more than to say that a language is a system for assembling small given units called 'words' into larger units called 'sentences' is an attempt to answer the general question, 'What is language?'. The one speaks of the structure of time; the other speaks of the structure of linguistic utterances. Dr. Read says that I answer the question whether time is a continuum of instants 'more or less in the negative' (p. 199); I thought I answered it with a resounding 'No'. He is much exercised by my taking it as part of the classical model that time is *composed* of durationless instants. I did so because the classical mathematical continuum is composed of real numbers, and I take the analogous idea to be integral to the classical model of

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time. All the same, Dr. Read really needed to draw a distinction. There is, first, the question whether a maximally precise specification of when an event without duration occurred—say the beginning of some process that has duration—would consist in a means of indicating a durationless instant. An affirmative answer would indeed presuppose that the specification could not be given by us, but would be a fact about how things were to a knowledge of which we could only approximate. But the point is that to such an affirmative answer to this question it would be irrelevant whether or not time is *composed* of such durationless instants. To regard it as composed of such instants is to think of those instants, together with the magnitudes of basic physical quantities at those instants, as fundamental; time itself, and change over time, would be to be explained in terms of instants and the states of the universe corresponding to them, just as the real line, and the behaviour of functions defined over it, are supposed to be explained in terms of individual real numbers and the values of functions for particular real numbers as argument. In fn. 16, p. 204, Dr. Read objects to treating anything as 'a foundation or "reduction basis"', that is, as explanatorily fundamental; but the proper order of explanation of our concepts is frequently of importance. It was, for instance, philosophically fruitful for Frege to ask whether 'direction' was to be explained in terms of 'parallel' or conversely. Dr. Read quotes from me the following remark (p. 205):

> Time is the measure of change: its existence simply consists of there being functions giving the magnitudes of other quantities at different *times*. So time is given as the totality of possible arguments of such functions: instants on the classical model, constitutive intervals on the fuzzy realist one. The arguments of such functions are the basic temporal units: it is of them that time is composed.

Of this, he comments that 'at best, it re-states (pretty unperspicuously) what we all already knew'. The point of the remark was to make clear what, in respect of different models of time, was meant by speaking of a unit of time. I did my best to give unmetaphorical substance to describing time as composed of units; the explanations do not seem to have made that clear to Dr. Read.

The choice of the preferred order of explanation determines what makes sense—what is logically conceivable. Now it is true that the absurd consequences I drew from the classical model depend upon this second ingredient of it, the assumption that time is in this sense composed of instants; on this model, the magnitude of a basic

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quantity at any one instant is logically independent of its magnitude at any other instant, just as the value of a function for one argument is logically independent of its value for another. Without the second ingredient of the model, these absurdities need not arise. But it is difficult to get a grasp of a variant model comprising the first ingredient of the classical one but not the second. Dr. Read might legitimately complain that I did not explore this alternative; but that would be alien to his philosophical style. He prefers to inveigh against the metaphorical character of the word 'composed', which contributes little to the question.

Dr. Read asks (p. 203) what could really *turn on* the answer to the questions I am considering. Well, that could be asked of the solution to many philosophical perplexities. Dr. Read implies that the problem with which I was concerned does 'not address the vexations whose dissolving is our real need' (p. 208, fn. 21). Well, it is a problem that has perplexed me for a great many years. It perplexed St. Augustine, too. The past is what has been present, the future what will be present: but the present is a mere durationless boundary between the past and the future, and a boundary can exist only in virtue of the existence of that which it bounds. That was Augustine's puzzle. If you are indifferent to philosophy, you will happily ignore it; if not, you will want to know the solution to it.

The classical, or super-realist, model of time embodies the way in which we have come to think about time; I believe it to be incoherent. There are many who could not give an account of the classical mathematical continuum which is, for the super-realist, the model for physical time. But, then, it took quite a time for mathematicians to formulate their conception of it exactly. I think that, were the inchoate conception of time entertained by the mathematically uninformed to be clearly articulated, it would prove to coincide with the classical model. It is also essentially the model of time used by physicists—at least of time relative to any given frame of reference. If we want to describe the world correctly, we must find a more coherent way of thinking of time than the classical model supplies.

That is just what Dr. Read denies. He is 'not content to let the Realist picture stand' (p. 206). He expresses himself (p. 199) as pretty sympathetic to a negative answer to the question 'Is time a continuum of instants?'. But he says of me, 'He seems to think that this negative reply entails giving an alternative theoretical account'; obviously, *h*e does not think so. I do not see how he could deny that we cannot determine the duration of any temporal interval T save to within a margin of error. His repudiation of realism presumably implies that he would deny that it follows that that duration will in fact be represented, in

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terms of seconds, by some real number, rational or irrational, undiscoverable by us. But what makes it reprehensible to ask what saying that the duration of T lies within the interval $(s - \epsilon, s + \epsilon)$ secs. does mean, if it does not mean that it has some specific value in that interval? Dr. Read's recommendation seems to be to get on with our 'normal social life' (p. 208) and not bother with such questions; but that is the recommendation you expect from someone who despises philosophy, not from one who practises it.

I am totally convinced that the model of the classical continuum is inappropriate, and indeed incoherent, as applied to physical time, and, consequently, that a different model is needed. I am very far from certain that any of the alternative models I described is the right one. I published my article in the hope that it would convince some people that the classical model is wrong, and stimulate some to contrive a better replacement than I had succeeded in proposing. Dr. Read feels sure that I am 'extremely uninterested' in time as experienced by us (p. 203). He should be chary of denying the existence of anything simply because it has not fallen within his experience. I am indeed very interested in our experience of time, but find it even more difficult to think about than is physical time. I read a paper in Edinburgh on the subject, but have not published it because I am still dissatisfied with it.

Dr. Read refers to my substitution of small temporal intervals for the durationless instants of the classical model. He represents this as an instance of replacing precise objects by vague ones. Of this, he says that it involves 'a predication to the object of what is ordinarily perspicuously understood to be a feature of the mode of presentation' (pp. 200–1). Does he suppose that this thought—that vagueness attaches only to verbal expressions, and not to what they are applied to—has never occurred to me? I was not discussing vague objects in general. But, if they are in question, how is it to be demonstrated that what we should naturally take as an only partially determinate description may not sometimes be the fullest, most determinate, description to be given? To deny this needs argument, not sneers. The term 'vague object', Dr. Read says, 'only makes sense if there is a genuine contrast class' (p. 201). He accuses me of not understanding Frege: perhaps he might have learned from Frege that a predicate is not rendered senseless by its applying to nothing, nor by its applying to everything. Are we back with 'ordinary language' philosophy and the paradigm case argument? Is someone who believes that there are no non-denumerable sets bound to deny that there are any denumerable sets, either, on the ground that the term 'denumerable' only makes sense if there is a

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genuine contrast class? The argument rests on a very crude conception of meaning.

Dr. Read goes on from this to assimilate my whole criticism of the classical model of time to an application of the general anti-realist thesis, and then to spurn the entire controversy between realists and anti-realists. I indeed categorized the classical model as not merely realist, but *super*-realist: but, of the alternative models I considered, only the last, constructive, model was anti-realist in character; the others were, as I called them, realist. I did not call them so because, as Dr. Read insinuates, I was trying 'to win respectability and deflect Realist opprobrium' (p. 202), but because they are realist according to my characterisation of realism. Perhaps his mistake was due to his erroneous identification of my constructive model with that I called 'modified fuzzy realism'; they are quite different. The fuzzy realist model, in both its plain and modified forms, allows that the true duration of a temporal interval may exceed our powers of discovery; only the constructive model holds that there is no more to its magnitude than what we can discover.

Dr. Read's repudiation of the dispute between realists and anti-realists is a generalization of his paradigm case argument about vague and precise objects: if the views of either side are nonsense, those of the other side must be nonsense, too. He characterizes realism, not very inappositely, thus (p. 206):

> The Realist says that the World contains more than we, constrained as we are by our alleged 'finitude', can understand or know. But the Realist thinks that we can look beyond those 'limits', in philosophy (metaphysics), to say something about what there is beyond what we can actually understand or know.

But he thinks that the anti-realist's view is 'barely differentiable' from that:

> The Anti-Realist ... says that the world contains nothing more than we find, within our 'limits'. But the Anti-Realist thinks that we can look beyond those 'limits', in philosophy (metaphysics), to say something about what there isn't beyond what we can actually understand or know.

Why is not Dr. Read content to say, 'The anti-realist thinks that *we cannot* look beyond those limits'? Because the anti-realist derives from his account of meaning—of how we think and what we understand—what the reality we think about is like. For instance, he is not content to deny that the law of bivalence holds for the statements we make or the propositions we grasp; he concludes that reality may

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have gaps in it. He does so on the ground that, if not all propositions are either true or false, then not all questions need have an answer. The realist may retort to him that he is talking only of reality as we conceive of it, or rather as he believes we are entitled to think of it; the anti-realist may ask with what justification one could talk about any more than that. But Dr. Read is convinced that we cannot talk about so much. For him, we have no business to consider how we ought to think of how things are. No one can say, on philosophical grounds, that we ought not to think of them in some particular way unless he can show that thinking of them in that way is nonsense; and then thinking of them in any other way will for the same reason be nonsense, too.

Dr. Read accuses the anti-realist of unclarity about whether he rejects realism as nonsensical or as false (pp. 202, 207). I argued that the classical model is incoherent, not just mistaken. I say the same about realism in general. It is clear that Dr. Read is opposed to the classical model of time, and does not believe that any question about when something took place has an absolutely precise answer, given, in terms of some unit, by a real number. He does not think that it means anything to assert that any question has such an answer. But he is much more annoyed by someone who agrees with him about this, but then asks, 'What, then, is the most complete answer to such a question?'. He believes that 'time is in the end no more mysterious than ... maps or tape-measures' (p. 209): we all know perfectly well how to answer questions of the form 'When did ... take place?', and that's all there is to the matter. Well, in ordinary life we might answer such a question by saying, 'It took place at exactly 10 past 11 a.m.: I looked at my watch'. If the questioner then asked, 'Are you sure it did not happen at a thousandth of a second before or after 11.10?', we should take this enquiry as silly. But why? Because we do not think our watches capable of timing events to that degree of accuracy, and because we do not aim at such a degree of accuracy in our everyday affairs; but not because the question made no sense. In such cases we leave the margin of error as understood; scientists give explicit margins of error. So there is a question to be answered about what form a maximally accurate specification of the time of occurrence would take: a question not to be brushed aside by saying, 'We all know perfectly well how to say when something happened'.

So, finally, we are able to answer whether or not Dr. Read is a philosophical philistine or nihilist: does he think that philosophy is all a tissue of nonsense, or does he think there are genuine philosophical problems, with genuine solutions? It seems that he is not a nihilist: in his final footnote (p. 208, fn. 21) he is careful to

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allow some work for the philosopher to do: 'Philosophising', he says, 'can clarify the "logical grammar" of scientific and non-scientific "language-games" involving the word "time" and related words'. But what Dr. Read abominates is metaphysics, which purports to give us a clearer conception of what the world we inhabit is like. It provokes Dr. Read into some banal rhetoric mocking it, such as '"Reality, 'out there', is literally like this" (whatever *that* means ...)' (p. 200). But how does Dr. Read know that a clarification of the logical grammar of our ordinary discourse involving time will not reveal that it implicitly embodies the classical conception of time? I suppose that it is because he is convinced that our ordinary language is in order as it is: it is beyond criticism, and certainly it does not incorporate any dubious metaphysical ideas. Well, it is obviously not beyond criticism, since it allows ambiguities: that is an evident defect in a language. The English language fails to make the distinction made in Latin by 'homo' and 'vir' or in German by 'Mensch' and 'Mann': and see what trouble that has caused. Those in the grip of some erroneous philosophical conception, such as that of the soul imprisoned in the body, are very likely to speak in ways that accord with that conception: it is very far from absurd to suspect a language of embodying an incorrect metaphysical idea.

Presumably a clarification of the logical grammar of the language-games that relate to ourselves, our actions, thoughts, intentions and awareness—a good philosophical treatment of the question, 'Do we have free will?', for example—will yield us a clearer conception of what we are like. Why, then, does it seem so certain to Dr. Read that a clarification of the logical grammar of the language-games that concern our physical environment cannot bring us a clearer conception of what physical reality is like? I surmise that the reason is that he suspects that a clarification of the logical grammar will make us see such questions as 'Do we have free will?' or 'Is time a continuum of instants?' as nonsensical, and deprive us of the desire to ask, let alone attempt to answer, them. But, if so, we shall be able to explain what is wrong with those questions; and we shall have achieved illumination about ourselves and about the structure of time. Rather than fulminating against me for asking, and trying to answer, the second of the two questions, Dr. Read would do much better to carry out that clarification of logical grammar which he believes would stop anyone from asking it; we could then see whether his labours really had that effect, and, if they did, could gain a clearer conception of what we mean, or should mean, when we ask at what time an event took place.

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