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Electro-acoustic shock structures in dusty plasmas

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Two types of electro-acoustic shock structures, namely dust-ion-acoustic (DIA) and dust-acoustic (DA) shock structures, formed in two different kind of dusty plasma systems have been theoretically investigated. The sources of dissipation, which are responsible for the formation of DIA and DA shock structures in these dusty plasma systems, are identified. The conditions for the formation of these shock structures and their new basic features are pinpointed. The implications of the results in experimental observations are also discussed.

1. Introduction

Two types of electro-acoustic waves, namely dust-ion-acoustic (DIA) waves (Shukla et al. 1992) and dust-acoustic (DA) waves (Rao et al. 1990), are observed theoretically (Rao et al. 1990; Shukla et al. 1992) as well as experimentally (Barkan et al. 1995, 1996) in unmagnetized dusty plasmas. The linear properties of both DIA and DA waves in dusty plasmas are now well understood from both theoretical (Rao et al. 1990; Shukla et al. 1992) and experimental (Barkan et al. 1995, 1996) points of view.

The nonlinear structures, particularly shock structures associated with these DIA and DA waves have now received a great deal of interest in understanding the localized electro-acoustic disturbances in laboratory and space dusty plasmas (Nakamura et al. 1999; Luo et al. 1999; Shukla and Mamun 2001; Mamun and Shukla 2002; Eliasson and Shukla 2004; Heinrich et al. 2009; Mamun and Cairns 2009; Mamun and Shukla 2009; Mamun and Shukla 2010). It has been shown that the presence of non-isothermal electrons following the vortex-like distribution (Schamel 1972, 1973), which occur in many space and laboratory dusty plasma situations (Schamel 1972, 1973; Mamun et al. 2004; Eliasson and Shukla 2004b, 2006), significantly modifies the basic features of the nonlinear electro-acoustic waves or introduces new features in them. On the other hand, a strongly coupled dusty plasma (Kaw and Sen 1998) containing highly positively or negatively charged dust, which have been observed in both space (Rosenberg and Mendis 1995; Horányi and Walch 1998) and laboratory devices (Fortov et al. 1996; Mohideen et al. 1998), has received a considerable interest because of its important role in phase transitions of interest, in processes occurring in upper atmosphere, planetary rings, etc. The present work is, therefore, concerned with the DIA shock waves in a dusty plasma containing charge fluctuating stationary dust, inertial warm ions, and non-isothermal electrons following the vortex-like distribution, and with the DA shock waves in a strongly coupled dusty plasma

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containing arbitrarily charged strongly coupled dust and weakly coupled electrons and ions following Boltzmann distribution.

The manuscript is organized as follows. The theory describing the basic features of the DIA (DA) shock structures is provided in Sec. 2 (Sec. 3). A brief discussion is finally presented in Sec. 4.

2. DIA shock structures

We consider an unmagnetized dusty plasma, whose constituents are charge fluctuating stationary dust, inertial warm ions, and non-isothermal electrons following the vortex-like distribution. The nonlinear dynamics of the DIA waves, whose phase speed is much smaller than the electron thermal speed, is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{2.1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\sigma_i}{n_i}\frac{\partial n_i}{\partial x}, \tag{2.2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1-\mu)n_e - n_i + \mu z_d, \tag{2.3}$$

where n_i is the ion number density normalized by its equilibrium value n_{i0}, u_i is the ion fluid speed normalized by the ion-acoustic speed $C_i = (T_e/m_i)^{1/2}$, ϕ is the electrostatic wave potential normalized by T_e/e, n_e is the electron number density normalized by its equilibrium value n_{e0}, z_d (normalized by its equilibrium value z_{d0}) is the number of electrons residing onto the dust grain surface, $\mu = z_{d0}n_{d0}/n_{i0}$, $\sigma_i = T_i/T_e$, T_e (T_i) is the free electron (ion) temperature in units of the Boltzmann constant k_B, e is the magnitude of the electron charge, and m_i is the ion mass. The time and space variables are in units of the ion plasma period $\tau_i = (m_i/4\pi n_{i0}e^2)^{1/2}$, and the Debye radius $\lambda_{Dm} = (T_e/4\pi n_{i0}e^2)^{1/2}$, respectively.

We assume that electrons follow the vortex-like distribution of Schamel (1972, 1973), which satisfies the electron Vlasov equation. Thus, we have $f_e = f_{ef} + f_{et}$, where (Schamel 1972, 1973)

$$f_{ef} = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(v^2 - 2\phi)\right] \quad \text{for} \quad |v| > \sqrt{2\phi}, \tag{2.4}$$

$$f_{et} = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\beta(v^2 - 2\phi)\right] \quad \text{for} \quad |v| \leqslant \sqrt{2\phi}, \tag{2.5}$$

in which v is the electron speed normalized by the free electron thermal speed $v_{Te} = (k_B T_e/m_e)^{1/2}$ (with m_e being the electron mass), and β is a parameter determining the number of trapped electrons. The magnitude of β is defined as (Schamel 1972, 1973) the ratio of the free electron temperature T_e to the trapped electron temperature T_{et}, i.e. $|\beta| = T_e/T_{et}$. We note that $\beta = 1$ ($\beta = 0$) represents a Maxwellian (flat-topped) distribution, whereas $\beta < 0$ represents a vortex-like excavated trapped electron distribution corresponding to an under-population of trapped electrons. The latter ($\beta < 0$) is our present interest. Now, integrating the electron distributions [given by (2.4) and (2.5)] over the velocity space, n_e for $\beta < 0$ can be expressed as (Schamel 1972, 1973)

$$n_e = I(\phi) + \frac{2}{\sqrt{\pi|\beta|}} W_D(\sqrt{-\beta\phi}), \tag{2.6}$$

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where $I(x) = [1 - \mathrm{erf}(\sqrt{x})] \exp(x)$ and $W_D(x) = \exp(-x^2) \int_0^x \exp(y^2) dy$. For $\phi \ll 1$ we can express (2.6) as (Schamel 1972, 1973)

$$n_e \simeq 1 + \phi - \frac{4(1-\beta)}{3\sqrt{\pi}} \phi^{3/2} + \frac{1}{2}\phi^2. \tag{2.7}$$

We consider a general (realistic) situation for dust charge state, i.e. z_d is not constant ($z_d \neq 1$), but varies with time [$z_d = 1 + \tilde{z}_d(t)$] accordingly;

$$\frac{\partial \tilde{z}_d}{\partial t} = -(I_e + I_{i0}), \tag{2.8}$$

where I_e (normalized by e/τ_i) is the current associated with the electron distribution functions defined by (2.4) and (2.5), and is given by Berbri and Tribeche (2009)

$$I_e = -\gamma \left[\left\{ 1 - \frac{1}{\beta^2} + \left(1 - \frac{1}{\beta} \right) (\phi - \alpha \tilde{z}_d) \right\} \exp(\alpha\beta) + \exp\{\beta(\phi - \alpha \tilde{z}_d)\} \right], \tag{2.9}$$

$$\gamma = \sqrt{8\pi} r_d^2 n_{d0} \lambda_{Dm} \left(\frac{m_i}{m_e} \right)^{\frac{1}{2}} \left(\frac{1-\mu}{\mu} \right) \exp(-\alpha\beta), \tag{2.10}$$

I_{i0} (normalized by e/τ_i) is the equilibrium ion current, and $\alpha = z_{d0} e^2 / T_e r_d$, and $I_{e0} = -I_{i0} = I_e(\phi = 0,\ \tilde{z}_d = 0)$ is the equilibrium electron current. We note that just to avoid the mathematical complexities, in (2.8) the ion current fluctuation, which is negligible in comparison the electron current fluctuation has been neglected, since $m_i/m_e \geqslant 1860$ and $T_i/T_e \ll 1$. Using (2.9) and $\beta(\phi - \alpha \tilde{z}_d) \ll 1$, we can express (2.8) as

$$\frac{\partial \tilde{z}_d}{\partial t} = \gamma \left[\left\{ \beta + \left(1 - \frac{1}{\beta} \right) \exp(\alpha\beta) \right\} (\phi - \alpha \tilde{z}_d) + \frac{\beta^2}{2} (\phi - \alpha \tilde{z}_d)^2 \right]. \tag{2.11}$$

To derive a dynamical equation for the nonlinear propagation of the DIA waves from (2.1)–(2.3) and (2.11) by using the reductive perturbation technique, we first introduce the stretched coordinates: $\xi = \epsilon^{1/2}(x - v_0 t)$, $\tau = \epsilon\, t$; expand n_i, u_i, and ϕ in a power series of ϵ: $n_i = 1 + \epsilon n_i^{(1)} + \epsilon^{3/2} n_i^{(2)} + \cdots$, $u_i = \epsilon u_i^{(1)} + \epsilon^{3/2} u_i^{(2)} + \cdots$, $\phi = \epsilon \phi^{(1)} + \epsilon^{3/2} \phi^{(2)} + \cdots$, $\tilde{z}_d = \epsilon \tilde{z}_d^{(1)} + \epsilon^{3/2} \tilde{z}_d^{(2)} + \cdots$; and develop different sets of equations in various powers of ϵ. The latter yield a modified Burgers equation (Mamun and Shukla 2010)

$$\frac{\partial \phi}{\partial \tau} + A\sqrt{\phi} \frac{\partial \phi}{\partial \xi} = C \frac{\partial^2 \phi}{\partial \xi^2}, \tag{2.12}$$

where superscripts on ϕ has been dropped out by using ϕ instead of $\phi^{(1)}$, and

$$A = \frac{(1-\beta)(1-\mu)}{\sqrt{\pi} v_0 (1 - \mu + \mu/\alpha)^2}, \tag{2.13}$$

$$C = \frac{\mu}{2\gamma\alpha^2 (1 - \mu + \mu/\alpha)^2 [\beta + (1-\beta)\exp(\alpha\beta)]}. \tag{2.14}$$

It is obvious that the dissipative term is due to the effect of dust charge fluctuation, and the strong nonlinear term is due to the effect of non-isothermal electrons following vortex-like distribution.

We are now interested in looking for the stationary shock wave solution of (2.12) by introducing $\zeta = \xi - u_0 \tau$, where u_0 is the shock wave speed normalized by C_i. Thus, the stationary shock wave solution of (2.12) is given by Mamun and Shukla (2010)

$$\phi = \phi_0 [1 + \tanh(\zeta/\delta)]^2, \tag{2.15}$$

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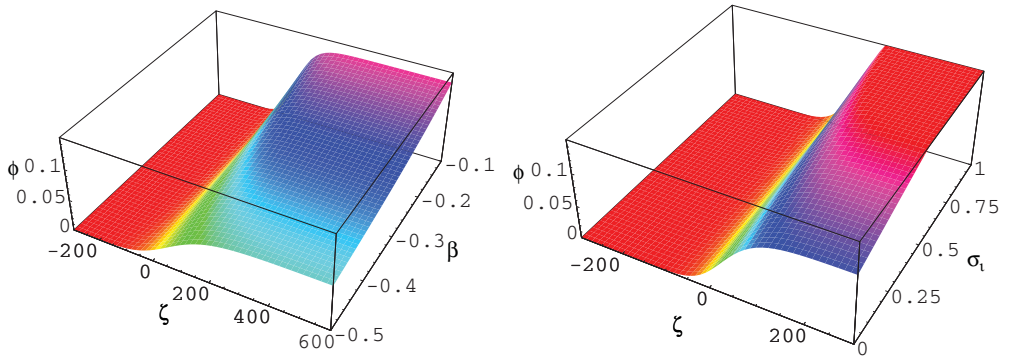


FIGURE 1. (Color online) Showing how the DIA shock structures change with β for $\sigma_i = 0.1$ (left panel) and with σ_i for $\beta = 0.3$ (right panel). The other parameters, which are common in both figures, are $\mu = 0.2$, $\alpha = 1.44$ and $u_0 = 0.1$.

where $\phi_0 = (3u_0/4A)^2$ and $\delta = 4C/u_0$ are, respectively, amplitude and thickness of the DIA shock structures. The DIA shock structures showing the variation of their amplitude and thickness with the variation of the trapped parameter (β) and ion-temperature (σ_i) for typical dusty plasma parameters are shown in Fig. 1.

3. DA shock structures

We consider a strongly coupled dusty plasma containing strongly correlated arbitrarily (positively or negatively) charged dust, and weakly coupled electrons and ions obeying the Boltzmann distribution. The dynamics of the nonlinear DA waves in such a strongly coupled dusty plasma is governed by the well known generalized hydrodynamic equations (Kaw and Sen 1998; Shukla and Mamun 2001; Mamun and Cairns 2009):

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \tag{3.1}$$

$$D_\tau \left(n_d D_t u_d + s n_d \frac{\partial \psi}{\partial x} + \sigma_d \frac{\partial n_d}{\partial x} \right) = \eta_l \frac{\partial^2 u_d}{\partial x^2}, \tag{3.2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = (\mu + s)\exp(\sigma_i \psi) - \mu \exp(-\psi) - s n_d, \tag{3.3}$$

where n_d is the dust number density normalized by its equilibrium value n_{d0}; u_d is the dust fluid speed normalized by $C_d = (Z_d T_i/m_d)^{1/2}$; ψ is the electrostatic wave potential normalized by T_i/e; s indicates the polarity of the dust, i.e. $s = 1$ ($s = -1$) for positively (negatively) charged dust; Z_d is the number of electrons (protons) residing onto the negative (positive) dust grain surface; t [x] time [space] variable normalized dust plasma period $\omega_{pd}^{-1} = (m_d/4\pi n_{d0} Z_d^2 e^2)^{1/2}$ [Debyelength $\lambda_D = (T_i/4\pi n_{d0} Z_d e^2)^{1/2}$]; m_d is the dust mass; $\mu = n_{i0}/Z_d n_{d0}$; $\sigma_d = \mu_d T_d/Z_d T_i$; $D_\tau = 1 + \tau_m \partial/\partial t$; $D_t = \partial/\partial t + u_d \partial/\partial x$; τ_m is the viscoelastic relaxation time normalized by ω_{pd}^{-1}; μ_d is the compressibility; and η_l is the longitudinal viscosity coefficient normalized by $m_d n_{d0} \lambda_D^2 \omega_{pd}$. There are various approaches for calculating these transport coefficients. These have been widely discussed in the literature (Kaw and Sen 1998; Shukla and Mamun 2001; Mamun and Cairns 2009).

To derive a dynamical equation for the DA shock waves, we first introduce the stretched coordinates (Mamun and Cairns 2009): $\xi = \epsilon(x - V_p t)$, $\tau = \epsilon^2 t$ (where V_p

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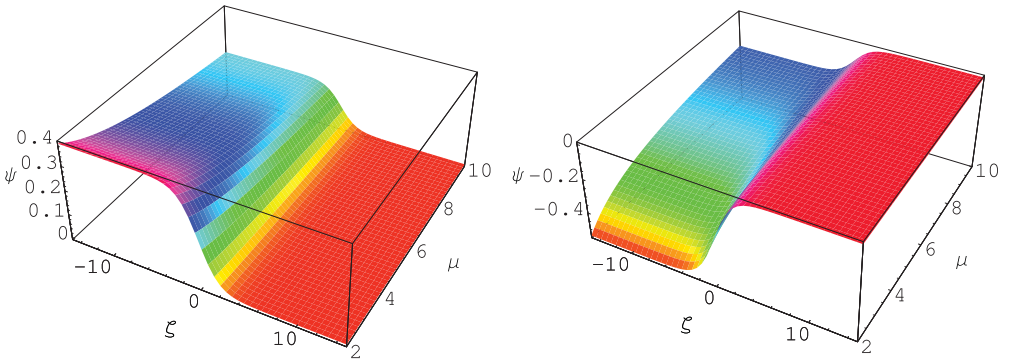


FIGURE 2. (Color online) Showing how the DA shock structures change with the variation of μ for positively charged dust (left panel) and negatively charged dust (right panel) for $\sigma_d = 10^{-3}$, $\sigma_i = 0.1$, $\eta_l = 1$ and $U_0 = 0.5$.

is the DA wave phase speed normalized by C_d); expand n_d, u_d, and ψ in power series of ϵ: $n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots$, $u_d = \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \cdots$, $\psi = \epsilon\psi^{(1)} + \epsilon^2\psi^{(2)} + \cdots$; and develop different sets of equations in various powers of ϵ. The latter (with a number of straight forward algebraic steps) reduce to a standard Burgers equation

$$\frac{\partial\psi}{\partial\tau} + A_d\psi\frac{\partial\psi}{\partial\xi} = D\frac{\partial^2\psi}{\partial\xi^2}, \tag{3.4}$$

where superscripts on ψ have been dropped out by using ψ instead of $\psi^{(1)}$, and

$$A_d = \frac{s}{V_p}\left[1 + \frac{\alpha_n}{2}\left(\frac{1+\sigma_i}{\alpha_n+\sigma_i}\right)^2 + \frac{3\sigma_d}{2s}\left(\frac{\sigma_i+\alpha_n}{1-\alpha_n}\right)\right], \tag{3.5}$$

$D = \eta_l/2$, $V_p = [s(1-\alpha_n)/(\sigma_i+\alpha_n) + \sigma_d]^{1/2}$, and $\alpha_n = n_{i0}/n_{e0} = \mu/(\mu+s)$. It is obvious that the dissipative term, i.e. the right-hand side of (3.4) is due to the strong correlation among the charged dust. It is clear from (3.5) that $A_d > 0$ for positively charged dust ($s = 1$ and $\alpha_n < 1$), and $A_d < 0$ for negatively charged dust ($s = -1$ and $\alpha_n > 1$).

We are now interested in looking for the stationary shock wave solution of (3.4) by introducing $\zeta = \xi - U_0\tau$, where U_0 is the DA shock wave speed normalized by C_d. The stationary shock wave solution of (3.4) is given by Mamun and Cairns (2009)

$$\psi = \psi_0[1 - \tanh(\zeta/\Delta)], \tag{3.6}$$

where $\psi_0 = U_0/A_d$ and $\Delta = 2D/U_0$ are, respectively, the amplitude and the thickness of the DA shock structures. It is obvious that the formation of such shock waves is due to the strong correlation among highly charged dust, and that the shock structures are formed with $\psi > 0$ ($\psi < 0$) for positively (negatively) charged dust, since $A_d > 0$ ($A_d < 0$) for positively (negatively) charged dust. It clear that the shock thickness (η_l/U_0) is directly proportional to η_l, and is inversely proportional to U_0 for both positively and negatively charged dust. The DA shock structures showing the variation of their amplitude and thickness with μ for positively and negatively charged dust are shown in Fig. 2.

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4. Discussion

The basic features of small amplitude DIA and DA shock structures formed in dusty plasmas are pinpointed by employing the reductive perturbation method without stretching any plasma parameter. The main results that have been obtained from this investigation can be pointed out as follows:

(*a*) The effect of dust grain charge fluctuation acts as a source of dissipation, and is responsible for the formation of the DIA shock structures.

(*b*) The effect of vortex-like electron distribution gives rise to a MB equation exhibiting stronger nonlinearity. It admits a new kind of shock solution showing the existence of the DIA shock structures with $\phi > 0$, larger amplitude, smaller thickness, and higher propagation speed.

(*c*) The effect of strong correlation among charged dust provides a source of dissipation, and is responsible for the formation of the DA shock structures.

(*d*) The DA shock structures with $\psi > 0$ ($\psi < 0$) are formed for positively (negatively) charged dust.

There is a large number of published articles where Korteweg-de Vries-Burgers equation was derived by using different stretched co-ordinates along with an additional stretching of any of the plasma parameters [viz. dust charging parameter (in case of DIA and DA shock waves), kinematic viscosity coefficient of the ion fluid (in case of DIA shock waves) or of the weakly coupled dust fluid (in case of DA shock waves) or the longitudinal viscosity coefficient of strongly coupled dusty plasma (in case of DA waves]. The latter (stretching of any of the plasma parameters) are, however, not correct from both mathematical and physical points of view. Therefore, in the present work, these physical and mathematical errors are overcome by using the appropriate stretched coordinates. There is also a significant number of published articles where the effect of dust charge fluctuation has been included. But the effect of dust charge fluctuation is not important for the DA waves at all since the DA wave time period (fraction of a second) is much larger than the dust charging time period (few micro-seconds). Thus, the effect of dust charge fluctuation is not included in the present investigation of the DA shock waves.

The basic properties of the DIA and DA shock structures discussed here should be useful in understanding the salient features of the electrostatic shock structures observed in laboratory dusty plasma experiments (Luo et al. 1999; Nakamura et al. 1999; Merlino and Goree 2004; Heinrich et al. 2009).

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