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REVIEW

Mathematical Methods in Risk Theory. By HANS BÜHLMANN (Springer-Verlag)

Let us start with a question that is not answered in the text. Why have Springer reprinted without modification a text first published in 1970? The answer to this question can be found in *ASTIN Bulletin*, **26** (2). It contains an article reporting the results of a survey on appropriate texts to be donated by ASTIN to universities and national actuarial associations in emerging countries. The list of favoured texts contains a number that are out of print, and the article informs us that some of these items will be reprinted. Bühlmann's book is in this category.

And so to another question — why did a book that is now clearly not up-to-date find such favour amongst the respondents to the above survey? The answer here is obvious — Bühlmann's book is the classic text on risk theory.

Given the book's age, it seems inappropriate to critically review it. There has been a huge amount of research in risk theory since 1970, and any book written today would contain important results from this period. I will, therefore, concentrate mostly on a short description of its contents.

The book is divided into two parts: Part I, 'The Theoretical Model'; and Part II 'Consequences of the Theoretical Model'. Part I contains three chapters: 'Probability Aspects of Risk', 'The Risk Process' and 'The Risk in the Collective'. In these chapters Bühlmann lays the mathematical foundations. The first chapter contains a review of random variables and basic distribution theory, with particular emphasis on distributions that are useful in modelling non-life claim numbers and amounts. The second half of the chapter deals with jointly distributed random variables, independence and correlation. Much of the content of this chapter can be found in standard texts on probability. However, Bühlmann keeps the discussion focussed mostly on insurance applications.

Chapter 2 is devoted to the risk process. Stochastic processes, in particular Markov processes, are discussed in fairly general terms before the focus turns to insurance applications — first counting processes for the number of claims, then the aggregate claims process itself (i.e. the interaction of the claim numbers and claim amounts). As well as considering the classical risk model (where claims occur as a homogeneous Poisson process), Bühlmann also discusses contagion models leading to binomial and negative binomial distributions for claim numbers.

The final chapter in Part I is devoted to the idea of a risk as belonging to a collective. The collective is simply a set of risks, each risk being characterised by a parameter θ. The important structure function, which can be thought of as representing the probability distribution of the θ values in the collective, is introduced, and weighted risk processes are discussed. The chapter then discusses the claim number and aggregate claims processes in the collective, thus extending ideas introduced in Chapter 2.

It is worth pointing out, at this stage, that it is assumed throughout the first two chapters that everything we require is known. There is no discussion of fitting models or parameters, a point which is clearly made in the preface. Chapter 3 is the first place where this assumption is abandoned.

The second part of the book deals with applications of the models and ideas developed in the first part. Chapter 4, 'Premium Calculation', starts with a brief discussion of four premium principles, then discusses the risk premium and the collective premium (in the spirit of risk as part of a collective, as introduced in Chapter 3). The remainder of this chapter is largely devoted to the credibility premium. At the time this chapter was written, most of the material it contains was relatively new, with Bühlmann himself being the leading figure in this area. Credibility theory is a field of research that has blossomed since then, and this part of the book should be seen in that context.

Chapter 5, 'Retentions and Reserves', is an interesting chapter, containing material largely ignored by authors of subsequent texts on risk theory. The main idea in this chapter is to find reinsurance retention levels for a portfolio consisting of n independent risks. The criterion for determining the retention levels is that the variance of the insurer's (net of reinsurance) profit should be minimised, subject to the constraint of a fixed expected (net) profit. Solutions are presented in the case of simple proportional and non-proportional reinsurance arrangements. The chapter concludes with an

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introduction to the idea of technical reserves, i.e. those required to take account of fluctuations in the surplus process. These ideas are more fully developed in the final chapter.

Chapter 6, 'The Insurance Carrier's Stability Criteria', is the longest in the book, containing four substantial sections. In the preface the author describes this chapter as taking up real operational problems. The problems considered in this chapter are: classical ruin theory; reinsurance retention levels (as in Chapter 5) using a ruin probability criterion; dividend policy; and utility theory. The section on ruin theory includes Lundberg's inequality, classical integro-differential equations and the application of methods from renewal theory. The section on reinsurance retention levels effectively focuses on the use of the adjustment coefficient, since direct analytical treatment of the problem does not appear possible. The section on dividend policy is concerned with paying dividends from the surplus process (both in discrete and continuous time), and considers a policy as being optimal if it maximises the expected discounted value of dividends. The section on utility theory discusses, amongst other things, the problem of Pareto-optimal risk exchanges.

The book concludes with a short appendix on the generalised Reimann-Stieltjes integral. It also contains a small number of exercises, but no solutions.

At the time it was first published, this book was rightly acclaimed by researchers and practitioners alike. Although the years have dated it, it is, nevertheless, a book that any serious student of risk theory should read. It contains many important ideas and techniques.

In 1997 Hans Bühlmann retires from academic life. It is appropriate to view this book simply as one of his many outstanding contributions to the field of actuarial science during a distinguished career.

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