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REGIME-SENSITIVE COINTEGRATION WITH AN APPLICATION TO INTEREST-RATE PARITY

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There exist a variety of reasons for the failure to find a unique cointegrating relationship between economic time series where one would normally be expected on the basis of economic theory. Among these are the testing procedure, the span of the data set, the choice of lag length in generating the test statistic, the presence of structural breaks, and the presence of cointegration only beyond some threshold. We propose the concept of regime-sensitive cointegration whereby the underlying series need not be cointegrated at all times. We show that cointegration can be switched off when a common stochastic trend is added. Alternatively, cointegration can be switched on or off because series normally believed to contain a unit actually do not. This implies that a linear combination of such variables need not be cointegrated. To illustrate the concept empirically, we test the hypothesis of interest-rate parity, and related hypotheses, using daily Eurorates for the United States and Canada.

Keywords: Cointegration, Structural Breaks, Interest-Rate Parity

1. INTRODUCTION

The statistical concept of cointegration has spawned a large literature and with good reason. Many economic theories are concerned with equilibrium relationships in which one or more series are expected to act as attractors to each other. In particular, many applications of cointegration tests can be found in the finance literature

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because such markets are believed to clear quickly in the absence of regulatory impediments and also because equilibrium relationships are well established and defined. Examples include testing for cointegration between nominal interest rates [e.g., Hall et al. (1992), Siklos and Wohar (1997)], real interest rates [Kugler and Neusser (1993), Goodwin and Grennes (1994), and Throop (1994)], and stock prices [Kasa (1992)]. By contrast, disequilibria may take a considerable amount of time to be reversed in the market for goods [e.g., as in tests of purchasing-power parity (PPP); Brenner and Kroner (1995) make the general point].

It may be surprising, therefore, that the existing body of empirical work has been less than completely successful at consistently finding such equilibrium relationships, that is, of rejecting the null of no cointegration in favor of the alternative of one or more cointegrating vectors.[1] A case in point is interest-rate-parity theory. Economic theory suggests that interest-rate differentials between countries can be explained either by inflation differentials, in the case of uncovered interest parity, or by the forward premium, in the case of covered interest parity. If inflation differentials or forward premia are covariance-stationary, then one would expect nominal interest rates for several countries to be cointegrated. Yet, on balance, the evidence on this score is mixed. McCallum (1994) convincingly argues that tests of nominal interest rate differentials are central to "orthodox" macromodels. Hence, understanding the time-series properties of interest-rate spreads, either across the maturity structure or, as is the focus of the empirical portion of the present paper, across countries for instruments with the same maturity, is likely to be important. Consider Figure 1, which plots the three-month Eurorate for Canada and the United States. The data are at the daily frequency for the period January 5, 1979, to July 31, 1995, inclusive. It is immediately clear that there are abrupt changes in the levels from time to time and that these conceivably can be large enough to overturn some underlying equilibrium relationship between the two series, as is illustrated later. Moreover, the time-series properties of the two rates also appear to change over time. This becomes clear on inspection of the spread between the two rates. Indeed, the resulting plot (not shown) suggests periods when the spread appears to be stationary whereas, at other times, the same spread is clearly nonstationary. Figure 2 plots the exchange rate vis-à-vis the U.S. dollar. The disinflation era is highlighted and, again, the behavior of the exchange rate is quite different following the formal announcement of inflation targets in Canada. It remains to be seen, of course, whether the available statistical tests can confirm the significance of the inflation control regime.

A variety of statistical explanations for not rejecting the null of no cointegration in empirical applications are possible. These include the test for cointegration itself [e.g., Engle and Granger (1987), Johansen (1995), Stock and Watson (1993)], the span of the data set [Hendry (1995) and Perron (1989)], the choice of lags in the testing equation [e.g., Banerjee et al. (1993)], the possibility of structural breaks in the relationship of interest [Gregory and Hansen (1996), Stock and Watson (1994)], and cointegration defined outside some bound only, as in the threshold cointegration model of Balke and Fomby (in press).[2]

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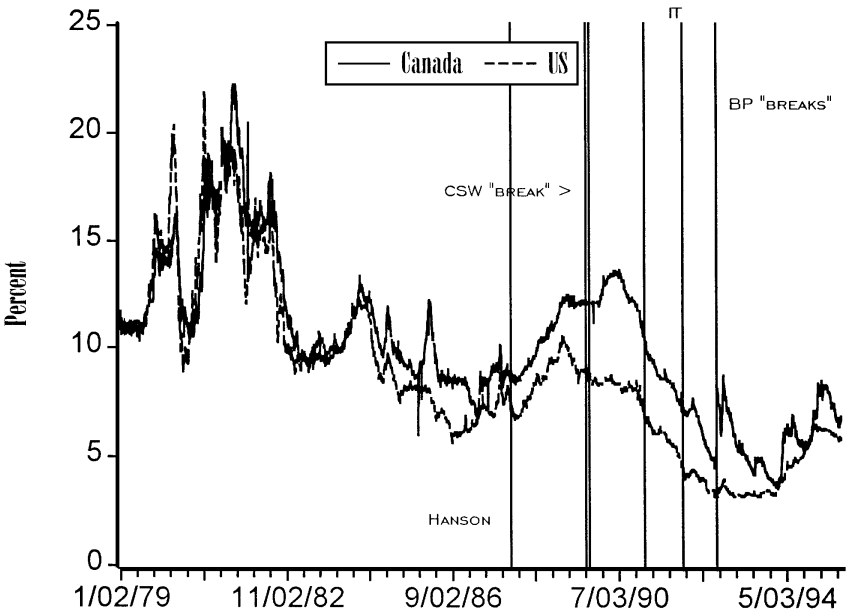


FIGURE 1. U.S. and Canadian Eurorates, daily: 1979–1995. The sample begins January 1, 1979, and ends July 31, 1995. CSW "break" refers to the timing of the break in the trivariate relationship between the two interest rates and the exchange rate. The break occurs in October 1989. Hanson refers to the date of the Hanson lecture delivered by the Governor of the Bank of Canada on January 8, 1988. IT refers to the February 26, 1991, announcement of inflation targets. BP "breaks" refers to the two breaks chosen by the Bai–Perron test. These occurred September 25, 1992, and December 18, 1990, in the bivariate relationship between U.S. and Canadian Eurorates. *Source:* BIS.

We propose the concept of regime-sensitive cointegration. The motivation for this concept is that, for some applications at least, it seems more realistic to assume that some series are cointegrated during some periods and not in others. Events or important changes in some of the institutional features of an economy can interrupt an underlying equilibrium-type relationship possibly for an extended period of time. This view stands in contrast with notions of time-varying cointegration in which the equilibrium relationship exists at all times, but where the underlying strength of the link is changeable, or with a relationship of interest as being an equilibrium one but interrupted by perhaps a one-time shift to capture the advent of a new regime. Instead, we suggest that economic relationships are best thought of as occasionally falling in or out of equilibrium because of some major policy shift or event. We show how existing tests can be used to discuss the concept of regime-sensitive cointegration.

Clearly, because what prompts economic relationships to switch into or out of an equilibrium path is some important shock, these need to be identified carefully.

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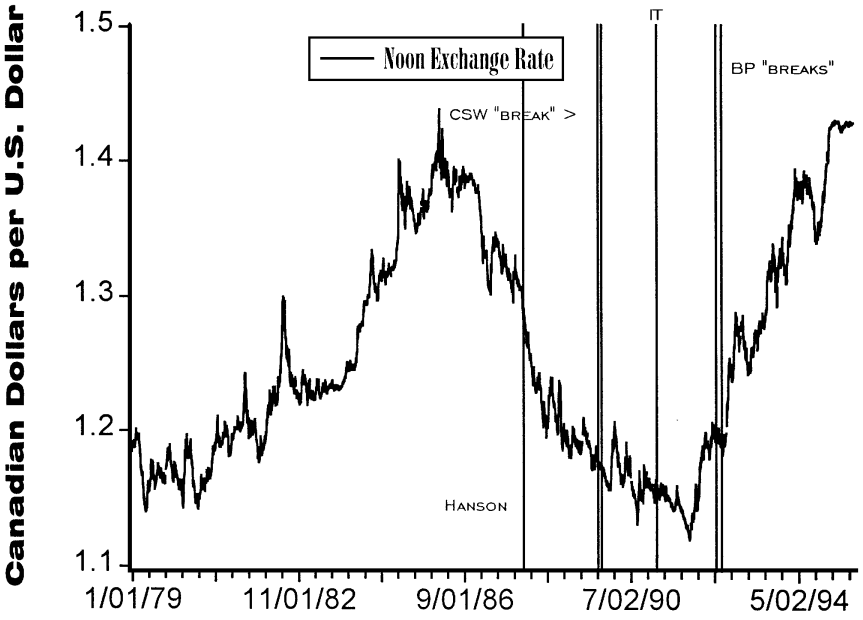


FIGURE 2. Exchange rate between the Canadian and U.S. dollars, daily: 1979–1995. The exchange rate is defined as the Canadian dollar price of foreign exchange. For sample and other details, see Figure 1. The BP "breaks" refer here to the trivariate relationship, which includes the exchange rate. The timing of the breaks are June 6 and July 28, 1992. The CSW break is the same as shown in Figure 1. *Source:* Bank of Canada.

Consequently, we utilize structural break tests proposed by Bai and Perron (1995), and Chu et al. (in press) to identify such regime shifts. There may be other ways to identify regime shifts [e.g., Hamilton (1994, Ch. 22)], but the object of the exercise being considered is to suggest that, at least for some cases, it is preferable to think of economic relationships as coexisting in parallel worlds with major shocks leading to switches into and out of equilibrium. Although it is desirable to let the data select when and where regime shifts occur, it also would seem appropriate to ask whether regimes identified by well-known events whose timing is known also exhibit idiosyncracies in time-series behavior.[3] The reason is that economic theory also should provide some guidance about whether some equilibrium relationship is being upset or not. As argued below, the adoption of an inflation-targeting regime in one country with close economic ties to another that does not openly adopt such targets, as in the U.S.–Canada example, possibly represents one such example.

The paper is organized as follows. Section 2 characterizes the basic approach to regime-sensitive cointegration. In Section 3, we provide an example that we believe illustrates the usefulness of the proposed concept, namely, the case of the spread between U.S. and Canadian interest rates. Section 4 concludes.

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2. CHARACTERIZING REGIME-SENSITIVE COINTEGRATION

Let us examine the long-run relationship between two time series X_t and Y_t such that both are integrated of order 1, denoted as $I(1)$. This, of course, implies the possibility that X_t and Y_t are cointegrated. However, suppose that cointegration exists only during certain periods; otherwise, the series are not cointegrated. For example, consider the following model:

$$X_t = AW_t + \lambda_{1t} Q_t + \alpha_1 Z_t, \tag{1}$$

$$Y_t = W_t + \lambda_{2t} Q_t + \alpha_2 Z_t, \tag{2}$$

where $W, Q_t \sim I(1)$ are not cointegrated, but $Z_t \sim I(0), \alpha_{it} \geq 0$, with $\alpha_1 - A\alpha_2 = 1$. If $\lambda_{1t} = \lambda_{2t} = 0$, then X_t, Y_t will be cointegrated; otherwise, this property will not hold in general, because there are two stochastic trends. Clearly, some major policy shift could create a condition under which λ_{1t}, $\lambda_{2t} \neq 0$, that is, when an interfering variable adds a stochastic trend to an existing equilibrium relationship and thereby upsets it. To illustrate, suppose that W_t and Q_t are each pure random walks and that Z_t is a stationary AR(1) model with coefficient ρ, so that $\Delta W_t = e_{W_t}$, $\Delta Q_t = e_{Q_t}$, and $Z_t = \rho Z_{t-1} + e_{Z_t}$, all e's being zero-mean white-noise processes and independent of each other. Multiplying (2) by A and subtracting from (1), one obtains, by construction,

$$Z_t = X_t - AY_t - (\lambda_{1t} - A\lambda_{2t}) Q_t. \tag{3}$$

Hence, X_t, Y_t, and Q_t are cointegrated as is clear on inspection of (3), but X_t and Y_t will only be cointegrated if $\lambda_{1t} - A\lambda_{2t} = 0$. If we now express (1) in first differences, we get

$$\Delta X_t = A\Delta W_t + \Delta(\lambda_{1t} Q_t) + \alpha_1 \Delta Z_t. \tag{4}$$

Equation (4) can, after some manipulation, be written as

$$\Delta X_t = Ae_{W_t} + Q_t \Delta\lambda_{1t} - \lambda_{1,t-1} \Delta Q_t + \alpha_1(\rho - 1) Z_{t-1}. \tag{5}$$

Note that equation (5) has an error correction representation if $\lambda_{1t} = 0$. Indeed, the error correction representation of (1) and (2) can be written as

$$\Delta X_t = \alpha_1(\rho - 1) Z_{t-1} + \eta_{Xt}, \tag{6}$$

$$\Delta Y_t = \alpha_2(\rho - 1) Z_{t-1} + \eta_{Yt}, \tag{7}$$

where $\eta_{Xt} = A\epsilon_{Wt} + \Delta\lambda_{1t}\epsilon_{Qt} + \alpha_1\epsilon_{Zt}$ and $\eta_{Yt} = \epsilon_{Wt} + \Delta\lambda_{2t}\epsilon_{Qt} + \alpha_2\epsilon_{Zt}$.

Consider now a second characterization of a model with regime-sensitive cointegration. Write

$$X_t = AW_t + \alpha_1 Z_t, \tag{8}$$

$$Y_t = W_t + \alpha_2 Z_t, \tag{9}$$

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where

$$Z_t = \theta_t Z_{t-1} + \epsilon_t, \tag{10}$$

$$W_t = W_{t-1} + \eta_t, \tag{11}$$

with $0 < \theta_t \leq 1$, and $\alpha_1 - A\alpha_2 = 1$.

If $\theta_t = 1$, then $Z_t \sim I(1)$, and so, X_t and Y_t are not cointegrated. If, on the other hand, $\theta_t < 1$, then $Z_t \sim I(0)$ and X_t and Y_t are cointegrated with cointegrating vector $(1, -A)$. In the present case, the source of the suspension of the cointegration property arises from the fact that X_t and Y_t do not actually contain a unit root (even though this might represent a convenient simplification). This is particularly appealing in the case of interest rates, which often are found to contain a unit root even if such a property is unappealing from a theoretical standpoint.[4] An important reason for the finding of a unit root in a series is the presence of some large break in the process. Obviously, a major policy shift (e.g., monetary vs. interest-rate control or the adoption of inflation targeting) is likely to be the culprit.

As before, it is possible to derive an error correction representation of (8) and (9). Again, let

$$W_t = W_{t-1} + \epsilon_{Wt},$$

$$Z_t = \theta_t Z_{t-1} + \epsilon_{Zt},$$

with $0 < \theta_t \leq 1$.

The error correction representation[5] is then

$$\Delta X_t = \alpha_1(\theta_t - 1)Z_{t-1} + \eta^*_{Xt}, \tag{12}$$

$$\Delta Y_t = \alpha_2(\theta_t - 1)Z_{t-1} + \eta^*_{Yt}, \tag{13}$$

where $\eta^*_{Xt} = \epsilon_{Zt} + A\epsilon_{Wt}$, and $\eta^*_{Yt} = \epsilon_{Zt} + \epsilon_{Wt}$.[6]

It is, therefore, possible to construct cases in which the cointegration property can be switched on or off as, for example, when there is a regime change. Section 3 illustrates that this is most likely to occur when some important policy shift takes place.

3. AN ILLUSTRATION: INTEREST PARITY BETWEEN THE UNITED STATES AND CANADA

It is well known that monetary policy in Canada is highly influenced by U.S. policy. In particular, U.S. interest rates represent a key determinant of Canadian rates [e.g., see Bank of Canada (1996, p. 18)]. Figure 3 plots annual inflation for the United States and Canada at the monthly frequency (the highest available). Although both countries' inflation rates fell more or less at the same time, the era of formal inflation targeting in Canada (since February 1991) is notable for three reasons. First, the disinflation in Canada seems relatively more pronounced. Second, inflation in Canada falls consistently below that of the United States.[7] Third, although the United States did not formally target inflation, U.S. monetary policy under Fed Chairman Alan Greenspan was geared toward inflation control.

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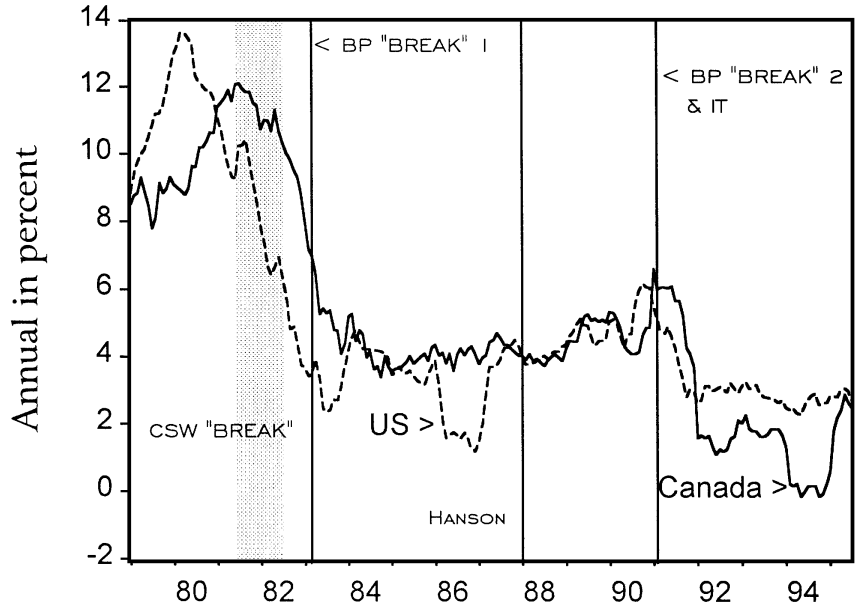


FIGURE 3. U.S. and Canadian annual inflation rates, monthly: 1979–95. Inflation is 100 times $\log P_t - \log P_{t-12}$, where P_t is the CPI at month t. All other terms are defined in Figure 1. The timing of the CSW break is during the period 1981:06–1982:07; the timing of the BP breaks are 1983:03 and 1991:02. *Source:* CANSIM and Federal Reserve Bank of St. Louis.

Although the Canadian dollar floats against the U.S. dollar, the Bank of Canada regularly intervenes in the market for foreign exchange [see Murray et al. (1997)]. This is clearly reflected, as pointed out earlier, by inspection of the comovements between U.S. and Canadian interest rates (see Figure 1).[8] As pointed out by Laidler and Robson (1993), the behavior of monetary policy was substantially different during John Crow's tenure as Governor of the Bank of Canada and reflects a "made in Canada" disinflation.[9] In fact, Laidler and Robson (1993, p. 89) trace the change in policy to Crow's Hanson Memorial lecture delivered January 8, 1988. On that day, the Governor of the Bank of Canada publicly affirmed the central bank's new policy of aiming for some form of price stability. This disinflation era, as dated from the Hanson lecture, is indicated by a vertical bar in Figure 1. It was not yet clear, however, what was meant by "price stability." The reason, as Laidler and Robson (1993, p. 121) also spell out, is that commitment to price stability occurred only gradually to be more or less formalized with the announcement of formal inflation targets in the February 1991 federal government budget (February 26, 1991).[10] Thus, the era of inflation targeting in Canada is presumed, at least initially, to begin with the budget speech announcement. However, although the late 1980's ushered in a substantially new monetary policy regime, a precise date for the actual policy shift cannot necessarily be selected precisely based on the events described above.

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TABLE 1. Unit root tests: Interest rates

Country	Test	Full[a]	Inflation target[b]	No target[c]
	(A) Unit root tests: Three-month Eurorates and spread			
United States	ADF	0.76(49)	−1.05(19)	−0.83(34)
	Phillips–Perron	−1.70(9)	−1.74(6)	−1.95(8)
Canada	ADF	−1.32(50)	−1.83(20)	−1.12(34)
	Phillips–Perron	−2.03(9)	−2.09(6)	−1.63(8)
Spread	ADF	−1.80(50)	−1.90(20)	−1.67(34)
	Phillips–Perron	−3.97(9)	−3.42(6)	−4.51(8)*
	(B) Efficient unit root tests			
United States	DF–GLS[d]			
	Constant (C)	−1.15	−0.30	−1.63
	Constant + Trend (CT)	−2.78	−0.03	−2.58
Canada	C	−1.74	−0.15	−2.11*
	CT	−0.19	−0.74	−2.11

[a] All data are daily Jan. 1, 1979, to July 31, 1995, inclusive, or January 1979–July 1995 for monthly data.
[b] Feb. 1, 1991, to July 31, 1995, or February 1991–July 1995 for monthly data.
[c] Jan. 1, 1979, to Jan. 31, 1991, or January 1979–January 1991 for monthly data.
[d] Critical values: $C = -1.95$, $CT = -2.89$. See Elliott et al. (1996, Table 1).
NOTES: $*$ = rejection at 5% level. The numbers in parentheses represent the number of lags used in the lag augmentation portion of the test equation.

3.1. Data and Estimated Models

In the empirical analysis that follows, we use daily three-month Eurorates for the United States and Canada for the period January 1, 1979, to July 31, 1995. This ensures that the underlying assets are comparable in terms of default risk and tax treatment. Exchange-rate data, shown in Figure 2, are from the Bank of Canada and reflect the daily noon spot rate.

McCallum (1994) argues in favor of an interest-rate spread equation of the form

$$(R - R^*)_t = \lambda \Delta S_t + \sigma (R - R^*)_{t-1} + \delta_t, \tag{14}$$

where $R - R^*$ is the spread between Canadian (R) and U.S. interest rates (R^*), ΔS_t is the (log) difference in the spot exchange rate, and δ_t reflects random policy influences. Alternatively, McCallum (1994) also considers the case in which the policy disturbances follow an AR(1) process. Both possibilities are captured by the regime-sensitive cointegration models outlined in the preceding section.

3.2. Empirical Evidence

3.2.1. Unit root tests. A variety of unit root tests were applied to the data. The results are contained in Tables 1 through 3. First, we verified whether a unit root exists in both the U.S. and Canadian interest rates using the augmented Dickey–Fuller (ADF) test as well as the Phillips–Perron procedure. As shown in Table 1, at conventional significance levels (5%), one cannot reject the null of a unit root in interest-rate levels for any of the samples considered. Turning to the spread between

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TABLE 2. Unit root tests: U.S. and Canadian inflation

Country	Test	Full[a]	Inflation target[b]	No target[c]
United States	ADF	−1.303(12)	−4.013*	−1.392(1)
	Phillips–Perron	−1.379(4)	−4.039*	−1.255(4)
Canada	ADF	−0.777(12)	−3.144	−1.296(4)
	Phillips–Perron	−1.258(4)	−2.326	−1.167(4)
Spread	ADF	−2.041(12)	−2.916*(3)	−2.141(2)
	Phillips–Perron	−2.412(4)	−2.412(3)	−2.071(4)

[a] All data are daily Jan. 1, 1979, to July 31, 1995, inclusive, or January 1979–July 1995 for monthly data.
[b] Feb. 1, 1991, to July 31, 1995, or February 1991–July 1995 for monthly data.
[c] Jan. 1, 1979, to Jan. 31, 1991, or January 1979–January 1991 for monthly data.
NOTE: ∗ = rejection at 5% level. The numbers in parentheses represent the number of lags used in the lag augmentation portion of the test equation.

TABLE 3. Unit root tests: Exchange rates

Test	Samples		
	Full[a]	Inflation target[b]	No target[c]
	(A) Exchange rate[d]		
ADF	−0.133 (44)	−3.10 (19)**	−0.84 (19)
Phillips–Perron	−1.04 (9)	−3.37 (6)*	−0.99 (9)
DF–GLS (C)	0.23	−1.73	−0.90
(CT)	−1.02	−1.74	−0.75
	(B) Forward premium[e]		
ADF	−1.33 (32)	−2.001 (37)	−1.359 (20)
Phillips–Perron	−2.32 (8)	−2.163 (6)	−1.453 (7)
DF–GLS (C)	−1.684	−0.671	−0.564
(CT)	−2.078	−2.98*	−2.113

[a] All data are daily Jan. 1, 1979, to July 31, 1995, inclusive, or January 1979–July 1995 for monthly data.
[b] Feb. 1, 1991, to July 31, 1995, or February 1991–July 1995 for monthly data.
[c] Jan. 1, 1979, to Jan. 31, 1991, or January 1979–January 1991 for monthly data.
[d] Defined as Canadian dollars per U.S. dollar.
[e] Defined as the 90-day forward rate less the noon spot-exchange rate.
NOTE: ∗ = rejection at 5% level; ∗∗ = rejection at 10% level. The numbers in parentheses represent the number of lags used in the lag augmentation portion of the test equation.

the two rates ($R^{\text{Can}} - R^{\text{US}}$), again the null of a unit root cannot be rejected, based on the Phillips–Perron test, except possibly for the no-inflation targeting regime.[11]

We also conducted a variety of unit root tests either by rolling the sample or by estimating the various unit-root-test equations recursively. Even if we omit the trend in every subsample, the null of a unit root cannot be rejected except for the period January 1, 1979–September 1, 1981 for the three-month U.S. interest rate, and the period February 1, 1985–March 6, 1987 for the U.S./Canadian exchange rate (not shown).[12]

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A difficulty with existing tests is that we cannot be certain, on economic grounds, whether or not a deterministic trend belongs, and the data plotted in Figures 1 and 2 reveal trendlike behavior. Recently, Elliott et al. (1996) developed a modified Dickey–Fuller test that has power against the standard alternative.[13] The essence of their procedure is to modify the treatment of the deterministic component to correct inefficient estimates of the trend in standard tests. This is accomplished via GLS estimation which Elliott et al. call the Dickey–Fuller–GLS test.[14]

Table 1B provides test results using the ERS approach for the interest-rate series. At the 5% significance level, one cannot, with one exception, reject the null of a unit root whether or not a deterministic trend is included for both the inflation-targeting and no-targeting regimes. However, one can reject at the 5% level of significance the null of a unit root for Canadian interest rates in the no-targeting era. This would seem to imply the possibility that a cointegrating relationship between U.S. and Canadian interest rates does not exist for that sample. Estimates of the DF–GLS test statistics for different samples (not shown) also lead to the same conclusion.[15]

Table 2 considers the behavior of inflation in the United States and Canada. If the policy of inflation control is credible and successful, then presumably the inflation rate cannot be $I(1)$. As revealed by the test statistics, inflation is $I(1)$ for the full sample,[16] but this clearly is not true for the inflation-control subsample in the United States while Canada's inflation rate remains $I(1)$. Note that, although the inflation-control period for Canada can be defined on the basis of known events, the same is not necessarily true for the United States where the Fed has always been viewed as being primarily, albeit informally, concerned with inflation. We return to a discussion of the time-series properties of U.S. and Canadian inflation in the next section.

Table 3A provides unit root tests for the Canadian–U.S. dollar exchange rate. The exchange rate for the full sample as well as the no-inflation targeting sample displays unit root behavior according to both unit root tests. By contrast, it is easy to reject the null hypothesis of a unit root, at conventional significance levels, for the inflation-targeting regime when a trend is included. Because the data appear to be stationary around a trend, this would be the equivalent of the case where $\lambda_i = 0$ in (1) and (2) augmented with a trend, whereas the unit root in the same series for the rest of the sample considered is consistent with $\lambda_i \neq 0$ for the same equations. Turning to the efficient unit root tests, we cannot reject the null of a unit root whether or not a deterministic trend is present. However, estimates over several subsamples (not shown) reveal that exchange rates are stationary around a linear deterministic trend during the initial informal phase of the inflation-control era, that is, following the Governor's 1988 speech but before the targets were formalized in 1991 (result not shown).

Clearly, the relationship between interest rates also will be influenced by the behavior of the relationship between spot and forward exchange rates. Hai et al. (1996) recently summarized and revisited some of the empirical regularities concerning the spread between these two series. Of particular relevance to this paper is whether the forward premium (see Figure 4) is stationary over the regimes considered.[17] The relevant unit root tests are provided in Table 3B. On the basis

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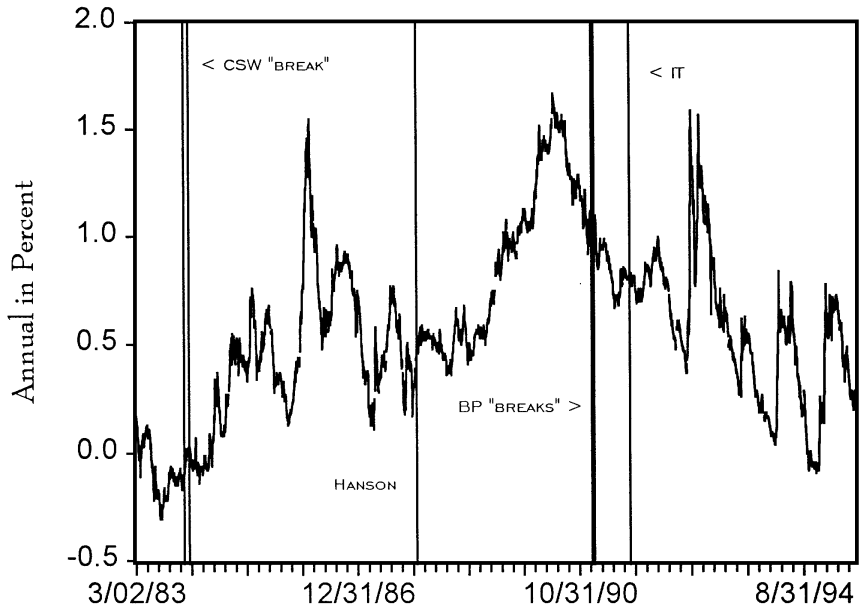


FIGURE 4. Covered 90-day forward premium (+) and discount (−) on U.S. dollars, daily: 1979–95. The timing of the CSW break is January 1984. The two BP breaks occurred January 16 and September 12, 1991. See also Figure 1 for other details. *Source:* Bank of Canada.

of ADF and Phillips–Perron tests (with no deterministic trend), there do not seem to be any differences across the samples. However, stationarity in the presence of a trend cannot be rejected for the inflation-targeting sample based on the ERS version of the unit root test.

3.2.2. Cointegration tests. Table 4A provides the Johansen maximal eigenvalue test of cointegration between the two interest rates considered. Cointegration is rejected in both the full sample and the sample in which no formal inflation targeting existed. However, only when the sample consists of the period when inflation was formally targeted does one find cointegration between the two Eurorates.[18] We speculate that one reason for these findings is the absence of a common stochastic trend, which upsets the cointegrating relationship in one regime. Clearly, the opposite would have to be true for the period in which there was no formal inflation targeting. As argued earlier, the obvious variable in the present context that could produce such a result is the exchange rate. Indeed, when the (log) exchange rate is added to the case considered in Table 4A we find cointegration in the trivariate system in both the full and non-inflation-targeting samples whereas there are two cointegrating vectors in the inflation control sample (not shown). Table 4B provides cointegration tests between daily forward and spot rates. On the basis of the results, one cannot reject the stationarity of the forward premium for the

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TABLE 4. Cointegration tests

Sample	Null hypothesis	Likelihood ratio
(A) Canadian and U.S. 3-month Eurorates[a]		
Cointegrating vector = [1 − 0.69];		
Error-correction coefficient = −0.012 (0.005)*		
Full	$r = 0$	10.23
	$r \leq 1$	1.59
No inflation targeting	$r = 0$	9.83
	$r \leq 1$	3.47
Inflation targeting	$r = 0$	15.56*
	$r \leq 1$	0.19
(B) Spot and forward exchange rates[b]		
Cointegrating vector = [1 − 0.42];		
Error-correction coefficient = −0.02 (−0.006)@		
Full	$r = 0$	9.89
	$r \leq 1$	0.16
No inflation targeting	$r = 0$	11.50
	$r \leq 1$	3.76
Inflation targeting	$r = 0$	24.19@
	$r \leq 1$	3.40

[a] Thirty lags were used in the VAR for the full and no-inflation-targeting samples; 15 lags were used in the inflation-targeting sample.
[b] Fifty lags were used for the full sample, 40 for the no-inflation-targeting sample, and 30 for the inflation-targeting sample.
NOTES: ∗ = rejection at 5% level; @ = rejection at 1% level.

inflation-targeting sample only. Otherwise there is unequivocal nonrejection of the null of no cointegration for the full and no-inflation-targeting samples. This is just another way of confirming the results of Table 4A that revealed the interest-rate differential to be stationary during the inflation-targeting era.

Another way of looking at the properties of the two interest-rate series is to consider empirically the possibility of the case that leads to equation (3). This would be the equivalent of concluding that there is a unit root in both interest rates when in fact this presumption is incorrect. Elliott (1995) argues that the normal two-step procedure of testing for cointegration (i.e., first test for a unit root and then test for cointegration) too often will lead the researcher to conclude that there is a unit root when there is none. The case considered in this paper is particularly relevant, as pointed out earlier, because most economists would prefer that interest rates not contain a unit root. In the present context, this also would amount to asking whether the spread between the two interest rates is $I(0)$. Elliott (1995) recommends that a unit root test and a test for a $(1, -1)$ cointegrating vector be jointly specified. In the bivariate framework considered here, Elliott's proposed test equation would be written

$$\Delta R_t^i = \mu + \theta_1 R_{t-1}^i + \theta_2 EC_{t-1} + \pi(L)\Delta R_{t-1}^i + \epsilon_t, \qquad \textbf{(15)}$$

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TABLE 5. Joint test for unit roots and $(1, -1)$ cointegrating vector [a]

Dependent variable	Test statistic[b]	Full	Inflation target	No inflation target
Canada	WALD	57.35+	12.89*	49.26+
	Likelihood ratio	57.01+	12.90*	48.92+
United States	WALD	10.45*	5.43	9.61**
	Likelihood ratio	10.45*	5.46	9.62**

[a] Five lags were used in the AR part of the test equation. For the full and no-inflation samples, EC = spread; for the inflation-targeting sample, EC = $R^{\text{Can}} - 0.69R^{\text{US}}$ [see Table 4(A)] although the same conclusion was reached when EC = spread is assumed.
[b] The 10% critical value is 8.002, 5% is 9.663, and 2.5% is 11.38 [Elliott (1995, Table 2)].
NOTES: $*$ = rejection at 5% level; $**$ = rejection at 10% level; $+$ = rejection at 2.5% level.

where (15) is the vector error correction (EC) form derived from the cointegrating relationship between the interest rates considered (i.e., $R^i = R^{\text{Can}}, R^{\text{US}}$). Elliott (1995) shows that the test for the joint hypothesis that R has a unit root and that R^{Can} and R^{US} are cointegrated with cointegrating vector $(1, -1)$ simply tests the null that $\theta_1 = 0$. His study also produces the relevant critical values for such a test. The evidence is in Table 5 in which it is seen that the joint hypothesis is rejected for the inflation targeting sample when the Canadian interest rate is the dependent variable but not when the U.S. interest rate is the dependent variable. The situation is somewhat different for the full sample and the no-inflation-targeting sample. The results are not inconsistent with the conclusion that an $I(0)$ spread is a feature of the inflation-targeting period only. Otherwise interest rates cannot be said to be $I(1)$.

As a further check on the significance of regime shifts in testing for cointegration, we also examined the relationship between U.S. and Canadian inflation rates (not shown). The relevant data are only available at the monthly frequency. Cointegration is essentially a property of the inflation-target sample, thereby confirming the stationarity of the spread between the two inflation rates during this period (see Table 2), whereas there is no linear combination between them that is stationary when inflation-control targets are not formally in existence. An issue with the tests based on inflation rates is whether these suffer from some finite-sample problems.[19] Accordingly, we generated a simulation experiment in which $\alpha_1 - A\alpha_2 = 1$, λ_{1t} and $\lambda_{2t} \neq 0$ in (1) and (2).[20] As a result, the null hypothesis is that X_t and Y_t are not cointegrated. Because the objective of the Monte Carlo experiment is not to determine whether the asymptotic critical values give misleading results in the small-dimension systems considered here only 1,000 artificial X_t and Y_t series for a sample of 200 observations were generated for the DGPs outlined in Section 2.[21] The results (not shown) reveal that, in just over one-third of the cases (377), we reach incorrect conclusions, that is, we find cointegration where cointegrated by construction does not exist.[22] Because the potential bias is toward a finding of cointegration when none is expected, it is not clear that the empirical results, which found no cointegration in the full sample, are due to finite-sample

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problems. Instead, the inflation data yield conclusions that are consistent with the interest-rate and exchange-rate results.

In interpreting the evidence accumulated so far, there is the risk that the choice of time periods when a switch between the cointegration and noncointegration properties occurs is *ad hoc* and, therefore, any testing of the sort conducted above arises from data miming.

Although the evidence above suggests that the nature of the relationship between U.S. and Canadian interest rates was severed and that the inflation-targeting era might have played an important role, the evidence so far does not produce a precise dating for the regime changes. We, therefore, considered two additional sets of tests to seek to more precisely define the *on* and *off* cointegration periods. The recent tests proposed by Bai and Perron (1995) and Chu et al. (in press) were implemented for this purpose.

3.3. Where Are the Structural Breaks?

Chu et al. (in press) propose sequential testing methods to detect standard breaks in economic relationships. They view the problem as involving one in which a researcher estimates a relationship for a sample, say of size N, and wishes to determine whether the model continues to be useful as new observations (say m) are added in the sense that the parameters remain stable. However, they argue that a retrospective view that reexamines the entire sample each time new data are added (i.e., for sample $N + m$ in the illustration above) leads to significant type-I errors. Instead, they propose tests that *monitor* the recursive residuals and fluctuations in parameter estimates sequentially based on the well-known CUSUM test.[23] The location of the breaks is indicated by vertical bars and these are labeled "CSW [Chu–Stinchcombe–White] break" in Figures 1 to 4. Figures 1 and 2 show the break when the trivariate relationship (that is, one that includes the exchange rate) is estimated. Other significant events, such as the Hanson lecture and the formal introduction of inflation targeting, also are shown. The break signaled by the CSW test occurs approximately midway between the formal and informal inflation-targeting events. In the case of the relationship between U.S. and Canadian inflation rates (see Figure 3) the CSW test selects a break around the time of the end of the recession in the two countries, that is, at the beginning of the disinflation phase. Finally, in Figure 4, the CSW test reveals a break in the early stages of a persistent rise in the forward premium.

There are potentially two drawbacks with the CSW approach. First, the test can only identify a single break. Second, the test assumes that the series are stationary which, as we have seen, may or may not be the case. Nevertheless, it is conceivable that the test works well even in the nonstationary case.

Although we have focused on the change in regime associated with the introduction of inflation-control targets, it is conceivable that more than one regime, as we have defined it, exists in the sample considered. For this reason, we also implement the test proposed by Bai and Perron (1995), who extend existing tests for a single

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break at an unknown location. The null hypothesis is that of no structural change vs. the alternative of multiple breaks. It should be emphasized, however, that their procedure is asymptotic in nature and that their power in finite samples is unknown.

The timing of the breaks is again shown in Figures 1 through 4 (labeled as BP "breaks"). Because of the size of the sample owing to the use of daily data, preliminary testing for the full sample revealed a large number of breaks. With the exception of the inflation data (see below) we restricted our attention to a sample covering the 1988–1992 period and restricted the maximum number of breaks to two.

In the bivariate interest-rate relationship (see Figure 1) both breaks chosen by the BP test straddle the inflation-targets announcement, whereas in the trivariate relationship (see Figure 2), both breaks occur in mid-1992, that is, approximately a year and a half after the inflation-control guidelines were introduced. Turning to the U.S./Canadian inflation relation (see Figure 3), the BP test selects one break late in the disinflation phase of the early 1980's but the second break coincides with the introduction of the inflation targets. Note that, in this case, we did not restrict the sample size when testing for breaks. Finally, Figure 4 shows that both breaks in the forward-spot relation occur just a few months before the IT era begins.

4. CONCLUSIONS

We suggest that theoretical considerations may make it unattractive to model a relationship either by imposing cointegration over the sample of interest or by ignoring it altogether. Instead, we propose the concept of regime-sensitive cointegration in which the cointegrating relationship can be switched on or off depending on the nature of the policy regime in place. Using interest-parity theory, we empirically explore the practical implementation of the regime-sensitive cointegration concept.

Although we focus on the role of the inflation-targets policy as a potential source of change in the cointegrating relationship, it is certainly conceivable that both the timing and the source of the regime switch occurred at a different time and for other reasons. Our aim was simply to point out that proper testing for cointegration requires more than an application of some existing test. Instead, researchers also need to think clearly about, for example, whether an additional stochastic trend might interfere with some specified equilibrium relationship, or even whether it is sensible to impose the unit root property on the series in question. Future work might consider how to combine the cointegration–no-cointegration hypothesis with the selection of the breakpoint or, alternatively, using information contained in the error-correction term to decide when to abandon the cointegration hypothesis. We leave these issues to future research.

NOTES

1. Put differently, it is often the case that empirical investigations find cointegration for a particular sample, for a model augmented with dummies or other variables introduced to reflect a particular

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institutional feature of the data set, and so on. Of course, this interpretation is based on published evidence that is likely to show more success at detecting cointegration than might actually be the case. Horvath and Watson (1995) also suggest that cointegration tests yield different results when a particular cointegrating vector, assumed to be known, is imposed.

2. Other problems or issues that could be added to the above list include the power of various cointegration tests and the role played by specifying the null of no cointegration. For more on these questions, see Hamilton (1994) and Hargreaves (1994).

3. This is the approach taken by those who, for example, subscribe to the events study methodology.

4. The reason, of course, is that there is, in practice, both an upper and a lower bound to nominal interest-rate movements.

5. $\Delta Z_t = Z_t - Z_{t-1} = \theta_t Z_{t-1} + \epsilon_{zt} - Z_{t-1} = (\theta_t - 1)Z_t - 1 + \epsilon_{zt}$

6. Note that, if we write (12) and (13) as

$$\Delta X_t = \gamma_1 \delta(Q_t) Z_{t-1} + \epsilon_{X_t}, \Delta Y_t = \gamma_2 \delta(Q_t) + \epsilon_{Y_t},$$

this produces a model similar to the one proposed by Balke and Fomby (1996).

7. Indeed, it is the longest continuous period during which U.S. inflation exceeds Canadian inflation since 1958. Previously, between 1958 and 1960, 1968 and 1971, and then again in 1979–80, Canadian inflation performance was superior to that of the United States.

8. The same situation holds when 1-, 6-, and 12-month Eurorates are employed.

9. However, most industrialized countries would follow suit, so that the disinflation phenomenon would become generalized, at least in the industrialized world.

10. This occurs approximately at the midpoint of the area to the right of the IT-labeled vertical line in Figure 1. Although the announcement was made in 1991, inflation targets (in the CPI) of 2–4% were in effect for 1993, reduced to 1.5–3.5% in 1994, and reduced to 1–3% range for 1995 to 1998 [e.g., see Siklos (1997)]. There were other events that might have a significant, if perhaps transitory, impact on the spread, such as the introduction of the Goods and Services Tax in January 1991, the failure of the Meech Lake Constitutional Accord in 1992, the failure of the Charlottetown Constitutional Agreement, and the subsequent national referendum in 1992.

11. The conclusions are not substantially altered when the inflation targeting regime is assumed to begin in January 1988.

12. If a trend is included, one cannot reject the null of a unit root even in these two cases.

13. Typically, the largest AR root is close to unity for many macroeconomic time series [Stock (1991)]. The Elliott–Rothenberg–Stock (ERS) test uses a local-to-unity detrending procedure to maximize power against a local alternative.

14. They also provide a test of the hypothesis $\alpha = 1$ against $\alpha = \alpha - (= 1 + c - / T$, where $T =$ sample size, and $c-$ is some fixed number, so that as $T \to \infty, \alpha \to 1$) in the model $y_t = d_t = u_t$, $u_t = \alpha u_{t-1} + v_t$, where d_t is the deterministic component in y_t, and where $\alpha = 1$ is a test of the unit root hypothesis in y_t.

15. The null of a unit root is rejected in the following samples (m/d/y): U.S. interest rate in the presence of a deterministic trend (1/19/81–9/14/83); Canadian interest rate with or without a deterministic trend (2/1/85–3/05/87); and the exchange rate with a deterministic trend (2/20/89–3/06/91).

16. This, of course, implies two unit roots in the (log) level of prices.

17. We investigate in Section 3.2.2 whether spot and forward rates are cointegrated.

18. These test results are confirmed by the estimation of vector error correction models (results not shown).

19. The nature of the markets examined, the size of the sample considered, and the number of variables in the models tests suggest that such problems will be negligible for data at the daily frequency.

20. The starting value for λ_{1t} was set to average 0.5, the mean of λ_{2t} was set at 0.2, and α_1 was set at 0.9 and α_2 was set to -0.05.

21. The number of observations is equivalent to the full inflation sample considered.

22. The Johansen technique was used to test for cointegration and the VAR's were estimated with four lags as in the unit-root and cointegration tests for inflation reported earlier.

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23. In conducting the test, the initial sample was assumed to be 200 observations, although other starting sample sizes were tried without any impact on the conclusions.

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