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Zonal flow in toroidal ion temperature gradient mode turbulence

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Abstract. The properties of zonal flows in the toroidal ion temperature gradient mode turbulence are investigated taking into account the polarization drift effects. The stability criterion and the characteristic oscillation frequency of the zonal flow are determined in terms of the spectra of turbulent fluctuations. The nonlinear evolution of zonal flows may lead to the formation of stationary long-lived coherent structures supporting stationary shear layers. These results indicate the existence of regions with reduced levels of anomalous transport attributed to zonal flows generalizing previous findings regarding zonal flows in electron drift turbulence.

1. Introduction

Low-frequency electrostatic turbulence driven by spatial gradients is believed to be the dominant source of anomalous transport in magnetically confined fusion plasma [1, 2]. During the last decade, special emphasis has been given on analytical and numerical investigations of the properties of the large-scale flows that are generated by the drift-type turbulence. The formation of such flows is commonly attributed to several mechanisms such as Reynolds stress, modulation or parametric instabilities. Their importance lies on the fact that as they interact with the background turbulence may regulate the levels of the turbulent transport. Furthermore, due to their relatively large scale, these structure may also drive large-scale transport events in magnetically confined plasmas, such as bursts or avalanches.

Zonal flows correspond to azimuthally symmetric band-like shear flows that spatially depend on the radial coordinate (the coordinate along the axis of plasma inhomogeneity) [3]. In tokamak plasmas, zonal flows have the ability to limit the radial size of turbulent eddies through the shear decorrelation mechanism [4], and hence to regulate turbulent transport. As a consequence, the high plasma confinement modes are usually attributed to the presence of the zonal flows. The spontaneous generation of large-scale anisotropic flows with attributed to drift-type turbulence has been experimentally observed in plasma discharges in various machines, for example in TEXT, RFX and DIII-D [5, 6].

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It becomes evident that for an adequate description and understanding of the transport phenomena observed in experiments a self-consistent description of turbulence that takes into account the nonlinear dynamics of both components in the turbulent spectra, namely the large-scale flows and the ambient small-scale turbulence is required. The interaction between these disparate scales of turbulent spectra can be described by employing the evolution equations for the mean flow and the wave-kinetic equation for an action-like invariant. As it was initially shown for the case of electron drift wave-zonal flow turbulence [7], and later for the flute-type turbulence [8], zonal flows can be spontaneously generated by the Reynolds stress [9], or due to resonant non-local (in **k**-space) interaction between the flow and the modulation of the small-scale turbulence [10]. The nonlinear evolution of these instabilities can lead to the formation of long-lived coherent structures in the drift-wave zonal flow system, constituting paradigms for intermittency in drift wave turbulence and manifesting itself in regions with a reduced level of anomalous transport.

During the last few years, a significant number of both theoretical and numerical investigations in plasma dynamics has focused on the effects related to the development of the toroidal ion temperature gradient mode (TITG) instability [11]. This is due to the successful interpretations of various experimental results—related to the observed levels of turbulent transport in tokamak plasmas—through the dynamics of the TITG mode. The associated instability appears due to the gradient of the ion temperature profile along the radial direction and in tokamaks it is mainly driven by the magnetic field curvature [12]. The generation of zonal flows by the TITG turbulence and the interaction of the flow with the small-scale fluctuations has been considered in [13, 14], taking into account the advective nonlinear terms but not the effects associated with the linear (finite Larmor radius (FLR) effects) and nonlinear polarization drifts.

On the basis of the reactive fluid model [15], the linear properties of the TITG mode were analytically investigated [16] by rigorously accounting for the FLR effects, which are actually induced by the polarization drifts. It was shown [16] that FLR effects may modify significantly the linear properties of the TITG instability—especially in the regions of peaked plasma density—decreasing the instability threshold and the wavelength of the associated marginally unstable modes. These results established new conditions for the release of the free energy—necessary for the formation of large-scale flows—especially when excited near the marginal conditions.

In this study we focus on the properties and the long-term evolution of the zonal flows attributed to the complex interplay with the short-scale TITG turbulence. Using the methodology developed in [7] for the drift turbulence, we derive the wave-action invariant for the TITG turbulence and derive the wave-kinetic equation in the presence of zonal flow, taking into account the polarization drift effects. The growth rate and the oscillation frequency of the zonal flow are determined in terms of the turbulent spectra while it is shown that the evolution of the zonal flow can lead to the formation of stationary coherent structures, which corresponds to propagating shear layer 'domain walls' between regions of different flow velocities. Such regions can be characterized as region of reduced plasma transport.

The rest of the article is organized as follows. In Sec. 2, we briefly present the model of equations that describe the evolution of the TITG instability, and in Sec. 3

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we describe the coupled dynamics of the TITG turbulence with zonal flows. Finally, a summary and conclusions are given in Sec. 4.

2. Model equations

The two-dimensional model equations that describe the dynamics of the TITG modes can be derived in the two-fluid plasma approximation [17] by using a low-frequency expansion based on the standard drift velocity ordering. Then, the ion continuity and energy equations, respectively, can be written in the following normalized form [14, 15]

$$\frac{\partial n_{\rm i}}{\partial t} - \left[\frac{\partial}{\partial t} - \tau(1+\eta_{\rm i})\frac{\partial}{\partial y}\right]\nabla_\perp^2\phi + \frac{\partial \phi}{\partial y} - \epsilon_n \frac{\partial}{\partial y}[\phi + \tau(n_{\rm i} + T_{\rm i})] = \{\phi, \nabla_\perp^2\phi - n_{\rm i}\}, \qquad (2.1)$$

and

$$\frac{\partial T_{\rm i}}{\partial t} - \frac{5}{3}\tau\epsilon_n\frac{\partial T_{\rm i}}{\partial y} + \left(\eta_{\rm i} - \frac{2}{3}\right)\frac{\partial \phi}{\partial y} - \frac{2}{3}\frac{\partial n_{\rm i}}{\partial t} = \left\{\phi, \frac{2}{3}n_{\rm i} - T_{\rm i}\right\}. \qquad (2.2)$$

The curly brackets on the right-hand side of (2.1) and (2.2) denote the Poisson bracket, defined by $\{A, B\} = \partial_x A \partial_y B - \partial_x B \partial_y A$.

The length and the time scales have been normalized by $\rho_{\rm s} = c_{\rm s}/\omega_{\rm ci}$ and $L_n/c_{\rm s}$, respectively, where $c_{\rm s}^2 = T_{\rm e}/m_{\rm i}$ is the ion sound velocity defined at the electron temperature, $\omega_{\rm ci} = eB_0/m_{\rm i}c$ is the ion gyro-frequency, and $L_g^{-1} = -d\ln g(r)/dr$ describes the inverse characteristic scale-length of inhomogeneity along the radial direction of the plasma parameter $g(r)$ where $g(r) = n(r), T_{\rm i}(r)$. The electrostatic potential $\delta\phi$ has been normalized as $\phi = e\delta\phi/T_{\rm e}\ L_n/\rho_{\rm s}$, the perturbed density δn as $n = \delta n/n_0\ L_n/\rho_{\rm s}$, and the perturbed ion temperature $\delta T_{\rm i}$ as $T_{\rm i} = \delta T_{\rm i}/T_{\rm i0}\ L_n/\rho_{\rm s}$. With the curvature R of the magnetic field lines and the ion temperature inhomogeneity scale length $L_{T_{\rm i}}$ we define $\epsilon_n = 2L_n/R$ and $\eta_{\rm i} = L_n/L_{T_{\rm i}}$ respectively, where L_n is the plasma inhomogeneity scale length. Furthermore, $\tau = T_{\rm i}/T_{\rm e}$ denotes the ratio of ion to electron temperature.

The dispersion relation for the TITG modes and the threshold of the associated instability, explicitly taking the FLR effects into account, have been presented in [16]. The real frequency $\omega^{\rm RE}(\mathbf{k}_\perp)$ and the growth rate $\gamma(\mathbf{k}_\perp)$ are given, respectively, by

$$\omega_k^{\rm RE} = \frac{k_y}{2(1+k_\perp^2)}\left\{1 - \epsilon_n - \frac{10}{3}\epsilon_n\tau - \left[\tau(1+\eta_{\rm i}) + \frac{5}{3}\tau\epsilon_n\right]k_\perp^2\right\},$$

and

$$\gamma_k = \frac{k_y}{2(1+k_\perp^2)}\sqrt{f(k_\perp^2)}.$$

The radical $f(k_\perp^2)$ has the form

$$f(k_\perp^2) = 4\epsilon_n\tau(\eta_{\rm i} - \eta_C) + 2\tau(1+\epsilon_n)(\eta_{\rm i} - \eta_B)k_\perp^2 - \tau^2(1 + \eta_{\rm i} - \tfrac{5}{3}\epsilon_n)^2 k_\perp^4 \qquad (2.3)$$

where the parameters η_B and η_C are

$$\eta_B = \frac{4\epsilon_n - 1}{\epsilon_n + 1} - \frac{5}{3}\frac{\epsilon_n^2}{\epsilon_n + 1}\left(1 - \frac{4}{3}\tau\right)$$

and

$$\eta_C = \frac{2}{3} + \frac{10}{9}\epsilon_n\tau + \frac{(\epsilon_n - 1)^2}{4\epsilon_n\tau}.$$

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From (2.3) it turns out that the TITG modes become unstable for $\eta_\mathrm{i} > \eta_\mathrm{ith}$ where the explicit threshold η_ith for the instability is [16]

$$\eta_\mathrm{ith}(\epsilon_n,\tau) = \begin{cases} \eta_* & \text{for } \epsilon_n < 1 \text{ and for } \epsilon_n > 1 \text{ when } 0 < \tau < \tau_*, \\ \eta_C & \text{for } \epsilon_n > 1 \text{ when } \tau > \tau_* \equiv 3(1-1/\epsilon_n)/2, \end{cases}$$

where

$$\eta_* = \frac{2}{3} + \frac{10}{9}\epsilon_n\tau.$$

In absence of FLR effects the threshold of the instability is η_C [15]. However, for $\epsilon_n < 1$ the FLR effects become rather important [16], leading to a decrease of the threshold and to the appearance of the marginal unstable modes at a shorter wavelength regime. The wavenumber of the marginal unstable mode, i.e. $\gamma(k_{\perp m}) = 0$, is equal to

$$k_{\perp m}^2 \approx \begin{cases} (1+\epsilon_n)(\eta_\mathrm{i}-\eta_B)/\tau(1+\eta_\mathrm{i}-\frac{5}{3}\epsilon_n)^2, & \text{when } \eta_\mathrm{ith} = \eta_*, \text{ and} \\ 0, & \text{when } \eta_\mathrm{ith} = \eta_C. \end{cases}$$

As a consequence, for $\eta_* < \eta_\mathrm{i} < \eta_C$ there is a stable region in the wavelength regime around $k_\perp = 0$. These results are crucial for the self-generation of large-scale flows by TITG turbulence as they establish new marginal conditions for the release of the available free energy, which is necessary for the formation of large-scale flows.

3. Coupled dynamics of TITG turbulence and zonal flows

To describe the dynamics of large-scale plasma flows that vary on a longer time scale compared with the small-scale fluctuations, a multiple scale expansion is usually employed assuming that there is a sufficient spectral gap separating the large- and the small-scale motions. In what follows, $[\tilde{\Phi}(\mathbf{r},t), \tilde{n}(\mathbf{r},t), \tilde{T}(\mathbf{r},t)]$ denote the small-scale fluctuations and $[\overline{n}(\mathbf{r},t), \overline{\Phi}(\mathbf{r},t), \overline{T}(\mathbf{r},t)]$ the large-scale fluctuations attributed to the zonal flow. In the Fourier space the scale separation is determined through the inequality $\mathbf{k} \gg \mathbf{q}$ where $\mathbf{k}$ and $\mathbf{q}$ denote the characteristic wavenumbers of the short- and large-scale fluctuations, respectively.

3.1. Dynamics of TITG turbulence in the presence of zonal flow

To describe self-consistently the model for TITG turbulence we have to take into account the influence of the ZF on the small-scale turbulence. The scale separation lead us to the following equations that govern the turbulent fluctuations

$$\frac{\partial\tilde{\phi}}{\partial t} - \left[\frac{\partial}{\partial t} - \tau(1+\eta_\mathrm{i})\frac{\partial}{\partial y}\right]\nabla_\perp^2\tilde{\phi} + \frac{\partial\tilde{\phi}}{\partial y} - \epsilon_n\frac{\partial}{\partial y}[(\tau+1)\tilde{\phi} + \tilde{T}_\mathrm{i}] = \{\overline{\Phi}, \nabla_\perp^2\tilde{\Phi} - \tilde{\Phi}_\mathrm{i}\}, \quad (3.1)$$

and

$$\frac{\partial\tilde{T}_\mathrm{i}}{\partial t} - \frac{5}{3}\tau\epsilon_n\frac{\partial\tilde{T}_\mathrm{i}}{\partial y} + \left(\eta_\mathrm{i} - \frac{2}{3}\right)\frac{\partial\tilde{\phi}}{\partial y} - \frac{2}{3}\frac{\partial\tilde{\phi}}{\partial t} = \left\{\overline{\Phi}, \frac{2}{3}\tilde{\Phi} - \tilde{T}_\mathrm{i}\right\}. \quad (3.2)$$

Here we have assumed for the short-scale fluctuations that electrons follow the Boltzmann distribution and that the low-frequency electrostatic oscillations $\omega \ll \omega_{ci}$ are quasi-neutral, i.e. $\tilde{n}_\mathrm{e} = \tilde{n}_\mathrm{i} = \tilde{\Phi}$. The propagation of the TITG modes in a weakly inhomogeneous plasma can be described by employing the wave-kinetic equation for the wave-action density $N_k(r,t)$ in the $\mathbf{r}$–$\mathbf{k}$ space. The source of the slow spatial and temporal variations is the zonal flow sustained by the short-scale turbulent

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perturbations. The wave-kinetic equation for the generalized wave action allows us to determine the modulations of $N_k(r,t)$ due the mean flow. The method of constructing the adiabatic invariant has been previously discussed in [7]. Expressing the fluctuating fields in terms of Fourier series in the **k**-space, we multiply (3.1) with α_k—a parameter to be determined later—and we add the resulted equation to (3.2). Using the standard method presented in [7], we determine the adiabatic invariant Ψ_k that obeys the equation

$$\frac{\partial \Psi_k}{\partial t} + i\omega_k \Psi_k = \{\Psi_k, \overline{\Phi}\}. \tag{3.3}$$

The invariant is given by

$$\Psi_k = T_k + [(1+k_\perp^2)\alpha_k - \tfrac{2}{3}]\Phi_k = T_k - L_k \Phi_k$$

where L_k is

$$L_k = \frac{2}{3} + \frac{1}{2\epsilon_n \tau}\left\{1 - \epsilon_n - \left[\tau(1+\eta_i) - \frac{5}{3}\tau\epsilon_n\right]k_\perp^2 + i\sqrt{f(k_\perp^2)}\right\}.$$

Using the linear response of ion temperature T_k over TITG electrostatic perturbations we can express the action density invariant Ψ_k in terms of the potential Φ_k as

$$\Psi_k = -i\frac{\sqrt{f(k_\perp^2)}}{\tau\epsilon_n}\Phi_k,$$

where $f(k_\perp^2)$ has been defined in (2.3). We should point out here that for the derivation of (3.3) the approximative relation $L_k \approx L_{k-q_x}$ was used, which is well justified for the scale separation limit $k \gg q_x$.

The Wentzel Krammer Brillouin (WKB)-type wave-kinetic equation that describes the evolution of the generalized action invariant $N_k(\mathbf{r},t) = |\Psi_k|^2$ due to the interaction between the zonal flow and the turbulent fluctuations, can be derived from (3.3) similar to the approach used in [7] and it is given by

$$\frac{\partial N_k}{\partial t} + \frac{\partial N_k}{\partial \mathbf{r}} \cdot \frac{\partial \omega_k^{NL}}{\partial \mathbf{k}} - \frac{\partial \omega_k^{NL}}{\partial \mathbf{r}} \cdot \frac{\partial N_k}{\partial \mathbf{k}} = \gamma_k N_k - \Delta\omega_k N_k^2. \tag{3.4}$$

Equation (3.4) has been analyzed for the case of drift wave turbulence [7], magnetic curvature driven (flute turbulence) [8] and TITG [14] turbulence but in the absence of polarization drift effects for the latter. The generalized action density for the TITG mode turbulence is given here by

$$N_k \equiv |\Psi_k|^2 = \frac{|f(k_\perp^2)|}{(\tau\epsilon_n)^2}|\Phi_k|^2 \tag{3.5}$$

and includes the linear and nonlinear effects associated with the polarization drift. The nonlinearly shifted frequency is defined through $\omega_k^{\mathrm{NL}} = \omega_k^{\mathrm{RE}} + k_y V_{0y}$, where the shift is due to the presence of the zonal flows, since

$$V_{0y} = -\tfrac{1}{2}(\nabla_x \bar{\Phi} \times \hat{z}). \tag{3.6}$$

The term $\Delta\omega_k N_l^2$ in the right-hand side of (3.4) represents the wave nonlinear damping via self-interaction of the TITG waves (inverse cascade), which balances the linear growth rate.

Considering small deviations of the spectrum function from the equilibrium, we may write the adiabatic action invariant as a sum of $N_k = N_k^0 + \tilde{N}_k$, where

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$N_k^0 = \langle N_k \rangle$ describes the equilibrium part of the turbulent spectrum and $\tilde{N}_k$ is the perturbed part. For the equilibrium part, we may consider a balance between the terms in the right-hand side of (3.4), which corresponds to the case of stationary turbulence and gives $N_k^0 \approx 2\gamma_k/\Delta\omega_k$. The quasilinear equation for N_k^0 has the following form

$$\frac{\partial N_k^0}{\partial t} - \left\langle \frac{\partial}{\partial X}(k_y V_{0y}) \frac{\partial \tilde{N}_k}{\partial k_x} \right\rangle = 0, \tag{3.7}$$

where the perturbed density of the 'quasiparticles' $\tilde{N}_k$ can be calculated by the linearized wave-kinetic equation. For a uniform equilibrium $\partial N_k^0/\partial \mathbf{r} = 0$, it becomes

$$\frac{\partial \tilde{N}_k}{\partial t} + \frac{\partial}{\partial k_x}(\omega_k + k_y V_{0y}) \frac{\partial \tilde{N}_k}{\partial X} - \frac{\partial}{\partial X}(\omega_k + k_y V_{0y}) \frac{\partial N_k^0}{\partial k_x} = -\gamma_k \tilde{N}_k. \tag{3.8}$$

In the local approximation, i.e. $\partial\omega_k/\partial\mathbf{r} = 0$, equation (3.8) can be solved by assuming that the large-scale variation of the action density is of the form $N_k \sim \exp(iq_x x - i\Omega t)$. This yields the resonant part of the distribution:

$$\tilde{N}_k^{\mathrm{res}} = \frac{\partial}{\partial X}(k_x V_{0y}) \frac{\partial N_k^0}{\partial k_x} R(\Omega, q_x, \delta\omega_k). \tag{3.9}$$

Here R is the response function defined by $R(\Omega, q_x, \delta\omega_k) = i/(\Omega - q_x V_{gx} + i\delta\omega_k)$, $\delta\omega_k$ is the total decorrelation frequency, which includes the linear growth rate and a nonlinear shift, and V_{gx} is the group velocity defined by $V_g x = \partial\omega_k/\partial k_x$. In a weakly nonlinear regime it is $R(\Omega, q, \delta\omega_k) \to \pi\delta(\Omega - q_x V_{gx})$, while for a wide fluctuating spectrum $R(\Omega, q_x, \delta\omega_k) \to 1/\delta\omega_k$.

Zonal flow regulates the TITG turbulence by the process of random shearing providing a mechanism for self-regulation and saturation of the turbulence. The process of the random shearing of the turbulence can be expressed in terms of the diffusion of the stationary spectra in the **k**-space. Indeed, substituting (3.9) into (3.7), the following diffusion-like equation for N_k^0 is obtained

$$\frac{\partial N_k^0}{\partial t} - \frac{\partial}{\partial k_x} D_{k_x} \frac{\partial N_k^0}{\partial k_x} = 0,$$

where D_{k_x} is the diffusivity in the radial wave number given by

$$D_{k_x} = (q_x^4 k_y^2/4) |\overline{\Phi}|^2 R(\Omega, q_x, \delta\omega_k).$$

3.2. Dynamics of zonal flows

The equations that govern the dynamics of the zonal flows generated by the TITG turbulence can be determined by averaging (2.1) and (2.2), and assuming that the short-scale fluctuations are quasineutral and follow the Boltzmann relation. This yields

$$\frac{\partial \overline{n}}{\partial t} - \frac{\partial}{\partial t} \overline{\nabla_\perp^2 \phi} = \overline{R^\Phi}, \tag{3.10}$$

and

$$\frac{\partial \overline{T_\mathrm{i}}}{\partial t} - \frac{2}{3} \frac{\partial \overline{n_\mathrm{i}}}{\partial t} = \overline{\{\tilde{T}_\mathrm{i}, \tilde{\Phi}\}}, \tag{3.11}$$

where $R^\Phi \equiv \{\tilde{\Phi}, \nabla^2 \tilde{\Phi}\}$ is the standard Reynolds force due to the polarization drift nonlinearity. To estimate the driving forces in the right-hand side of (3.10) and (3.11)

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we consider the excitation of zonal flows with characteristic wave-vector $\mathbf{q}(q_x, q_y) = \mathbf{q}(q_x, 0)$. Then assuming the triad interactions $\mathbf{k} + \mathbf{k}' + \mathbf{q} \approx 0$ in the $\mathbf{k}$-space and averaging (3.11) we obtain

$$\frac{\partial \overline{T}_{\mathrm{i}q_x}}{\partial t} - \frac{2}{3}\frac{\partial \overline{n}_{q_x}}{\partial t} = 0, \tag{3.12}$$

under the approximation $T_{q-k} \approx T_{-k}$, which is well justified for the considered scale separation limit $q \ll k - q \approx k$. Averaging the continuity equation for the zonal flows it turns out that in the leading order the density variation associated with the zonal flow does not evolve in time, i.e. $\partial \overline{n}_{q_x}/\partial t = 0$. Thus, in what follows it will be considered that both terms in the left-hand side of (3.12) are equal to zero. Averaging now the Raynold stress forces in (3.10) we obtain

$$\frac{\partial \overline{\Phi}_{q_x}}{\partial t} = \int k_x k_y |\tilde{\Phi}_k|^2 \, d^2k, \tag{3.13}$$

where it can be seen that the zonal flow is generated locally by the TITG turbulence through the turbulent Raynold stress. Equation (3.13) together with (3.4) and (3.5) constitutes a self-consistent model of TITG zonal flow turbulence, which rigorously accounts for the FLR effects induced by the polarization drift.

3.2.1. Stability and oscillation frequency of ZF. Using (3.5) and (3.6), we get a relation from (3.13) which connects the zonal flow velocity with the spectra of the short-scale fluctuations,

$$\frac{\partial V_{0y}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial x}\int k_x k_y \zeta(k_\perp) N_k \, d^2k. \tag{3.14}$$

Here $\zeta(k_\perp)$ is defined by

$$\zeta(k_\perp) = \frac{(\tau\epsilon_n)^2}{|f(k_\perp^2)|}. \tag{3.15}$$

Closure conditions for the N_k modulations in terms of the mean flow are provided by the wave-kinetic equation (3.4). Inserting (3.9) into (3.14) and assuming the large-scale variations to be of the form $V_{0y} \sim \exp(iq_x x)$, we obtain

$$\frac{\partial V_{0y}}{\partial t} = q_x^2 D_{xx} V_{0y},$$

where D_{xx} is given by

$$D_{xx} = -\frac{1}{2}\int k_x k_y^2 \frac{\partial N_k}{\partial k_x}\zeta(k_\perp) R(\Omega, q_x, \delta\omega_k) \, d^2k \tag{3.16}$$

and determines the stability of the zonal flow since $\gamma_{zf} = q_x^2 D_{xx}$. As in the drift wave turbulence, where it is usually given by $k_x(\partial N_k^0/\partial k_x) < 0$, the zonal flow may become unstable according to the integral (3.16). This instability is interpreted as a result of the resonant interaction between zonal flow and the small-scale modulations of the turbulence.

3.2.2. Oscillation frequency of zonal flow. For low-frequency perturbations, $\Omega \ll q_x V_{gx}$, we can take into account the non-resonant response $\tilde{N}_k^{(1)}$ of the turbulent spectra over the perturbations of the induced zonal flow. The solution of the

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linearized wave-kinetic equation (3.4) takes the form

$$\tilde{N}_k^{(1)} = k_y V_{0y} \left(\frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial N_k^0}{\partial k_x}.$$

Substituting the later expression into (3.14), we obtain the oscillation frequency of the zonal flow, $\Omega_{\rm zf} \approx -u_x q_x$, where u_x is defined by

$$u_x = \frac{1}{2} \int k_x k_y^2 \left(\frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial N_k^0}{\partial k_x} \zeta(k_\perp)\, d^2k.$$

3.2.3. Nonlinear evolution of zonal flow. However, as the amplitude of the zonal flow grows, nonlinear effects become significant. Using the derived expression of $\tilde{N}_k^{(1)}$, we determine iteratively from (3.4), the next order nonlinear response $\tilde{N}_k^{(2)}$ for the non-resonant interactions,

$$\tilde{N}_k^{(2)} = \frac{1}{2}(k_y V_{0y})^2 \left(\frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial}{\partial k_x} \left[\left(\frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial N_k^0}{\partial k_x} \right].$$

Including the total response $\tilde{N}_k = \tilde{N}_k^{\rm res} + \tilde{N}_k^{(1)} + \tilde{N}_k^{(2)}$ into (3.14), we obtain, similar to the case of the drift wave-zonal flow turbulence in [7], a nonlinear equation that describes the evolution of the zonal flow:

$$u_x \frac{\partial^2}{\partial x^2} V_{0y} + b_x \frac{\partial^2}{\partial x^2} V_{0y}^2 - D_{xx} \frac{\partial^3}{\partial x^3} V_{0y} = \frac{\partial}{\partial t} \frac{\partial}{\partial x} V_{0y}. \tag{3.17}$$

The coefficient b_x in the nonlinear term is given by

$$b_x = \frac{1}{4} \int k_x k_y^3 \zeta(k_\perp) \left(\frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial}{\partial k_x} \left[\left(\frac{\partial \omega_k}{\partial k_x} \right)^{-1} \frac{\partial N_k^0}{\partial k_x} \right] d^2k.$$

Equation (3.17) admits localized solutions. For the family of stationary solutions of the type $V_{0y}(x - u_{0x}t)$, the simplest solution can be found by considering the boundary conditions $V_{0y} \to V_{1y}$, $V_{0y}' = 0$ for $x \to -\infty$, and $V_{0y} \to V_{2y}$, $V_{0y}' = 0$, for $x \to \infty$. A characteristic solution of (3.17) is given by

$$V_{0y} = \tfrac{1}{2}\{V_{1y} + V_{2y} + (V_{1y} - V_{2y}) \tanh[x\, b(V_{1y} - V_{2y})/2D_{xx}]\}$$

and is known as a 'switching' wave or 'kink' soliton. It describes the transient region between two different values of the flow, in contrast to the solution with same asymptotic value which is called the 'bell' soliton [7].

The cooperative effects of the wave motion, steeping and instability, give rise to the possibility of the formation of stationary or moving kink solitons. The values of the parameters that determine the characteristic lengths of these structures are determined by the value of the group velocity and by the spectral fluctuations. The simple analysis given here demonstrates the self-organization properties of the TITG mode zonal flow coupled system.

4. Conclusions

The properties of zonal flows were investigated and determined by using a kinetic-wave equation coupled with averaged fluid equations that describe TITG turbulence. The resonant interaction between the variations of the mean flow and the

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turbulent spectra may lead to zonal flow destabilization. The nonlinear evolution of the large flows can lead to the formation of stationary coherent structures in the transition layer between surfaces of different flow velocities, significantly modifying the transport properties of the turbulent plasma.

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