J. Plasma Physics (1998), *vol.* 60, *part* 2, *pp.* 265–273. Printed in the United Kingdom

© 1998 Cambridge University Press

Dust acoustic solitons in a magnetized dusty plasma

H. K. MALIK, K. D. TRIPATHI and S. K. SHARMA
Department of Physics, Indian Institute of Technology, New Delhi 110016, India

(Received 21 July 1997 and in revised form 30 December 1997)

Propagation characteristics of dust acoustic solitons are studied in a magnetized dusty plasma consisting of ions, electrons and negatively charged dust grains. It is found that rarefactive solitons (solitons associated with negative potentials) are possible in such a plasma. The effects on the soliton characteristics of magnetic field strength, wave propagation angle, and densities and temperatures of the plasma particles are analysed.

1. Introduction

There has been a great deal of interest in dusty plasmas because of their importance in space environments such as asteroid zones, cometary tails, planetary rings and magnetospheres, and the lower part of the Earth's ionosphere (Whipple *et al.* 1985; de Angelis *et al.* 1988; Goertz 1989; Kurth 1991; Gendrin 1991), as well as in laboratory devices (Sheehan *et al.* 1990; Garscadden 1991; Carlile *et al.* 1991).

Waves in the dusty plasmas have been extensively studied theoretically by several researchers (de Angelis *et al.* 1988; Rao *et al.* 1990; D'Angelo 1990; Melandsø *et al.* 1993; Chow and Rosenberg, 1995; Mamun *et al.* 1996; Das *et al.* 1996). Rao *et al.* (1990) showed that novel dust acoustic waves can propagate both linearly and nonlinearly in dusty plasmas consisting of electrons and ions with Boltzmann distributions and negatively charged heavy dust grains. Their theoretical investigation also showed that the nonlinear dust acoustic waves can propagate as solitons of either positive or negative potential. Dust acoustic waves have also been examined experimentally by Barkan *et al.* (1996) in a dusty plasma with significant depletion of electrons by the dust.

Usually the dust grains in a plasma are negatively charged owing to plasma currents, ultraviolet irradiation, field emission and electron attachment, etc. However, in the presence of disturbances, the charge on the dust-grain surface becomes a time-dependent quantity and starts to fluctuate. Considering these fluctuations of the dust charge, Ma and Liu (1997) have studied the propagation of dust acoustic solitons in a dusty plasma with cold dust grains. Their analysis is for an unmagnetized dusty plasma. However, it will be interesting to see how the dust acoustic soliton characteristics are modified in a magnetized plasma with a finite temperature of dust grains. Therefore, in the present paper, we study dust acoustic solitons in a magnetized dusty plasma consisting of finite-temperature dust grains, ions and electrons. For the sake of simplicity, we assume that the dust grains have a constant negative charge. Considering the linear analysis, we present the dispersion relation for the dust acoustic waves

https://doi.org/10.1017/S0022377898006977 Published online by Cambridge University Press

with the effect of the magnetic field. Furthermore, a nonlinear treatment predicts that only rarefactive solitons (solitons associated with negative potentials) are possible in such a plasma.

2. Basic equations and dispersion relation

As we have mentioned, we consider a magnetized dusty plasma consisting of finite-temperature ions, electrons, and extremely heavy dust grains of density n_d, uniform mass m_d, negative charge $-Ze$ and temperature T_d. The ions, of density n_i and temperature T_i, and electrons, of density n_e and temperature T_e, are assumed to follow Boltzmann distributions. However, the dynamics of the dust fluid is governed by the continuity and momentum equations. Under these conditions, we write the following basic fluid equations for a magnetized dusty plasma:

$$\frac{\partial n_d}{\partial t}+\nabla\cdot(n_d\,\mathbf{v})=0, \tag{1}$$

$$m_d\,n_d\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\,\mathbf{v}\right]-Zen_d\,\nabla\phi+Zen_d(\mathbf{v}\times\mathbf{B})+\gamma T_d\,\nabla n_d=0. \tag{2}$$

$$n_i=n_{i0}\exp\left(-e\frac{\phi}{T_i}\right), \tag{3}$$

$$n_e=n_{e0}\exp\left(e\frac{\phi}{T_e}\right), \tag{4}$$

$$\epsilon_0\nabla^2\phi-en_e-Zen_d+en_i=0, \tag{5}$$

where $\mathbf{v}$ is the dust-fluid velocity and ϕ is the potential. Equations (1) and (2) are respectively the continuity and momentum equations of the dust fluid. It can be seen from (2) that the last two terms appearing owing to the magnetic field effect and the pressure of the dust particles with specific heat ratio γ ($\gamma=3$ for the one-dimensional case).

We carry out the usual normal-mode analysis of (1)–(5) to obtain the dispersion relation for linear low-frequency waves. We consider the magnetic field to be along the z direction, i.e. $\mathbf{B}=B\hat{\mathbf{z}}$, and the wave propagation to be almost perpendicular to the direction of the magnetic field, so that the electrons and ions can preserve charge neutrality. Furthermore, all the perturbations are assumed to be of the form $\exp[i(kx-\omega t)]$. Under these conditions, and with $n_{i0}=n_{e0}+Zn_{d0}$ for quasineutrality in equilibrium, we find the following linear dispersion relation:

$$\omega^2=\beta^2k^2C_s^2\left(1+\frac{k^2\lambda_{De}^2}{1+\eta\delta}\right)^{-1}+\frac{k^2\gamma T_d}{m_d}+\Omega_d^2, \tag{6}$$

where

$$\beta^2=\frac{Z(\delta-1)}{1+\eta\delta},\qquad C_s=\left(\frac{T_e}{m_d}\right)^{1/2},$$

$$\eta=\frac{T_e}{T_i},\qquad \delta=\frac{n_{i0}}{n_{e0}},\qquad \lambda_{De}=\left(\frac{\epsilon_0\,T_e}{n_{e0}\,e^2}\right)^{1/2}.$$

https://doi.org/10.1017/S0022377898006977 Published online by Cambridge University Press

Ω_d is the dust cyclotron frequency. The specific-heat ratio γ can be taken as 3 for one-dimensional motion.

It can be seen from (6) that ω^2 is always positive, since $\delta > 1$ for the present plasma system. Also, for an unmagnetized plasma with cold dust, this relation reduces to that obtained by Rao *et al.* (1990) to show the acoustic nature of the waves in a dusty plasma. Furthermore (6) shows that the dust particles undergo an acoustic-type oscillation with weak dispersive effect for finite wavelengths, but the Lorentz force constitutes a new restoring force giving rise to the term Ω_d^2; the second term is due to the thermal effect of the dust particles.

In the next section, we consider the weakly nonlinear and dispersive effects to study the propagation of dust acoustic waves in the present plasma system, using the Korteweg–de Vries (KdV) model.

3. Korteweg–de Vries (KdV) model

To derive the Korteweg–de Vries (KdV) equation, we use a normalized form of (1)–(5), with the same normalization as used by Lakshmi and Bharuthram (1994) with the effective temperature T_{eff} given by

$$\frac{Z}{T_{\text{eff}}} = \frac{n_{e0}}{T_e} + \frac{n_{i0}}{T_i}.$$

Here, n_{e0} and n_{i0} are the equilibrium densities of electrons and ions respectively, normalized by the effective density Zn_{d0}, with n_{d0} the equilibrium dust density. The velocity $\mathbf{v}$ is normalized by the dust acoustic speed $(T_{\text{eff}}/m_d)^{1/2}$, time by the inverse dust plasma frequency $\omega_{pd}^{-1} = (\epsilon_0 m_d/n_{d0} Z^2 e^2)^{1/2}$, space by the dust Debye length $\lambda_{Dd} = (\epsilon_0 T_{\text{eff}}/n_{d0} Z^2 e^2)^{1/2}$, and the potential ϕ by T_{eff}/Ze. The wave propagation is considered to be in the (x, z) plane at an angle θ to the direction of the magnetic field. Since the number of degrees of freedom for the present case is 2, we take the specific heat ratio γ as 2. Therefore the following are the normalized basic equations:

$$\frac{\partial n_{d'}}{\partial t} + \frac{\partial}{\partial x}(n_{d'} v_x) + \frac{\partial}{\partial z}(n_{d'} v_z) = 0, \tag{7a}$$

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_z \frac{\partial v_x}{\partial z} - \frac{\partial \phi}{\partial x} + a v_y + \frac{2\sigma}{n_{d'}} \frac{\partial n_{d'}}{\partial x} = 0, \tag{7b}$$

$$\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_z \frac{\partial v_y}{\partial z} - a v_x = 0, \tag{7c}$$

$$\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_z \frac{\partial v_z}{\partial z} - \frac{\partial \phi}{\partial z} + \frac{2\sigma}{n_{d'}} \frac{\partial n_{d'}}{\partial z} = 0, \tag{7d}$$

$$n_i = n_{i0} \exp(-\alpha_i \phi), \tag{7e}$$

$$n_e = n_{e0} \exp(\alpha_e \phi), \tag{7f}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} - n_e - n_{d'} + n_i = 0. \tag{7g}$$

https://doi.org/10.1017/S0022377898006977 Published online by Cambridge University Press

In these equations, a is the ratio of the dust cyclotron frequency to the dust plasma frequency, $n_{d'} = Zn_d$, $\sigma = T_d/T_{\text{eff}}$ and $\alpha_j = T_{\text{eff}}/ZT_j$, with $j = e, i$.

To study the propagation of the dust acoustic wave, the basic equations (7) are solved for stationary nonlinear solutions using the reductive perturbation technique with ξ as the spacelike coordinate and τ as the timelike coordinate. These stretched coordinates and the expansions of the quantities $n_{d'}$, n_e, n_i, ϕ, v_x, v_y and v_z are taken to be the same as used by Mishra *et al.* (1994) for a magnetized plasma consisting of negative ions, electrons and positive ions. The use of these coordinates and expansion of the dependent quantities in (7) yields the following relation for the phase velocity S of the wave in (ξ, τ) space:

$$S = \left[1 + 2\left(\frac{n_{e0}\,T_d}{ZT_e} + \frac{n_{i0}\,T_d}{ZT_i}\right)\right]^{1/2} \cos\theta. \tag{8}$$

It is evident from (8) that the phase velocity of the dust acoustic wave depends on the densities and temperatures of the plasma particles, the number of electrons Z residing on the dust grains, and the wave propagation angle θ. It is clear from this equation that the phase velocity of the wave decreases when more electrons are depleted by the dust grains. However, it increases when the positive-ion density n_{i0} (or electron density n_{e0}) is increased. It can also be seen that the phase velocity has weak and opposite dependences on the dust and ion (or electron) temperatures. It increases for higher values of the dust temperature but decreases with T_i (or T_e). It is strongly affected by the wave propagation angle θ, and decreases for larger values of θ.

The reductive perturbation treatment of (7) gives various equations in first- and second-order quantities of the densities $n_{d'}$, n_e, n_i, velocities v_x, v_y, v_z, and potential ϕ. Elimination of the second-order quantities $n_{d2'}$, n_{e2}, n_{i2}, v_{x2}, v_{y2}, v_{z2} and ϕ_2 from these equations with the help of the phase-velocity relation (8) gives the following KdV equation in ϕ_1:

$$\frac{\partial\phi_1}{\partial\tau} + \alpha\phi_1\frac{\partial\phi_1}{\partial\xi} + \beta\frac{\partial^3\phi_1}{\partial\xi^3} = 0. \tag{9}$$

The coefficients of nonlinearity (α) and dispersion (β) appearing in the KdV equation are given by

$$\alpha = \frac{\cos^2\theta}{2S}\left[\frac{n_{i0}}{(n_{i0} + n_{e0}\,T_i/T_e)^2} - \frac{n_{e0}}{(n_{e0} + n_{i0}\,T_e/T_i)^2} - 1\right] - S, \tag{10a}$$

$$\beta = \frac{\cos^2\theta}{2S} + \frac{S^3\tan^2\theta}{2a^2}. \tag{10b}$$

To solve the KdV equation, we convert it into a single-variable differential equation in ν by using the transformation $\nu = \xi - V\tau$, where V is a constant velocity. We now integrate this equation using the boundary conditions that ϕ_1 and its all derivatives with respect to ν vanish when $|\nu| \to \pm\infty$. This integration finally yields the following soliton solution:

$$\phi_1(\nu) = \phi_m\,\text{sech}^2\left(\frac{\nu}{W}\right), \tag{11}$$

https://doi.org/10.1017/S0022377898006977 Published online by Cambridge University Press

where ϕ_m and W are the peak soliton amplitude and the soliton width respectively, given by

$$\phi_m = \frac{3V}{\alpha}, \tag{12}$$

$$W = 2\left(\frac{\beta}{V}\right)^{1/2}. \tag{13}$$

For the soliton solution (11), the energy E of the soliton, in terms of its amplitude and width, is found to be (Malik 1996)

$$E = \tfrac{4}{3}\phi_m^2 W. \tag{14}$$

4. Results and discussion

In the present analysis, we have obtained expressions for the phase velocity, peak amplitude ϕ_m, width W and energy E of the dust acoustic soliton. On the basis of these expressions, we discuss the propagation characteristics of the soliton. One very important point can be noted from (10a), which gives the expression for the nonlinearity coefficient α: α is always negative for the present system of a dusty plasma with constant negatively charged dust grains. Das and Tagare (1975) and Nakamura and Tsukabayashi (1984) showed for the negative-ion-containing plasmas that rarefactive solitons propagate in the plasma when the nonlinearity coefficient α is negative. Since α is always negative in our system, we also conclude that only rarefactive solitons (solitons associated with negative potential, i.e $\phi_m < 0$) can propagate in a magnetized dusty plasma having finite-temperature ions, electrons and dust grains of uniform mass and the same negative charge.

On the basis of (12)–(14), we plot some graphs to see the behaviour of the dust acoustic soliton in a magnetized dusty plasma. We give typical values to various parameters appearing in the expression for the amplitude, width and energy of the soliton. The values of Z ($\approx 10^4$) and m_d/m_i ($\approx 10^{12}$) have been inferred from observations as nominal parameters for Saturn's E ring or G ring (Goertz 1989; Rosenberg 1993).

Figure 1 shows the opposite behaviours of the peak soliton amplitude $|\phi_m|$ and soliton width W with electron number density n_{e0}. Here $\theta = 30°$, $a = 0.02$, $V = 0.2$, $Z = 2 \times 10^4$, $T_d/T_e = 0.001$, $T_d/T_i = 0.01$ and $T_i/T_e = 0.1$. It is clear from the figure that the soliton amplitude decreases but the width increases for higher electron number density n_{e0}. However, the soliton width varies slowly with n_{e0} as compared with the amplitude. This can be understood from (12) and (13), which show that the amplitude is inversely proportional to the nonlinearity coefficient α and the width is directly proportional to the square root of the dispersion coefficient β. Since β depends weakly on n_{e0} as compared with α (see (10a, b)), the width varies slowly with n_{e0} in comparison with the amplitude. Furthermore, we note that to satisfy the quasineutrality condition ($n_{i0} = n_{e0} + 1$, in normalized form), the positive-ion density n_{i0} also increases with increasing n_{e0}. Therefore the soliton amplitude and width should behave in the same manner with n_{i0} and n_{e0}. However, the effect of the number of electrons Z residing on the dust grains is found to be opposite to that of n_{e0} (or n_{i0}).

https://doi.org/10.1017/S0022377898006977 Published online by Cambridge University Press

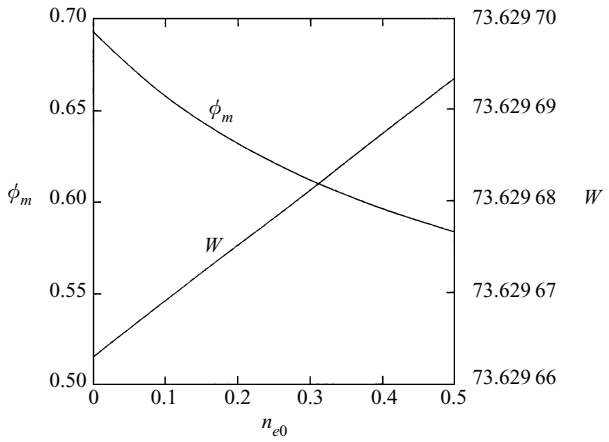


Figure 1. Variation of the peak soliton amplitude $|\phi_m|$ and the soliton width W with electron number density n_{e0}. Here $\theta = 30°$, $a = 0.02$, $V = 0.2$, $Z = 2 \times 10^4$, $T_d/T_e = 0.001$, $T_d/T_i = 0.01$ and $T_i/T_e = 0.1$.

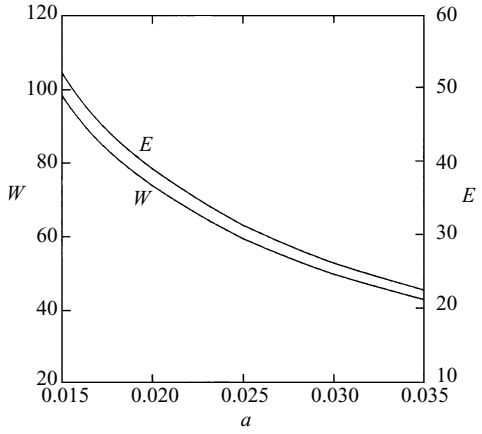


Figure 2. Dependence of the soliton width W and the soliton energy E on the ratio a of the dust cyclotron frequency to the dust plasma frequency, for $n_{e0} = 0.2$ and for the other parameters the same as in Fig. 1.

The effect of the magnetic field strength on the soliton width W and energy E is exhibited in Fig. 2, which shows the variation of W and E with the ratio a of the dust cyclotron frequency to the dust plasma frequency for $n_{e0} = 0.2$ and for the other parameters the same as in Fig. 1. It is clear from the figure that the soliton width and energy decrease with increasing a. This also conveys the meaning that, under a stronger magnetic field effect, the soliton propagates with smaller width and lower energy in a dusty plasma. Furthermore, it can be seen from (10a) that α is independent of a, and hence the soliton amplitude is not affected by the magnetic field strength (see (12)).

The dependence of the peak soliton amplitude $|\phi_m|$ and energy E on the wave propagation angle θ is shown in Fig. 3, for $n_{e0} = 0.2$ and for the other parameters the same as in Fig. 1. This figure shows that the soliton amplitude and energy increase with increasing wave propagation angle. This means that

https://doi.org/10.1017/S0022377898006977 Published online by Cambridge University Press

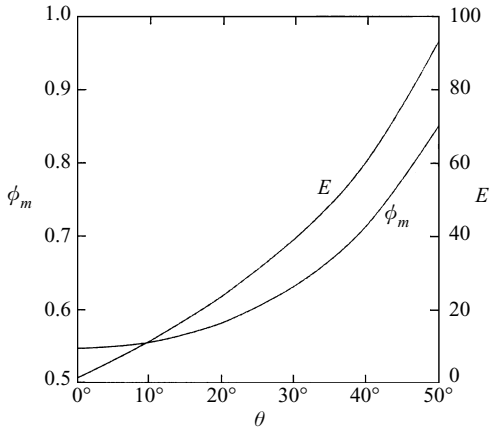


Figure 3. Dependence of the peak soliton amplitude $|\phi_m|$ and the soliton energy E on wave propagation angle θ, for $n_{e0} = 0.2$ and for the other parameters the same as in Fig. 1.

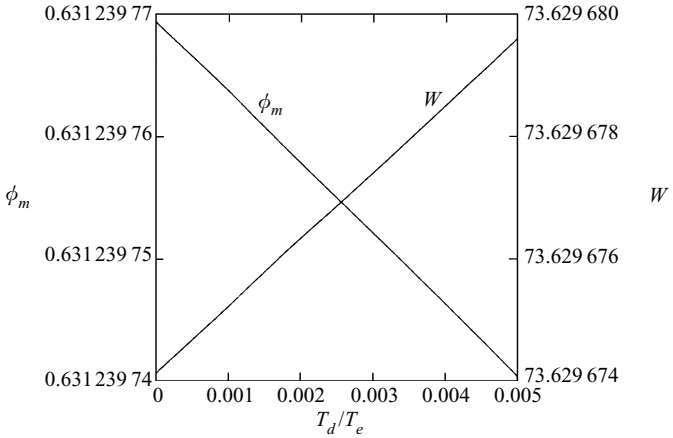


Figure 4. Weak dependence of the peak soliton amplitude $|\phi_m|$ and the soliton width W on the dust-to-electron temperature ratio T_d/T_e, for $n_{e0} = 0.2$ and for the other parameters the same as in Fig. 1.

the wave propagating at larger angles to the direction of the magnetic field corresponds to a higher-amplitude and more-energetic soliton in a magnetized dusty plasma.

The opposite behaviours of the soliton amplitude $|\phi_m|$ and soliton width W with the dust-to-electron temperature ratio T_d/T_e are shown in Fig. 4, for $n_{e0} = 0.2$ and for the other parameters the same as in Fig. 1. It can be seen from this figure that the soliton amplitude decreases but the width increases for higher values of T_d/T_e; however, they have very weak dependences on the dust temperature. The effects of the ion-to-electron temperature ratio T_i/T_e on the soliton amplitude and energy are shown in Fig. 5. It is clear from this figure that the soliton amplitude and energy decrease with increasing T_i/T_e. This figure also shows that these soliton characteristics are strongly dependent on the ion temperature in a magnetized dusty plasma.

https://doi.org/10.1017/S0022377898006977 Published online by Cambridge University Press

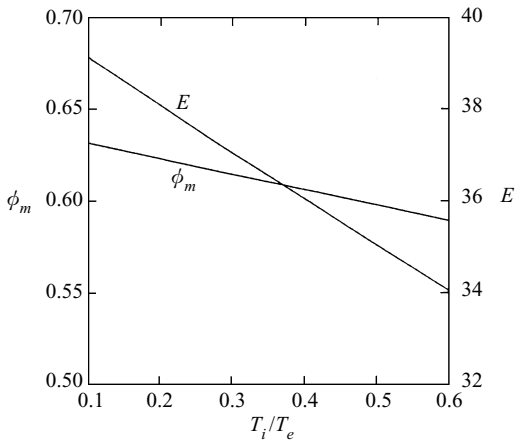


Figure 5. Variation of the peak soliton amplitude $|\phi_m|$ and the soliton energy E with the ion-to-electron temperature ratio T_i/T_e, for $n_{e0} = 0.2$ and for the other parameters the same as in Fig. 1.

5. Conclusions

We have derived a linear dispersion relation for dust acoustic waves and a KdV equation for dust acoustic solitons in a magnetized dusty plasma consisting of finite-temperature ions, electrons, and dust grains of uniform mass and the same negative charge. It is seen that solitons associated with negative potentials (rarefactive solitons) can propagate in this plasma system. Expressions for the soliton phase velocity, peak soliton amplitude, soliton width and soliton energy have been obtained in order to study soliton behaviour in a dusty plasma.

It is found that a soliton with smaller width and lower energy propagates in a dusty plasma under the effect of a stronger magnetic field. For larger wave propagation angles, the amplitude and energy of the soliton go to higher values. The dust-to-electron (or ion) temperature ratio has weak effects on the soliton characteristics. For higher dust temperature, the soliton propagates with smaller amplitude, greater width and lower energy. The soliton behaviours with positive-ion (or electron) density and the number of electrons residing on the dust grains are found to be opposite in nature. The soliton amplitude and energy decrease, but the width increases, with increasing density of ions (or electrons).

Acknowledgement

The financial support for this work provided by the Department of Science and Technology (DST), Delhi is gratefully acknowledged.

References

Barkan, A., D'Angelo, N. and Merlino, R. L. 1996 *Planet. Space Sci.* **44**, 239.
Carlile, R. N., Geha, S., O'Hanlon, J. F. and Stewart, J. C. 1991 *Appl. Phys. Lett.* **59**, 1167.
Chow, V. W. and Rosenberg, H. 1995 *Planet. Space Sci.* **43**, 613.

https://doi.org/10.1017/S0022377898006977 Published online by Cambridge University Press

D'Angelo, N. 1990 *Planet. Space. Sci.* **38**, 1143.

Das, A. C., Misra, A. K. and Goswami, K. S. 1996 *Phys. Plasmas* **3**, 457.

Das, G. C. and Tagare, S. G. 1975 *Plasma Phys.* **17**, 1025.

de Angelis, U., Formisano, V. and Giordano, M. 1988 *J. Plasma Phys.* **40**, 399.

Garscadden, A. 1991 Particles in Plasma. Invited Lecture, 20th International Conference on Phenomena in Ionized Gases, Pisa, Italy.

Gendrin, R. 1991 Ionized Environments of Planets and Stars: Actual Problems. Invited Lecture, 20th International Conference on Phenomena in Ionized Gases, Pisa, Italy.

Goertz, C. K. 1989 *Rev. Geophys.* **27**, 271.

Kurth, W. S. 1991 *Rev. Geophys. Suppl.* 1075.

Lakhsmi, S. V. and Bharuthram, R. 1994 *Planet. Space. Sci.* **42**, 875.

Ma, J. X. and Liu, J. 1997 *Phys. Plasmas* **4**, 253.

Malik, H. K. 1996 *Phys. Rev.* **E54**, 5844.

Mamun, A. A., Cairns, R. A. and Shukla, P. K. 1996 *Phys. Plasmas* **3**, 702.

Melandsø, F., Aslaksen, T. and Havnes, O. 1993 *Planet. Space Sci.* **41**, 321.

Mishra, M. K., Chhabra, R. S. and Sharma, S. R. 1994 *J. Plasma Phys.* **52**, 409.

Nakamura, Y. and Tsukabayashi, I. 1984 *Phys. Rev. Lett.* **52**, 2356.

Rao, N. N., Shukla, P. K. and Yu, M. Y. 1990 *Planet. Space Sci.* **38**, 543.

Rosenberg, M. 1993 *Planet. Space Sci.* **41**, 229.

Sheehan, D. P., Carilo, M. and Heidbrink, W. 1990 *Rev. Sci. Instrum.* **61**, 3871.

Whipple, E. C., Northrop, T. G. and Mendis, D. A. 1985 *J. Geophys. Res.* **90**, 7405.

https://doi.org/10.1017/S0022377898006977 Published online by Cambridge University Press