Appendix

A.1 Tephigram

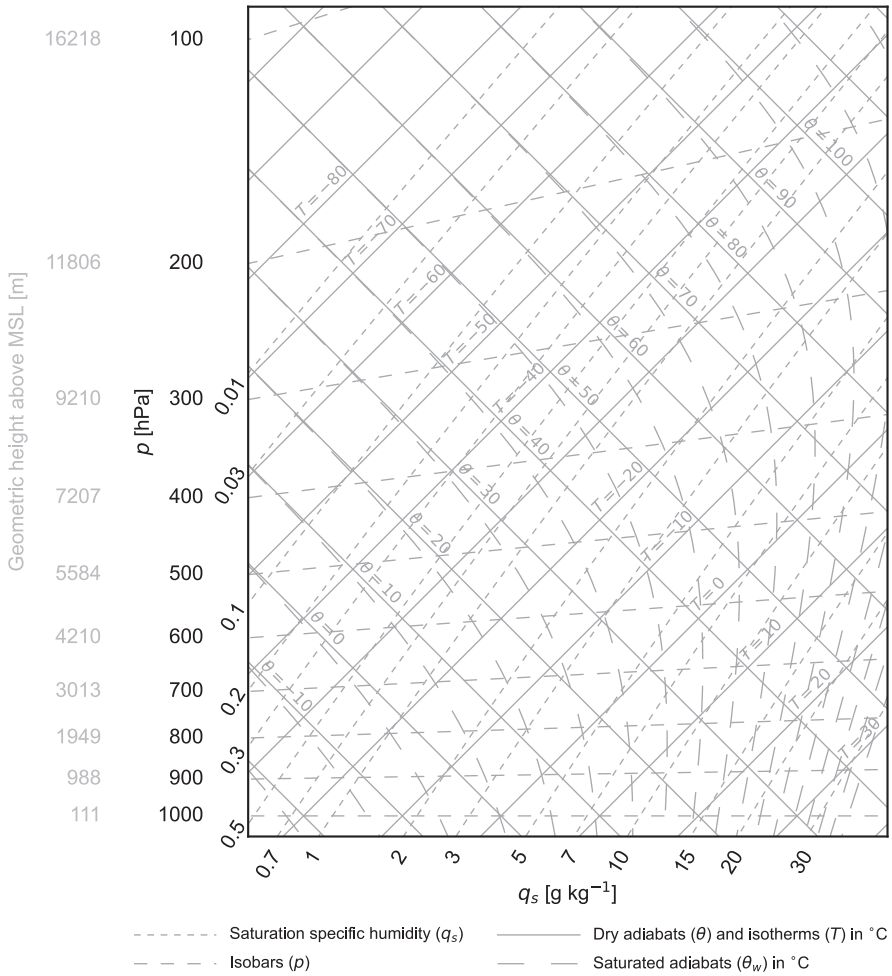


Figure A.1 An electronic version of this tephigram can be found at: www.cambridge.org/lohmann_mahrt. The source code to plot tephigrams can be obtained from: https://zenodo.org/records/18405425.

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A.2 Psychrometric Chart

The calculation of RH_w, specific humidity q_v, or T_d from measurements of the environmental (dry-bulb) T and isobaric wet bulb temperature T_{iw} is an iterative process because both T and q_v are changing until the equilibrium T_{iw} is found. Therefore, often charts like the one shown in Figure A.2 are created for meteorologists to infer q_v, RH_w, or T_d from T_w or vice versa.

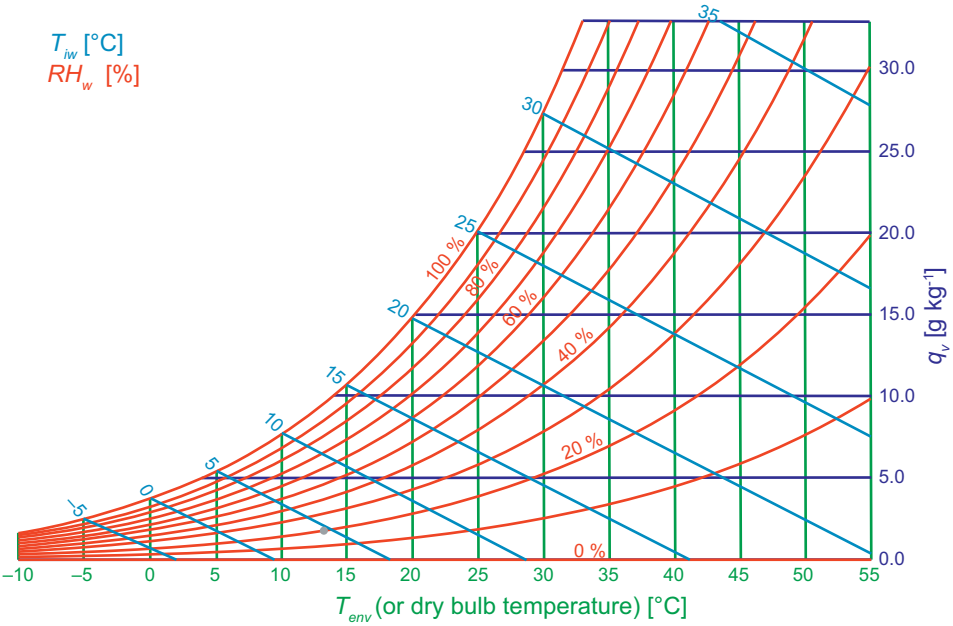


Figure A.2 Example of a psychrometric chart at sea level pressure of 1013.25 hPa. The dark green vertical lines refer to the environmental (dry-bulb) temperature, the blue lines to the isobaric wet bulb temperature T_{iw}, the purple horizontal lines to the specific humidity q_v, and the red curved lines to the relative humidity with respect to water (RH_w). For the example in Figure 3.17 with the temperatures at 1000 hPa of $\theta = T = 13.5\,°C$ and $\theta_w = T_{iw} = 4.8\,°C$, we obtain $RH_w = 20\,\%$, $q_v = 2\,g\,kg^{-1}$, as illustrated by the gray circle.

A.3 Parameterizations of Thermodynamic Properties as a Function of Temperature and Pressure

For a detailed description of the thermodynamic properties of water, see the website of the IAPWS[1] (Wagner and Pruss, 2002). Relevant data bases, where other thermodynamic data/properties can be looked up at: https://webbook.nist.gov/chemistry/.

[1] www.iapws.org

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A.3.1 Density of Liquid Water as a Function of Temperature

A parameterization of the liquid water density as a function of temperature in K at standard pressure ($p_0 = 1000\,\text{hPa}$) is given by Marcolli (2017):

$$\begin{aligned}\rho_l(T, p_0) = {} & 1864.3535 - 72.5821489 \cdot T + 2.5194368 \cdot T^2 \\ & - 4.9000203 \cdot 10^{-2} \cdot T^3 + 5.860253 \cdot 10^{-4} \cdot T^4 - 4.5055151 \cdot 10^{-6} \cdot T^5 \\ & + 2.2616353 \cdot 10^{-8} \cdot T^6 - 7.3484974 \cdot 10^{-11} \cdot T^7 + 1.4862784 \cdot 10^{-13} \cdot T^8 \\ & - 1.6984748 \cdot 10^{-16} \cdot T^9 + 8.3699379 \cdot 10^{-20} \cdot T^{10}.\end{aligned} \tag{A.1}$$

Here, ρ_l is in units of $\text{kg}\,\text{m}^{-3}$, and the validity range is from 50 K to 393 K. This expression can be used in a parameterization of the liquid water density that also considers changes in the environmental pressure as given by Marcolli (2020):

$$\rho_l(T, p) = \rho_l(T, p_0) + \kappa(T) \cdot p + \frac{\partial \kappa(T)}{\partial p} \cdot p^2, \tag{A.2}$$

valid in the range from −110 MPa to 399 MPa and 203.15 K to 333.15 K. Here, κ (in units of MPa^{-1}) describes the compressibility, given as:

$$\kappa(T) = 0.487 - 0.004368 \cdot (T - 273.15) + 0.00007235 \cdot (T - 273.15)^2, \tag{A.3}$$

and

$$\frac{\partial \kappa(T)}{\partial p} = -0.0003805 + 6.639 \cdot 10^{-6} \cdot (T - 273.15) - 9.688 \cdot 10^{-8} \cdot (T - 273.15)^2, \tag{A.4}$$

in units of MPa^{-2}.

A.3.2 Density of Ice as a Function of Temperature

A parameterization of ρ_i as a function of temperature in °C for the temperature range −180 °C to 0 °C is given by Pruppacher and Klett (1997):

$$\rho_i(T) = 916.7 - 1.75 \cdot 10^{-4} \cdot T - 5 \cdot 10^{-4} \cdot T^2. \tag{A.5}$$

A.3.3 Specific Heat of Dry Air as a Function of Temperature

A parameterization of the specific heat of dry air in $\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$ at standard pressure (101 300 Pa) and T in K is given by Tsilingiris (2008):

$$\begin{aligned}c_p(T) = {} & 1034.09 - 0.284887 \cdot T + 0.7816818 \cdot 10^{-3} \cdot T^2 \\ & - 0.4970786 \cdot 10^{-6} \cdot T^3 + 0.1077024 \cdot 10^{-9} \cdot T^4.\end{aligned} \tag{A.6}$$

This expression is valid between 250.15 K and 1050.15 K.

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A.3.4 Specific Heat of Water Vapor as a Function of Temperature

A parameterization of the specific heat of water vapor in $\mathrm{J\,kg^{-1}\,K^{-1}}$ at standard pressure (101 300 Pa) and T in K is given by Tsilingiris (2008):

$$c_{pv}(T) = 1869.10989 - 0.2578421578 \cdot (T - T_0) + 0.01941058941 \cdot (T - T_0)^2, \tag{A.7}$$

where $T_0 = 273.15\,\mathrm{K}$. This expression is valid between 273 K and 393.15 K.

A.3.5 Specific Heat of Liquid Water as a Function of Temperature

A parameterization of the specific heat of pure water c_l in $\mathrm{J\,kg^{-1}\,K^{-1}}$ at standard pressure (10^5 Pa) and T in °C is given by Millero et al., (1973):

$$\begin{aligned} c_l(T) = {} & 4217.4 - 3.72083 \cdot T + 0.1412855 \cdot T^2 \\ & - 2.654387 \cdot 10^{-3} \cdot T^3 + 2.093236 \cdot 10^{-5} \cdot T^4. \end{aligned} \tag{A.8}$$

This expression is valid between 0 °C and 40 °C.

A.3.6 Specific Heat of Ice as a Function of Temperature

A parameterization of the specific heat of ice c_i in $\mathrm{J\,kg^{-1}\,K^{-1}}$ at standard pressure (10^5 Pa) and T in K is given by Murphy and Koop (2005):

$$c_i(T) = \frac{-2.0572 + 0.14644 \cdot T + 0.06163 \cdot T \exp\left[-(\frac{T}{125.1})^2\right]}{M_w} \tag{A.9}$$

where M_w is the molecular weight of water. This expression is valid for $T > 20$ K.

A.3.7 Dynamic Viscosity of Air as a Function of Temperature

A parameterization of the dynamic viscosity of air μ in $\mathrm{kg\,m^{-1}\,s^{-1}}$ is given in Dixon (2007):

$$\mu(T) = \frac{1.458 \cdot 10^{-6} \cdot T^{1.5}}{T + 110.4}. \tag{A.10}$$

This expression is valid between 100 K and 800 K.

A.3.8 Interfacial Tensions as a Function of Temperature

A temperature dependence of the surface tension of ice in water $\sigma_{i,w}$ in $\mathrm{mN\ m^{-1}}$ is given by Zobrist et al. (2007), based on measurements as:

$$\sigma_{i,w} = 0.01 \cdot \left[3.298 + 1.2048\frac{T - T_0}{T_0} - 46.705\left(\frac{T - T_0}{T_0}\right)^2\right]. \tag{A.11}$$

It is valid in the temperature range 229 K to 238 K, but can be extrapolated to 273.15 K. Please note that the accurate determination of $\sigma_{i,w}$ is difficult, and different $\sigma_{i,w}$ parameterizations exist (Ickes et al., 2015).

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A temperature dependence of the surface tension of water in pure water vapor, based on Vargaftik et al. (1983) and updated by IAPWS, is given by[2]:

$$\sigma_{w,v} = B \cdot \tau^{\mu} \cdot (1 + b \cdot \tau), \tag{A.12}$$

where $\sigma_{w,v}$ is given in $\mathrm{mN\,m^{-1}}$, $\mu = 1.256$, $b = -0.625$ and $B = 235.8\,\mathrm{mN\,m^{-1}}$. Moreover, $\tau = 1 - T/T_c$, with $T_c = 647.096\,\mathrm{K}$ and T being the absolute temperature. Eq. (A.12) is valid for $273.16\,\mathrm{K} \geq T \geq 647.096\,\mathrm{K}$, but has also been shown to provide reasonable values for supercooled water down to temperatures of about 248 K (Vinš et al., 2015). Throughout this book, we assume $\sigma_{w,v} \approx \sigma_{w,a}$.

A temperature dependence of the surface tension of ice in pure water vapor is given by Hale and Plummer (1974):

$$\sigma_{i,v} = (141 - 0.15 \cdot T) \cdot 0.001, \tag{A.13}$$

where $\sigma_{i,v}$ is given in $\mathrm{mN\,m^{-1}}$ and T is given in K. This expression is valid approximately between 273.15 K and 233.15 K.

A.4 Chemical and Physical Properties

Other chemical and physical properties can be found in data bases, including the following:

https://hbcp.chemnetbase.com/
https://webbook.nist.gov/chemistry/
www.nist.gov/pml/productsservices/physical-reference-data

[2] www.iapws.org/relguide/Surf-H2O.html

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