J. Plasma Physics (2000), *vol.* 63, *part* 3, *pp.* 239–253. Printed in the United Kingdom
© 2000 Cambridge University Press

Dielectric tensor and electromagnetic modes in magnetized non-ideal plasmas

V. M. RYLYUK,[1] I. M. TKACHENKO[2] and J. ORTNER[3]

[1] Department of Theoretical Physics, University of Odesa, Dvorjanskaja 2, 270100, Odesa, Ukraine

[2] Departament de Matematica Aplicada, ETSII, Universitat Politecnica, Cami de Vera s/n, E-46022 Valencia, Spain

[3] Institut für Physik, Humboldt Universität zu Berlin, Invalidenstr. 110, D-10115 Berlin, Germany

(Received 15 February 1999 and revised form 29 September 1999)

Abstract. The effect of Coulomb correlations on the spectrum of electromagnetic waves propagating in a non-ideal magnetized fully ionized hydrogen plasma is studied. We employ the dielectric tensor constructed by means of the classical theory of moments without using perturbation theory.

1. Introduction

The dielectric tensor for a cold non-ideal magnetized plasma was constructed by Ortner et al. (1994) by applying the classical theory of moments. The aim of the present paper is to use the expressions obtained in that work for the analysis of the spectra of electromagnetic waves propagating in such a system. Here we shall regard the damping of the modes in question as negligibly small (nevertheless, see Sec. 5). The validity of this assumption can be confirmed from experimental data. We note from the very beginning that damping can be essential and must be taken into account near cyclotron resonances. But here the thermal motion of the particles leading to spatial dispersion must also be taken into consideration. Thus our results are valid only far from cyclotron resonances.

The dispersion equation for electromagnetic waves in an anisotropic medium is

$$\det\left(\varepsilon_{\mu\nu} - N^2 T_{\mu\nu}\right) = 0, \tag{1.1}$$

where $\varepsilon_{\mu\nu}$ are the components of the dielectric tensor, $N = \omega_0/\omega$ is the scalar refractive index, $\omega_0 = |\mathbf{k}|\, c$, c is the velocity of light, $T_{\mu\nu} = \delta_{\mu\nu} - k_\mu k_\nu / |\mathbf{k}|^2$, $\delta_{\mu\nu}$ is the Kronecker delta and k_μ is the μth Cartesian component of the wave vector $\mathbf{k}$ ($\mu = 1, 2, 3$).

In a Cartesian system of coordinates with the z axis parallel to the external magnetic field $\mathbf{B}$, and neglecting thermal motion, the dielectric tensor can be expressed as

$$\varepsilon_{\mu\nu} = \begin{pmatrix} \varepsilon_\perp & ig & 0 \\ -ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \tag{1.2}$$

where $\varepsilon_\perp$ and $\varepsilon_\parallel$ are the transverse and longitudinal (with respect to the external

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

magnetic field **B**) components of the tensor. The dispersion equation then simplifies, and one has

$$AN^4 + BN^2 + C = 0, \tag{1.3}$$

where

$$A = \varepsilon_\perp \sin^2\theta \ + \ \varepsilon_\parallel \cos^2\theta, \tag{1.4a}$$

$$B = -\varepsilon_\perp\varepsilon_\parallel(1 + \cos^2\theta) \ - \ (\varepsilon_\perp^2 - g^2)\sin^2\theta, \tag{1.4b}$$

$$C = \varepsilon_\parallel(\varepsilon_\perp^2 - g^2), \tag{1.4c}$$

and θ is the angle between the wave vector **k** and the external magnetic field, **B**.

Since it follows from (1.3) that

$$N^2 = \frac{1}{2A}[-B \pm (B^2 - 4AC)^{1/2}], \tag{1.5}$$

we conclude that two different kinds of waves of a given frequency and with different refractive indices can propagate in a magnetized plasma: ordinary and extraordinary waves. For a given wavevector **k** (1.3) defines the frequencies $\omega_i(\mathbf{k}, 0)$ of these waves. It is impossible to solve (1.3) analytically in the general case. Therefore we give here analytical solutions of this equation in some limiting cases. In doing so, we shall also find the frequency dependence of the refractive index. For this purpose, we use the explicit expressions for the components of the dielectric tensor of a system under consideration found in Ortner et al. (1994) within the framework of the theory of moments.

Neglecting the damping (i.e. the functions $h_\perp$and $h_\parallel$), and within the first approximation in the ratio $(m/M)^{1/2}$ (m and M being the electron and the ion masses) the components of the dielectric tensor can be represented as

$$\varepsilon_\perp(\omega) = 1 - \omega_p^2 \frac{\omega^2 - \boldsymbol{\Omega}_\perp^2}{(\omega^2 - \boldsymbol{\Omega}_\perp^2)^2 - \omega^2\omega_B^2}, \tag{1.6a}$$

$$\varepsilon_\parallel(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - \boldsymbol{\Omega}_\parallel^2}, \tag{1.6b}$$

$$g(\omega) = \omega_p^2 \frac{\omega\omega_B}{(\omega^2 - \boldsymbol{\Omega}_\perp^2)^2 - \omega^2\omega_B^2}, \tag{1.6c}$$

where $\omega_p = (4\pi ne^2/m)^{1/2}$ is the plasma frequency, $-e$, m, and n being the charge mass, and density of the electrons; $\omega_B = eB/mc$ is the electron cyclotron frequency. The positive magnitudes $\boldsymbol{\Omega}_\perp$ and $\boldsymbol{\Omega}_\parallel$ take into account the Coulomb correlations between the particles, and are expressible via the second frequency moment of the Hermitian part of the magnetized plasma conductivity tensor (Ortner et al. 1994), so that

$$\boldsymbol{\Omega}_\perp^2 = \frac{\omega_p^2}{2} \sum_{\mathbf{q}\neq 0} S_{ei}(\mathbf{q}) \frac{q_\perp^2}{q^2}, \tag{1.7a}$$

$$\boldsymbol{\Omega}_\parallel^2 = \omega_p^2 \sum_{\mathbf{q}\neq 0} S_{ei}(\mathbf{q}) \frac{q_\parallel^2}{q^2}, \tag{1.7b}$$

$S_{ei}(\mathbf{q})$ being the partial electron–ion static structure factor, and $q_\perp$ and $q_\parallel$ are the projections of the vector **q** on the directions perpendicular and parallel to the external magnetic field. An analysis of these magnitudes is given in Sec. 5; we

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

mention here only that in the ideal-plasma limit both $\mathbf{\Omega}_\perp$ and $\mathbf{\Omega}_\parallel$ tend to 0. We also wish to emphasize that electron–ion correlation is the factor that guarantees the existence of non-vanishing parameters $\mathbf{\Omega}_\perp$ and $\mathbf{\Omega}_\parallel$. Note that the expressions (1.6) for the dielectric tensor components coincide (to first order in the ratio $(m/M^{1/2})$) with those from the quasilocalized-charge model developed by Kalman and Golden (1990).

2. Waves in non-ideal magnetized plasmas

2.1. Wave propagation parallel to the external magnetic field

First consider the propagation of electromagnetic waves parallel to the direction of the external magnetic field. Putting $\theta = 0$ in (1.3) and (1.4), one obtains the following dispersion relations:

$$\varepsilon_\parallel = 0, \tag{2.1a}$$

$$N_\pm^2 = \varepsilon_\perp \pm g. \tag{2.1b}$$

The solution of (2.1a) describes Langmuir oscillations in a cold non-ideal plasma, polarized parallel to the external magnetic field $\mathbf{B}$ and influenced by the magnetic field only through the dependence of the magnitude $\mathbf{\Omega}_\parallel^2$ on the magnetic field. (Note that the latter is independent of the magnetic field in a classical plasma.) Thus we have for the frequency of the Langmuir oscillations:

$$\omega^2 = \omega_p^2 + \mathbf{\Omega}_\parallel^2. \tag{2.2}$$

With the use of (1.6), (2.1b) can be transformed into

$$N_\pm^2 = 1 - \frac{\omega_p^2}{\omega^2 - \mathbf{\Omega}_\perp^2 \pm \omega\omega_B}\,, \tag{2.3}$$

The two signs in (2.3) correspond to two independent transverse waves circularly polarized in the plane perpendicular to the external magnetic field: a left (or ordinary) wave and right (or extraordinary) wave. In general, the solutions of the dispersion equations (2.3) are quite cumbersome. In order to get simple analytical expressions, it is useful to consider limiting cases.

First of all, we turn to the situation most typical of a non-ideal plasma, namely the case $\mathbf{\Omega}_\perp > \omega_B$. Very strong magnetic fields are needed to satisfy the opposite inequality. If, for example, the electron density varies within the limits 10^{18}–10^{21} cm^{-3} and the degree of non-ideality is estimated as $\mathbf{\Omega}_\perp/\omega_p \approx 0.01$–$0.1$ then the above inequality becomes invalid for magnetic fields $|\mathbf{B}| > B_0 \approx (10^5$–$10^8)$ G. Magnetic fields of this strength exist under astrophysical conditions (neutron stars and white dwarfs).

As the first limiting case, consider the high-frequency part of the spectrum of the wave propagating along the external magnetic field: $\omega \gg \omega_B$. Putting $|\mathbf{B}| \ll B_0$ (the case of ordinary magnetic field strengths), we can neglect the contribution $\pm\omega\omega_B$ in (2.3), describing the influence of the plasma gyrotropy, and find the corresponding zeroth-order approximation spectrum

$$\tilde{\omega}^2 = \tfrac{1}{2}\{\omega_p^2 + \omega_0^2 + \mathbf{\Omega}_\perp^2 + [(\omega_p^2 + \omega_0^2 + \mathbf{\Omega}_\perp^2)^2 - 4\omega_0^2\mathbf{\Omega}_\perp^2]^{1/2}\}. \tag{2.4}$$

Taking into account the plasma gyrotropy as a small perturbation, one obtains the

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

following spectra of ordinary and extraordinary waves:

$$\omega^2 = \tilde{\omega}^2 \left(1 \pm \frac{\omega_B}{\tilde{\omega}} \frac{\tilde{\omega}^2 - \omega_0^2}{2\tilde{\omega}^2 - \omega_0^2 - \omega_p^2 - \Omega_\perp^2} \right) . \tag{2.5}$$

It is easily seen that as $\Omega_\perp \to 0$, this result transforms into the well-known spectra for an ideal plasma.

Consider now the case of weak non-ideality, or the case of very strong magnetic fields, i.e. of $\Omega_\perp \ll \omega_B$. Then, within the zeroth-order approximation in non-ideality, (2.3) transforms into

$$\frac{\omega_0^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega\left(\omega \pm \omega_B\right)} . \tag{2.6}$$

In the high-frequency limit, this expression turns into the well-known result (Akhiezer et al. 1975; Aleksandrov et al. 1988)

$$\tilde{\omega}^2 = \left(\omega_p^2 + \omega_0^2\right) \left[1 \pm \frac{\omega_p^2 \omega_B}{\left(\omega_p^2 + \omega_0^2\right)^{3/2}} \right] , \tag{2.7}$$

whereas, on accounting for the correlation corrections, we get the following expression for the spectra of ordinary and extraordinary waves in a weakly non-ideal magnetized plasma:

$$\omega^2 = \tilde{\omega}^2 \left[1 + \frac{2}{\tilde{\omega}} \frac{\left(\tilde{\omega}^2 - \omega_0^2\right) \Omega_\perp^2}{\tilde{\omega}^3 \pm \omega_B \left(\tilde{\omega}^2 + \omega_0^2\right)} \right] , \tag{2.8}$$

where $\tilde{\omega}^2$ satisfies (2.6). In the case of a weak magnetic field, $\omega_B \ll \omega_p$, (2.8), with account taken of (2.7), can be represented as

$$\omega^2 = \left(\omega_p^2 + \omega_0^2\right) \left\{ 1 \pm \frac{\omega_p^2}{\left(\omega_p^2 + \omega_0^2\right)^{3/2}} \left[\omega_B \pm \frac{\Omega_\perp^2}{\left(\omega_p^2 + \omega_0^2\right)^{1/2}} \right] \right\} . \tag{2.9}$$

Consider now the low-frequency region of the spectra, i.e. $\omega \ll \omega_B$. In an ideal magnetized plasma, the solution of (2.6) then produces a spiral wave – a helicon or whistler, the frequency of which is equal to (Akhiezer et al. 1975)

$$\bar{\omega} = \frac{\omega_0^2 \omega_B}{\omega_p^2 + \omega_0^2} , \tag{2.10}$$

and tends to zero as $|\mathbf{B}| \to 0$. Weak non-ideality modifies this expression into

$$\omega = \bar{\omega} + \frac{\omega_0^2 \Omega_\perp^2 - \bar{\omega}^2 \Omega_\parallel^2}{\omega_0^2 \omega_B} . \tag{2.11}$$

2.2. Wave propagation orthogonal to the external magnetic field

Consider further electromagnetic waves propagating perpendicularly with respect to the external magnetic field. Equations (1.3) and (1.4) with $\theta = \frac{1}{2}\pi$ then lead to the dispersion relations

$$N_1^2 = \varepsilon_\parallel , \tag{2.12a}$$

$$N_1^2 = \frac{\varepsilon_\perp^2 - g^2}{\varepsilon_\perp} . \tag{2.12b}$$

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

Equation (2.12a) describes an ordinary electromagnetic wave that is purely transverse and polarized along the external magnetic field. Using (1.6), one obtains an explicit expression for its frequency:

$$\omega^2 = \tfrac{1}{2}\{\omega_p^2 + \omega_0^2 + \boldsymbol{\Omega}_\parallel^2 + [(\omega_p^2 + \omega_0^2 + \boldsymbol{\Omega}_\parallel^2)^2 - 4\omega_0^2\boldsymbol{\Omega}_\parallel^2]^{1/2}\}\,. \tag{2.13}$$

Equation (2.12b) describes an extraordinary longitudinal–transverse electromagnetic wave. To study its dispersion relation, we consider the limiting cases of weak and strong coupling. Neglecting first the frequencies $\boldsymbol{\Omega}_\perp$ and $\boldsymbol{\Omega}_\parallel$ in (1.6), one obtains in the zeroth-order approximation in the coupling, the following well-known expression (Akhiezer et al. 1975; Aleksandrov et al. 1988):

$$\tilde{\omega}^2 = \tfrac{1}{2}\{\omega_B^2 + 2\omega_p^2 + \omega_0^2 \pm [4\omega_p^2\omega_B^2 + (\omega_B^2 - \omega_0^2)^2]^{1/2}\}. \tag{2.14}$$

Taking into account the frequency $\boldsymbol{\Omega}_\perp$ as a small correction, after straightforward but cumbersome calculations, one finds the dispersion relation for the extraordinary wave propagating in a weakly non-ideal plasma in the direction orthogonal to the external magnetic field:

$$\omega^2 = \tilde{\omega}^2\left[1 + \frac{\boldsymbol{\Omega}_\perp^2}{\tilde{\omega}^4}\frac{2\left(\tilde{\omega}^2 - \omega_p^2\right)\left(\tilde{\omega}^2 - \omega_0^2\right) - \omega_p^2\omega_0^2}{2\left(\tilde{\omega}^2 - \omega_p^2\right) - \omega_0^2 - \omega_B^2}\right]. \tag{2.15}$$

with $\tilde{\omega}$ from (2.14).

In the case of strong coupling, i.e. when $\boldsymbol{\Omega}_\perp > \omega_B$, we consider the influence of the magnetic field as a small perturbation, and find

$$\omega^2 = \tilde{\omega}^2\left\{1 + \frac{\left(\tilde{\omega}^2 - \omega_0^2\right)\omega_B^2}{\left(2\tilde{\omega}^2 - \omega_p^2 - \omega_0^2 - \boldsymbol{\Omega}_\perp^2\right)\left(\tilde{\omega}^2 - \omega_p^2 - \boldsymbol{\Omega}_\perp^2\right)}\right\}, \tag{2.16}$$

where $\tilde{\omega}^2$ is to be taken from (2.4). Note that in the limiting case $|\mathbf{B}| \to 0$, (2.4) and (2.13) coincide.

2.3. Arbitrary direction of wave propagation

Having considered both parallel and perpendicular cases of wave propagation, let us now turn to a general case of wave propagation at an arbitrary angle θ with respect to the external magnetic field. To this end, we represent the square of the refractive index from (1.5) as

$$N_\pm^2 = \frac{2E}{2F - G\sin^2\theta \pm D^{1/2}}\,. \tag{2.17}$$

Here

$$E = \omega_\parallel^2\left(\omega_\perp^4 - \omega^2\omega_B^2\right), \tag{2.18a}$$

$$F = \omega_\parallel^2\left(\omega_\perp^4 + \omega_p^2\omega_\perp^2 - \omega^2\omega_B^2\right), \tag{2.18b}$$

$$G = \omega_p^2\left(\omega^2\omega_B^2 + \delta\omega_\perp^2\right), \tag{2.18c}$$

$$D = (2\omega_p^2\omega_\parallel^2\omega\omega_B)^2\cos^2\theta + G^2\sin^4\theta\,, \tag{2.18d}$$

with

$$\omega_\parallel^2 = \omega^2 - \omega_p^2 - \boldsymbol{\Omega}_\parallel^2, \tag{2.19a}$$

$$\omega_\perp^2 = \omega^2 - \omega_p^2 - \boldsymbol{\Omega}_\perp^2, \tag{2.19b}$$

$$\delta = \boldsymbol{\Omega}_\perp^2 - \boldsymbol{\Omega}_\parallel^2\,. \tag{2.19c}$$

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

Let us first study the high-frequency branch of the spectrum, i.e. the dispersion law for electromagnetic waves in a strongly non-ideal plasma when the inequality $\boldsymbol{\Omega}_\perp, \boldsymbol{\Omega}_\parallel > \omega_B$ holds. In this case, we can consider the Coulomb interaction to be so strong that the cyclotron motion in the external magnetic field is virtually destroyed (see the discussion following (3.11) below). Thus the latter no longer influences the particle motion, and the distortions of interparticle correlations due to the external magnetic field will be 'weak' with respect to the isotropic-plasma case. This permits us to consider the equality $\boldsymbol{\Omega}_\perp = \boldsymbol{\Omega}_\parallel = \boldsymbol{\Omega}$ valid and to put $\delta = 0$ in (2.19). In this case, the expression for the square of the refractive index from (2.17) can be transformed into

$$N_\pm^2 = \frac{1-\epsilon^2}{N_i^{-2} - \epsilon^2 + \epsilon K_\mp(\epsilon)}\,, \tag{2.20}$$

where

$$\epsilon = \frac{\omega\omega_B}{\omega_i^2}\,, \tag{2.21a}$$

$$\omega_i^2 = \omega^2 - \omega_p^2 - \boldsymbol{\Omega}^2\,, \tag{2.21b}$$

$$K_\mp(\epsilon) = \frac{\omega_p^2}{2\omega_i^2}[-\varepsilon\sin^2\theta \mp (4\cos^2\theta + \varepsilon^2\sin^4\theta)^{1/2}]\,, \tag{2.21c}$$

and

$$N_i^{-2} = 1 + \frac{\omega_p^2}{\omega_i^2} \tag{2.22}$$

is the inverse refractive index in the absence of an external magnetic field.

If the latter can be regarded as a small perturbation ($\epsilon \ll 1$), we obtain the following dispersion law for both ordinary and extraordinary electromagnetic waves:

$$\omega^2 = \bar\omega^2\left\{1 + \frac{\bar\omega^2 - \omega_0^2}{\bar\omega_i^2 + \bar\omega^2 - \omega_0^2}\left[\frac{\omega_B}{\bar\omega}\frac{\lambda_\pm(\theta)}{2}\Lambda_\pm(\theta) + \left(\frac{\omega_B}{\bar\omega_i}\right)^2\right]\right\}. \tag{2.23}$$

Here

$$\bar\omega_i^2 = \bar\omega^2 - \omega_p^2 - \boldsymbol{\Omega}^2, \tag{2.24a}$$

$$\lambda_\pm(\theta) = -\frac{\bar\omega\omega_B}{\omega_i^2}\sin^2\theta \pm \left[4\cos^2\theta + \left(\frac{\bar\omega\omega_B}{\omega_i^2}\right)^2\sin^4\theta\right]^{1/2}, \tag{2.24b}$$

$$\Lambda_\pm(\theta) = 1 \pm \frac{\bar\omega^2 - \omega_0^2}{\bar\omega_i^2 + \bar\omega^2 - \omega_0^2}\frac{\mu\omega_B}{2}\,|\cos\theta|\,, \tag{2.24c}$$

$$\mu = \frac{\bar\omega_i^2 - 4\bar\omega^2}{\bar\omega_i^2\bar\omega}\,. \tag{2.24d}$$

and the frequency $\bar\omega$ satisfies (2.22):

$$\omega^2 = \tfrac{1}{2}\{\omega_p^2 + \omega_0^2 + \boldsymbol{\Omega}^2 + [(\omega_p^2 + \omega_0^2 + \boldsymbol{\Omega}^2)^2 - 4\omega_0^2\boldsymbol{\Omega}^2]^{1/2}\}. \tag{2.25}$$

The dispersion law (2.23) is of a form permitting the limiting transformations as $\theta \to 0$ and $\theta \to \frac{1}{2}\pi$. In particular, $\lambda_\pm(\theta = 0) = \pm 2$, and (2.23) in the approximation linear in the cyclotron frequency ω_B coincides with (2.5) with $\boldsymbol{\Omega}_\perp$ put equal to $\boldsymbol{\Omega}$.

If $\theta = \frac{1}{2}\pi$ then $\lambda_-(\theta = \frac{1}{2}\pi) = -2\bar\omega\omega_B/\omega_i^2$ and $\lambda_+(\theta = \frac{1}{2}\pi) = 0$, and (2.23) coincides (in the approximation linear in the cyclotron frequency ω_B) either with (2.13) with $\boldsymbol{\Omega}_\parallel = \boldsymbol{\Omega}$, or with (2.16) but with $\boldsymbol{\Omega}_\perp = \boldsymbol{\Omega}$.

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

It follows from (2.23) that if the angle θ is close to $\frac{1}{2}\pi$ (quasi-orthogonal propagation), the dispersion relations (2.13) and (2.16) (in the approximation quadratic in the cyclotron frequency ω_B) do not change. On the other hand, if the angle θ takes arbitrary values from the interval $[0, \pi]$ but is not close to $\frac{1}{2}\pi$, (2.23) can be shown to take the form

$$\omega^2 = \bar{\omega}^2 \left\{ 1 + \frac{\bar{\omega}^2 - \omega_0^2}{\bar{\omega}_i^2 + \bar{\omega}^2 - \omega_0^2} \left[\pm \frac{\omega_B}{\bar{\omega}} |\cos\theta| + \left(\frac{\omega_B}{\bar{\omega}} \right)^2 M(\theta) \right] \right\}, \tag{2.26}$$

with

$$M(\theta) = \frac{5\bar{\omega}^2 - \omega_0^2}{\bar{\omega}_i^2 + \bar{\omega}^2 - \omega_0^2} \frac{\cos^2\theta}{2} - \frac{\bar{\omega}^2}{\bar{\omega}_i^2} (1 - \tfrac{3}{2}\sin^2\theta). \tag{2.27}$$

We now turn to the problem of dispersion of electromagnetic waves in a weakly non-ideal plasma, i.e. when $\boldsymbol{\Omega}_\perp \ll \omega_B$. We convert (2.17) into

$$N_\pm^2 = 1 - \frac{A}{Q \pm C^{1/2}}, \tag{2.28}$$

where

$$A = 2\omega_p^2(\omega_\parallel^2 - \delta \sin^2\theta), \tag{2.29a}$$

$$Q = 2\omega_\parallel^2(\omega^2 - \boldsymbol{\Omega}_\perp^2) - W_0 \sin^2\theta, \tag{2.29b}$$

$$W_0 = \omega^2\omega_B^2 + \delta(\omega^2 + \omega_p^2 - \boldsymbol{\Omega}_\perp^2)\sin^2\theta, \tag{2.29c}$$

$$C = 4\omega_\parallel^4\omega^2\omega_B^2 \cos^2\theta + (\omega^2\omega_B^2 + \delta\omega_\perp^2)^2 \sin^4\theta. \tag{2.29d}$$

We first take the frequency $\boldsymbol{\Omega}_\perp$ into account as a small correction, while considering $\boldsymbol{\Omega}_\parallel$ to be of arbitrary magnitude. After quite long transformations, one can get for this wave

$$\omega^2 = \omega_\pm^2 \left[1 + 2\frac{\boldsymbol{\Delta}_\pm^{(1)}(\theta)}{\boldsymbol{\Psi}_\pm^{(1)}(\theta)} \boldsymbol{\Omega}_\perp^2 \right]. \tag{2.30}$$

Here

$$\boldsymbol{\Delta}_\pm^{(1)}(\theta) = 2(\bar{\omega}^2 - \omega_p^2)(\bar{\omega}^2 - \omega_0^2) + W_1 \sin^2\theta, \tag{2.31a}$$

$$W_1 = (\bar{\omega}^2 - \omega_0^2)\left[\bar{\omega}^2 + \omega_p^2 \mp \frac{\bar{\omega}\omega_B}{H_0}(\bar{\omega}^2 - \omega_p^2)\sin^2\theta \right] - 2\bar{\omega}^2\omega_p^2, \tag{2.31b}$$

$$\boldsymbol{\Psi}_\pm^{(1)}(\theta) = 4\bar{\omega}^4(\bar{\omega}^2 - \omega_p^2) \pm 2\bar{\omega}\omega_B W_2 + 2\bar{\omega}^2 W_3 \sin^2\theta, \tag{2.31c}$$

$$W_2 = \bar{\omega}^2 H_0^{1/2} - \frac{2}{H_0^{1/2}}(\bar{\omega}^2 - \omega_p^2)^2(\bar{\omega}^2 - \omega_0^2)\cos^2\theta, \tag{2.31d}$$

$$W_3 = W_1 - \bar{\omega}^2\omega_B^2, \tag{2.31e}$$

$$H_0 = 4(\bar{\omega}^2 - \omega_p^2)^2\cos^2\theta + \bar{\omega}^2\omega_B^2 \sin^4\theta, \tag{2.31f}$$

and the frequency $\omega_\pm$ satisfies (2.28) with $\boldsymbol{\Omega}_\perp$ put equal to zero. The formulae (2.31) can be used to observe that the corrections of order of $\boldsymbol{\Omega}_\perp^2$ in (2.30) and (2.8) coincide at $\theta = 0$, and, besides, that the frequency $\omega_\pm$ in this case satisfies (2.6). Indeed, if $\boldsymbol{\Omega}_\perp = 0$ and $\theta = 0$, $\boldsymbol{\Omega}_\parallel$ disappears from (2.28), and it transforms into (2.6).

In another limiting case, corresponding to $\theta = \frac{1}{2}\pi$, we also obtain two solutions. The first, $\omega^2 = \omega_+^2$, follows immediately, since $\boldsymbol{\Delta}_\pm^{(1)}(\theta = \frac{1}{2}\pi) = 0$. Then the exact solution (2.13) stems for ω_+^2 from (2.28) with $\boldsymbol{\Omega}_\perp = 0$ and $\theta = \frac{1}{2}\pi$.

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

The second solution can be obtained for the lower sign in (2.30), and it can be shown to coincide with the dispersion law (2.15).

Thus, indeed, (2.30) together with (2.31) contains all particular limiting cases considered before, and provides the dispersion laws for both ordinary and extra-ordinary waves propagating in a weakly non-ideal magnetized plasma at an arbitrary angle with respect to the external magnetic field. An expansion of the magnitude $\omega_\pm$ in (2.30) in powers of $\Omega_\parallel^2$ then leads to the final form of the dispersion law in the case when $\Omega_\perp, \Omega_\parallel \ll \omega_B$:

$$\omega^2 = \bar{\omega}^2 \left[1 + 2\frac{\Delta_\pm^{(1)}(\theta)}{\Psi_\pm^{(1)}(\theta)} + 2\frac{\Delta_\pm^{(2)}(\theta)}{\Psi_\pm^{(2)}(\theta)} \right] . \tag{2.32}$$

Here

$$\Delta_\pm^{(2)}(\theta) = 2\bar{\omega}^2(\bar{\omega}^2 - \omega_0^2 - \omega_p^2 \cos^2\theta) - W_4, \tag{2.33a}$$

$$W_4 = (\bar{\omega}^2 - \omega_0^2)[(\bar{\omega}^2 + \omega_p^2)\sin^2\theta \mp \frac{\bar{\omega}\omega_B}{H_0^{1/2}}(\bar{\omega}^2 - \omega_p^2)(1 + \cos^2\theta)] , \tag{2.33b}$$

$$\Psi_\pm^{(2)}(\theta) = 4\bar{\omega}^4(2\bar{\omega}^2 - 2\omega_p^2 - \omega_0^2) - 2\bar{\omega}^4\omega_B^2 \sin^2\theta \pm W_5, \tag{2.33c}$$

$$W_5 = \bar{\omega}\omega_B(\bar{\omega}^2 + \omega_0^2)H_0^{1/2} + \frac{(\bar{\omega}^2 - \omega_0^2)\bar{\omega}^3\omega_B\bar{h}}{H_0^{1/2}} , \tag{2.33d}$$

$$\bar{h} = 8(\bar{\omega}^2 - \omega_p^2)\cos^2\theta + \omega_B^2 \sin^4\theta, \tag{2.33e}$$

In addition, the frequency $\bar{\omega}$ in (2.32) and (2.33) satisfies the equation

$$\frac{\omega_0^2}{\bar{\omega}^2} = 1 - \frac{2\omega_p^2(\bar{\omega}^2 - \omega_p^2)}{2\bar{\omega}^2(\bar{\omega}^2 - \omega_p^2) - \bar{\omega}^2\omega_B^2 \sin^2\theta \pm W_6} , \tag{2.34}$$

with

$$W_6 = \bar{\omega}\omega_B[4(\bar{\omega}^2 - \omega_p^2)^2 \cos^2\theta + \bar{\omega}^2\omega_B^2 \sin^4\theta]^{1/2} , \tag{2.35}$$

which stems from (2.28) and (2.29) on putting $\Omega_\perp = \Omega_\parallel = 0$.

The dispersion laws for the modes in an ideal magnetized plasma follow from (2.34) (Akhiezer et al. 1975). In particular, it can be observed from (2.33) and (2.34) that $\Delta_\pm^{(2)}(\theta = 0) = 0$, and (2.32) coincides under these conditions with (2.8).

In the same manner, at $\theta = \frac{1}{2}\pi$, $\Delta_-^{(2)}(\theta = \frac{1}{2}\pi) = 0$, and (2.32) transforms into (2.15). On the other hand, $\Delta_+^{(2)}(\theta = \frac{1}{2}\pi) = 2\omega_0^2\omega_p^2$ and $\Psi_\pm^{(2)}(\theta) = 4\bar{\omega}^4\omega_0^2$. Since, as we have seen, $\Delta_+^{(1)}(\theta = \frac{1}{2}\pi) = 0$, we find from (2.32) the dispersion relation for a purely orthogonal electromagnetic wave in a weakly ionized plasma:

$$\omega^2 = \bar{\omega}^2 \left(1 + \frac{\omega_p^2}{\bar{\omega}^4}\Omega_\parallel^2 \right) , \tag{2.36}$$

where $\bar{\omega}^2 = \omega_p^2 + \omega_0^2$. It can easily be observed that this same results also follows from (2.13) if $\Omega_\parallel^2$ is small.

3. Spiral waves in non-ideal plasmas

3.1. Correlation corrections

Consider first the dispersion relation of a spiral wave (helicon) in a weakly non-ideal plasma. In this case, we limit ourselves to the frequency range $\omega \ll \omega_B$, where ω_B is

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

not supposed to be small with respect to ω_p. Then in the zeroth-order approximation in the non-ideality, we get from (2.28)

$$\bar{\omega} = \frac{2\omega_p^2\omega_0^2\omega_B\,|\cos\theta|}{2\omega_p^2(\omega_p^2+\omega_0^2)+\omega_0^2\omega_B^2\sin^2\theta}\,. \tag{3.1}$$

The non-ideality regarded as a small correction then leads to the following expression for the whistler frequency:

$$\omega = \bar{\omega} + \frac{\Omega_\parallel^2 u_1 + \Omega_\perp^2 u_2}{2\omega_0^2\omega_B\,|\cos\theta|}\,, \tag{3.2}$$

where

$$u_1 = \omega_0^2\sin^2\theta - 2\bar{\omega}^2\cos^2\theta, \tag{3.3a}$$

$$u_2 = \omega_0^2(1+\cos^2\theta) - 2\bar{\omega}^2\sin^2\theta. \tag{3.3b}$$

Equation (3.2) is valid if the angle is not close to $\frac{1}{2}\pi$. At $\theta = 0$, it coincides with the result (2.11).

3.2. Whistling sound

Consider now a long-wavelength limiting form of dispersion of electromagnetic waves in plasmas with strong interparticle interaction, i.e. when the inequality $\Omega_\perp > \omega_0\omega_B/\omega_p$ holds. In the zeroth-order approximation in the external magnetic field, there follows directly from (2.28) and (2.29) an expression for a 'sound wave' in a non-ideal plasma,

$$\omega = \omega_s = \omega_0\tilde{\varepsilon}^{-1/2}, \tag{3.4}$$

where $\tilde{\varepsilon}$ is a 'generalized static dielectric permeability'

$$\tilde{\varepsilon} = 1 + \frac{U}{V}, \tag{3.5a}$$

$$U = 2\omega_p^2(\omega_p^2 + \Omega_\parallel^2\cos^2\theta + \Omega_\perp^2\sin^2\theta)\,, \tag{3.5b}$$

$$V = 2\Omega_\perp^2(\omega_p^2+\Omega_\parallel^2) + \delta\sin^2\theta[\Omega_\perp^2 - \omega_p^2 \mp (\omega_p^2+\Omega_\perp^2)]\,; \tag{3.5c}$$

here δ is given by (2.19c). If the influence of the external magnetic field can be considered as a small perturbation, we obtain the following expression for both ordinary and extraordinary *whistling sound waves* (introduced in Ortner et al. 1998), which can propagate in strongly non-ideal plasmas at an arbitrary angle with respect to the external magnetic field:

$$\omega = \omega_s\left[1 \pm \left(1 - \frac{1}{\tilde{\varepsilon}}\right)\frac{W}{2V}\right], \tag{3.6}$$

with

$$W = \pm\left[H_1^{1/2} + \delta(\omega_p^2+\Omega_\perp^2)\sin^2\theta\left(\frac{\omega_s^2\omega_B^2}{H_1^{1/2}}\sin^2\theta - 1\right)\right] - \omega_s^2\omega_B^2\sin^2\theta, \tag{3.7a}$$

$$H_1 = 4\omega_s^2\omega_B^2(\omega_p^2+\Omega_\parallel^2)^2\cos^2\theta + [\delta(\omega_p^2+\Omega_\perp^2)\sin^2\theta]^2. \tag{3.7b}$$

We are now in a position to consider some limiting cases.

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

3.2.1. Longitudinal propagation. In particular, if the whistling sound propagates along the external magnetic field, i.e. if $\theta = 0$, (3.6) simplifies to (Tkachenko et al. 1998)

$$\omega = v_\perp k \pm \zeta_\perp k^2, \tag{3.8}$$

with the whistling sound velocity

$$v_\perp = \frac{u}{(1 + u^2/c^2)^{1/2}}\,, \tag{3.9}$$

$$u = c\boldsymbol{\Omega}_\perp/\omega_p \tag{3.10}$$

being a quantity with dimensions of velocity, and

$$\zeta_\perp = \frac{\omega_B \omega_p^2 c^2}{2\left(\omega_p^2 + \boldsymbol{\Omega}_\perp^2\right)^2}\,, \tag{3.11}$$

Magnetic sound waves in ideal plasmas have a similar dispersion relation, but with the Alfvén velocity u_A determined by the ion cyclotron and ion plasma frequencies instead of $\boldsymbol{\Omega}_\perp$ and ω_p respectively. In this sense, the whistling sound only resembles the Alfvén mode due to the low-frequency oscillations of ions in a magnetic field, since it is related to the motion of strongly correlated electrons of the plasma. The external magnetic field in this case produces only a small perturbation in the dispersion relation.

The physical reason for this result is that strong interparticle correlations destroy the cyclotron motion, and the ideal-gas approximation as a zeroth-order approximation is no longer appropriate. Indeed, already for an electron number density of the order of $n \approx 10^{18}$ cm^{-3}, a temperature $T \approx 10^4$ K and $B \approx 10^5$ G, the relation between the cyclotron frequency and the mean collision frequency in the plasma $\omega_T = v_T n^{1/3}$, (where $v_T = (k_B T/m)^{1/2}$ is the thermal velocity) becomes of the order of unity, so that an electron cannot traverse even one cyclotron orbit without colliding with another particle. Therefore the correct account of the non-ideality effects within the framework of the method of moments (Ortner et al. 1994), allows one to conclude that the low-frequency part of the spectra changes qualitatively, and the reason for this variation is related not to the influence of the external magnetic field, but to the strong interparticle Coulomb interaction.

3.2.2. Transverse propagation. Just as in the longitudinal case, at $\theta = \frac{1}{2}\pi$, (3.4)–(3.7) produce two more solutions:

$$\omega_+ = v_\parallel k, \tag{3.12}$$

with

$$v_\parallel = c\boldsymbol{\Omega}_\parallel/\omega_p\,, \tag{3.13}$$

and

$$\omega_- = v_\perp k - \zeta_\parallel k^3\,, \tag{3.14}$$

where

$$\zeta_\parallel = \frac{\omega_B^2 c^3 \boldsymbol{\Omega}_\perp}{2(\omega_p^2 + \boldsymbol{\Omega}_\perp^2)^{5/2}} \tag{3.15}$$

These results for the low-frequency part of the spectra of electromagnetic waves in strongly non-ideal plasmas can be given an alternative interpretation. Observe

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

that at $\theta = 0$ and $\theta = \frac{1}{2}\pi$, the whistler sound frequency can be represented as

$$\omega_s = \omega_0[\varepsilon_\perp(0)]^{-1/2}, \tag{3.16a}$$

$$\omega_s = \omega_0[\varepsilon_\parallel(0)]^{-1/2}, \tag{3.16b}$$

respectively, where $\varepsilon_\perp(0)$ and $\varepsilon_\parallel(0)$ are the static values of the dielectric functions (1.6a,b). It follows from (3.16) that in non-ideal plasmas, the whistler sound wave possesses the character of a vacuum electromagnetic wave screened by strongly correlated electrons of the medium. The plasma plays for the whistler sound the role of a dielectric crystal with refractive indices $[\varepsilon_\perp(0)]^{1/2}$ and $[\varepsilon_\parallel(0)]^{1/2}$. On the other hand, this wave does not exist in an ideal plasma: its frequency goes to zero as the plasma non-ideality decreases; see (3.9), (3.10) and (3.12)–(3.14), and also the next section.

4. Refractive indices of non-ideal magnetized plasmas

In this section we shall try to stress the characteristic features of the magnetized non-ideal plasma refractive index that distinguish it from that of a magnetized ideal plasma.

The zeros of the refractive indices determine the long-wavelength limit ($k \to 0$) for different waves. From (2.1), (2.3) and (2.12), we found three zeros:

$$\omega_\pm^{(0)} = \tfrac{1}{2}\{\mp\omega_B + [\omega_B^2 + 4(\omega_p^2 + \boldsymbol{\Omega}_\perp^2)]^{1/2}\}, \tag{4.1a}$$

$$\omega_3^{(0)} = (\omega_p^2 + \boldsymbol{\Omega}_\parallel^2)^{1/2}. \tag{4.1b}$$

corresponding to longitudinal ($N_\pm$) and transverse (N_2) propagation.

The frequencies that satisfy the relation $A(\omega, \mathbf{k}) = 0$, (see (1.4)) define the plasma resonances. Note that one of the refractive indices in (1.5) diverges as the frequency approaches the resonance value, $N_+^2 = -B/A$, while the second remains finite, $N_-^2 = -C/B$.

Within our approximation for the dielectric tensor, the condition $A(\omega, \mathbf{k}) = 0$, (1.4) produces a cubic equation with respect to ω^2. As a result, we obtain three resonance frequencies for non-ideal plasmas. Conversely, in ideal magnetized plasmas ($\boldsymbol{\Omega}_\perp = \boldsymbol{\Omega}_\parallel = 0$), only two resonances exist (we neglect the ion motion!). For the case of propagation almost parallel to the external magnetic field ($\theta \ll 1$), we found for the refractive indices three poles:

$$\omega_\pm^{(p)} = \tfrac{1}{2}[\mp\omega_B + (\omega_B^2 + 4\boldsymbol{\Omega}_\perp^2)^{1/2}], \tag{4.2a,b}$$

$$\omega_3^p = (\omega_p^2 + \boldsymbol{\Omega}_\parallel^2)^{1/2}\left(1 + \frac{\theta^2}{2}\frac{\omega_p^2\omega_B^2}{\omega_p^4 - \omega_p^2\omega_B^2 - \boldsymbol{\Omega}_\parallel^2\omega_B^2}\right). \tag{4.2c}$$

For the case of transverse propagation, the three resonances are

$$(\omega_\pm^{(p)})^2 = \boldsymbol{\Omega}_\perp^2 + \tfrac{1}{2}\{\omega_p^2 + \omega_B^2 \mp [(\omega_p^2 + \omega_B^2)^2 + 4\omega_B^2\boldsymbol{\Omega}_\perp^2]^{1/2}\}, \tag{4.3a,b}$$

$$\omega_3^{(p)} = \boldsymbol{\Omega}_\parallel. \tag{4.3c}$$

Having determined the zeros and poles, and taking into account the zero-frequency behaviour $N_\pm^2(\omega = 0) = 1 + \omega_p^2/\boldsymbol{\Omega}_\perp \equiv N_0^2$, and of the high-frequency relation $N_\pm^2(\omega \to \infty) \to 1$, the refractive indices can be plotted. In Fig. 1, the squares of the refractive indices for the case of inclined wave propagation are shown versus frequency (Tkachenko et al. 1998).

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

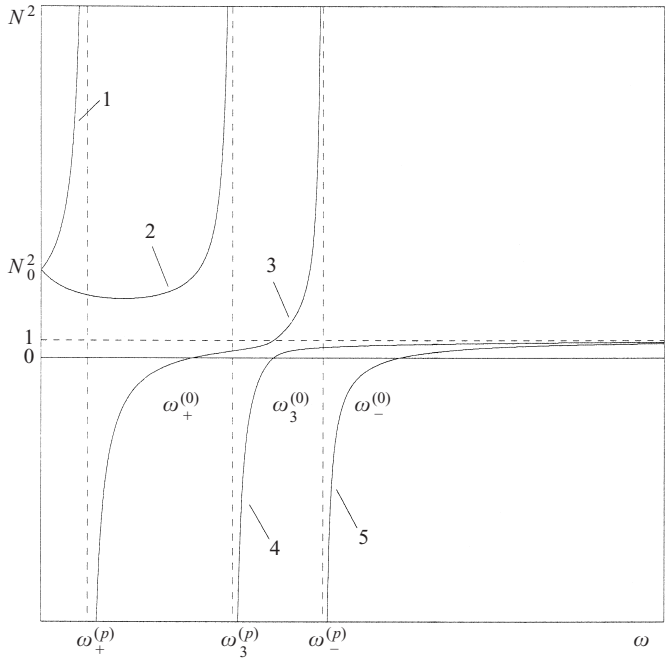


Figure 1. Frequency dependence of the refractive indices of coupled magnetized plasmas for an angle $0 < \theta < \pi$ (in arbitrary units). Curves 1 and 2 are for strongly coupled plasma whistling sound waves, curve 3 is for the slow extraordinary wave, curve 4 is for the ordinary wave, and curve 5 is for the fast extraordinary wave.

The branches of propagation ($N^2(\omega) > 0$) determine the eigenfrequencies ω_k. The dispersion relation for the different waves propagating in non-ideal plasmas is shown in Fig. 2 (Tkachenko et al. 1998). The eigenfrequencies ω_k are determined by (1.3), which, within the present approximation, is the fifth-order equation (with respect to ω^2). Thus, in a non-ideal magnetized plasma, there can exist five eigenmodes: ordinary and extraordinary whistling sound waves, and the slow extraordinary, the ordinary and the fast extraordinary waves. In contrast, in an ideal plasma, with neglect of the Alfvén wave, only four eigenfrequencies can be found. From Fig. 2, we observe how in the case of a non-ideal plasma the ideal-plasma helicon wave splits into two branches, which we have called non-ideal plasma whistling sound waves.

5. Estimate of the frequencies $\Omega_\parallel$ and $\Omega_\perp$

We limit ourselves to a simple estimate of the frequencies $\mathbf{\Omega}_\parallel$ and $\mathbf{\Omega}_\perp$ without including their magnetic field dependence, within the random-phase approximation (RPA), and in the hydrogen plasma model.

As is clear from Ortner et al. (1994), both correlation parameters are directly related to the zero separation value of the electron–ion correlation function,

$$h_{ei}(0) = \sum_{q \neq 0} S_{ei}(q), \tag{5.1}$$

where $S_{ei}(q)$ is the electron–ion structure factor. The latter can be estimated in a

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

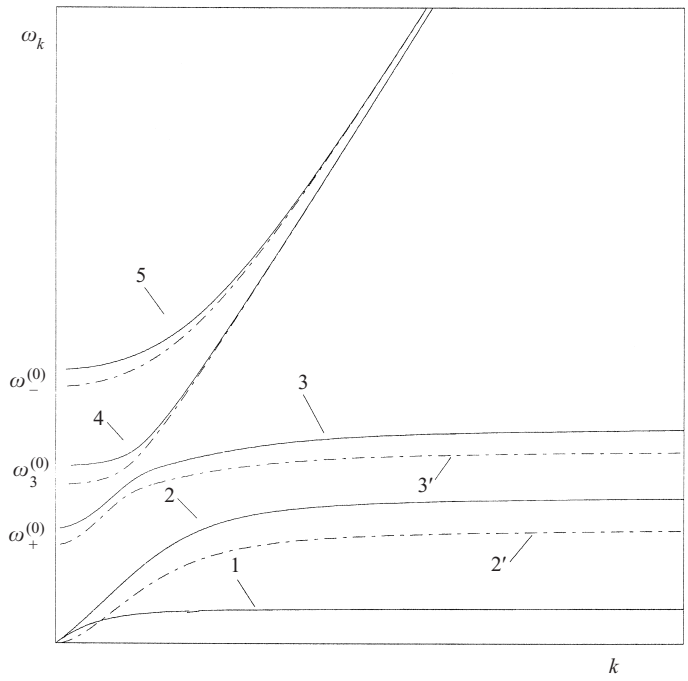


Figure 2. Effect of Coulomb correlations on the spectrum of electromagnetic waves in a magnetized plasma (in arbitrary units). Curves 1–5 are for waves in coupled plasmas (see Fig. 1), curve 2′ is for the helicon wave of an ideal plasma and curve 3′ is for the slow extraordinary wave in an ideal plasma ($0 < \theta < \pi$). Full curves are for a non-ideal plasma and dot-dashed curves are for an ideal plasma.

Coulomb system as (Abrikosov et al. 1965)

$$S_{ei}(q) = \frac{4\pi e^2}{n\beta q^2} \frac{\Pi_e(q)\Pi_i(q)}{\varepsilon(q)}. \tag{5.2}$$

Here $\Pi_e(q), \Pi_i(q)$, and $\varepsilon(q)$ are the static electronic and ionic polarization operators, and the dielectric function respectively, n is the number density of electrons (ions), $-e$ is the electronic charge, and $\beta^{-1} = k_B T$ is the system temperature in energy units. The ions can be considered as classical particles, so that we put

$$\Pi_i(q) = n\beta. \tag{5.3}$$

For $\Pi_e(q)$, we employ a rational interpolation (Adamyan and Tkachenko 1983)

$$\Pi_e(q) = \frac{\gamma^4/4\pi e^2}{q^2 + \gamma^4 \lambda_D^2}, \tag{5.4}$$

constructed to satisfy both long- and short-wavelength limiting conditions:

$$4\pi e^2 \Pi_e\,(q = 0) \equiv \lambda_D^2 = 4\pi e^2 n\beta \tag{5.5}$$

and

$$4\pi e^2 \Pi_e(q \to \infty) \approx \frac{\gamma^4}{q^2} \tag{5.6}$$

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

with

$$\gamma^4 = \frac{16\pi n e^2 m}{\hbar^2}. \tag{5.7}$$

With the substitution $q = 2k_F z$, the integral in (5.1) simplifies to

$$h_{ei}(0) = A \int_0^\infty \frac{z^2\, dz}{z^4 + Bz^2 + \frac{1}{6}A}, \tag{5.8}$$

with $A = 4\alpha r_s/\pi$, $\alpha = (4/9\pi)^{1/3} = 0.521$ and

$$B = \left(\frac{\pi}{3}\right)^{1/3} \frac{A}{4\Gamma} + \frac{A}{6\Theta}. \tag{5.9}$$

The usual notation has been introduced here: k_F is the Fermi wavenumber, $r_s = a/a_B$ is the Brueckner parameter, i.e. the Wigner–Seitz distance a in units of the Bohr radius, $\Gamma = \beta e^2/a$, and $\Theta = (\beta E_F)^{-1}$, E_F being the Fermi energy. Note that $r_s = \Gamma\Theta/0.543$.

The integral in (5.8) is calculated immediately (Prudnikov et al. 1981):

$$h_{ei}(0) = 2\alpha r_s \times \begin{cases} \dfrac{\sqrt{2}}{[B + (QS)^{1/2}]^{1/2} + [B - (QS)^{1/2}]^{1/2}} & (Q > 0), \\ \dfrac{1}{S^{1/2}} & (Q \leqslant 0), \end{cases} \tag{5.10}$$

where $S = B + (\frac{2}{3}A)^{1/2}$ and $Q = B - (\frac{2}{3}A)^{1/2}$.

In the case of weakly coupled plasmas with $\Gamma \to 0$,

$$h_{ei}(0) \approx 3.4\Gamma\Theta^{1/2} \approx 13.1 \left(\frac{1\,\mathrm{eV}}{T}\right)^{1/2}, \tag{5.11}$$

with the temperature T measured in eV. Note that the frequencies $\Omega_\parallel$ and $\Omega_\perp$ coincide in this approximation:

$$\Omega_\parallel = \Omega_\perp = \Omega = \omega_p[\tfrac{1}{3}h_{ei}(0)]^{1/2}, \tag{5.12}$$

so that

$$\Omega(\Gamma \to 0) \approx 3.6\omega_p \left(\frac{1\,\mathrm{eV}}{T}\right)^{1/4}. \tag{5.13}$$

On the other hand, in very dense systems,

$$h_{ei}(0) \approx 4.2529\Gamma^{1/2}\Theta, \tag{5.14}$$

and the coupling frequency can become greater than the plasma frequency:

$$\Omega(n \to \infty) \approx 1.1906\omega_p\Gamma^{1/4}\Theta^{1/2}.$$

The damping effects neglected throughout this paper can be represented roughly by the ideal-gas collision frequency

$$\omega_T = \left(\frac{k_B T}{m}\right)^{1/2} n^{1/3} = 0.35816\frac{\omega_p}{\Gamma^{1/2}}.$$

We could really neglect damping, and the whistling sound mode could propagate, provided that ck were much greater than ω_T. Then the inequality

$$\Omega > \frac{\omega_0\omega_B}{\omega_p}$$

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press

would reduce to

$$\frac{\Gamma^3\Theta^2}{\xi^4} > 10^{-2}, \tag{5.15}$$

where $\xi = \omega_B/\omega_p$, for $n \simeq 10^{22}$ cm^{-3} and $B \approx 10^5$ G $\xi \approx 3 \times 10^{-4}$, so that the inequality (5.15) is easily satisfied as soon as r_s and Γ are of the order of unity.

Remember that the magnetic effects represent only small corrections to the whistling sound frequency; see (3.8) and (3.14).

6. Conclusions

In this paper, the dispersion laws for electromagnetic waves in cold magnetized plasmas have been analyzed. Our analysis has been based on the expression for the plasma dielectric tensor obtained from the classical theory of moments without using perturbation parameters. Thus the cases of weak and strong Coulomb coupling have been considered, and the mode spectra determined for an arbitrary propagation angle with respect to the external magnetic field. A qualitative distinction between systems with weak and strong Coulomb coupling has been established in their low-frequency electromagnetic wave propagation spectra. It has been shown that the weakly coupled plasma helicon branch splits in non-ideal plasmas into two whistling sound branches. The thermodynamic dependence of the coupling parameters has been estimated.

Acknowledgements

This work was partly financed by the Deutsche Forschungsgemeinschaft and the Polytechnic University of Valencia, Spain.

References

Abrikosov, A. A., Gor'kov and L. P. Dzyaloshinski, I. Ye. 1965 *Quantum Field Theoretical Methods in Statistical Physics*. Pergamon, Oxford.

Adamyan, V. M. and Tkachenko, I. M. 1983 High-frequency electric conductivity of a collisional plasma. *Teplofiz. Vys. Temp.* **21**, 417 [*High Temp.* (*USA*) **21**, 307].

Akhiezer, A. I., Akhiezer, I. A., Polovin, R. V., Sitenko, A. G. and Stepanov, K. N. 1975 *Plasma Electrodynamics*. Pergamon, Oxford.

Aleksandrov, A. F., Bogdankevich, L. S. and Rukhadze, A. A. 1988 *Fundamentals of Plasma Electrodynamics*. Vyssh. Shkola, Moscow (in Russian).

Kalman, G. and Golden, K. I. 1990 Response function and plasmon dispersion for strongly coupled Coulomb liquids *Phys. Rev. A* **A41**, 5516.

Ortner, J., Rylyuk, V. M. & Tkachenko, I. M. 1994 Reflectivity of cold magnetized plasmas. *Phys. Rev.* **E50**, 4937.

Ortner, J., Rylyuk, V. M. & Tkachenko, I. M. 1998 Electromagnetic modes in cold magnetized strongly coupled plasmas. In *Strongly Coupled Coulomb Systems* (ed. G. J. Kalman, K. Blagoev and J. M. Rommel), p. 699. Plenum Press, New York.

Prudnikov, A. P., Brychkov, Yu. A. and Marichev, O. I. 1981 *Integrals and Series*, Vol. 1. Nauka, Moscow (in Russian).

Tkachenko, I. M., Ortner, J. and Rylyuk, V. M. 1998 Electromagnetic modes in cold magnetized strongly coupled plasmas. *Phys. Rev.* **E57**, 4846.

https://doi.org/10.1017/S0022377899008211 Published online by Cambridge University Press