J. Plasma Physics (1997), *vol.* 57, *part* 2, *pp.* 247–257. Printed in the United Kingdom
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Effect of a stochastic electron background on low-m tearing modes

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(Received 22 February 1995 and in revised form 22 September 1995)

The effect of a stochastic electron background on the stability of low-m tearing modes is studied. The coherent direct interaction approximation is applied to the drift-kinetic equation in order to include the effects of current density diffusion, electron density diffusion and magnetic field diffusion in the tearing mode equations. It is found that for the $m = 1$ mode in the collisionless regime the magnetic field diffusion destabilizes the mode further, while in the collisional regime these diffusions may not significantly affect the mode; for the $m \geqslant 2$ modes in the collisional and semicollisional regimes the electron density diffusion stabilizes the modes, while in the collisionless regime the effects of current density diffusion and electron density diffusion may balance each other without significantly affecting the modes.

1. Introduction

The tearing instability is one of the most important instabilities in toroidal plasmas (Furth *et al.* 1963; Rutherford 1973; Hazeltine *et al.* 1975; Drake and Lee 1977). For a tearing mode in a cylindrical geometry, since the equilibrium current density usually decreases monotonically along the minor radius of the tokamak, the free energy associated with the equilibrium current density gradient inside the mode-rational surface introduces vortex motion in the plasma around this surface in order to drive the tearing instability. The released free energy is partly absorbed outside the mode-rational surface, and what is left over is converted into Ohmic heating near this surface. The rate of release of the free energy is proportional to the rate of increase of the perturbed current parallel to the equilibrium magnetic field near the mode-rational surface. The perturbed current density induces space-charge density wherever $k_\parallel$ is non-zero, where $k_\parallel$ is the component of the wavevector of the tearing mode parallel to the equilibrium magnetic field. Since $k_\parallel = 0$ at the mode-rational surface and increases monotonically along the minor radius from this surface, the induced space-charge density also increases monotonically. This space-charge density, in turn, by virtue of its kinetic and Coulomb forces, suppresses the perturbed current density. Consequently, the perturbed current density decreases monotonically along the minor radius from the mode-rational surface, and is limited to a narrow region around this surface. It is quite obvious that if there is any mechanism by which the background electron orbits become stochastic, the tearing instability may be significantly affected (Meiss *et al.* 1982; Esarey *et al.* 1983). In this paper we study the evolution of low-m tearing modes in a background of stochastic electrons. To do this, we assume that lower-hybrid

https://doi.org/10.1017/S0022377896004655 Published online by Cambridge University Press

turbulence is present in the plasma; such turbulence is likely to be present in tokamak discharges owing to external excitations, as in the case of lower-hybrid heating experiments. The electrons that are in resonance with some of the lower-hybrid modes act as the stochastic electron background. In this background the rate of increase of the perturbed current density and broadening of the current channel width of the tearing mode clearly depend on electron density diffusion (EDD) and current density diffusion (CDD). Since there is an electron flux towards the mode-rational surface due to EDD, the electron density inside the current channel width of the mode increases further. This effect further increases the kinetic and Coulomb forces, which in turn further suppress the perturbed current density and the current channel width of the mode. Hence EDD acts against release of the free energy. On the other hand, CDD enhances the rate of release of the free energy by increasing the perturbed current density and broadening the current channel width. Thus EDD and CDD act against each other. Besides, the nonlinear interaction of the stochastic electrons with the inhomogeneous magnetic field of the tearing mode contributes to magnetic field diffusion (MFD). This enhances the rate of release of the free energy by increasing the perturbed current density and broadening the current channel width of the mode further. In the present work we study these three effects on stability of the $m = 1$ and $m \geqslant 2$ modes.

The plan of the paper is as follows. In Sec. 2 we derive the renormalized tearing mode equations in the background of lower-hybrid turbulence by considering nonlinear wave–particle interaction but ignoring wave–wave coupling. In Sec. 3 these equations are solved by employing the variational principle for the Fourier-transformed perturbed current density, and the dispersion relations so obtained are solved for the $m = 1$ and $m \geqslant 2$ modes in various limits. The results are discussed in Sec. 4.

2. Renormalized tearing mode equations

We consider the slab geometry of the tokamak with equilibrium electron density gradient along the x axis and the equilibrium shear magnetic field, $\mathbf{B} = B_0(\hat{\mathbf{z}} + (x/L_s)\,\hat{\mathbf{y}})$, where $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are the unit vectors along the y and z axes, and L_s is the shear length of the magnetic field. We study the effect of high-frequency and high-wavenumber lower-hybrid wave turbulence on the low-m tearing modes. We include only the wave–particle nonlinearities in the tearing mode equations, while discarding the wave–wave coupling terms. As a consequence, the energy exchange between the tearing mode and the lower-hybrid modes is not considered in the calculations. The potential field of the lower-hybrid turbulence can be expressed as $\phi = \Sigma_{\mathbf{k}',\omega'} \phi_{\mathbf{k}',\omega'} \exp[i(\mathbf{k}'.\mathbf{X} - \omega' t)] + \text{c.c.}$ It is assumed that the background turbulence can affect dynamics of the tearing mode only within the inner region around the mode-rational surface where the magnetic field lines decouple from the plasma. The perturbed distribution function $\tilde{\mathbf{f}}_{\mathbf{k},\omega}$ pertaining to the tearing mode can be expressed as $\tilde{f}_{\mathbf{k},\omega}(x) \exp[i(k_y y + k_z z - \omega t)] + \text{c.c.}$ The electromagnetic fields of the tearing mode are represented by $\mathbf{E}_{\mathbf{k},\omega} = -\nabla\phi_{\mathbf{k},\omega} - c^{-1}\,\partial \mathbf{A}_{\|\mathbf{k},\omega}/\partial t$ and $\mathbf{B}_{\mathbf{k},\omega} = \nabla \times \mathbf{A}_{\|\mathbf{k},\omega}$, where the subscript $\|$ indicates the component parallel to the equilibrium magnetic field. The drift-kinetic equation can be considered for the electrons in order to study the tearing mode as well as the lower-hybrid modes:

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$$\frac{\partial f}{\partial t}-\frac{e}{m}\frac{\partial}{\partial v_\parallel}\left[E_\parallel f+\frac{1}{c}(\mathbf{V}_\perp\times\mathbf{B})_\parallel f\right]+\boldsymbol{\nabla}\left[\left(v_\parallel\hat{\mathbf{b}}+\frac{c}{B_0}\mathbf{E}\times\hat{\mathbf{b}}+v_\parallel\frac{\mathbf{B}}{B_0}\right)f\right]=-\nu_c f, \quad (1)$$

where e, m, ν_c and $\hat{\mathbf{b}}$ are respectively the electron charge, the electron mass, the electron–ion collision frequency and the unit vector along $\mathbf{B}_0$. The quantity $\mathbf{V}_\perp$ is the perturbed electron velocity perpendicular to $\mathbf{B}_0$, and $\mathbf{B}$ is the perturbed magnetic field. While considering the above equation for the lower-hybrid mode, we should add the polarization drift, $(e/m)\,(i\omega'/\Omega_e^2)\,\mathbf{E}_{\perp\mathbf{k}',\omega'}$, where Ω_e is the electron–cyclotron frequency, to the $(c/B_0)\,\mathbf{E}\times\hat{\mathbf{b}}$ drift in the second set of square brackets. Linearizing (1), we obtain an equation for the perturbed distribution function $\tilde{f}_{\mathbf{k},\omega}$:

$$-i(\omega+i\nu_c-k_\parallel v_\parallel)\tilde{f}_{\mathbf{k},\omega}+N_{\mathbf{k},\omega}=-\frac{ef_0}{T_e}v_\parallel E_{\parallel\mathbf{k},\omega}-i\omega_{*e}\frac{ef_0}{T_e}L_{\mathbf{k},\omega}, \quad (2)$$

$$N_{\mathbf{k},\omega}=\left[-\frac{e}{mc}(\mathbf{V}_\perp\times\mathbf{B})_\parallel\frac{\partial f_0}{\partial v_\parallel}-\frac{c}{B_0}(\boldsymbol{\nabla}L\times\hat{\mathbf{b}}\,.\,\boldsymbol{\nabla}f)\right]_{\mathbf{k},\omega},$$

$$k_\parallel=\mathbf{k}\cdot\hat{\mathbf{b}},\quad \omega_{*e}=-k_y\frac{cT_e}{eB_0}\frac{1}{f_0}\frac{\partial f_0}{\partial x},\quad L=\phi-\frac{v_\parallel}{c}A_\parallel,$$

where $N_{\mathbf{k},\omega}$ represents the nonlinear part of the equation, f_0 is the Maxwellian distribution function and T_e is the electron temperature. Following the renormalization procedure (Dupree and Tetrault 1978; Diamond and Rosenbluth 1981; Meiss *et al.* 1982), we replace $N_{\mathbf{k},\omega}$ with the renormalized $N_{\mathbf{k},\omega}$ in the presence of the background lower-hybrid turbulence:

$$N_{\mathbf{k},\omega}\to-D_{\mathbf{k},\omega}\frac{e}{mc}\frac{\partial f_0}{\partial v_\parallel}\frac{\partial^2 A_{\parallel\mathbf{k},\omega}}{\partial x^2}-D_{\mathbf{k},\omega}\frac{\partial^2\tilde{f}_{\mathbf{k},\omega}}{\partial x^2}$$

$$+\sum_{\mathbf{k}',\omega'}\left(\frac{c}{B_0}\right)^2(k_y')^2\,|\phi_{\mathbf{k}',\omega'}|^2\,h_{\mathbf{k}-\mathbf{k}',\omega-\omega'}\frac{1}{\phi^*_{\mathbf{k}',\omega'}}\tilde{f}^*_{\mathbf{k}',\omega'}\frac{\partial^2 L_{\mathbf{k},\omega}}{\partial x^2}, \quad (3a)$$

$$D_{\mathbf{k},\omega}=\sum_{\mathbf{k}',\omega'}\left(\frac{c}{B_0}\right)^2(k_y')^2\,|\phi_{\mathbf{k}',\omega'}|^2\,h_{\mathbf{k}-\mathbf{k}',\omega-\omega'}, \quad (3b)$$

$$h_{\mathbf{k}-\mathbf{k}',\omega-\omega'}=[-i(\omega-\omega')+i(k_\parallel-k_\parallel')\,v_\parallel$$
$$-D_{\mathbf{k}-\mathbf{k}',\omega-\omega'}(\tilde{f}_{\mathbf{k}-\mathbf{k}',\omega-\omega'})^{-1}\,(k_y')^{-2}(\mathbf{k}'\times\hat{\mathbf{b}}\cdot\boldsymbol{\nabla})\,(\mathbf{k}'\times\hat{\mathbf{b}}\cdot\boldsymbol{\nabla}\tilde{f}_{\mathbf{k}-\mathbf{k}',\omega-\omega'})]^{-1}, \quad (3c)$$

where $D_{\mathbf{k},\omega}$ is the diffusion coefficient, $h_{\mathbf{k}-\mathbf{k}',\omega-\omega'}$ is the renormalized propagator and $\tilde{f}_{\mathbf{k}-\mathbf{k}',\omega-\omega'}$ is the perturbed electron distribution function pertaining to the lower-hybrid mode. Equation (2) can be easily solved by transformation into the moment equations, and the entire information required to study the stability of the tearing modes, in all limits, can be retained in the moment equations. They are

$$i\omega n_{e\,\mathbf{k},\omega}=i\omega_{*e}\frac{n_0\,e\phi_{\mathbf{k},\omega}}{T_e}-ir\tilde{k}_\parallel'\frac{1}{e}j_{e\parallel\mathbf{k},\omega}-D_{\mathbf{k},\omega}\frac{\partial^2 n_{e\mathbf{k},\omega}}{\partial r^2}$$

$$+\sum_{\mathbf{k}',\omega'}\left(\frac{c}{B_0}\right)^2(k_y')^2\,\phi_{\mathbf{k}',\omega'}h_{\mathbf{k}-\mathbf{k}',\omega-\omega'}(n^*_{e\mathbf{k}',\omega'})\frac{\partial^2\phi_{\mathbf{k},\omega}}{\partial r^2}, \quad (4a)$$

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$$i(\omega+i\nu_c)j_{e\|\mathbf{k},\omega}+i\tfrac{1}{2}(v_e^2\tilde{k}'_\|)\,r(en_{e\mathbf{k},\omega})+D_{\mathbf{k},\omega}\left[n_0\frac{e^2}{mc}\frac{\partial^2 A_{\|\mathbf{k},\omega}}{\partial r^2}+\frac{\partial^2 j_{e\|\mathbf{k},\omega}}{\partial r^2}\right]$$

$$-\sum_{\mathbf{k}',\omega'}\left(\frac{c}{B_0}\right)^2(k'_y)^2\,\phi_{\mathbf{k}',\omega'}h_{\mathbf{k}=\mathbf{k}',\omega-\omega'}\left[j^*_{e\|\mathbf{k}',\omega'}\frac{\partial^2\phi_{\mathbf{k},\omega}}{\partial r^2}+\frac{1}{2}\frac{e}{c}v_e^2\,n^*_{e\mathbf{k}',\omega'}\frac{\partial^2 A_{\|\mathbf{k},\omega}}{\partial r^2}\right]$$

$$=-n_0\frac{e^2}{m}E_{\|\mathbf{k},\omega}+i\omega_{*e}\,n_0\frac{e^2}{mc}A_{\|\mathbf{k},\omega}, \quad (4b)$$

where

$$n_{e\mathbf{k}',\omega'}=\phi_{\mathbf{k}',\omega'}\left[n_0\frac{e}{m}\frac{(k'_\perp)^2}{\Omega_e^2}-n_0\frac{e}{m}\frac{(k'_\|)^2}{(\omega')^2}\right],$$

$$j_{e\|\mathbf{k}',\omega'}=n_0\frac{e^2}{m}\frac{k'_\|}{\omega'}\phi_{\mathbf{k}',\omega'}, \quad n_e=\int dv_\| f, \quad j_{e\|}=-e\int dv_\|\,v_\| f,$$

$$v_e=\left(\frac{2T_e}{m}\right)^{1/2}, \quad k_\|=\tilde{k}'_\|\,r, \quad \tilde{k}'_\|=\frac{k_y}{L_s}, \quad r=x-x_s,$$

x_s is the radius of the mode-rational surface, and n_0 is the equilibrium electron number density. Since $j_{e\|\mathbf{k}',\omega'}$ is an odd function of $k'_\|$, the sum over all $\mathbf{k}'$ in the first term of the second set of square brackets in (4b) becomes zero. The last term in (4a) and the second term in the second set of square brackets in (4b) are of the order of $(k'_\|\,v_e/\omega')^2$ (or $(\rho_e\,k'_\perp)^2$) times the other nonlinear terms, and these two terms are dropped from further calculations because, for the lower-hybrid modes, $(k'_\|\,v_e/\omega')^2\ll 1$ and $(\rho_e\,k'_\perp)^2\ll 1$, where ρ_e is the electron Larmor radius. The first and second terms in the first set of square brackets of (4b) contribute respectively to the MFD and the CDD, and the third term on the right-hand side of (4a) contributes to the EDD. Substituting $j_{e\|\mathbf{k},\omega}=-(c/4\pi)\,(\partial^2/\partial r^2-k^2)\,A_{\|\mathbf{k},\omega}$ into (4a, b), one has

$$\left(i\omega+D_{\mathbf{k},\omega}\frac{\partial^2}{\partial r^2}\right)n_{e\mathbf{k},\omega}=i\omega_{*e}\frac{n_0\,e\phi_{\mathbf{k},\omega}}{T_e}+i\frac{c}{4\pi e}\tilde{k}'r\left(\frac{\partial^2}{\partial r^2}-k^2\right)A_{\|\mathbf{k},\omega}, \quad (5a)$$

$$i(\omega+i\nu_c)\left(\frac{\partial^2}{\partial r^2}-k^2\right)\mathrm{A}_{\|\mathbf{k},\omega}-\frac{1}{2}\frac{4\pi}{c}(\tilde{k}'_\|\,v_e^2)\,(ir)\,(en_{e\mathbf{k},\omega})$$

$$-D_{\mathbf{k},\omega}k_0^2\frac{\partial^2 A_{\|\mathbf{k},\omega}}{\partial r^2}+D_{\mathbf{k},\omega}\frac{\partial^2}{\partial r^2}\left(\frac{\partial^2}{\partial r^2}-k^2\right)A_{\|\mathbf{k},\omega}$$

$$=i(\omega-\omega_{*e})\,k_0^2A_{\|\mathbf{k},\omega}-i(\tilde{k}'_\|\,c)\,k_0^2(r\phi_{\mathbf{k},\omega}), \quad (5b)$$

where $k_0=\omega_{pe}/c$, $k^2=k_y^2+k_z^2$ and ω_{pe} is the electron plasma frequency. Ignoring the nonlinear contribution to the ion equation, we simply write the familiar equation for the ion density perturbation of the tearing mode (Drake and Lee 1977):

$$i\omega\,n_{i\mathbf{k},\omega}=i\omega_{*e}\frac{n_0\,e\phi_{\mathbf{k},\omega}}{T_e}+n_0\frac{e}{M}\frac{1}{\Omega_i^2}\nabla_\perp^2\,\phi_{\mathbf{k},\omega}[i(\omega-\omega_{*i})], \quad (5c)$$

where $\omega_{*i}=-(T_i/T_e)\,\omega_{*e}$, T_i is the ion temperature, M is the ion mass and Ω_i is the ion cyclotron frequency. Equations (5a–c) constitute the required set of equations, which are valid only within the inner region, to study the $m=1$ and

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$m \geqslant 2$ modes in the background of lower-hybrid turbulence. In the outer regions the plasma dynamics satisfies the ideal magnetohydrodynamic equations. In the absence of the nonlinear terms, the above equations are equivalent to those used previously to study the linear tearing modes (Hazeltine and Ross 1978). In the following section these equations are solved by applying a variational principle.

3. Variational solution

We take the Fourier transform

$$(A(p); n(p); \phi(p)) = \int_{-\infty}^{\infty} dr\, e^{-ipr} (A_{\|\mathbf{k},\omega}; n_{\mathbf{k},\omega}; \phi_{\mathbf{k},\omega}), \tag{6}$$

where p is the wavenumber along x. Then the diffusion coefficient can be expressed as

$$D = \sum_{\mathbf{k}',\omega'} \frac{(c/B_0)^2 (k'_y)^2 F(k'_x(x), k'_y, k'_\|(x))}{i(\omega' - k'_\| v_\|) + p^2 D}, \tag{7}$$

where $F \equiv |\phi_{\mathbf{k}',\omega'}|^2$. In (7) we have replaced the diffusion coefficients with the test-particle diffusion coefficient D, and have employed the Markovian approximation, $\mathbf{k}' \gg \mathbf{k}$ and $\omega' \gg \omega$. We may assume that the background lower-hybrid spectrum is sufficiently wide that the autocorrelation time of the turbulent field is smaller than the particle decorrelation time (Dupree 1967; Diamond and Rosenbluth 1981). Then the electrons that are in resonance with some of the lower-hybrid modes are responsible for the turbulent electron diffusion. Assuming that the background lower-hybrid modes are closely packed, we may convert the summation over k'_z into an integral:

$$\begin{aligned} D &= \sum_{\mathbf{k}'_\perp} \frac{R}{v_\|} \int dk'_z \left(\frac{c}{B_0}\right)^2 (k'_y)^2 \frac{iF(k'_x(x), k'_y, k'_\|(x))}{(k'_z + (x/L_s) k'_y - \omega'/v_\|) + ip^2 D/v_\|} \\ &= \sum_{\mathbf{k}'_\perp} \frac{\pi R}{v_\|} \left(\frac{c}{B_0}\right)^2 (k'_y)^2 F(k'_x(x), k'_y, k'_\|(x) = \omega/v_\|), \end{aligned} \tag{8}$$

where R is the major radius of the tokamak. Taking the Fourier transform of (5), we can obtain a relation for $\phi(p)$ in terms of $A(p)$ by employing the quasi-neutrality condition, $n_e(p) \approx n_i(p)$. Substituting the relations for $\phi(p)$ and $n_e(p)$ into the Fourier transform of (5*b*), we obtain a second-order differential equation in $A(p)$:

$$\begin{aligned} &-i(\omega - \omega_{*e}) \left(\frac{k_0}{x_A}\right)^2 \frac{\partial}{\partial p} \left\{ \frac{\partial J(p)}{\partial p} \frac{1}{p^2 + k_\perp^2} \left[1 - \frac{iDp^2}{\omega} + \frac{i\omega_{*e} k_D^2 D}{(\tilde{k}'_\| c)^2 x_A^2} - \frac{\omega_{*e}}{(\omega - \omega_{*e})} \frac{Dp^2}{i\omega} \right] \right\} \\ &+ \frac{(\tilde{k}'_\| v_e)^2}{2i\omega} \frac{\partial}{\partial p} \left[\frac{\partial J(p)}{\partial p} \left(1 + \frac{Dp^2}{i\omega} \right) \right] - i(\omega + i\nu_c) J(p) \\ &+ Dp^2 J(p) + Dk_0^2 J(p) - i(\omega - \omega_{*e}) \frac{k_0^2 J(p)}{p^2 + k^2} = 0, \end{aligned} \tag{9}$$

$$J(p) = (p^2 + k^2) A(p), \quad x_A^2 = \frac{\omega(\omega - \omega_{*i})}{(\tilde{k}'_\| v_A)^2}, \quad v_A = \frac{B_0}{(4\pi n_0 M)^{1/2}}, \quad k_D^2 = \frac{4\pi n_0 e^2}{T_e},$$

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where $J(p)$ is the Fourier-transformed perturbed current density and v_A is the Alfvén speed. It has been assumed in (9) that $|i\omega| > Dp^2$. Note that the integration range in the Fourier transform (6) is from $-\infty$ to ∞, but the basic equations (5) are valid only within the current-channel width. The full-range Fourier transform therefore makes sense only for the localized modes. Following Hazeltine *et al.* (1979), however, the information required for (9) to be valid even for the non-local modes can be included by making the replacement

$$\frac{1}{p^2+k^2} \rightarrow \frac{1}{p^2+k^2} - \frac{\pi}{2}\left(\frac{1}{k}+\frac{2}{\Delta'}\right)[\delta(p+ik)+\delta(p-ik)] \tag{10}$$

in the last term of (9), where

$$\Delta' = \frac{1}{A_{\|\mathbf{k},\omega}(0)} \frac{\partial A_{\|\mathbf{k},\omega}}{\partial r}\bigg|_{r=0-}^{r=0+} \tag{11}$$

is the discontinuity in the logarithmic derivative of the exterior magnetic field perturbation across the current-channel width, and δ is the Dirac delta function. Since (9) is self-adjoint, we can define the functional

$$\begin{aligned} S[J(p)] = \int_{-\infty}^{\infty} dp \Bigg\{ & i(\omega-\omega_{*e})\left(\frac{k_0}{x_A}\right)^2 \left(\frac{\partial J(p)}{\partial p}\right)^2 \frac{1}{p^2+k_\perp^2}\left[1 - \frac{iDp^2}{\omega} + \frac{i\omega_{*e}k_D^2 D}{(\tilde{k}_\|' c)^2 x_A^2}\right. \\ & \left. - \frac{\omega_{*e}}{\omega-\omega_{*e}}\frac{Dp^2}{i\omega}\right] - \frac{(\tilde{k}_\|' v_e)^2}{2i\omega}\left[\frac{\partial J(p)}{\partial p}\right]^2\left(1+\frac{Dp^2}{i\omega}\right) - i(\omega+i\nu_c)[J(p)]^2 \\ & + Dp^2[J(p)]^2 + Dk_0^2[J(p)]^2 - i(\omega-\omega_{*e})\frac{k_0^2[J(p)]^2}{p^2+k^2}\Bigg\}, \end{aligned} \tag{12}$$

which is variational, i.e. $\delta S = 0$ reproduces (9), and its value is zero, i.e. $S[J(p)] = 0$. We may choose a trial function $J(p) \propto \exp(-\frac{1}{2}\alpha p^2)$. Hence the current-channel width is approximately $|\alpha|^{1/2}$. Substituting (10) and the trial function into (12) and applying the condition $ik\alpha^{1/2} \ll 1$, we have

$$\begin{aligned} \frac{S(\alpha)}{\pi^{1/2}} = {} & i(\omega-\omega_{*e})\left(\frac{k_0}{x_A}\right)^2\left[\alpha^{3/2} - \frac{iD}{2\omega}\alpha^{1/2} + \frac{i\omega_{*e}k_D^2 D}{(\tilde{k}_\|' c)^2 x_A^2}\alpha^{3/2}\right. \\ & \left. - \frac{\omega_{*e}}{\omega-\omega_{*e}}\frac{D}{2i\omega}\alpha^{1/2}\right] - \frac{(\tilde{k}_\|' v_e)^2}{2i\omega}\left(\frac{1}{2}\alpha^{1/2} + \frac{3}{4}\frac{D}{i\omega}\alpha^{-1/2}\right) \\ & - i(\omega+i\nu_c)\alpha^{-1/2} + \tfrac{1}{2}D\alpha^{-3/2} + Dk_0^2\alpha^{-1/2} + 2ik_0^2(\omega-\omega_{*e})\alpha^{1/2} \\ & + \frac{2\pi^{1/2}}{\Delta'}k_0^2\, i(\omega-\omega_{*e}). \end{aligned} \tag{13}$$

For mathematical simplicity, for both the $m = 1$ mode and the $m \geqslant 2$ modes in the collisional regime, since $|\omega| \gg \omega_{*e}$, the third and fourth terms in the square brackets in (13) are ignored. The simultaneous equations $S(\alpha) = \partial S/\partial\alpha = 0$, yield the dispersion relations for the $m = 1$ and $m \geqslant 2$ modes.

3.1. The $m = 1$ mode

This mode is usually studied under the assumption that $\Delta' a \rightarrow \infty$, where a is the radius of the plasma column. The current-channel width of the mode is basically limited by the scalar potential field, while thermal effects can be

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ignored. In the absence of the diffusion terms, the function $S(\alpha)$ is an extremum at $\alpha = \alpha_1 \equiv -x_A^2$. It is assumed that the stochastic electron background may affect the extremum value only by changing the growth rate of the mode, while the basic structure of the relation remains unchanged. The dispersion relation for the $m = 1$ mode at this extremum value is $S(\alpha_1) = 0$, i.e.

$$i(\omega-\omega_{*e})\,k_0^2\,x_A^2 = -\left[i(\omega+i\nu_c)+\frac{\omega-\omega_{*e}}{\omega}\frac{1}{2}Dk_0^2-Dk_0^2+\frac{1}{2}\frac{D}{x_A^2}\right]. \tag{14}$$

For simplicity, substituting $x_A^2 \approx -(\omega+i\nu_c)/k_0^2(\omega-\omega_{*e})$ in the last term on the right-hand side and rearranging, we have

$$\omega(\omega-\omega_{*e})\,(\omega-\omega_{*i}) \approx -\left(\frac{c}{\omega_{pe}}\tilde{k}'_{\parallel}\,v_A\right)^2(\omega+i\nu_c)\left[1+\frac{1}{2}\frac{\omega-\omega_{*e}}{\omega}\frac{Dk_0^2}{i(\omega+i\nu)}\right.$$
$$\left.-\frac{Dk_0^2}{i(\omega+i\nu_c)}-\frac{1}{2}\frac{\omega-\omega_{*e}}{\omega+i\nu_c}\frac{Dk_0^2}{i(\omega+i\nu_c)}\right]. \tag{15}$$

The second term on the right-hand side has appeared owing to EDD, the third term has appeared owing to MFD, and the fourth term has appeared owing to CDD. The dispersion relation is solved in the collisionless and collisional regimes.

3.1.1. Collisionless $m = 1$ mode: $\omega_{*e} \ll |\omega| \gg \nu_c$. In this regime the frequency, the growth rate and the current-channel width of the mode are

$$\left.\begin{aligned}&\omega_r \approx \tfrac{1}{2}(\omega_{*e}+\omega_{*i}),\quad \gamma_1 \approx \tilde{\gamma}_1\left(1-\frac{1}{2}\frac{Dk_0^2}{\tilde{\gamma}_1}+\frac{Dk_0^2}{\tilde{\gamma}_1}+\frac{1}{2}\frac{Dk_0^2}{\tilde{\gamma}_1}\right),\\&|\alpha_1(\nu_c\to 0)|^{12} \approx \frac{(\omega_r^2+\gamma_1^2)^{1/2}}{\tilde{k}'_{\parallel}\,v_A},\quad \tilde{\gamma}_1 = \frac{c}{\omega_{pe}}\tilde{k}'_{\parallel}\,v_A.\end{aligned}\right\} \tag{16}$$

Thus the effects of EDD and CDD balance each other, while the MFD enhances the growth rate and the current-channel width further. There can be a significant growth rate only when $Dk_0^2/\tilde{\gamma}_1 > 1$, but the present results are not valid at these values because they are obtained by employing a perturbative treatment. However, these results indicate roughly that the instability may reach a state at which the MFD significantly enhances the growth rate and the current-channel width of the mode.

3.1.2. Collisional $m = 1$ mode: $\omega_{*e} \ll |\omega| \gg \nu_c$. In this regime the frequency, the growth rate and the current-channel width of the mode are

$$\omega_r \approx \tfrac{1}{3}(\omega_{*e}+\omega_{*i}),\quad \gamma_2 \approx \tilde{\gamma}_2\left(1-\frac{1}{2}\frac{Dk_0^2}{\nu_c}+\frac{Dk_0^2}{\nu_c}+\frac{1}{2}\frac{Dk_0^2}{\nu_c}\frac{\tilde{\gamma}_2}{\nu_c}\right)^{1/3},$$
$$|\alpha_1(|\omega|/\nu_c\to 0)|^{1/2} \approx \frac{(\omega_r^2+\gamma_2^2)^{1/2}}{\tilde{k}'_{\parallel}\,v_A},\quad \tilde{\gamma}_2 = \left(\frac{c}{\omega_{pe}}\tilde{k}'_{\parallel}\,v_A\right)^{2/3}\nu_c^{1/3}. \tag{17}$$

The collisional effect significantly suppresses the CDD. Since the EDD and the MFD are of the same order of magnitude, these effects may balance each other so that they do not significantly affect the instability.

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3.2. The $m \geqslant 2$ modes

These modes are usually solved under the assumption that $\Delta' a \approx 1$. Equation (13) is solved in the collisional, collisionless and semicollisional regimes.

3.2.1. Collisional $m \geqslant 2$ modes: $\omega_{*e} \ll |\omega| \ll \nu_c$. In this regime the current channel width is much larger than the ion-Larmor radius. Then, the scalar potential field basically limits the current channel width, while thermal effects can be ignored. The term $2ik_0^2(\omega-\omega_{*e})\alpha^{1/2}$ is usually much smaller than the one that is associated with $\alpha^{3/2}$. Hence we ignore the term $2ik_0^2(\omega-\omega_{*e})\alpha^{1/2}$ as well as the MFD term $k_0^2 D\alpha^{-1} \sim k_0^2 Dp^2$. Considering these simplifications, (13) becomes

$$\frac{S(\alpha)}{\pi^{1/2}} \approx i(\omega-\omega_{*e})\left(\frac{k_0}{x_A}\right)^2\left(\alpha^{3/2}-\frac{iD}{2\omega}\alpha^{1/2}\right)-i(\omega+i\nu_c)\alpha^{-1/2}+\tfrac{1}{2}D\alpha^{-3/2} + \frac{2\pi^{1/2}}{\Delta'}k_0^2[i(\omega-\omega_{*e})]. \tag{18}$$

In the absence of the diffusion terms, the function is an extremum at

$$\alpha = \alpha_2 \equiv \left[-\left(\frac{x_A}{k_0}\right)^2\frac{\omega+i\nu_c}{3(\omega-\omega_{*e})}\right]^{1/2}.$$

As in the case of the $m = 1$ mode, the dispersion relation at this extremum value is $S(\alpha_2) = 0$, i.e.

$$\left.\begin{aligned}\omega(\omega-\omega_{*i})(\omega-\omega_{*e})^3 &= i\left(1+\frac{\omega}{i\nu_c}\right)^3\Gamma^5\left[1-\frac{3^{1/2}}{8}\left(\frac{\omega-\omega_{*e}}{\omega+i\nu_c}\right)^{1/2}\frac{ik_0}{x_A}\frac{iD}{\omega}\right.\\ &\quad\left.+\frac{3^{3/2}}{8}\left(\frac{\omega-\omega_{*e}}{\omega+i\nu_c}\right)^{1/2}\frac{ik_0}{x_A}\frac{iD}{\omega+i\nu_c}\right]^4,\\ \Gamma^5 &= \left(\frac{2\Delta' a}{\pi^{1/2}3^{3/4}}\right)^4\frac{(\tilde{k}'_\parallel a)^2}{\tau_A^2\tau_s^3},\quad \tau_A = \frac{a}{v_A},\quad \gamma_s = \left(\frac{\omega_{pe}}{c}\right)^2\frac{a^2}{\nu_c}.\end{aligned}\right\} \tag{19}$$

The second term on the right-hand side has appeared owing to EDD, and the third term has appeared owing to CDD. The frequency, the growth rate and the current-channel width of the modes are

$$\left.\begin{aligned}\omega_r &\approx \tfrac{1}{4}(3\omega_{*e}+\omega_{*i}),\\ \gamma_3 &\approx \Gamma\left[1-\frac{3^{1/2}}{8}\frac{D}{\nu_c}\frac{k_0(\tilde{k}'_\parallel v_A)}{\Gamma}\left(\frac{\Gamma}{\nu_c}\right)^{1/2}\frac{\nu_c}{\Gamma}+\frac{3^{3/2}}{8}\frac{D}{\nu_c}\frac{k_0(\tilde{k}'_\parallel v_A)}{\Gamma}\left(\frac{\Gamma}{\nu_c}\right)^{1/2}\right]^{4/5},\\ |\alpha_2(|\omega|/\nu_c\to 0)| &\approx \left(\frac{\nu_c}{\gamma_3}\right)^{1/4}\left[\frac{(\omega_r^2+\gamma_3^2)^{1/2}}{3^{1/2}k_0^2(\tilde{k}'_\parallel v_A)}\right]^{1/2}.\end{aligned}\right\} \tag{20}$$

The effect of CDD is significantly suppressed by the collisional effect, while the EDD stabilizes the modes. They may be significantly stabilized whenever the EDD is sufficiently large:

$$D \approx \frac{8}{3^{1/2}}\frac{\nu_c^{1/2}\Gamma^{3/2}}{k_0(\tilde{k}'_\parallel v_A)}. \tag{21}$$

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Though the present results are not exactly valid at this level of diffusion, they do roughly indicate that, whenever the EDD inside the current-channel width reaches a level that satisfies (21), these modes may be significantly stabilized.

3.2.2. Collisionless $m \geqslant 2$ modes: $\omega_{*e} < |\omega| > \nu_c$. In the collisionless and semicollisional regimes the current-channel width of these modes is much smaller than the ion Larmor radius. Then thermal effects basically limit the current-channel width, while the effect of the scalar potential can be ignored. For these modes, we usually have $(\tilde{k}'_\parallel v_e)^2/8\omega \gg k_0^2(\omega - \omega_{*e})$; hence in (13) we ignore the terms $k_0^2(\omega-\omega_{*e})\alpha^{1/2}$ and $k_0^2 D\alpha^{-1/2}$. Then the function (13) becomes

$$\frac{S(\alpha)}{\pi^{1/2}} \approx -\frac{(\tilde{k}'_\parallel v_e)^2}{2i\omega}\left(\frac{1}{2}\alpha^{1/2} + \frac{3}{4}\frac{D}{i\omega}\alpha^{-1/2}\right) - i(\omega + i\nu_c)\alpha^{-1/2} + \tfrac{1}{2}D\alpha^{-3/2} + \frac{2\pi^{1/2}}{\Delta'}k_0^2[i(\omega-\omega_{*e})]. \tag{22}$$

In the absence of the diffusion terms, the function $S(\alpha)$ is an extremum at $\alpha = \alpha_3 \equiv -4\omega(\omega + i\nu_c)/(\tilde{k}'_\parallel v_e)^2$. The extremum value satisfies the consistency condition, i.e. $\mathrm{Re}\,\alpha > 0$, only when $\mathrm{Im}\,\omega > \mathrm{Re}\,\omega$. As in the previous cases, the dispersion relation at this extremum value is $S(\alpha_3) = 0$, i.e.

$$\omega - \omega_{*e} = i\tilde{\gamma}_0\left(\frac{\omega + i\nu_c}{\omega}\right)^{1/2}\left[1 - \frac{3}{16}\frac{(\tilde{k}'_\parallel v_e)^2 D}{i\omega^2(\omega + i\nu_c)} + \frac{1}{16}\frac{(\tilde{k}'_\parallel v_e)^2 D}{i\omega(\omega + i\nu_c)^2}\right], \tag{23}$$

where $\tilde{\gamma}_0 = \Delta'(\tilde{k}'_\parallel v_e)/2\pi^{1/2}k_0^2$. The second term on the right-hand side has appeared owing to EDD, and the third term has appeared owing to CDD. In the collisionless regime the frequency, the growth rate and the current-channel width of the modes are

$$\left.\begin{aligned} &\omega_r \approx \omega_{*e}, \quad \gamma_4 \approx \tilde{\gamma}_0\left[1 - \frac{3}{16}\frac{(\tilde{k}'_\parallel v_e)^2 D}{\gamma_0^3} + \frac{1}{16}\frac{(\tilde{k}'_\parallel v_e)^2 D}{\gamma_0^3}\right], \\ &|\alpha_3(\nu_c \to 0)|^{1/2} \approx \frac{2(\omega_{*e}^2 + \gamma_4^2)^{1/2}}{\tilde{k}'_\parallel v_e}. \end{aligned}\right\} \tag{24}$$

Thus the effects of EDD and CDD are of the same order of magnitude; hence they may balance each other so that they do not affect the instability.

3.2.3. Semicollisional $m \geqslant 2$ modes: $\omega_{*e} < |\omega| < \nu_c$. In this regime (23) yields the frequency, the growth rate and the current-channel width of the modes:

$$\left.\begin{aligned} &\omega_r \approx \tfrac{2}{3}\omega_{*e}, \quad \gamma_5 \approx \tilde{\gamma}_5\left[1 - \frac{3}{16}\frac{(\tilde{k}'_\parallel v_e)^2 D}{\tilde{\gamma}_5^2 \nu_c} + \frac{1}{16}\frac{(\tilde{k}'_\parallel v_e)^2 D}{\tilde{\gamma}_5^2 \nu_c}\frac{\tilde{\gamma}_5}{\nu_c}\right]^{2/3}, \\ &|\alpha_3(|\omega| < \nu_c)|^{1/2} \approx \frac{2(\omega_{*e}^2 + \gamma_5^2)^{1/2}}{\tilde{k}'_\parallel v_e}\left(\frac{\nu_c}{\gamma_5}\right)^{1/2}, \quad \tilde{\gamma}_5 = \tilde{\gamma}_0^{2/3}\nu_c^{1/3}. \end{aligned}\right\} \tag{25}$$

As in the collisional regime, the CDD is significantly suppressed by the collisional effect, while the EDD stabilizes the modes. They may be significantly

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stabilized whenever the EDD inside the current-channel width is sufficiently large: roughly

$$D \approx \frac{16}{3}\frac{\tilde{\gamma}_5^2 \nu_c}{(\tilde{k}'_\parallel v_e)^2}. \tag{26}$$

4. Results and discussion

It is clear from the present calculations that the EDD stabilizes the tearing modes, while the CDD and the MFD enhance the instability of the modes further. The effect of MFD is significant only for the $m = 1$ mode, while it is negligible for the $m \geqslant 2$ modes. For the $m = 1$ mode, in the collisionless regime the effects of EDD and CDD balance each other; however, the MFD enhances the growth rate and the current-channel width of the mode. In the collisional regime the CDD is considerably reduced owing to suppression in spatial variation of the perturbed current density by a strong collisional effect. On the other hand, the EDD and the MFD are still effective, and are of the same order of magnitude. Hence these effects may balance each other so that they do not significantly affect the instability.

For the $m \geqslant 2$ modes, in the collisional regime the CDD is considerably reduced owing to a strong collisional effect, while the EDD stabilizes these modes. Physically, whenever the EDD inside the current-channel width is sufficiently large, a critical state in the evolution of these modes may be reached so that the rate of increase of the perturbed current density due to the release of free energy is significantly suppressed by the increment in the scalar potential field due to EDD. In the collisionless regime the effects of EDD and CDD are of the same order of magnitude. Hence these effects may balance each other without significantly affecting these modes. In the semicollisional regime the effect of CDD is smaller than the effect of EDD owing to a collisional effect. Hence, as in the collisional regime, the EDD stabilizes these modes.

Finally, it is appropriate to comment on the neglect of the nonlinear contribution to the ion equation ($5c$). The ions in the lower-hybrid modes behave as if they are unmagnetized charge particles. The ions that are in resonance with some of the lower-hybrid modes act as a stochastic ion background. The velocity of the stochastic ions is much smaller than the velocity of the stochastic electrons, and hence the turbulent ion diffusion is much smaller than the turbulent electron diffusion. Therefore the neglect of the turbulent ion diffusion term in ($5c$) is clearly justified.

Acknowledgements

The authors would like to acknowledge Professor Abhijit Sen for his immense help at key points in the calculations, Professor B. Buti for her useful encouragement and the referees for their helpful suggestions. This work was supported by the Council of Scientific and Industrial Research, India.

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