STIFFNESS ANALYSIS OF LINEAR GUIDEWAYS WITHOUT PRELOAD

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ABSTRACT

Theoretical and experimental investigations of the stiffness of non-clearance linear guideways without preload were conducted. A theoretical approach that applied Hertz's contact theory to steel balls to derive the contact angle equation was proposed. The theoretical analysis showed that the stiffness of the linear guideway under variable vertical loads changes nonlinearly with the external load. Therefore, an experimental setup was proposed in which the stiffness curves of three blocks, differing in geometrical shape and assembled with steel balls, were measured under increased vertical load until the guideway began to plastically deform. The results showed that variation in both the width and the thickness of the block does not significantly affect stiffness. A comparison of the theoretical and experimental results revealed a relative error of 4.5%, indicating the correctness of the theoretical model. Based on theoretical equations, the main parameters that affect stiffness are conformity f_m, coefficient of contact deformation c_δ, reference diameter of the ball D_w, number of load-carrying rows i, number of load-carrying balls in one row Z, and initial contact angle α_0. The results are useful for determining the static/dynamic behavior and rigidity of linear guideways in the design stage.

Keywords: Linear guideway system, Stiffness, Zero preload, Hertz's contact theory.

1. INTRODUCTION

The linear guideway is one of the key components of a high-speed machine. Guideways with rolling balls are popular and effective owing their low wear and friction force, as compared to conventional sliding contact guides. The stiffness of a linear guideway is the key parameter for the static/dynamic behaviors of a machine. In most cases, the designer will select a zero-preload setting for the linear guideway system to ensure light friction performance and high durability. Zero (or light) preload is achieved by inserting rolling elements with the same diameter as the clearance between the block and rail grooves into the space between them, as shown in Fig. 1.

The classical solution for the contact stress and the deformation of two elastic bodies making contact at a single point was established by Hertz [1] in 1895. Johnson [2] showed that groove deformation increases with the load applied on the ball and that the contact stiffness of the rolling element interface increases in a nonlinear manner with the external load. Harris [3] and Brändlein [4] showed methods of elastic deformation and contact pressure in rolling bearing analyses based on Hertz's theory. Shimizu [5] reported a theoretical stiffness model of a linear guideway that assuming the block and rail were rigid and expect for the contact points with the balls. Ninomiya and Kato [6] proposed a stiffness equation for the linear guideway system and discussed methods to improve stiffness. Hsu [7] reported a series of theoretical studies in which the stiffness of a linear guideway under variable interference conditions of oversized steel balls were discussed.

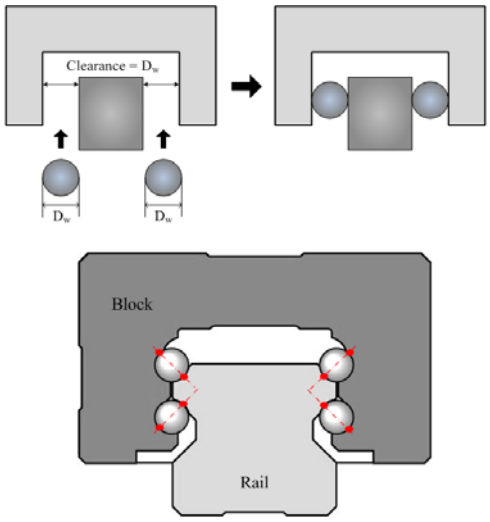


Fig. 1 Zero preload and contact configurations of a linear guideway

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Previous research on the stiffness of linear guideways has focused on either linear spring models or FE analysis. For example, Ohta and Tanaka [8] investigated the vertical stiffness of preloaded linear guideway-type ball bearings by incorporating the flexibility of the carriage and rail, as well as estimated deformations of the flexible model through finite element (FE) analysis. When they considered the Hertzian contacts between the balls and the carriage or rail in comparison with the results of the experiment, they found relative differences of 9 ~ 21%. Most of these and other investigations have been based either on the assumption that the contact angle is fixed or on a less systematic theoretical approach. Therefore, a nonlinear analysis of the contact mechanics of a linear guideway by considering changes in the contact angle under a load is worthy of investigation.

In this study, a theoretical approach that applies Hertz's contact theory to steel balls to derive the contact angle equation and vertical deflection equation are proposed. The vertical stiffness was calculated theoretically and verified by experimental measurements. Particular attention was paid to studying the stiffness of the linear guideway of rolling balls, angular contacts, face-to-face arrangement, and zero preload.

2. EXPERIMENTAL SETUP AND METHOD OF ANALYSIS

2.1 Experimental Setup for Measuring the Stiffness of a Linear Guideway

The experimental setup is shown in Fig. 2. To measure the deformations of the linear guideway (*i.e.*, vertical deflections on the order of micrometers), a highly rigid frame is used to minimize the deformation upon the application of a vertical load. A load cell (model no. UWD-25klb,) was used to measure the load on the block. The trapezoidal screw is used to apply downward force, which is increased by using a torque wrench to rotate the screw. The vertical deflection δ_y was obtained by using a Linear Variable Differential Transformer (LVDT) lever probe with a TESATONIC length-measuring instrument (model no. TT10). The probe was equipped with a 0.1-micrometer and a 1 micrometer, one micrometer has the measure range within +/−5 micrometer and the other is within +/−50 micrometer. The lever probe was positioned atop the central part of the block. Three types of blocks were assembled and tested with the same rail in the experiments. The specifications of the linear guideway are listed in Table 1. Model *U* was a standard-type block with a thickness of 12mm and a width of 60mm. Model *R* was 3mm thicker, and model *A* was 30mm wider. All the test blocks were prepared to investigate the effect of varying the block geometry on stiffness. Components such as the balls, blocks, and rail were made of steel with the following properties: Young modulus E = 207GPa, Poisson ratio ν = 0.30, and density ρ = 7800kg/m^3. The plastic parts of the block had the following properties: Young modulus E = 3.57GPa, Poisson ratio ν = 0.30, and density ρ = 1400kg/m^3.

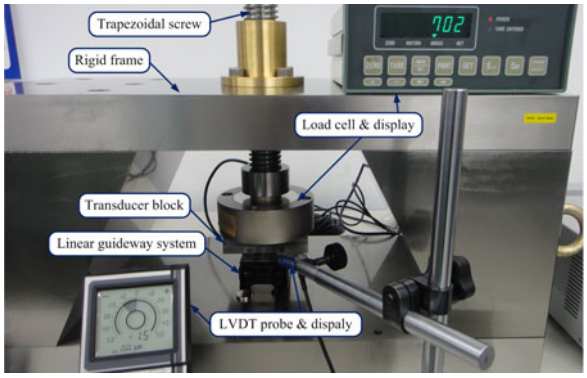
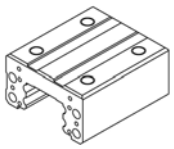
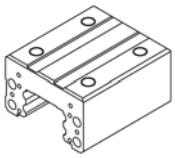
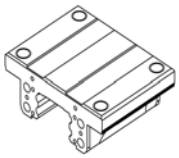
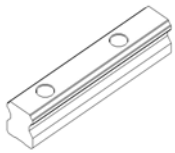


Fig. 2 Setup of experimental apparatus

Table 1 Specifications of tested linear guideways

	Nominal contact angle	45°
	Number of raceway grooves	4
	Conformity	0.52
	Total number of balls assembled in block	144
Model U	Number of balls in the raceway grooves	48
	Number of loaded balls in the raceway grooves	24
	Reference ball diameter	5.55625mm
	Block length	67.4mm
Model R	Distances between ball center	28mm
	Block thickness (Model *U*)	12mm
	Block thickness (Model *R*)	15mm
	Block thickness (Model *A*)	12mm
	Block width (Model *U*)	60mm
Model A	Block width (Model *R*)	60mm
	Block width (Model *A*)	90mm
	Rail width	28mm
	Rail height	26mm
	Rail length	100mm
Rail		

2.2 Theoretical Modeling

2.2.1 Change in Contact Angle of Linear Guideway After Loading

A simplistic model with a symmetric arrangement was employed (Fig. 3). The ball-raceway configurations with and without a vertical load applied to the block are shown by the solid and dotted-dashed lines, respectively. In the figure, points *A* and *B* are the centers of curvature for the raceway grooves of the block and rail, respectively. The raceway groove radii of the block and rail are r_b and r_e, respectively. Point *E* is the center of the

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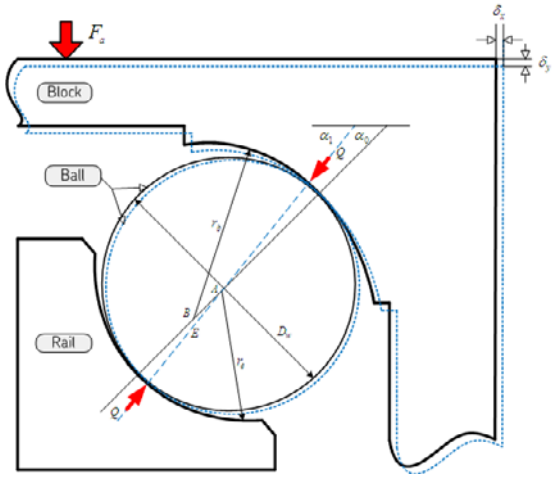


Fig. 3 Geometry of angular-contact linear guideway under vertical load

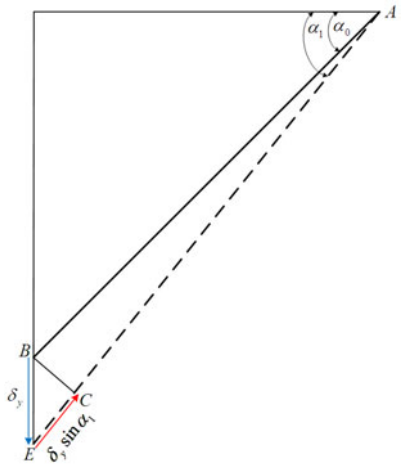


Fig. 4 Mathematical modeling of changes in the center of curvature on the block

raceway groove of the block after application of the vertical load. Reference balls of diameter D_w (the normal ball diameter) are inserted into the space between the block and rail grooves. The following assumptions were made.

1. All contact deformations occur in the elastic range.
2. Only a vertical load F_a is applied at the initial contact angle α_0
3. A vertical load causes vertical deflection δ_y and horizontal deflection δ_x changing the contact angle to α_1.
4. The total deformation δ_T is determined by the compression force Q at the ball.
5. The same steel material is used for the rail, block, and ball.

The theoretical analysis model employed in this study is a revised version of the rolling bearing analysis discussed in [9]. The static balance of the vertical deflection δ_y and horizontal deflection δ_x at the block is changed by the vertical load F_a. The ball in the raceway groove of the linear guideway is compressed by Q, and the total deformation δ_T at the ball can be expressed as follows.

$$\delta_y \sin\alpha_1 = \delta_T \left(1 - \frac{1}{\frac{\delta_y \tan\alpha_1}{\delta_x} - 1} \right) \tag{1}$$

In a previous study [9], the horizontal deflection did not significantly affect the change in the contact angle. For mathematical modeling purposes, a portion of Fig. 3 is enlarged in Fig. 4. In this study, the center of curvature of the block shifted vertically and the contact angle changed. The distance between the centers of the raceway grooves of the block and rail is L_g, *i.e.*, the distance between A and B. L_g can be expressed as follows.

$$\overline{AB} = L_g = r_b + r_e - D_w \tag{2}$$

The center of curvature of the block arc shifted from point B (zero external load) to point E (external load F_a), and the contact angle changed from α_0 to α_1. The distance between the centers of curvature increased from $\overline{AB}$ to $\overline{AE}$. Assuming a point C in $\overline{AE}$ such that $\overline{AB} = \overline{AC} = L_g$, we can express $\overline{AE}$ as follows.

$$\overline{AE} = \overline{AC} + \overline{CE} = L_g + \delta_y \sin\alpha_1 \tag{3}$$

Assuming that $\overline{BC}$ and $\overline{AC}$ are oriented at a right angle, the relation can be expressed as follows.

$$\overline{AE}\cos\alpha_1 = \overline{AB}\cos\alpha_0 \tag{4}$$

Substituting Eq. (3) into Eq. (4) yields the following equation.

$$\frac{\cos\alpha_0}{\cos\alpha_1} = 1 + \frac{\delta_y \sin\alpha_1}{L_g} \tag{5}$$

By using Hertz's theory [9,10], the total deformation δ_T with a point contact consisting of the same material and subjected to compression force Q can be obtained as follows:

$$\delta_T = c_\delta \frac{Q^{2/3}}{D_w^{1/3}} = c_\delta D_w \left(\frac{Q}{D_w^2} \right)^{2/3} \tag{6}$$

where c_δ is the coefficient of contact deformation. In the linear guideway, the radii of the groove at the block and the rail are identical and can be expressed as follows:

$$r_e = r_b = f_m D_w \tag{7}$$

where f_m is a design parameter called the conformity. In this study, a conformity of 0.52 and a coefficient of contact deformation c_δ of 4.514×10^{-4} were used in the analysis.

A previous study [3] indicated that the vertical load could be expressed as a function of the compression force as follows:

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$$F_a = Z \cdot Q \cdot \sin\alpha_1 \tag{8}$$

where Z is the number of load-carrying balls. If application of the face-to-face arrangement in the linear guideway is considered and i is the number of load-carrying rows, Eq. (8) can be expressed as follows.

$$Q = \frac{F_a}{i \cdot Z \cdot \sin\alpha_1} \tag{9}$$

After substituting Eqs. (1), (2), (6), (7), and (9) into the governing Eq. (5), the equation can be expressed as follows.

$$\frac{\cos\alpha_0}{\cos\alpha_1} - 1 = \frac{c_\delta}{2f_m - 1}\left(\frac{F_a}{i \cdot Z \cdot \sin\alpha_1 \cdot D_w^{\,2}}\right)^{2/3}\left(1 - \frac{1}{\frac{\delta_y \tan\alpha_1}{\delta_x} - 1}\right) \tag{10}$$

Assuming that the horizontal deflection δ_x is insignificant and can be ignored, Eq. (10) can be expressed as

$$\frac{\cos\alpha_0}{\cos\alpha_1} - 1 = \frac{c_\delta}{2f_m - 1}\left(\frac{F_a}{i \cdot Z \cdot \sin\alpha_1 \cdot D_w^{\,2}}\right)^{2/3} \tag{11}$$

2.2.2 Vertical Deflection of the Linear Guideway

Figure 5 shows the relation of the externally applied and transmitted load with the deflection of the linear guideway. The ball-raceway configurations with and without a vertical load applied on the block are shown by the dotted-dashed and solid lines, respectively. The distance between the centers of curvature increased from $\overline{AB}$ to $\overline{AE}$. Relations $\overline{AB}$ and $\overline{AE}$ is given by

$$\overline{AB} = r_e + r_b - D_w \tag{2}$$

$$\overline{AE} = r_e + r_b - D_1 \tag{12}$$

In Eq. (12), D_1 is the diameter after the compression force contacts the ball, and δ_T is the total deformation at the ball, where D_1 can be expressed as follows.

$$D_1 = D_w - \delta_T \tag{13}$$

Substituting Eq. (13) into Eq. (12) yields the following equation.

$$\overline{AE} = r_e + r_b - D_w + \delta_T \tag{14}$$

Assuming that the horizontal deflection is insignificant and can be ignored, the distance between the centers of curvature increases from $\overline{AB}$ to $\overline{AE}$. The relation between $\overline{AB}$ and $\overline{AE}$ can be expressed as follows.

$$\overline{AE}\cos\alpha_1 = \overline{AB}\cos\alpha_0 \tag{4}$$

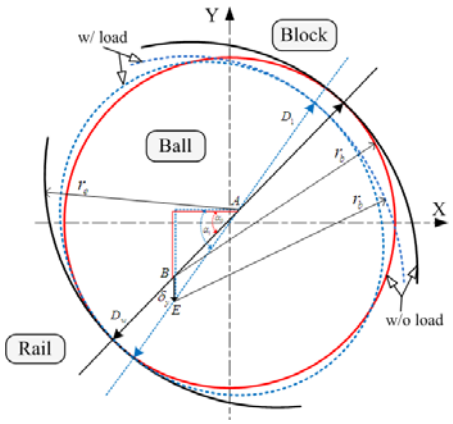


Fig. 5 Model of vertical deflection in the linear guideway system under vertical load

Substituting Eqs. (14) and (2) into Eq. (4) yields the following equation.

$$(r_e + r_b - D_w + \delta_T)\cos\alpha_1 = (r_e + r_b - D_w)\cos\alpha_0 \tag{15}$$

Substituting Eq. (7) into Eq. (15), we can express the total deformation δ_T as follows.

$$\delta_T = \frac{D_w(2f_m - 1)(\cos\alpha_0 - \cos\alpha_1)}{\cos\alpha_1} \tag{16}$$

As shown in Fig. 5, the vertical deflection δ_y in the linear guideway under vertical load should satisfy the following equation.

$$\delta_y = \overline{AE}\ \sin\alpha_1 - \overline{AB}\ \sin\alpha_0 \tag{17}$$

To solve the governing equation, δ_y can be expressed as follows.

$$\delta_y = \overline{AB}(\sin\alpha_1 - \sin\alpha_0) + \delta_T \sin\alpha_1 \tag{18}$$

Substituting Eqs. (2) and (16) into Eq. (18) yields the following equation.

$$\delta_y = D_w(2f_m - 1)(\sin\alpha_1 - \sin\alpha_0) + \left(\frac{D_w(2f_m - 1)(\cos\alpha_0 - \cos\alpha_1)}{\cos\alpha_1}\right)\sin\alpha_1 \tag{19}$$

2.2.3 Procedures for Calculating Vertical Stiffness

Considering the vertical load and vertical deflection, we can express the vertical stiffness of the linear guideway, K_s, as

$$K_s = \frac{F_a}{\delta_y}\ . \tag{20}$$

A flowchart of the calculation procedure is shown in Fig. 6. The detailed calculation procedure is described below.

Step 1. The downward force F_a, conformity f_m, reference ball diameter D_w, coefficient of contact deformation c_δ, number of load-carrying rows i, number of load-carrying balls Z, and initial contact angle α_0 are given. Then, the changed contact angle α_1 is calculated by using Eq. (11).

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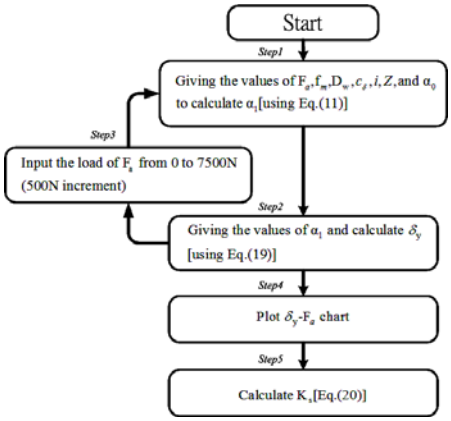


Fig. 6 Flowchart of vertical stiffness calculation

Step 2. From the obtained value of α_1, the vertical deflection δ_y is calculated by using Eq. (19).

Step 3. Steps 1 and 2 are repeated for numerous values of vertical deflection δ_y by increasing the downward load F_a from 0 to 7500N.

Step 4. The $\delta_y - F_a$ curve is plotted.

Step 5. The vertical stiffness K_s is calculated by using Eq. (20).

3. RESULTS AND DISCUSSION

The measured and theoretically analyzed results of the vertical load-deflection curves are shown in Fig. 7. Clearly, there is good agreement between the results of the theoretical model and the experimental measurements. Figure 7 also indicates that the variation in width and the variation in thickness at the block do not significantly affect the vertical stiffness in the experiments. Figure 8 shows a comparison of the theoretical and measured stiffness values for blocks of different geometry. This figure indicates that the stiffness of the linear guideway increases when the applied load increases. This observation shows the nonlinear characteristic of the stiffness of the linear guideway under a variable vertical load. In contrast to previous investigations on linear guideways [5,6,8], the ball is simulated as a linear or nonlinear spring. The experimental and theoretical results in this study agree more closely with Hertz's contact mechanics theory. These results show the nonlinear characteristic of the deflection and stiffness when a vertical load is applied. The nonlinearity of the rigidity and static/dynamic behavior of a linear guideway are two important properties in the design stage. In this study, the stiffness values of the theoretical model and experimental measurements for a 6000-N load are as follows.

$K_s^t = 352.9\ N/\mu m$ Calculated by theoretical model

$K_s^A = 336.3\ N/\mu m$ Experimentally measured-Model A

$K_s^U = 368.7\ N/\mu m$ Experimentally measured-Model U

$K_s^R = 336.3\ N/\mu m$ Experimentally measured-Model R

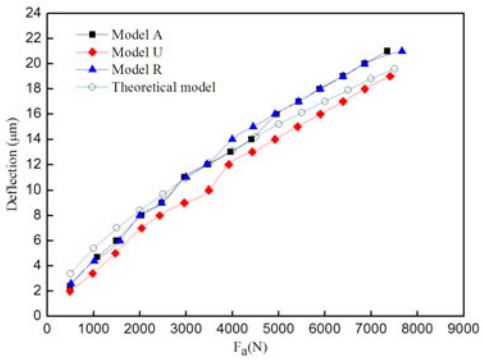


Fig. 7 Vertical deflection as a function of vertical load

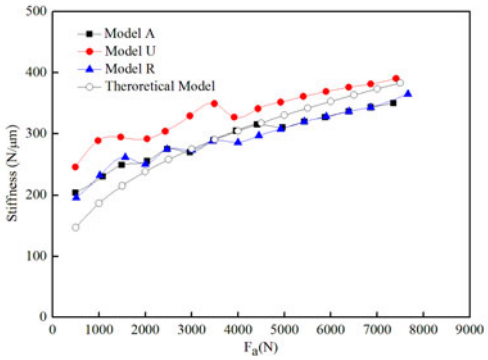


Fig. 8 Comparison of experimental and theoretical vertical stiffness

A comparison of the stiffness values obtained from experimental and theoretical analyses reveals that the relative maximum difference is 4.5%. This suggests that the theoretical model is reliable. However, a review of the parameters in Eqs. (11) and (19) yielded the following suggestions to achieve a high stiffness for linear guideways.

f_m: The curvature of the grooves should be close to the that of the balls (contact deformation is reduced and rigidity increased according to how close these two curvatures. However, the normal stress are increased on contact location)

c_δ: The material in the rail, block and ball should have a high modulus of elasticity. Higher modulus of elasticity reduced the contact deformation and improve the rigidity.

D_w: The rolling element diameter or interference fit can be increased to increase the loading capability.

i: The number of load-carrying rows should be increased (loading capability increased).

Z: The number of load-carrying balls in each row should be increased (loading capability increased).

α_0: The contact angle should be increased (although stiffness in sideward direction will decrease).

It is noted that in Fig. 7, model U always has the smallest deflection. As shown in Table 1, the block thickness of model U is smaller than model R and the block width of model U is smaller than model A. These make the block of model U is weakest of all types. This may makes the block bended more easily.

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This bending increases the pressure angle of the ball. The increasing of the pressure angle makes the model *U* is stiffer than the other kinds of linear guideway.

4. CONCLUSIONS

The following conclusions are drawn from the experimental and calculated results.

1. An equation for the change in contact angle based on Hertz's contact theory and a vertical deflection equation based on a rigid body model of block and rail were introduced to investigate the stiffness of the linear guideway.
2. Theoretical expressions of the stiffness of the linear guideway were presented. The numerical results agreed closely with the experimentally measured stiffness.
3. The study demonstrated nonlinearity either in the deflection or in the stiffness when a vertical load was applied to the zero preload linear guideway.
4. This study provides an accurate estimation procedure to determine the deflection and stiffness of linear guideways. In addition, it indicates significant parameters that determine the vertical stiffness of linear guideways.

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