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NOISY SUNSPOTS AND BANK RUNS

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In the existing literature, panic-based bank runs are triggered by a commonly acknowledged and observed sunspot signal. There are only two equilibrium realizations resulting from the commonly observed sunspot signal: Everyone runs or no one runs. I consider a more general and more realistic situation in which consumers observe noisy private sunspot signals. If the noise in the signals is sufficiently small, there exists a proper correlated equilibrium for some demand deposit contracts. A full bank run, a partial bank run (in which some consumers panic whereas others do not), or no bank run occurs, depending on the realization of the sunspot signals. If the probabilities of runs are small, the optimal demand deposit contract tolerates full and partial bank runs.

Keywords: Sunspot Equilibrium, Correlated Equilibrium, Imperfect Coordination, Imperfect Information

1. INTRODUCTION

In a panic-based bank run, the bank fails when a sufficiently large number of consumers withdraw their deposits.[1] The bank may survive if the number of withdrawals is small. A bank run in which all consumers panic and withdraw is a special case, and that a bank always fails whenever there is a run is also special. However, the focus of the existing literature has been on these two situations. This is the result of assuming a commonly observed extrinsic signal (sunspot) as the trigger of the panic-based runs.

Without the sunspot signals, bank runs cannot exist in equilibrium—if consumers anticipate a bank run, they will not deposit at the bank, whereas if consumers anticipate no bank run, they will deposit and will not run ex post. The assumption of a commonly observed sunspot signal in the existing literature allows consumers to always observe the same sunspot information and to perfectly coordinate their actions by running on the bank in exactly the same states of nature. In this way, the fact that all agents are observing the same signal makes it easier to construct a bank run in equilibrium, and by construction, these runs are necessarily full runs in which all consumers withdraw and the bank fails.

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Arguably, having all consumers observe the same sunspot signal is a strong assumption. Sunspots are purely extrinsic variables that do not affect any of the fundamentals; it is hard to identify a single sunspot that everyone agrees is important. The resulting full bank run from the perfectly correlated information structure is also too special. Suppose instead that one assumes noisy sunspot information. It is important to check whether the theory of panic-based bank runs is robust to this change in assumptions. I perform such a check in this paper. I relax the assumption of a commonly observed sunspot signal and adopt a more general extrinsic randomizing device. Then I show how much correlation in the signals is needed to generate bank runs in equilibrium and determine the size of the set of sunspot signals that can generate bank runs.

I consider an economy in which consumers receive noisy private sunspot signals. Receiving his own sunspot signal, a consumer tries to infer the signals that others observe and the actions they will take. These private sunspot signals can be viewed as a representation of the difference in information sources available to consumers. For example, consumers reading the same newspaper or coming from a similar background share the same information sources and tend to make the same decision. Kelly and O'Grada (2000) provide relevant empirical evidence. Their study of the behavior of recent Irish immigrant depositors in two banking panics in the 1850s shows that depositors' social networks, more specifically, their counties of origin in Ireland, are the prime determinant of their withdrawal decisions.[2] The present paper, by employing private sunspot signals, provides a formal model to capture this situation: One's judgment of the economy is based on one's own information source. How does an individual consumer make his decisions, knowing others might have different information and might make different judgments, which may affect his own payoff? Can consumers hold different beliefs and make different decisions in equilibrium? It turns out that there are economies in which the optimal banking contract tolerates full runs and partial runs depending on the realization of the sunspot variable.

Partial runs are the more general and more realistic predictions generated by the noisy sunspot signals introduced in this paper: In a partial bank run, some but not all consumers withdraw their deposits and the banks do not necessarily fail.[3] Although partial bank runs (in particular, the partial runs based on fundamental uncertainties) can be generated through other mechanisms identified in the literature [see Allen and Gale (1998), Goldstein and Pauzner (2005), and Ennis and Keister (2009), among others, for partial runs], this paper suggests that the imperfectly correlated information structure can be a convenient modeling tool.[4]

For simplicity, there are only two groups of consumers in the model. Consumers in the same group observe the same sunspot signal, which takes a value of either 0 or 1. Signals are imperfectly correlated. They can be viewed as a public signal plus some noise. This is the minimum structure required for the analysis of imperfect coordination. I find that if the noise in the signals that consumers obtain is sufficiently small, there exists a proper correlated equilibrium for some demand deposit contracts. As the noise becomes large, the set of demand deposit contracts that

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can generate the correlated equilibrium shrinks. In this correlated equilibrium, a full run, a partial run, or no run occurs, depending on the realization of the signals. The probabilities of full bank runs and partial bank runs are determined by the joint distribution of the sunspot signals. If the probabilities of full bank runs and partial bank runs are below some threshold, the optimal demand deposit contract will be accepted, and full and partial runs will be tolerated.[5]

The approach of private sunspot signals shares similarities with the approach of global games [see Carlsson and van Damme (1993), Morris and Shin (1998), and Goldstein and Pauzner (2005)]. Strategic complementarity and noisy signals are two critical elements in both. In the global games approach, consumers receive noisy signals about the fundamentals. Consumers run on the bank if their signals are above some threshold and wait otherwise. When noise becomes extremely small, the equilibrium outcome is uniquely determined by the fundamentals; sunspot signals do not play a role in determining the outcomes. This paper obtains a different result: In an environment with no fundamental uncertainty, if the noise in the sunspot signals becomes extremely small, then we are back to the traditional public sunspot information setup in which two equilibrium outcomes exist: a full bank run and no bank run, depending on the realization of the sunspot signal. The difference is driven by the fact that the global games approach assumes dominance regions for parameter values of the fundamentals. If the parameter values are in the upper (lower) dominance region, an individual consumer's strongly dominant strategy is to wait (withdraw) even if all others withdraw (wait) because the fundamentals are extremely strong (weak).[6] Then, iterated deletion of dominated strategies results in a unique equilibrium. The dominance regions for parameter values of the fundamentals do not exist in the sunspot signals, which by definition are unrelated to fundamentals. Extending the information structure to include a continuum of signals will not eliminate the multiplicity because it does not help introduce the dominance regions to the economy.[7]

Another major line of literature on bank runs attributes the origin of the runs to uncertainty in fundamentals. The view based on fundamental-based bank runs argues that bank runs occur when consumers learn negative information about the bank's portfolio returns or about the aggregate liquidity shock.[8] The view based on panic-based runs argues that the bank and bank runs are inherently intertwined because bank contracts provide short-term liquidity, whereas the portfolio matures only in the long term.[9] As a result, a panic-based run is self-fulfilling even in the absence of fundamental uncertainty. Although the runs in this paper are panic-based, the sunspot signals can be viewed as the uncertainty in the fundamentals taken to the limit [as in Manuelli and Peck (1992)]. Focusing on panic-based runs serves two purposes: (i) to check the robustness of the models on panic-based bank runs and (ii) to explain the importance of consumer information networks in determining their behavior.

The remainder of the paper is organized as follows: Section 2 introduces the model setup. Section 3 discusses the equilibrium in the deposit game. Section 4 provides some conclusions.

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2. THE MODEL

The basic setup of the model is similar to that in Diamond and Dybvig (1983), except that consumers observe private sunspot signals. There are three periods, $t = 0, 1, 2$, and a measure 1 of consumers in the economy. Period 0 is called ex ante; periods 1 and 2 are ex post. Each consumer is endowed with 1 unit of the consumption good in period 0 and nothing in periods 1 and 2. A consumer is an impatient type with probability α and a patient type with probability $1-\alpha$. Denote a consumer's type by τ, $\tau \in \{\text{impatient, patient}\}$. Impatient consumers derive utility only from consumption in period 1. Their utility is described by $u(c_1)$, where c_1 is the consumption received in period 1. Patient consumers consume in the last period. If a patient consumer receives consumption in period 1, he can store it costlessly and consume it in period 2. Thus, a patient consumer's utility is described by $u(c_1 + c_2)$, where c_2 is the consumption received in period 2. The coefficient of relative risk aversion (CRRA) of the utility function, $-xu''(x)/u'(x)$, is greater than 1 for $x \geq 1$. The utility function is normalized to 0 at $x = 0$; that is, $u(0) = 0$. Whether a consumer is patient or impatient is his private information and is revealed to the individual consumer in period 1.

Consumers have two technologies for saving: investment and storage. One unit of consumption good invested in period 0 yields one unit in period 1 and $R > 1$ units in period 2. One unit of consumption good placed in storage yields one unit of consumption good in the following period.

The banking market is competitive. The representative bank takes deposits from the consumers and makes the investment. For convenience, I assume that consumers deposit all or none of their endowments at the bank. The bank offers a demand deposit contract to the consumers and faces a sequential service constraint [Wallace (1988)].[10] The bank pays a fixed amount of the consumption good to consumers who withdraw in period 1, until it runs out of resources, and distributes the remaining resources equally (if there are any) among the consumers who wait until the last period.[11] Denote the payments in periods 1 and 2 by c^1 and c^2, respectively. Then

$$c^2 = \max\left\{\frac{1 - nc^1}{1 - n}R, 0\right\},$$

where n is the measure of consumers who withdraw in period 1.

Consumers are grouped into two groups. Group i observes a sunspot signal $\theta_i \in \{0, 1\}$, $i = 1, 2$. The joint distribution of θ_1 and θ_2 is denoted by $\Pr(\theta_1, \theta_2) = p_{\theta_1\theta_2}$ [e.g., $\Pr(\theta_1 = 1, \theta_2 = 1) = p_{11}$], $\sum_{\theta_2=0}^{1}\sum_{\theta_1=0}^{1} p_{\theta_1\theta_2} = 1$. Consumers in the same group observe the same sunspot information and conjecture the sunspot signal that the other group observes. In the rest of the paper, I use the terminology "private signals" to mean "private signals to groups." The private signals can be thought as follows: There is a public sunspot signal. However, consumers cannot observe the public sunspot perfectly. Instead, each group observes the public sunspot with noise, or a private sunspot signal. Without loss of generality, I assume that $p_{01} = p_{10} = \rho$ to facilitate the presentation, where ρ measures

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noise in signals. When $\rho = 0$, we have a public sunspot signal, as in the existing literature. Group i has a measure of n_i consumers. By the law of large numbers, the measure of impatient consumers in group i is αn_i. Consumers know ex ante that there are two groups, but until period 1 they do not know in which group they will be.

The sequence of timing of the banking game is as follows.

Period 0: Bank announces the contract.
Consumers make deposit decisions.
Period 1: Consumers get to know their own consumption types and which groups they are in, and they observe the realizations of their sunspot signals. Consumers make withdrawal decisions.
Period 2: Bank allocates the remaining resources to the rest of the consumers.

A consumer's strategy specifies a decision to accept or reject the contract in period 0 as a function of the bank's contract and a decision to withdraw or wait in period 1 as a function of the consumer's consumption type, which group he or she is in, and the value of the sunspot signal he or she observes. The decision in period 2 is simple: A consumer withdraws if he or she has not done so in period 1. Denote a consumer's strategy by $y = (y_0(c^1), y_1(\tau, i, \theta_i))$, where y_0 and y_1 are the decisions in periods 0 and 1, respectively.

I look for a subgame perfect Nash–Aumann equilibrium in which (1) the sunspot signals constitute a correlated equilibrium [following Aumann (1987)] in period 1 wherever possible given a contract, and (2) the correlated equilibrium is subgame perfect in the sense that consumers accept the contract in period 0 if and only if the expected utility under the contract is higher than under autarky.

3. DEPOSIT GAME

3.1. Commonly Observed Sunspot Signal: A Benchmark Case

In autarky, impatient consumers consume 1 unit of consumption good in period 1, whereas patient consumers consume R units in period 2. A consumer's expected utility in autarky is

$$\hat{W}^{\mathrm{AUT}} = \alpha u(1) + (1 - \alpha)u(R).$$

Instead of being in autarky, consumers can deposit in a bank in period 0 and receive payments in period 1 or 2, depending on when they withdraw. A feasible demand deposit contract satisfies the participation constraint

$$\frac{1 - \alpha c^1}{1 - \alpha} R \geq c^1, \tag{1}$$

which says that the consumers who wait until the last period should be paid more than those who withdraw in period 1, if the impatient consumers are the only ones who withdraw their deposits in period 1. In other words, if a contract satisfies (1), the strategy $\{y_1(\text{patient}) = \text{wait}, y_1(\text{impatient}) = \text{withdraw}\}$ constitutes a Nash

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equilibrium in period 1. The ex ante expected utility in such an equilibrium is

$$W^{\text{no-run}}(c^1) = \alpha u(c^1) + (1-\alpha)u\left(\frac{1-\alpha c^1}{1-\alpha}R\right).$$

Let M denote the set that contains all contracts described by c^1 that satisfy (1). Solving (1), we get $M = [0, R/(1-\alpha+\alpha R)]$, which is the set that includes all feasible banking contracts in the traditional bank run literature.

A banking contract in M admits bank runs if there is a Nash equilibrium in period 1 in which all consumers play $\{y_1 = \text{withdraw}\}$ regardless of their consumption types. A contract that allows a bank run is called a run-admitting contract. Otherwise, the contract is run-proof. Let M^{RA} and M^{RP} denote the sets that contain all run-admitting contracts and all run-proof contracts, respectively. Both sets are subsets of M, and they are complements to each other. In this simple model, a contract in M is run-admitting if $c^1 > 1$ and run-proof if $c^1 \leq 1$. Because the CRRA of the utility function is greater than 1, the best run-proof contract provides $c^1 = 1$, which results in the same allocation as in autarky.

Given a run-admitting contract, in the event that all people withdraw, each consumer receives payment with probability $1/c^1$. The ex ante expected utility in a run is

$$W^{\text{run}}(c^1) = \frac{1}{c^1}u(c^1),$$

which is strictly lower than $\hat{W}^{\text{AUT}}$ by concavity of the utility function. Hence, if consumers anticipate that a bank run will occur in period 1, then they will not accept such a contract in period 0, and a bank run cannot be the outcome of a subgame perfect Nash equilibrium.

Nevertheless, panic-based runs seem to be a relevant phenomenon.[12] To model this type of bank run, the existing literature relies on a sunspot signal to coordinate the actions of consumers. Whether a bank run occurs after consumers accept the contract is modeled as dependent on a publicly observed sunspot variable $\theta \in \{0, 1\}$. Let $p_\theta = \Pr(\theta)$. The sunspot can generate bank runs if there is a subgame perfect Nash equilibrium in which consumers play the following strategy: $\{y_0(c^1) = \text{accept},\ y_1(\text{patient}, \theta = 0) = \text{wait},\ y_1(\text{patient}, \theta = 1) = \text{withdraw},\ y_1(\text{impatient}, \theta = 0, 1) = \text{withdraw}\}$ and the contract is optimal in the sense that it maximizes consumers' expected utility.

Given the sunspot information structure, the probability of a run is p_1. The optimal run-admitting contract solves

$$\max_{c^1} p_1 W^{\text{run}}(c^1) + p_0 W^{\text{no-run}}(c^1)$$
$$\text{s.t. } c^1 \in M^{\text{RA}}.$$

If the probability of bank runs is small, the optimal run-admitting contract yields higher expected utility than autarky so it will be accepted, and bank runs can occur in equilibrium [Cooper and Ross (1998); see also Ennis and Keister (2006)].

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3.2. Noisy Sunspot Signals

The assumption in the existing literature that all consumers observe exactly the same signal makes it easier to construct a run in equilibrium: Consumers are coordinated perfectly to take the same actions by running on the bank in the same states of nature. However, we do not have a good theory to explain where the sunspot variables come from and which sunspot variables consumers use as their guidance for actions. Nor do we know whether all consumers agree on the same sunspot signal and if they do not, how different their signals are. In this section, I relax the assumption of a commonly observed signal and show how consumers are coordinated on actions if their signals are different and how much symmetry there must be in the signals to still generate bank runs in equilibrium.

Sets of contracts. I look for an equilibrium in which consumers are coordinated by the noisy private sunspot signals in period 1 given a contract. In particular, I look for a correlated equilibrium in period 1 that is based on the joint distribution of the sunspot signals. I exclude the degenerate cases in which either everyone runs or no one runs. In this sense, I use "correlated equilibrium" for "proper correlated equilibrium." I focus on the case in which consumers in the same group take the same action. A contract allows a correlated equilibrium in period 1 if consumers play the strategy $\{y_1(\text{patient}, i, \theta_i = 1) = \text{run},\ y_1(\text{patient}, i, \theta_i = 0) = \text{wait},\ y_1(\text{impatient}, i, \theta_i = 0, 1) = \text{run},\ i = 1, 2\}$. The contract must be individually rational in the sense that it is accepted only if the expected utility from depositing in the bank is at least equal to that under autarky. The contract also must be optimal in the sense that it maximizes the consumers' ex ante expected utility among all the feasible contracts. Because the best run-proof contract results in the same allocation as autarky, I assume throughout the rest of the paper that consumers stay in autarky if a run-proof contract is provided. So the analysis in the rest of the paper is focused on the run-admitting contracts.

Generally speaking, because sunspots do not affect the fundamentals, people can interpret the signals in any way they please. Thus, an imperfect information structure can allow for more than one correlated equilibrium. I focus on the case where consumers in both groups interpret a signal of value 1 as the sign to withdraw and 0 as the sign to wait. If the exogenous uncertainty is viewed as the uncertainty in fundamentals taken to the limit, then it can be understood that people usually have common views on which signal is favorable and which is not.

For an impatient consumer, withdrawal in period 1 is his dominant strategy because he only values consumption in that period. The correlated equilibrium strategies for a patient consumer in group i in period 1 require the following conditions:

$$\Pr(\theta_{-i} = 0 \mid \theta_i = 1)\min\left\{\frac{1}{(n_i + \alpha n_{-i})\, c^1}, 1\right\}u(c^1) + \Pr(\theta_{-i} = 1 \mid \theta_i = 1)\frac{1}{c^1}u(c^1)$$
$$\geq \Pr(\theta_{-i} = 0 \mid \theta_i = 1)u\left(\max\left\{\frac{1 - (n_i + \alpha n_{-i})\, c^1}{(1 - \alpha)n_{-i}}R, 0\right\}\right), \qquad (2)$$

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$$\begin{aligned}
&\Pr(\theta_{-i}=1\,|\,\theta_i=0)u\left(\max\left\{\frac{1-(n_{-i}+\alpha n_i)\,c^1}{(1-\alpha)\,n_i}R,0\right\}\right)\\
&\quad+\Pr\left(\theta_{-i}=0\,|\,\theta_i=0\right)u\left(\frac{1-\alpha c^1}{1-\alpha}R\right)\\
&\quad\geq\Pr(\theta_{-i}=1\,|\,\theta_i=0)\min\left\{\frac{1}{(n_{-i}+\alpha n_i)c^1},1\right\}u(c^1)\\
&\quad+\Pr(\theta_{-i}=0\,|\,\theta_i=0)u(c^1). \qquad (3)
\end{aligned}$$

Inequality (2) is the condition that a patient consumer in group i runs on the bank given the probability that some, but not all, patient consumers wait until the last period. When a patient consumer in group i observes $\theta_i = 1$, he knows that with conditional probability $\Pr(\theta_{-i} = 1|\theta_i = 1)$, the other group (denoted by $-i$) observes $\theta_{-i} = 1$ and withdraws and that with probability $\Pr(\theta_{-i} = 0|\theta_i = 1)$, the other group waits. The left-hand side of (2) is the expected utility if he withdraws, given that his group members withdraw and the members in the other group follow the signal they observe. The minimum operator captures the fact that when group i runs on the bank whereas group $-i$ does not, there are a measure $n_i + \alpha n_{-i}$ of consumers who withdraw, but the bank can serve at most a measure $\frac{1}{c^1}$ of consumers. Because consumers are served in a random order and there is a sequential service constraint, the probability that a consumer gets paid is $\min\{1/c^1(n_i + \alpha n_{-i}), 1\}$. The right-hand side of (2) is the expected utility if he waits, whereas the others follow their signals. The maximum operator captures the fact that a consumer's payoff in period 2 depends on whether there are funds left at the bank after period 1. Similarly, (3) is the condition that a patient consumer in group i waits given the probability that some, but not all, patient consumers run on the bank.

Let $M^{\mathrm{CE}}(p_{11}, \rho)$ denote the set of contracts that satisfy (2) and (3) for a given distribution of θ's. Because a contract that satisfies (3) always satisfies (1), whereas the reverse is not true when ρ is strictly positive,[13] and because a contract in $M^{\mathrm{CE}}(p_{11}, \rho)$ must also satisfy (2), the set $M^{\mathrm{CE}}(p_{11}, \rho)$ is a subset of M^{RA}. If $\rho = 0$, then (2) is automatically satisfied, inequality (3) is reduced to (1), and we have $M^{\mathrm{CE}}(p_{11}, 0) = M^{\mathrm{RA}}$.

If a contract in $M^{\mathrm{RA}}\backslash M^{\mathrm{CE}}$ is provided, the sunspot signals fail to coordinate the actions of consumers. A consumer's strategy is not contingent on the sunspot realizations. Our equilibrium concept of correlated equilibrium does not apply to these contracts. Here, I follow Diamond and Dybvig (1983) and assume that consumers play a Nash equilibrium. Given a contract in $M^{\mathrm{RA}}\backslash M^{\mathrm{CE}}$, there are two Nash equilibria in period 1 : $\{y_1 = \text{withdraw}\}$ and $\{y_1(\text{patient}) = \text{wait}, y_1(\text{impatient}) = \text{withdraw}\}$. Because a consumer's expected payoffs differ in these two equilibria, assuming a particular equilibrium strategy will affect the bank's choice of the optimal contract, which yields the highest expected payoff among all feasible contracts given the equilibrium strategies of the consumers. In

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this paper, I assume that consumers play $\{y_1 = \text{withdraw}\}$. The discussion of the alternative case can be found in the working paper version of this paper.

Ex ante expected utilities. In this section, I calculate the ex ante expected utility given a contract and the equilibrium strategies. The ex ante expected utility is critical for two reasons: (i) A competitive bank provides the contract that maximizes the consumers' ex ante expected utility; (ii) consumers accept the contract if and only if doing so is a better choice than staying in autarky.

In a correlated equilibrium, there are three possible outcomes in period 1: If $\theta_1 = \theta_2 = 1$, all consumers withdraw; if $\theta_1 = 0$ and $\theta_2 = 1$, or if $\theta_1 = 1$ and $\theta_2 = 0$, only a fraction of patient consumers withdraw; no patient consumer withdraws when $\theta_1 = \theta_2 = 0$. Define a full bank run, partial bank run, and no bank run as follows:

DEFINITION 1. *In period 1, if all consumers withdraw deposits, then a full bank run occurs; if some, but not all, patient consumers withdraw, then a partial bank run occurs; if no patient consumers withdraw, then no bank run occurs.*

Given the distribution of the sunspots, the probabilities of having a full run and no run are p_{11} and p_{00}, respectively, and the probabilities of having partial runs driven by groups 1 and 2, respectively, are ρ. Throughout this paper, I assume $p_{00} < 1$ to rule out the trivial case in which bank runs never occur. Given the probabilities, the expected utility of a consumer in period 0 if he accepts contract c^1 is

$$W^{\text{CE}}(c^1) = p_{11}W^{\text{run}}(c^1) + \rho W^{\text{p-run-1}}(c^1) + \rho W^{\text{p-run-2}}(c^1) + p_{00}W^{\text{no-run}}(c^1),$$

where

$$W^{\text{p-run-i}}(c^1) = (n_i + \alpha n_{-i}) \min\left\{\frac{1}{c^1(n_i + \alpha n_{-i})}, 1\right\} u\left(c^1\right) + (1-\alpha)n_{-i}u\left(\max\left\{\frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)n_{-i}}R, 0\right\}\right)$$

is the expected utility when a partial run driven by group i occurs. In a partial run, a consumer withdraws with probability $n_i + \alpha n_{-i}$ (i.e., the probability that he is impatient or in group i), and if he withdraws, the probability that a consumer gets paid is $\min\{1/c^1(n_i + \alpha n_{-i}), 1\}$. The consumer waits with probability $(1-\alpha)n_{-i}$ (i.e., the probability that he is patient and in group $-i$), and his payoff in period 2 depends on whether there are funds left at the bank after the partial run.

The bank chooses a contract to offer to the consumers. A consumer accepts the contract only if the contract is individually rational, which means the expected utility under the contract is higher than under autarky. A contract in $M^{\text{RA}} \backslash M^{\text{CE}}$ is not individually rational because by the assumed equilibrium strategy $\{y_1 = \text{withdraw}\}$, the ex ante expected utility is $W^{\text{run}}(c^1)$, which is lower than $\hat{W}^{\text{AUT}}$ for $c^1 > 1$. The bank thus has choices only in the set M^{CE}.

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The best contract in M^{CE} solves

$$\hat{W}^{\mathrm{CE}} = \max_{c^1} W^{\mathrm{CE}}(c^1) \qquad \textbf{(PCE)}$$
$$\text{s.t. } c^1 \in M^{\mathrm{CE}}.$$

The noisy private sunspot signals can generate bank runs if (i) there exists at least one contract in M^{CE} that yields higher ex ante utility than autarky, so that it will be accepted, and (ii) it is the optimal contract in terms of ex ante expected utility, given the equilibrium strategies that consumers play if a contract in $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ is provided. Formally, we have the following definition.

DEFINITION 2. *Given the equilibrium strategies of the consumers if a contract in* $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ *is provided, we say the distribution of the sunspot variables* $\Pr(\theta_1, \theta_2)$ *can generate bank runs if there exists an individually rational contract* $c^1 \in M^{\mathrm{CE}}$*, under which (1) there is a correlated equilibrium in period* 1 *in which consumers play the strategy* $\{y_1(\text{patient}, i, \theta_i = 1) = \textit{withdraw}, y_1(\text{patient}, i, \theta_i = 0) = \textit{wait}, y_1(\text{impatient}, i, \theta_i = 0, 1) = \textit{withdraw}, i = 1, 2$*, and (2) the contract is optimal.*

The next step is to show the conditions of the information structure that are needed to generate equilibrium bank runs. We care about two features of the information structure: (i) the size of noise in the signals, which determines the size of M^{CE}, and (ii) the probabilities of bank runs, which determine the consumers' ex ante expected utility given a contract. I will first derive the properties of M^{CE} and of the value function of $\hat{W}^{\mathrm{CE}}$.

Properties of M^{CE} *and* $\hat{W}^{\mathrm{CE}}$. Lemma 1 shows that $M^{\mathrm{CE}}(p_{11}, \rho)$ shrinks when the signals become more noisy.

LEMMA 1. *Given* p_{11}*, let* $\rho' > \rho$*. We have* $M^{\mathrm{CE}}(p_{11}, \rho') \subseteq M^{\mathrm{CE}}(p_{11}, \rho)$*.* $M^{\mathrm{CE}}(p_{11}, \rho') = M^{\mathrm{CE}}(p_{11}, \rho)$ *if and only if* $M^{\mathrm{CE}}(p_{11}, \rho) = \varnothing$*.*

The strategies of the consumers are complementary. If the probability that other consumers will take a certain action increases, then an individual consumer's incentive to take a different action decreases. With the increases in ρ, it is more difficult to generate a run as well as no run, because when an individual consumer receives a signal that tells him or her to take a certain action, he or she knows that the other group is more likely to take a different action. Hence, as ρ increases, there are fewer contracts consistent with the definition of a correlated equilibrium. In extreme cases, no contract allows a correlated equilibrium. In the example below, $M^{\mathrm{CE}}(p_{11}, \rho)$ becomes empty when ρ gets large.

Example 1

$u(c) = [(c+b)^{1-\gamma} - b^{1-\gamma}]/(1-\gamma)$, $b = 0.5$, $\gamma = 2$. $n_1 = 0.45$, $n_2 = 0.55$, $\alpha = 0.4$, $R = 1.5$. $p_{11} = 0.001$. Let ρ vary between 0 and 0.4995. Figure 1 shows how the set of $M^{\mathrm{CE}}(p_{11}, \rho)$ varies with ρ. In this example, $M^{\mathrm{CE}}(p_{11}, \rho)$

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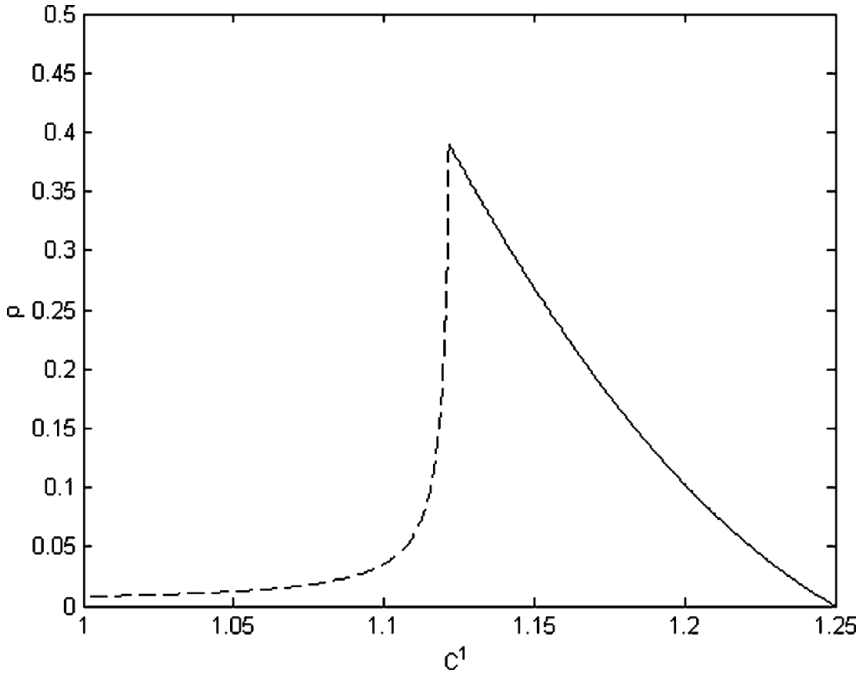


FIGURE 1. An example of M^{CE}.

is a convex set: For a given ρ, there exists an interval of c^1. All contracts in that interval allow a correlated equilibrium in period 1. The points on the dashed line and on the solid line are the lowest and the highest c^1 in $M^{\mathrm{CE}}(p_{11}, \rho)$ for the corresponding ρ, respectively. The set of $M^{\mathrm{CE}}(p_{11}, \rho)$ shrinks with ρ. When $\rho > 0.39$, no contract allows a correlated equilibrium.

Another way to show that consumers can be coordinated by their private signals in period 1 is to demonstrate that for any run-admitting contract, there is an upper bound of ρ below which the contract permits a correlated equilibrium in period 1. In most cases, the upper bound is strictly positive.

LEMMA 2. *Given a run-admitting contract, there exists* $\varepsilon(p_{11}, c^1) \geq 0$ *such that if* $\rho \leq \varepsilon(p_{11}, c^1)$*, then* $c^1 \in M^{\mathrm{CE}}(p_{11}, \rho)$*. We have* $\varepsilon(p_{11}, c^1) = 0$ *if and only if (1)* $c^1 = R/(1 - \alpha + \alpha R)$*, or (2)* $p_{11} = 0$*, and* $c^1 < \max\{\frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)n_{-i}} R, i = 1, 2\}$.

When constraint (1) is binding [i.e., $c^1 = R/(1 - \alpha + \alpha R)$], only the minimum incentive for a patient consumer to wait is satisfied. Any increase in the measure of consumers running on the bank, or any increase in the probability of more than α measure of consumers running on the bank, breaks down the participation incentive compatibility constraint. Thus, the contract that offers $c^1 = R/(1 - \alpha + \alpha R)$ does not allow a correlated equilibrium unless $\rho = 0$.

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If $p_{11} = 0$, then when a consumer in group i receives a signal to withdraw, he knows for sure that the other group receives the signal to wait. The incentive for him to withdraw is that if all of his group members run, the resources remaining at the bank for period 2 will yield a lower payment than what he can get from immediate withdrawal. However, if all other consumers in group i run on the bank, yet the bank still survives and pays a higher amount to those who wait [i.e., $c^1 < \frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)n_{-i}} R$], then a consumer in group i will not withdraw even though his signal tells him to do so. So a correlated equilibrium breaks down. Therefore, for these contracts, the probability that group i runs on the bank whereas the other group does not has to be zero.[14] Also note that Lemma 2 implies that given $p_{11} = 0$, if the difference in group sizes increases, then more contracts do not allow a correlated equilibrium.

Furthermore, Lemma 2 says that given $p_{11} = 0$, only the contracts that provide sufficiently high c^1 can allow noise. If a bank provides a contract that permits a correlated equilibrium given $p_{11} = 0$, the highest possible expected utility (which is calculated assuming no bank run occurs) is solved by the following problem:

$$\hat{W}^{\mathrm{CNR}} = \max_{c^1} W^{\text{no-run}}(c^1) \qquad \textbf{(PNR)}$$

$$\text{s.t. (1) and } c^1 \geq \max\left\{\frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)n_{-i}} R, i = 1, 2\right\}.$$

Although no run is assumed, the value of $\hat{W}^{\mathrm{CNR}}$ is not necessarily higher than $\hat{W}^{\mathrm{AUT}}$ because the choice set of contracts is limited.[15] In other words, if the probability of full runs is zero, there may not exist an individually rational contract that can tolerate noise in signals. So partial runs cannot be generated by the noisy private sunspots in these economies if full runs never occur. Example 2 illustrates this case.

Example 2

$u(c) = c/(1 + 1.2c)$. $n_1 = n_2 = 0.5$, $\alpha = 0.2$, $R = 1.5$. In this example, $\hat{W}^{\mathrm{AUT}} = 0.5195$, whereas $\hat{W}^{\mathrm{CNR}} = 0.5193$ with $c^1 = 1.1538$.

However, in other economies, partial runs can occur even if $p_{11} = 0$. Using the utility function and the parameters in Example 1, we find that $\hat{W}^{\mathrm{AUT}} = 1.4333$, whereas $\hat{W}^{\mathrm{CNR}} = 1.4338$ with $c^1 = 1.1236$.

Given p_{11}, the value function of problem PCE strictly decreases in ρ for two reasons: First, the set of contracts that allow a correlated equilibrium shrinks when noise increases, so the choice set is smaller. Second, because the expected utility under a partial run is lower than that under no run, the same contract yields lower expected utility if the probabilities of partial runs are larger and the probability of no run is smaller.

LEMMA 3. *Given p_{11}, the value function of problem PCE is strictly decreasing in ρ for $M^{\mathrm{CE}}(p_{11}, \rho) \neq \varnothing$.*[16]

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Note that given ρ, the value function of PCE is not necessarily decreasing in p_{11}. Although the probability of having a full bank run is lowered when p_{11} decreases, the choice set of M^{CE} may shrink.

Equilibrium bank runs. Now we are ready to state the condition under which the noisy private signals can generate bank runs. Again, I consider the case in which consumers play $\{y_0 = \text{reject}, y_1 = \text{withdraw}\}$ if a contract in $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ is provided. Given the strategies, the bank will not choose to offer such a contract ex ante. Hence, if the best contract in M^{CE} can generate higher utility than autarky, it will be accepted by the consumers and the consumers will be coordinated by the sunspots ex post. The following proposition shows that the noisy sunspot signals can generate bank runs if the probabilities of full runs and partial runs are below certain thresholds.

PROPOSITION. *Let consumers play the strategy*

$$\{y_0(c^1) = \textit{reject}, y_1 = \textit{withdraw}\}$$

if a contract in $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ *is provided. There exist* $\overline{p}_{11} > 0$ *and* $\overline{\rho}(p_{11}) \geq 0$ *such that for* $p_{11} \leq \overline{p}_{11}$ *and* $\rho \leq \overline{\rho}(p_{11})$, *the distribution of the sunspot variables can generate bank runs. Furthermore,* $\overline{\rho}(p_{11}) > 0$ *for* $0 < p_{11} < \overline{p}_{11}$ *and* $\overline{\rho}(0) > 0$ *if* $\hat{W}^{\mathrm{CNR}} > \hat{W}^{\mathrm{AUT}}$.

Proof. Given p_{11}, if $\rho = 0$, the problem is the same as the traditional public sunspot equilibrium problem. There is a unique cutoff level of $p_{11} \in (0, 1)$, above which autarky is better than the banking economy and below which the optimal contract tolerates bank runs [see Cooper and Ross (1998)]. Denote this cutoff level by $\overline{p}_{11}$.

Given p_{11}, the value function of PCE is strictly decreasing in ρ, by Lemma 3. Hence, $\rho(\overline{p}_{11}) = 0$.

Now consider $p_{11} > 0$. The value of PCE given $0 < p_{11} < \bar{p}_{11}$ and $\rho = 0$ is strictly higher than that in autarky. Holding p_{11} constant, by the monotonicity of the value function in ρ, we can find a cutoff probability $\overline{\rho}(p_{11})$, below which the contract allowing for a correlated equilibrium is better than autarky and above which otherwise.

The value function of PCE is not necessarily continuous because the choice set can be nonconvex. In the next step, we prove that $\overline{\rho}(p_{11})$ is not equal to 0 for $0 < p_{11} < \overline{p}_{11}$. Let $\hat{c}^1$ denote the solution to problem PCE with $0 < p_{11} < \overline{p}_{11}$ and $\rho = 0$. $\hat{c}^1$ cannot be $R/(1 - \alpha + \alpha R)$ [see Ennis and Keister (2006)]. By Lemma 2, $\hat{c}^1$ allows a correlated equilibrium for $\rho \leq \varepsilon(p_{11}, \hat{c}^1)$, where $\varepsilon(p_{11}, \hat{c}^1) > 0$. The expected utility at $\hat{c}^1$ for $\rho \leq \varepsilon(p_{11}, \hat{c}^1)$ is continuous in ρ. So ρ can be increased at least to $\min\{\overline{p}_{11} - p_{11}, \varepsilon(p_{11}, \hat{c}^1)\}$ and $\hat{c}^1$ can still be better than autarky. Thus, when $0 < p_{11} < \overline{p}_{11}$, $\overline{\rho}(p_{11}) > 0$.

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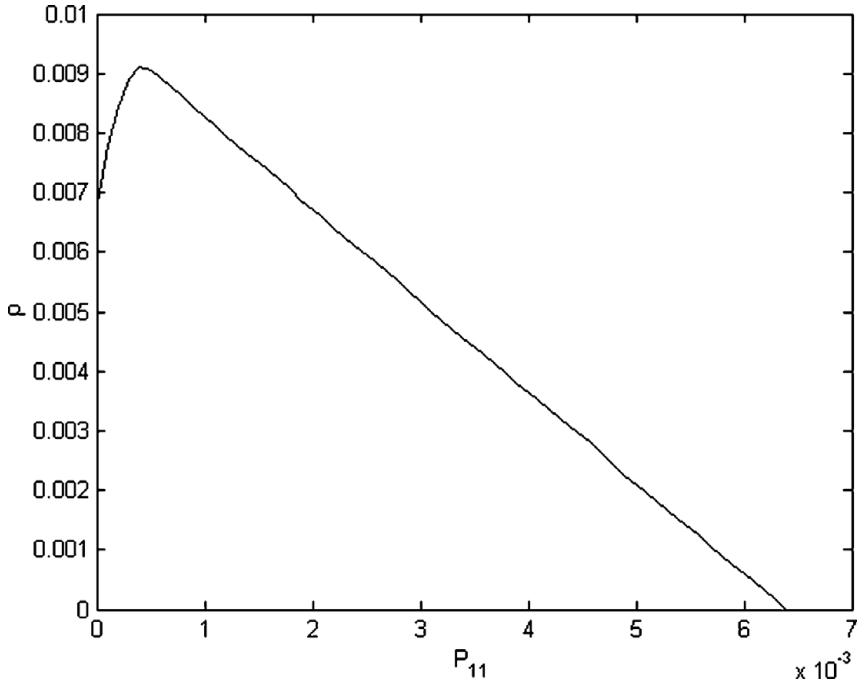


FIGURE 2. An example of $\bar{\rho}(p_{11})$.

Now consider $p_{11} = 0$. The upper bound of W^{CE} is $\hat{W}^{\mathrm{CNR}}$. If $\hat{W}^{\mathrm{CNR}} > \hat{W}^{\mathrm{AUT}}$, we can apply the above argument to show there exists a $\overline{\rho}(0) > 0$ such that if $\rho < \overline{\rho}(0)$, the distribution of the sunspot variables can generate bank runs. ■

The following example draws $\overline{\rho}$ as a function of p_{11}.

Example 3

Continue Example 1. In this example, $\overline{p}_{11} = 0.0064$. For $p_{11} < \overline{p}_{11}$, the upper bound of the noise in the signal, $\overline{\rho}$, is drawn as a function of p_{11} in Figure 2. $\overline{\rho}$ is not monotone in p_{11}. When p_{11} increases from zero, on one hand, the probability of having a full run increases, which lowers the value of PCE, so the probability of partial runs needs to be lowered to keep the value of PCE above $\hat{W}^{\mathrm{AUT}}$. On the other hand, the conditional probability of $\Pr(\theta_{-i} = 1|\theta_i = 1)$ also increases, which means the signals are more correlated and the size of the choice set of contracts becomes larger. So the value of PCE is increased, and a higher probability of partial runs will be allowed. Whether $\overline{\rho}$ is increasing or decreasing in p_{11} depends on the strength of these two forces. In this example, there is a critical p_{11}, below which $\overline{\rho}$ is increasing in p_{11} and above which $\overline{\rho}$ is decreasing in p_{11}.

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The proposition states results similar to those of Cooper and Ross (1998) and Peck and Shell (2003) for the case of partial runs. If the probabilities of runs (both full and partial) are small, then bank runs can be equilibrium phenomena. There are two reasons behind this. First, consumers are willing to accept a run-admitting contract if the chance of having bad outcomes is small. This is common in the literature. Second, the probabilities of partial runs also measure the noise in the signals, which determines the size of the choice set of contracts. The smaller the noise is, the better the consumers are coordinated. Consequently, there are more contracts allowing the correlated equilibrium and the bank has a larger choice set of M^{CE}. Because a decrease in noise pushes the frontier of the choice set outward, if the optimal contract is on the frontier, smaller noise results in a better contract and higher welfare. This mechanism is unique to the noisy sunspot signals structure. In a public sunspot information economy, the bank's choice set is M^{RA}, which is invariant to the probability of (full) runs.

The proposition also implies that partial runs can exist in an economy in which full bank runs never occur. In this sense, partial runs are more general phenomena of financial distress. Banks do not necessarily close in a partial run. One way to understand the partial run is to think about runs on several branches of a nationwide bank. Studies show that historically, branch-banking systems (e.g., the Canadian banks) tend to be less prone to the effects of panics than unit-banking systems (e.g., the U.S. banks) [Calomiris and Gorton (1991)]. The common view is that the branch-banking system can better absorb the risk imposed by the runs on individual branches. This paper provides another plausible explanation: Depositors of a bank with nationwide branches come from more diversified backgrounds. Coordination among these depositors is more difficult due to, for example, geographic separation or different news sources. As a result, the probability of a full run is reduced and the bank is more likely to survive.

Similar arguments apply and similar results obtain in the case with the alternative assumption that consumers play $\{y_0 = \text{accept}, y_1(\text{patient}) = \text{wait}, y_1(\text{impatient}) = \text{withdraw}\}$ if a contract in $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ is provided. In this case, the best contract in M^{CE} must compete with autarky as well as the best contract in $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$, assuming no bank run under such contract. The cutoff p_{11} is $\overline{p}_{11}$, as in the Proposition, because with $\rho = 0$, the set of $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ is empty. Given $p_{11} < \overline{p}_{11}$, if noise is small, the set of M^{CE} is large but $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ is small by Lemma 2. The expected utility under the best contract in $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ is low because the choice set is very limited. Therefore, the best contract in M^{CE} can be optimal if $\rho(p_{11})$ is small. With the increase in noise, on one hand, the set of M^{CE} shrinks, so value function of PCE decreases. On the other hand, the set of $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ expands and the expected utility under the best contract in $M^{\mathrm{RA}} \backslash M^{\mathrm{CE}}$ increases. By the argument of continuity, a cutoff $\rho(p_{11})$ can be found such that if ρ is below the cutoff, the optimal contract can tolerate full and partial runs. Proof of this alternative case and a numerical example can be found in the working paper version of this paper.

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Discussion. A partial bank run here results from the noise in the sunspot signals. In the traditional Diamond–Dybvig setup with a publicly observed sunspot, a partial run can happen if we allow mixed strategies. In a mixed strategy Nash equilibrium, the fraction of patient consumers who run on the bank is uniquely determined by the contract.[17] In contrast, in a correlated equilibrium, the fraction of consumers who run on the bank is determined by the information structure. A partial run can occur with different probabilities due to different information structures for the same contract. In this sense, a correlated equilibrium is more general than a Nash equilibrium.

A more general way to analyze a consumer's withdrawal decision is to model it as dependent on the withdrawals made by others as well as the consumer's private information. Gu (2010) studies fundamental-based bank runs as a result of herd behavior.[18] The withdrawal decisions are assumed to be made simultaneously in this paper. However, the correlated equilibrium (and partial runs) can survive even if we allow consumers to withdraw after they observe withdrawals by others.

This paper focuses on a simple demand deposit contract. However, the results in the paper obtain in a broad class of banking mechanisms, including partial suspension of convertibility [Wallace (1990)] in the Peck and Shell (2003) framework; an example can be found in the working paper version of this paper.

4. CONCLUSION

In this paper, I extend the analysis of panic-based runs to include noisy private sunspot signals. I explain how individual consumers make withdrawal decisions when there are uncertainties about the signals and decisions of other consumers. I show that in an economy with noisy sunspot signals, there exists a correlated equilibrium in a class of demand deposit contracts for the case in which the noise is sufficiently small. In this equilibrium, a full bank run, a partial bank run, or no bank run occurs, depending on the realization of the sunspot signals. If the probabilities of full bank runs and partial bank runs are below certain thresholds, the optimal banking contract tolerates full runs and partial runs.

This paper is also a robustness test of sunspot-triggered, panic-based bank runs. Not all sunspot signals can generate bank runs in equilibrium. The set of contracts that allow the correlated equilibrium diminishes if the signals become more noisy. The set can be empty if the noise in the signals is high enough.

The model I consider in this paper assumes that the information structure and the division of groups are given exogenously. What happens if depositors can choose groups? It is obvious that there is an equilibrium such that all depositors choose the same group so that they are coordinated perfectly. However, it is possible that depositors choose different groups in the equilibrium. It happens when the measures of the depositors in the two groups are adjusted to the point that the ex ante expected utilities of the two groups are equal. In this sense, noisy sunspot signals can be an equilibrium choice, and the heterogeneous beliefs about the bank's stability can also be an endogenous choice.

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NOTES

1. The classic work on panic-based bank runs includes Diamond and Dybvig (1983) and Peck and Shell (2003). Some empirical studies show that many banking crises cannot be explained by macroeconomic data. See Boyd et al. (2001). These findings give some empirical support to theories of panic-based runs. Ennis (2003) provides a panic-based bank run model that can account for the historical correlation of bank runs with poor economic fundamentals.

2. Solomon also provides good interpretations of imperfect correlated sunspots and examples. See Solomon (2003, 6–7; 2004, p. 7) for details.

3. For example, after Hurricane Katrina, several banks in the New Orleans area lost large amounts of cash due to deposit withdrawals. However, runs on these banks gradually ceased and the banks survived. Although the withdrawals were partly due to high fundamental demand for liquidity after the disaster, some bankers thought a panic-based run was taking place. Under pressure from Louisiana bankers, "the FDIC issued a press release Wednesday (September 7, 2005) that was clearly directed at soothing nervous bank customers," emphasizing that "customers' money was safe." (See "In D.C., Vows Banks Won't Fail, but No Specifics," by Heller Michele, *American Banker*, 9/8/2005, Vol. 170, Issue 173.)

4. The partial runs in Allen and Gale (1998) are generated by the fact that the long-term investment is totally illiquid. Runs are necessarily partial because there are funds available in the last period regardless of whether a bank run happens in the previous period. Ennis and Keister (2009) show that the bank runs are necessarily partial in the Green and Lin (2003) model of financial intermediation with correlated consumption types—only those who are early in the line have the incentive to run because the bank would increase the payments to them due to its undue optimism about the number of future withdrawals. As the bank's undue optimism disappears when the end of the line is approached, consumers' incentive to run also disappears. In Goldstein and Pauzner's (2005) work, the partial runs are generated in a global game where some consumers run on the bank as their private and imperfect information about the bank's portfolio return induces them to do so.

5. Postlewaite and Vives (1987) show that given the possibility that the bank runs happen ex post, the demand deposit contract that admits the first-best allocation is usually not optimal. Cooper and Ross (1998) show that with a perfect randomizing device, if the probability of bank runs is small, then the optimal demand deposit contract admits a run outcome. Peck and Shell (2003) illustrate that within a broad class of banking mechanisms including partial suspension of convertibility, the optimal contract can tolerate bank runs if the probability is small.

6. Goldstein and Pauzner (1995) create the upper dominance region by assuming that the short-term return is correlated with the long-term return. When returns are high, even if all other depositors withdraw, the bank survives because there are sufficient funds accrued in the short term to meet all withdrawal requests.

7. In a recent paper, Angeletos (2008) studies an investment problem with strategic complementarities in an economy with a continuum of noisy private sunspot signals. He finds that there exists a cutoff value of the sunspot signals above which agents invest, and below which they do not. In other words, sunspots determine the equilibrium outcome.

8. See Allen and Gale (1994, 1998), Gorton (1988), and Calomiris and Gorton (1991), among others, for theoretical models and empirical evidence on fundamental-based runs.

9. The classic papers on this topic are Bryant (1980) and Diamond and Dybvig (1983).

10. Wallace (1988) explains how an environment in which consumers are isolated from each other but in contact with the bank implies the sequential service constraint. Here, I follow the literature and assume that consumers make decisions simultaneously but contact the bank sequentially.

11. The assumption of a demand deposit contract simplifies the presentation. In the working paper version of this paper, I consider a banking contract that allows partial suspension of convertibility. The results still hold: In some economies, a correlated equilibrium exists, and full bank runs and partial runs are tolerated.

12. See Footnote 2.

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13. To see this, note that if $(n_{-i} + \alpha n_i)c^1 \geq 1$, inequality (3) becomes

$$\Pr(\theta_{-i} = 0 \,|\, \theta_i = 0)u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) \geq \Pr(\theta_{-i} = 1 \,|\, \theta_i = 0)\frac{u\left(c^1\right)}{(n_{-i}+\alpha n_i)\,c^1} + \Pr(\theta_{-i} = 0 \,|\, \theta_i = 0)u(c^1),$$

which is a more restrictive condition than (1). If $(n_{-i} + \alpha n_i)c^1 < 1$, inequality (3) becomes

$$\Pr\left(\theta_{-i} = 1 \,|\, \theta_i = 0\right) u\left(\frac{1-(n_{-i}+\alpha n_i)\,c^1}{(1-\alpha)\,n_i}R\right) + \Pr\left(\theta_{-i} = 0 \,|\, \theta_i = 0\right) u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) \geq u(c^1).$$

Because

$$u\left(\frac{1-(n_{-i}+\alpha n_i)\,c^1}{(1-\alpha)n_i}R\right) < u\left(\frac{1-\alpha c^1}{1-\alpha}R\right),$$

the LHS of the inequality is less than

$$u\left(\frac{1-\alpha c^1}{1-\alpha}R\right).$$

Again, the inequality states a more restrictive condition than (1).

14. A special case occurs when $p_{11} = p_{00} = 0$ and $n_1 = n_2 = 0.5$. The only contract that allows a correlated equilibrium requires

$$c^1 = \frac{1-(0.5+0.5\alpha)c^1}{0.5(1-\alpha)}R.$$

In the equilibrium, depositors can perfectly infer the sunspot signal the other group observes, but they will always make different decisions. Put more generally, there exists a rational equilibrium in some economies in which depositors with the same information can have different opinions regarding the consequences of their information.

15. If $\hat{W}^{\mathrm{CNR}} < \hat{W}^{\mathrm{AUT}}$, the optimal contract subject only to constraint (1) provides c^1 lower than the c^1 that solves problem PNR.

16. The proof of Lemma 3 can be found in the working paper version of this paper.

17. In a mixed strategy equilibrium, the probability that a patient consumer runs on the bank, denoted by p, solves $(1-[\alpha+(1-\alpha)p]c^1)/(1-\alpha)(1-p) = c^1$. By the law of large numbers, the fraction of the patient consumers who run on the bank is $(1-\alpha)p$.

18. In Gu (2010), a full bank run occurs as a result of a herd of withdrawals when all depositors withdraw due to unfavorable signals and/or unfavorable observations on withdrawals. Likewise, a nonrun occurs as a result of a herd of nonwithdrawals. Before either herd begins, depositors follow their private signals. Thus, there are cases in which some depositors withdraw even though a full run does not occur later.

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APPENDIX

Proof of Lemma 1. Let $M^{\mathrm{CE}}(p_{11}, \rho) \neq \varnothing$. Prove the proposition in two steps. First, I show that for any c^1 in $M^{\mathrm{CE}}(p_{11}, \rho')$, it is also in $M^{\mathrm{CE}}(p_{11}, \rho)$. Second, I determine that there exists some c^1 in $M^{\mathrm{CE}}(p_{11}, \rho)$ but not in $M^{\mathrm{CE}}(p_{11}, \rho')$.

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Discuss cases by parameters. Let $(\alpha n_1 + n_2)c^1 \leq 1$ and $(\alpha n_2 + n_1)c^1 \leq 1$. Rewrite inequalities (2) and (3) for both groups. We have

$$\rho\left[u(c^1) - u\left(\frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)\,n_{-i}}R\right)\right] \geq -p_{11}\frac{1}{c^1}u(c^1), \tag{A.2}$$

$$-\rho\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u\left(\frac{1-(\alpha n_i + n_{-i})c^1}{(1-\alpha)\,n_i}R\right)\right] - \rho\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1)\right]$$

$$\geq -(1-p_{11})\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1)\right], \tag{A.3}$$

for $i = 1, 2$. The RHS of (A.2) is negative. If

$$u(c^1) - u\left(\frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)n_{-i}}R\right) \geq 0,$$

any change in ρ does not affect the sign of (A.2). If

$$u(c^1) - u\left(\frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)\,n_{-i}}R\right) < 0,$$

a decrease in ρ raises the LHS. Therefore, if inequality (A.2) holds for ρ', it also holds for $\rho \leq \rho'$. In (A.3), each term in the brackets is positive and the LHS is decreasing in ρ. Therefore, if inequality (A.3) holds for ρ', it also holds for $\rho \leq \rho'$.

In the second step, I show that for $\rho' \geq \rho$, there exists a contract c^1 that is in $M^{\mathrm{CE}}(p_{11}, \rho)$ but not in $M^{\mathrm{CE}}(p_{11}, \rho')$. Suppose that there is $c^1 \in M^{\mathrm{CE}}(p_{11}, \rho)$ such that at least one of the inequalities is binding for group 1 and/or 2. Increasing ρ to ρ' breaks down the inequality. So such a contract is not in $M^{\mathrm{CE}}(p_{11}, \rho')$. Next, I show that there exists at least one contract in $M^{\mathrm{CE}}(p_{11}, \rho)$ such that at least one inequality is binding. If $\rho = 0$, inequality (A.3) is binding at $c^1 = R/(1-\alpha+\alpha R)$. If $\rho > 0$, then (A.3) is violated at $c^1 = R/(1-\alpha+\alpha R)$. Because $M^{\mathrm{CE}}(p_{11}, \rho)$ is not empty, inequality (A.3) holds for some contracts that provide $c^1 > 1$. By the continuity of the utility function, the inequality must be binding at some $c^1 < R/(1-\alpha+\alpha R)$.

If $M^{\mathrm{CE}}(p_{11}, \rho) = \varnothing$, at least one of the inequalities is not satisfied for any feasible c^1. It is easy to see that increasing ρ will not restore the inequalities.

In the same way, we can prove the other three cases in which (1) $(\alpha n_1 + n_2)c^1 > 1$ and $(n_1 + \alpha n_2)c^1 \leq 1$, or (2) $(\alpha n_1 + n_2)c^1 \leq 1$ and $(n_1 + \alpha n_2)c^1 > 1$, or (3) $(\alpha n_1 + n_2)c^1 < 1$ and $(n_1 + \alpha n_2)c^1 < 1$. ■

Proof of Lemma 2. We discuss the case in which $(\alpha n_1 + n_2)c^1 \leq 1$ and $(\alpha n_2 + n_1)c^1 \leq 1$. The same reasoning applies to the other cases. Let

$$\overline{v}(c^1) = \max\left\{u\left(\frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)\,n_{-i}}R\right), \quad i = 1, 2\right\}$$

and

$$\underline{v}(c^1) = \min\left\{u\left(\frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)\,n_{-i}}R\right), \quad i = 1, 2\right\}.$$

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Rewriting (A.2) and (A.3), by simple algebra, we have $\varepsilon(p_{11}, c^1)$, the upper bound of ρ, as follows:

$$\varepsilon(p_{11}, c^1)$$
$$= \begin{cases} \min\left\{ \dfrac{p_{11}\dfrac{1}{c^1}u(c^1)}{\overline{\upsilon}(c^1) - u(c^1)}, \dfrac{(1-p_{11})\left[u\left(\dfrac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1)\right]}{2u\left(\dfrac{1-\alpha c^1}{1-\alpha}R\right) - \underline{\upsilon}(c^1) - u(c^1)} \right\}, & \text{if } \overline{\upsilon}(c^1) > u(c^1), \\ \dfrac{(1-p_{11})\left[u\left(\dfrac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1)\right]}{2u\left(\dfrac{1-\alpha c^1}{1-\alpha}R\right) - \underline{\upsilon}(c^1) - u(c^1)}, & \text{otherwise.} \end{cases}$$

It is easy to see that $\rho = 0$ if and only if (1) $c^1 = R/(1-\alpha+\alpha R)$, or (2) $p_{11} = 0$ and $\overline{\upsilon}(c^1) > u(c^1)$ [i.e.,

$$c^1 < \max\left\{u\left(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R\right), u\left(\frac{1-(n_2+\alpha n_1)c^1}{(1-\alpha)n_1}R\right)\right\}\Bigg].$$

■

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