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COINTEGRATION AND SETTLEMENT OF COMMODITY FUTURES CONTRACTS

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Weekly live cattle prices in various markets would be expected to share common trends, i.e., they cannot be driven by separate nonstationarities because at some point the prices will diverge sufficiently for it to be economic to cross-ship the cattle, or at least the beef. This paper extends previous bivariate work to a multivariate analysis that is capable of modeling the multiple market linkages in prices for many geographical regions. The empirical estimation represents the application of an innovation on Aoki's Linear Systems State Space model that allows determination of long- and short-run dynamics common to multiple series. The common dynamics permit characterization of the multiple markets with a limited number of states (sufficient statistics for the past), resulting in dynamic arbitrage relations between the series. A nonparametric test is used to evaluate the value of expected arbitrage forecasts implied by the structure of the model. The arbitrage relationship also is employed to generate efficient discounts/premiums for either physical delivery or cash settlement of futures contracts. The proposed settlement mechanism accounts for spatial arbitrage opportunities and therefore better represents the true geographical discounts faced by traders in individual markets.

Keywords: Commodity Futures, State-Space Time Series, Cointegration

1. INTRODUCTION

Commodity futures markets play important roles in U.S. agriculture, ranging from risk shifting for hedgers to price stabilization for consumers. They provide potential benefits to a wide range of society. The possibility of making delivery of the actual commodity as a means of offsetting a short futures position links these futures contracts to actual market conditions. In the case of all agricultural commodities except feeder calves, the mechanism for delivery on agricultural commodity futures contracts is physical exchange of the commodity. Delivery can be made at any of

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several spot markets designated as delivery sites, with some of these sites being at par, and the others being at a discount or a premium. Delivery is at the discretion of the short position holder and can be exercised any time during the contract expiration month. However, the bulky and/or perishable nature of agricultural commodities makes such transactions logistically difficult with potentially high transactions costs.

The mechanics of physical delivery can be confusing to the casual observer. Delivery becomes an option for the seller of a futures contract in the expiration month of the contract. However, it seldom will be chosen because offsetting the futures position is usually more profitable. Although delivery is infrequent, the opportunity to make or take delivery always exists and, if markets are not operating efficiently, delivery will occur to force the appropriate degree of convergence between the cash and futures prices. Identical cash and futures prices are unlikely because of transportation cost differences and market characteristics, but there is an equilibrium market relation between these prices, institutionalized in the discount and premium structure and arbitraged by actual delivery. Because delivery incurs substantial transaction costs, disproportionate amounts of delivery in a market are evidence of an improperly functioning market, often the result of an inflexible discount and premium structure unable to keep up with new developments. The net result is excessive costs to the industry and those who purchase its product.

Delivery on futures contracts is made by the seller providing a negotiable instrument to the buyer. This usually takes the form of a warehouse or stockyard receipt or a shipping certificate. Note that delivery does not require the seller to actually transport the commodity to the point of sale. The seller can purchase from a warehouse or stockyard the needed quantity of the commodity.[1] As a result, a group of animals or a bin of grain at a certified delivery point may be delivered many times without physically moving. Each delivery generally would be made at a different price until an equilibrium is reached.

The exchange designates which cash market locations can be used as delivery sites. Geographical discounts and premiums often are set by the exchange. That is, the settlement price for physically delivered contracts is the short futures established by the seller less a discount or plus a premium for the place where delivery is made. Some or all of the sites may trade on par with the futures, which is itself a discount/premium structure. The premiums and discounts are set on the basis of historical evidence and do not account for factors such as seasonality and the distance of the delivery point from the center of production; in general, they are flat rates that remain constant for relatively long periods of time. Thus, for example, as production shifts geographically, this institutionalized discount/premium structure tends to encourage arbitrage even if it was initially set optimally. Also, the cash and futures represent distinctly different assets. Although they are highly correlated, they differ stochastically rather than by a constant as the exchange-determined premiums and discounts would suggest. The ad hoc nature of the current discounts, premiums, and pars may result in a market failure: Undesirable delivery

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sometimes will occur because the premium or discount is smaller than the market would dictate.

Delivery represents an attempt to arbitrage in two dimensions, space and time. The spatial considerations are easier to conceptualize because they are related to the basis between the delivery market and the futures market. That is, delivery is triggered when the basis (futures minus cash) exceeds the ad hoc discount plus the associated transaction costs. The time aspect is related to the fact that the short futures position of the seller was established some time in the past. Therefore, the decision to deliver also is based on the current futures and cash prices relative to the price established previously in the futures transaction.

Commodity prices in some spatially different cattle markets have been shown by Goodwin and Schroeder (1991) to be cointegrated, i.e., they follow a common trend. Intuitively, this must be so because these markets generate prices for nearly homogeneous commodities and generally will differ only by grading differences, local demand and supply, transportation, and other transaction costs of moving the commodity from one point to another. Arbitrage opportunities would ensure that these prices ultimately move together. Goodwin and Schroeder found that, in general, the distance between two markets was inversely related to the level of relatedness in the prices. However, their bivariate models were incapable of modeling multimarket linkages. To model the full spatial linkages between spot markets, it is necessary to formulate a multivariate model that includes many price series. In fact, the commodity futures price also should be included in the list of cointegrated prices because it represents a highly correlated asset and, because delivery is possible, its price will not infinitely diverge from those of the spot markets. Heuristically, we would expect the futures price to be highly correlated with the dominant common trend in spot prices because the cash markets will tend to converge to the futures as contract expiration nears.

An arbitrary set of discounts, premiums, and pars cannot represent the cointegrated nature and the spatial arbitrage opportunities until after the market has begun to exploit them: Excess deliveries trigger the adjustment of the discounts and premiums by the exchanges. For the futures market, and by association the spot markets, to operate efficiently, any deliveries should be made on a price that reflects the current spatial relationships and not those that existed in the past.

This paper suggests a possible method of utilizing the cointegration between a subset of spot prices and the nearby futures price to develop price indices that reflect stochastic equilibrium relations (stochastic arbitrage opportunities) between the various spatially diverse markets. These indices are used to generate weekly estimates of the appropriate discounts or premiums associated with individual markets. Sections 2 and 3 develop the methodology used to obtain parameter estimates and derive the arbitrage price indices. In Section 4, an application to live cattle is presented. The accuracy of this model is assessed from the standpoint of predictive accuracy using uncompromised postsample observations and a set of nonparametric tests of forecast value. The arbitrage price indices for live cattle are discussed in Section 5. Section 6 of the paper is reserved for a conclusion and summary.

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2. STATE-SPACE MODEL WITH COINTEGRATION

There are several time-series methodologies by which a model can be determined and estimated. For this study, the linear system state-space representation due to Aoki (1990) was chosen.[2] The state-space representation differs from the original development by Granger (1983) and Engle and Granger (1987) demonstrating the potential loss of information from differencing individual series in a multivariate time-series setting and leading to the notion of cointegration, which was developed in an ARMA framework. Even so, the state-space technique is well suited to modeling cointegrated series compared to other time-series techniques, because the state-space representation provides a methodology that identifies the number and form of the common trends of the cointegrated series, whereas the more commonly used time-series methods lack this capacity.

Arrange the centered and scaled series of spot and futures prices into an $(m \times 1)$ vector $\boldsymbol{y}_t$. Let the $(n \times 1)$ vector $\boldsymbol{z}_t$ denote the unobservable states, and the m-element vector $\boldsymbol{e}_t$ indicate serially uncorrelated error terms. Aoki's (1990) state-space model can be written in two matrix equations called the state and observation equations, respectively:

$$\boldsymbol{z}_{t+1|t} = A\boldsymbol{z}_{t|t-1} + B\boldsymbol{e}_t, \tag{1}$$

$$\boldsymbol{y}_t = C\boldsymbol{z}_{t|t-1} + \boldsymbol{e}_t. \tag{2}$$

In the above equation pair, $\boldsymbol{z}_{t+1|t}$ is a vector of conditional means of unobservable states and A, B, and C are matrices of coefficients to be estimated. This state-space representation and the more commonly specified ARMA representation are well known to be equivalent; see, e.g., Havenner and Aoki (1991). In fact, any state-space representation has an equivalent ARMA representation and vice versa. Stationarity is not imposed on the state equation. Consequently, unit (or even larger-magnitude) roots may arise in A so that the states are nonstationary. If the series are cointegrated, A will have fewer unit roots than there are series being modeled. The data-driven determination of the nonstationarity and the model specification described below guarantee that cointegration will be found naturally, if it is there. The state-space technique estimates roots in A off the point null, dropping the sharp distinction between unit roots and large roots. Although the point-null hypothesis of real unit roots is of interest in various settings, the primary focus of this endeavor is to enhance modeling capability while permitting both stationarity and general nonstationarity (including complex roots) in prices. Thus, the real interest lies in characterizing the spectrum of slow and fast dynamics in the system. Aoki and Havenner (1991, Sect. 4) detail this aspect of the modeling procedure in their paper.

To separate the slow and fast dynamic components, decompose $\boldsymbol{y}_t$ into long-run (trend) and short-run (cycle) dynamics. The first can be represented by a set of states $\boldsymbol{\tau}_{t|t-1}$ and is associated with a set of large eigenvalues denoted $\lambda_1(A)$. The second, denoted $\boldsymbol{y}_t^*$, is a stationary transform of $\boldsymbol{y}_t$ that retains the fast dynamics (small eigenvalues) and takes the place of the error term in the state and observation

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equation pair. The state-space model for this decomposition is

$$\boldsymbol{\tau}_{t+1|t} = A_\tau \boldsymbol{\tau}_{t|t-1} + B_\tau \boldsymbol{y}_t^*, \tag{3}$$

$$\boldsymbol{y}_t = C_\tau \boldsymbol{\tau}_{t|t-1} + \boldsymbol{y}_t^*. \tag{4}$$

The above model is referred to as the trend model because it specifies the slow dynamics, which include any common trends. Cointegration implies that the m series can be characterized with fewer than m nonstationary states; in this event, anything in the null space of C_τ' is a cointegrating matrix because it annihilates the nonstationary components $\boldsymbol{\tau}_{t|t-1}$, leaving a stationary residual $\boldsymbol{y}_t^*$. The residual from the trend model is used as the input variable into another state-space representation designed to model the high-frequency cycles. The cycle model is

$$\boldsymbol{\eta}_{t+1|t} = A_\eta \boldsymbol{\eta}_{t|t-1} + B_\eta \boldsymbol{e}_t, \tag{5}$$

$$\boldsymbol{y}_t^* = C_\eta \boldsymbol{\eta}_{t|t-1} + \boldsymbol{e}_t. \tag{6}$$

The state-space procedure employed arranges estimates of the autocorrelations in the form of a Hankel matrix (block band counterdiagonal) and uses the singular value decomposition to determine the rank of this matrix. The rank of the Hankel matrix thus determined can be shown to be equal to the number of states (minimum sufficient statistics) required to characterize the series. The number of autocorrelations included in the Hankel matrix is a function of the lag parameter N, which balances sampling error at longer lags against model generality. Thus, in state-space time-series analysis, model specification consists of selecting the series to be modeled, setting the lag parameter N, and determining the number of states required to characterize the dynamics.

The procedure for fitting the above trend/cycle model is a two-step process.[3] First, fit the trend model to the slow system dynamics, obtaining the residuals $\hat{\boldsymbol{y}}_t^*$. The second step is to again use the methods of model selection discussed above to fit the cycle model to the high-frequency dynamics remaining in $\hat{\boldsymbol{y}}_t^*$.

To facilitate further analysis and forecasting, the trend and cycle models can be stacked to form a combined state-space model of the full spectrum of dynamics. To form the combined model, substitute (5) into (3) and (4) rearrange and stack them with (5) and (6). The result is

$$\begin{pmatrix} \boldsymbol{\tau}_{t+1|t} \\ \boldsymbol{\eta}_{t+1|t} \end{pmatrix} = \begin{pmatrix} A_\tau & B_\tau C_\eta \\ 0 & A_\eta \end{pmatrix} \begin{pmatrix} \boldsymbol{\tau}_{t|t-1} \\ \boldsymbol{\eta}_{t|t-1} \end{pmatrix} + \begin{pmatrix} B_\tau \\ B_\eta \end{pmatrix} \boldsymbol{e}_t, \tag{7}$$

$$\boldsymbol{y}_t = \begin{pmatrix} C_\tau & C_\eta \end{pmatrix} \begin{pmatrix} \boldsymbol{\tau}_{t|t-1} \\ \boldsymbol{\eta}_{t|t-1} \end{pmatrix} + \boldsymbol{e}_t, \tag{8}$$

which can be written [in the form originally introduced in (1) and (2)] as

$$\boldsymbol{z}_{t+1|t} = A^* \boldsymbol{z}_{t|t-1} + B^* \boldsymbol{e}_t, \tag{9}$$

$$\boldsymbol{y}_t = C^* \boldsymbol{z}_{t|t-1} + \boldsymbol{e}_t. \tag{10}$$

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3. MARKET EQUILIBRIUM CONDITIONS

When the number of states is less than the number of series in the model—which occurs with cointegrated series and common dynamics—it is possible to exploit the structure of the observation matrix C^* to construct a portfolio of contracts with an expected arbitrage relation. This relation can be used to construct price indices on which to base efficient settlement of delivered futures contracts, or alternatively for cash settlement.

The expected arbitrage relation requires the creation of a new asset that is a linear combination of the existing assets. The weights of the linear combination are determined by the coefficient matrix C^* which has dimension $(m \times n)$. This matrix is partitioned horizontally such that the upper submatrix, call it C_1, is square with dimensions $(n \times n)$, and the lower submatrix, subsequently denoted C_2, is $(m - n \times n)$, where m is the number of series and n is the number of states chosen in the specification of (1) and (2). Partition $\boldsymbol{y}_t$ and $\boldsymbol{e}_t$ conformably:

$$\begin{pmatrix} \boldsymbol{y}_{1t} \\ \boldsymbol{y}_{2t} \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \boldsymbol{z}_{t|t-1} + \begin{pmatrix} \boldsymbol{e}_{1t} \\ \boldsymbol{e}_{2t} \end{pmatrix},$$

which implies

$$\boldsymbol{z}_{t|t-1} = C_1^{-1}\boldsymbol{y}_{1t} - C_1^{-1}\boldsymbol{e}_{1t}, \tag{11}$$

so that, for $\varepsilon_t = \boldsymbol{e}_{2t} - C_2C_1^{-1}\boldsymbol{e}_{1t}$,

$$\begin{aligned} \boldsymbol{y}_{2t} &= C_2C_1^{-1}\boldsymbol{y}_{1t} + \varepsilon_t \\ &\equiv D\boldsymbol{y}_{1t} + \varepsilon_t. \end{aligned} \tag{12}$$

Whenever the number of states (n) is less than the number of series (m) an arbitrage relationship can be formed between n of the markets and the remaining $n - m$ markets.[4] This relationship is analogous to those utilized in the Arbitrage Pricing Theory (APT) in that it represents a multifactor return-generating process in which the factors (or indices) are the states. Analogous to the factors in traditional APT, the states in the above model also affect returns on more than one asset and are the sources of covariance between the asset prices in the model. The traditional APT factors are static, however, whereas these are dynamic. It is not uncommon in state-space models for the number of series to exceed the number of states so that these dynamic arbitrage relations exist and have meaning in a variety of contexts; see Cerchi and Havenner (1988) for another application, and see Cerchi (1989) for relations to alternative financial models.

The coefficients in D represent unique information about pairs of markets. This information is attributable to relative buyer concentrations, volumes of trade, transportation costs, etc. The elements of D represent the weights used to construct the new portfolio of spot and/or futures market holdings. If we partition $\boldsymbol{y}_t$ such that the futures price is in $\boldsymbol{y}_{1t}$, then each row of D contains the vector of weights associated with an arbitrage portfolio of $m - n$ spot markets and the futures against each of

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the n remaining spot markets. This provides the starting point for constructing the settlement price indices which incorporate the implied arbitrage opportunities and result in the optimal premiums and discounts.

Because the arbitrage relations determine only relative portfolio weights, D must be normalized so that the elements in each row of D sum to unity. This is accomplished by dividing each weight by the sum of the weights in its row, i.e., creating

$$d_{ik}^{*} = \frac{d_{ik}}{\sum_{j=1}^{m} d_{ij}}, \tag{13}$$

where d_{ik}^{*} is the ikth element of a new matrix D^{*}. The scaled weights are the inputs for the proposed price indices.

In practice, the relationship between $\boldsymbol{y}_{1t}$ and $\boldsymbol{y}_{2t}$ implied by (12) and D^{*} is

$$\hat{\boldsymbol{y}}_{2t} = \hat{D}^{*}\boldsymbol{y}_{1t}. \tag{14}$$

The regional markets are sometimes thin and can be dominated by particular individual transactions, so it is advantageous to average over multiple geographic prices when setting a settlement price for a specific market. This is possible whenever the number of states is less than $m - 1$, because in that case there will be multiple partitions of $\boldsymbol{y}_t$, each assigning some markets to $\boldsymbol{y}_{1t}$ to be used in determining settlement indices for the markets in $\boldsymbol{y}_{2t}$. Specifically, there will be $(m-1)!/[(m-n)!(n-1)!]$ different partitions which each yield an index for a given market price holding the futures in $\boldsymbol{y}_{1t}$ at all times, allowing use of the average of the calculated indices to construct the discounts and premiums. By doing so, we are averaging over the different events in various markets, including such things as large transactions and different regional qualities (in contrast to the single hypothetical quality of the futures).[5] Averaging over several regional markets, as well as the futures, also virtually eliminates the potential for local events, which may be potentially manipulable, to create unfair settlements.

The main advantage of the proposed settlement price index is that it reflects the multimarket arbitrage opportunities that exist between a spot market, other spot markets, and the futures, and therefore saves the economic system the costs of undesirable delivery due to legitimate basis differences not being taken into account in setting premiums, discounts, and pars. The discounts and premiums implied by the arbitrage indices are the difference between the index for the spot market and the futures. This time-varying structure more directly reflects the character of the markets and the particular industry than the previous fixed relations to the futures. Although the state-space model is statistically somewhat sophisticated, given the parameter estimates, it is a simple matter for all interested parties to validate the settlement index.

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4. STATE-SPACE MODEL OF LIVE CATTLE PRICES

To demonstrate the implementation of the above model, series for weekly Choice yield grade 1 and 2 (900–1100 lb) steer prices for seven markets and the nearby Chicago Mercantile Exchange (CME) futures price for live cattle were used.[6] The cash prices are Friday closings and the futures price is the Friday settlement price for the next contract due to expire. Thus, as a contract expires, the next chronological contract price enters the data. The sample is from January 1973 through May 1987 for a total of 752 weeks and was compiled by the Chicago Board of Trade. The model was estimated[7] using the first 10 years of data (520 weeks) with the remaining 232 weeks kept uncompromised for model validation. The seven spot markets chosen were the seven designated delivery points as of 1987 when the sample terminates.[8] In practice, it would not be necessary for these markets to be used; they simply represent a convenient set of markets. In theory, any set of markets should be cointegrated. The eight series were subjected to Dickey-Fuller tests with the result that the random-walk hypothesis could not be rejected at any reasonable significance level. The calculated Dickey-Fuller statistics were -1.50, -1.44, -1.48, -1.48, -1.44, -1.59, -1.54, and -1.70. Test statistics[9] smaller that -2.86 would be necessary to reject the null hypotheses of a random walk at the 0.05 level of significance. Thus, individually, the series would not be forecasted easily and the conventional wisdom of several years ago would have dictated first differencing each series.

The lag parameter was set to 1 for the trend model because this implies the lowest-possible-order determinantal polynomial for each series and to model the slow dynamics requires only immediate lags. (With eight series, even setting the lag parameter to 1 allows complex model dynamics because the implied univariate models for each series potentially include lags up to 8.) To determine the rank of the Hankel matrix, and thus the number of states required in the model, an examination was made of the ratios of successive singular values to the first singular value (a scale-free measure analogous to a condition number). Including the first, the ratios for the trend model were: 1.0, 0.0014, 0.0013, 0.0002, 0.0001, 0.0001, 0.0000, and 0.0000. Obviously, there is a large break between the first and second singular values. As a result, a trend model with a single state was chosen, i.e., the data support one set of slow dynamics common to all eight series.

The lag parameter was set to 2 for the cycle model specification. This means that the implied univariate model for each detrended price series can include lags up to 16 periods. This allows the identification of relatively faster dynamics remaining in the detrended series. The ratios of the singular values to the first singular value for the cycle model were: 1.0, 0.7357, 0.5417, 0.1449, 0.1053, 0.0959, 0.0502, and 0.0480. One large change occurs between the third and fourth singular values. This would suggest a cycle model with three states. Models with more states were investigated using only the in-sample data but were found to be inferior. Two of the cycle states gave rise to complex roots in A, incorporating smooth cycles in the model.

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TABLE 1. Coefficient estimates

	A^*		
0.9893	−0.0256	−0.0219	−0.0332
0.0	0.6815	0.0274	−0.1194
0.0	−0.1607	0.7295	0.0571
0.0	−0.2987	−0.0224	0.5030
	$B^{*\prime}$		
0.0047	−0.1846	−0.3466	−0.0717
−0.0147	0.2092	−0.1800	−0.0634
0.0017	−0.3759	0.2227	−0.2612
−0.0121	0.2454	−0.1471	0.0711
−0.0485	−0.0616	0.1107	0.1046
−0.0101	−0.2197	0.0708	−0.0281
0.0086	0.1990	−0.1427	0.2106
−0.0303	0.2517	0.4049	0.0719
	C^*		
−11.9128	−0.1583	−0.3069	0.4894
−12.3892	0.5909	−0.3792	0.1250
−11.6381	−0.6512	0.2908	0.3607
−12.2591	0.5883	−0.2648	0.2912
−11.8430	−0.1423	−0.1815	0.4296
−11.6531	−0.4809	0.2767	0.6612
−12.0363	0.6091	−0.4088	0.4275
−11.8364	0.8246	1.0634	0.2254

Order: Sioux City, Amarillo, Joliet, Dodge City, Omaha, Peoria, Greeley, and Futures. The state initial conditions were 0.9834, 0.6435, 1.1372, and 0.3456.

The trend and cycle models were joined to form the model depicted in (7) and (8). This combined model has four states, one from the trend model and three from the cycle model. Estimates of the coefficient matrices A^*, B^*, and C^* for the combined model are presented in Table 1. The matrix A^* determines the dynamic response of the model. The eigenvalues of A^* are inherited from the trend and cycle models because A^* is block triangular. An examination of the moduli of the eigenvalues of A^* (0.9893, 0.7682, 0.7682, and 0.3834) reveals a spectrum of dynamics ranging from very slow to relatively fast. It is not necessary for us to categorize the dynamics into stationary and nonstationary classes to produce accurate forecasts with the state-space method, and so we will not test formally for unit roots. Informally, it appears that one nonstationarity exists that is associated with the eigenvalue 0.9893 which clearly tracks very slow dynamics. This is followed by some intermediate dynamics associated with the two complex eigenvalues and finally a high-frequency component reflected in the smallest eigenvalue. This full range of responses is unusual in our experience; one practical consequence is that it would be difficult indeed to successfully model these series by individually differencing them first.

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The discovery of only one potential nonstationarity for the eight series leads to the conclusion that the prices in fact are cointegrated. However, this comes as no surprise. To reiterate earlier assertions, the price series of interest represent closely related commodities, and the arbitrage opportunities between locations will tend to force the prices together. Therefore, the cattle price series cannot infinitely diverge and must be cointegrated. Dickey-Fuller tests were conducted on the trend model residuals to confirm that the nonstationary components of price series were removed. These tests are analogous to the cointegration test presented by Granger (1986, p. 219); see also Engle and Yoo (1991). The eight test statistics computed from this set of regressions were −18.8, −16.4, −18.7, −17.6, −20.5, −17.8, −17.3, −13.9. In all cases, there is an overwhelming rejection of the random-walk hypothesis, suggesting that the price series are indeed cointegrated.

The in-sample summary statistics for this model are presented in Table 2. Backcasting was performed to obtain reasonable estimates of the initial states. The residuals do not move systematically around zero; rather, they appear to be random. The correlations between actual and predicted values are very high and the mean absolute deviations and root-mean-squared errors are small. This suggests an excellent fit of the in-sample data. Figures 1 and 2 illustrate the out-of-sample forecasts. These graphs show the 232 postsample observations, the single-period forecasts, and the residuals for two series, the Live Cattle Futures contract and the Joliet Steers Price. They are representative of the eight markets modeled.

The model forecasts conducted through the 232 weeks of postsample data were evaluated on root-mean-squared error and also subjected to Henriksson and Merton (1981) nonparametric tests (discussed below). The summary statistics for postsample performance are given in the rightmost column of Table 2. We observe no degradation of the in-sample results: The out-of-sample correlations and root-mean-squared errors remain good.

The Henriksson and Merton (HM) test provides an alternative to mean-squared-error forecast evaluation. The HM test ignores information about the sizes of the errors and simply tests whether or not the forecasts predict the correct direction of change. By eliminating magnitude information and using a nonparametric test, it is possible to obtain a test statistic with a known finite sample distribution under weak and plausible assumptions. The null hypothesis of the test is[10]

$$H_0 : \text{Prob}(\boldsymbol{y}_{it} - \boldsymbol{y}_{it-1} > 0 \mid \hat{\boldsymbol{y}}_{it} - \boldsymbol{y}_{it-1} > 0) + \text{Prob}(\boldsymbol{y}_{it} - \boldsymbol{y}_{it-1} \leq 0 \mid \hat{\boldsymbol{y}}_{it} - \boldsymbol{y}_{it-1} \leq 0) = 1 \quad \textbf{(15)}$$

for each of the $i = 1, 2, \ldots, 8$ series in turn. To carry out the test, define

$N_1 =$ the number of observations where prices actually rose,
$N_2 =$ the number of observations where prices actually did not rise,
$N = N_1 + N_2 =$ the total number of observations,
$n_1 =$ the number of successful predictions given that prices rose,
$n_2 =$ the number of unsuccessful predictions given that prices did not rise,
$n = n_1 + n_2 =$ the number of times price rises were forecast.

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Table 2. Summary statistics in levels

Statistic	In sample	Out of sample
Sioux City steers price ($/cwt)		
Mean	52.57	61.61
Root-mean-squared error	1.19	1.35
Average error	0.027	0.329
Mean absolute deviation	0.904	1.073
Simple correlation	0.990	0.924
Amarillo steers price ($/cwt)		
Mean	53.36	63.12
Root-mean-squared error	1.23	1.15
Average error	0.030	0.036
Mean absolute deviation	0.938	0.902
Simple correlation	0.990	0.939
Joliet steers price ($/cwt)		
Mean	52.46	61.28
Root-mean-squared error	1.34	1.46
Average error	0.025	0.672
Mean absolute deviation	0.858	1.144
Simple correlation	0.991	0.917
Dodge City steers price ($/cwt)		
Mean	52.93	63.03
Root-mean-squared error	1.25	1.44
Average error	0.028	0.665
Mean absolute deviation	0.938	1.135
Simple correlation	0.990	0.922
Omaha steers price ($/cwt)		
Mean	52.54	61.21
Root-mean-squared error	1.19	1.04
Average error	0.027	0.016
Mean absolute deviation	0.897	0.814
Simple correlation	0.990	0.947
Peoria steers price ($/cwt)		
Mean	52.34	60.25
Root-mean-squared error	1.22	1.14
Average error	0.024	−0.230
Mean absolute deviation	0.923	0.922
Simple correlation	0.989	0.946
Greeley steers price ($/cwt)		
Mean	53.05	62.68
Root-mean-squared error	1.24	1.36
Average error	0.028	0.368
Mean absolute deviation	0.948	1.063
Simple correlation	0.990	0.920
Live cattle futures ($/cwt)		
Mean	53.28	62.70
Root-mean-squared error	1.56	1.62
Average error	0.022	0.477
Mean absolute deviation	1.216	1.226
Simple correlation	0.983	0.879

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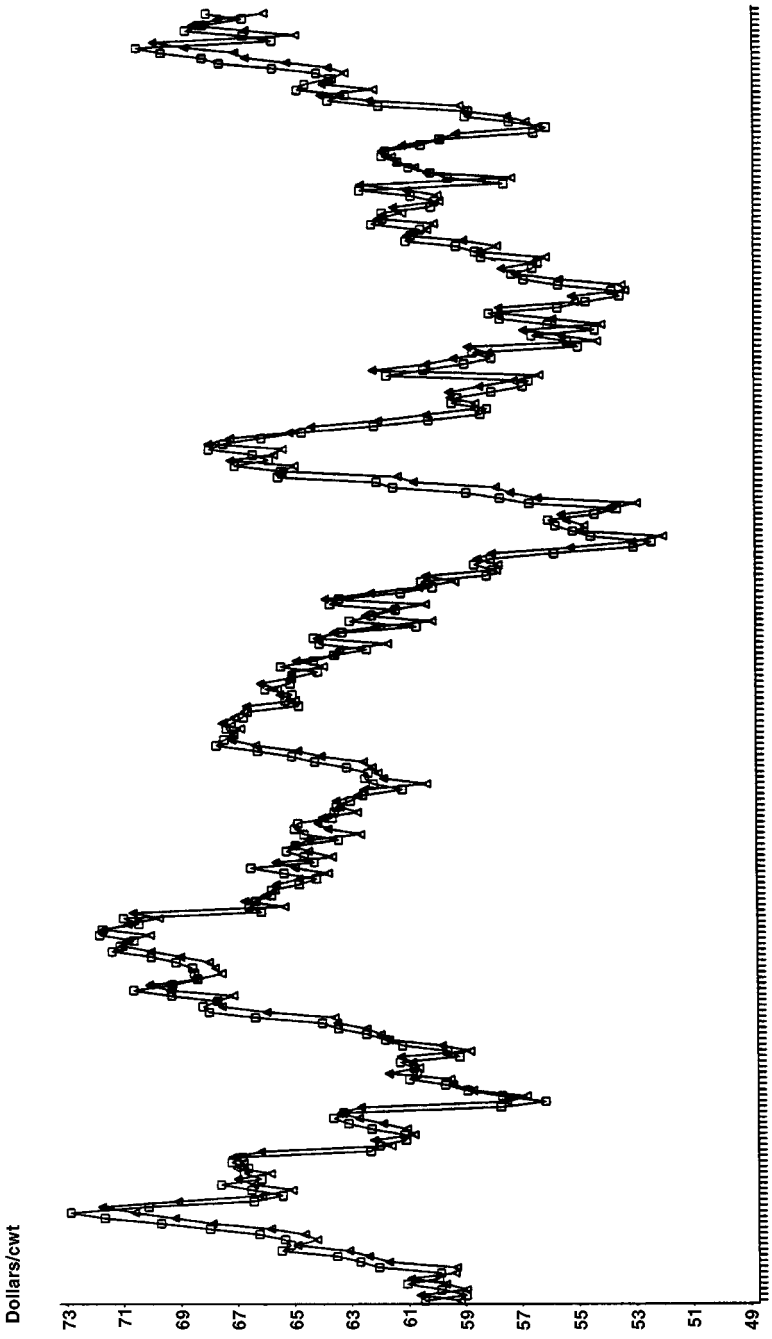


FIGURE 1. Actual and postsample forecasts of the CME live cattle price (weekly data, Jan. 1983–May 1987). □, actual; and Δ, forecast.

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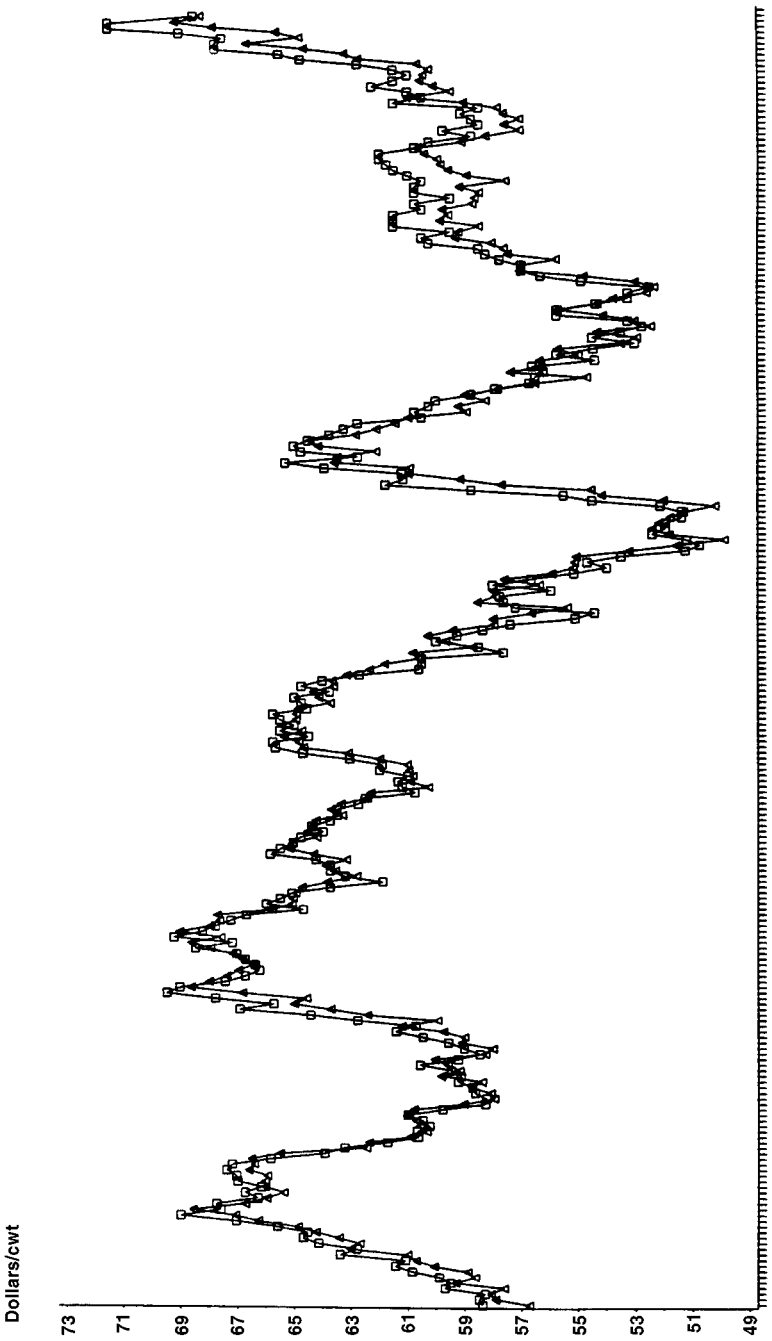


FIGURE 2. Actual and postsample forecasts of Joliet choice steer prices (weekly data, Jan. 1983–May 1987). □, actual; and △, forecast.

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Then, the finite-sample confidence level, c, is given by

$$c = 1 - \sum_{x=n_1}^{\min(N_1,n)} \binom{N_1}{x}\binom{N_2}{n-x} \Big/ \binom{N}{n}; \tag{16}$$

see Henriksson and Merton (1981).

The values for the HM test were $N_1 = 903$, $N = 1856$, $n_1 = 419$, and $n = 684$. These results lead to a confidence level of 1.0 that we can reject the null hypothesis that the forecasts are not informative. We feel that this is an extraordinary result, given the random-walk natures of the series being modeled.

5. ARBITRAGE PRICE INDICES FOR LIVE CATTLE

The matrices D_i^* were calculated for all of the possible partitions of $\boldsymbol{y}$ where the CME futures is kept in $\boldsymbol{y}_1$. Notice that some weights may be negative, indicating short positions in the corresponding market in the arbitrage portfolio. The arbitrage price index associated with an element of $\boldsymbol{y}_2$ is constructed using the corresponding row of D^*. The price indices were calculated for the 232 weeks of postsample data for all 35 possible partitions. Statistics for the difference between the current futures and the cash market indices are indicative of the relative strengths of the markets as compared to the futures, which is the actual commodity being traded. In all cases, the two values tend to converge whenever the difference between them becomes relatively great. This demonstrates the cointegration and arbitrage forces at work in the marketplace. It also represents the market forces that tend to reduce basis variability when the market is performing efficiently. Thus, the constructed arbitrage price index would be expected to mitigate some of the adverse effects associated with futures contract delivery if implemented to replace the current system of arbitrary discount/premium/par schedules.

Graphs of the indices and discounts/premiums are not included, but are available from the authors on request. The estimated differentials between arbitrage prices and the futures price demonstrate the model's capacity to capture the characteristics of the markets. For example, without exception, Joliet is estimated to trade at a discount to the futures. The mean absolute value of this discount is \$7.98 per cwt. This suggests a market with serious problems, and as it turns out, this market recently closed. The low volume indicates that disequilibrium prices might prevail, and subsequently, delivery as a percentage of total volume in Joliet should be high relative to other markets. To test this hypothesis, we spoke with Edna Kucinski of the CME Delivery and Inspections Division. As an example, she quoted the following delivery statistics for the month of October 1987: Joliet 54, Peoria 55, Dodge City 46, and Sioux City 103 contracts. At first, this would appear to violate the hypothesis tendered above. However, one must consider the deliveries relative to the total volume of the markets. According to the USDA Market News Livestock, Meat, and Wool Reports for the month of October 1987 the volumes of cattle sold in these markets were 2,271, 9,520, 30,333, and 14,991 head, respectively. Thus,

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although the total number of contracts delivered was essentially the same for Joliet, Peoria, and Dodge City, the delivery as a percentage of total volume was 4 times higher in Joliet than in Peoria, 13 times higher than in Dodge City, and 7 times higher than in Sioux City: The par setting on deliveries at Joliet resulted in relatively many more deliveries.[11] Had the exchange set an appropriate discount, then Joliet deliveries would not have been encouraged and both the price suppressing impacts of delivery as well as the transaction costs associated with delivery could have been avoided.

The estimated discounts and premiums imply that delivery at four of the cash markets during the postsample period should have, on the average, been at a discount and three at par.[12] The average discounts for Sioux City, Amarillo, Joliet, and Peoria were 2.37, 2.06, 7.32, and 2.13, respectively. The average differential for the three markets deemed to settle around par were −0.21 for Dodge City, 1.06 for Omaha, and −1.66 for Greeley.

Note also that 20 different indices of the nearby futures price paired with a given spot market price could be constructed from (12) by placing the CME futures price in $\boldsymbol{y}_{2t}$, the spot price in $\boldsymbol{y}_{1t}$, and generating all of the potential partitions of $\boldsymbol{y}_t$. The average of these indices forms an objective benchmark should cash settlement be chosen to replace physical delivery as the mechanism for arbitraging excessive basis deviations.

6. CONCLUSION

We have demonstrated a methodology for developing arbitrage price indices for futures settlement or cash settlement which are more efficient than simple indices constructed in an ad hoc fashion. The construction of these indices is based on the cointegration and spatial market arbitrage opportunities inherent in agricultural commodity markets. In general, ignoring these characteristics of the market in cash settlement or delivery would limit the futures contract in its ability to restore spatial market equilibrium and lead to undesirable and costly delivery.

An application to the live cattle market provided the empirical justification and demonstrated the construction of such indices. Aoki's state-space model was used to develop the cointegrated multivariate model of spot and futures prices. Because the validity of the settlement indices depends critically on the accuracy of the underlying model, the model was carefully tested using 232 weeks of data not used in any way at all in model construction. Postsample mean squared errors associated with one-week forecasts were small and the results of Henriksson and Merton nonparametric tests suggest that the forecasts are informative, validating the model as a basis for the proposed settlement indices.

The indices constructed have several advantages over less complicated alternatives. The new indices are time varying, and the discount and premium structure is based on spatial market arbitrage relations. Thus, they should tend to improve the efficient operation of live cattle markets. The incentives for delivery in any given location will be negated because the index for that market will adjust quickly as

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a result of possible disequilibria. By averaging over arbitrage relations linking to a large number of markets the new indices reduce the potential for manipulation of the indices while maintaining the integrity of the marketplace. Physical delivery, with its large associated transaction costs, is an indication of failure of the market to incorporate all relevant events in the contract clearing mechanism. The proposed settlement indices are designed to reduce the incidence of delivery and thus the market overhead costs by incorporating temporal and regional effects in an optimal manner.

NOTES

1. This method has an added advantage. Purchasing the commodity reduces the risk of not meeting the quality standards set down in the futures contract. This eliminates quality discounts that undoubtedly would arise if the commodity were shipped from a farm or other point of origin, but it does not render physical delivery costless: The delivering agent still pays for a variety of services such as feed and care, housing, weighing, grading, and commission fees.

2. See also Aoki and Havenner (1989, 1991).

3. Havenner and Aoki (1988) show that the estimators, based on the singular value decomposition, can be interpreted as instrumental-variable estimators.

4. This relationship does not depend on the cointegration of $\boldsymbol{y}_t$, i.e., the trend-cycle model. However, cointegration preserves the unforecastable trend(s).

5. Ross (1976) follows a similar tack in developing the APT model. That is, in his step 2, he invokes the law of large numbers to suggest that averaging over a large number of assets all with white-noise random errors leads to the elimination of the stochastic component. Here we are averaging over the prices of the different constructed portfolios for each geographical market.

6. For related work on a different data set, see Foster et al. (1995).

7. The program underlying this work was written in the Speakeasy© matrix language. A Gauss© translation is also available; see "User Guide for SSATS 2.0 Procedure Module" distributed through Gauss.

8. The seven delivery points were Amarillo, Dodge City, Greeley, Joliet, Omaha, Peoria, and Sioux City. In 1987, these all traded at par, but this was not the case for earlier years when some discounts were in effect.

9. Case 2, t analog.

10. Henriksson and Merton (1981) show that a necessary condition for the forecasts to have value is that the sum of these two conditional probabilities not be equal to 1, and a sufficient condition is that the sum exceed 1. For example, a perfect forecaster will have each probability individually equal to 1, summing to 2.

11. Anyone short in live cattle will monitor the futures price to potentially offset the position, and also monitor all regional markets to potentially deliver. If a regional price adjusted for the fixed premium/discount and the transaction costs of delivery is below the futures price, it is cheaper to deliver. When the fixed premium/discount is set incorrectly, excess delivery occurs.

12. The costs of delivery (grading discounts, shrinkage, and additional transportation costs, etc.) are estimated to be approximately \$2.00 per cwt. Thus, if the average discount/premium was between $\pm$ \$2, then that market was deemed to be on par on the average. This does not mean that the market should be arbitrarily traded that way; it is just a means of distinguishing the relative price differentials.

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