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On the planar stability of rigid-link binary walking robots

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SUMMARY

A binary walking robot moves as a result of bi-state actuator transitions. Because of the bi-state nature of binary joints, many research results about continuous walking robots cannot be applied to binary walking robots directly. In this paper, a new and simple model of rigid-link binary walking robot is proposed, around which related concepts are introduced, and formulas are derived. Based on this model, general characteristics and limitations of periodic gaits are discussed, and the stability qualities of several straight-line walking periodic gaits are studied in both pitch-greater-than-stroke and stroke-greater-than-pitch cases. Valuable results are obtained from the analysis, which should be followed in the design of rigid-link binary walking robots.

KEYWORDS: Walking robots; Planar stability; Rigid link; Periodic gaits.

1. INTRODUCTION

Because of their low cost and convenience of control, binary manipulators are receiving more and more attention. Since the 1990s, systematic research has been done by Chirikjian and coworkers in forward kinematics and optimal design of binary manipulators,[1] workspace generation,[2–5] inverse kinematics,[6–7] trajectory planning,[8] and inverse problem solving.[9] And a series of efficient algorithms have been developed. On the other hand, walking robots have attracted a lot of research effort because of their flexible locomotion performance and terrain adaptability. In order to take advantage of the merits of both, a combination of binary manipulators and walking robots resulted in the new idea of binary walking robots. So far many researchers have studied the gait stability of continuously-actuated walking robots. McGhee and Frank first analytically studied the gait stability, and derived the equation of the longitudinal stability margin for quadruped wave gaits.[10] Song and Waldron introduced an efficient analytical approach and derived a general equation for the longitudinal stability margin for the 2n-legged wave gait.[11–12] Nagy and Desa studied the gait stability of walking machines from the perspective of energy.[13] The research has been extended to dynamic stability[14] and crab gait stability,[15] and various walking robots were designed to implement those theoretical results.[16–23] However, many of the valuable results from continuous walking robots are found unsuitable to be applied to binary walking robots directly, because of the bi-state nature of binary joints.

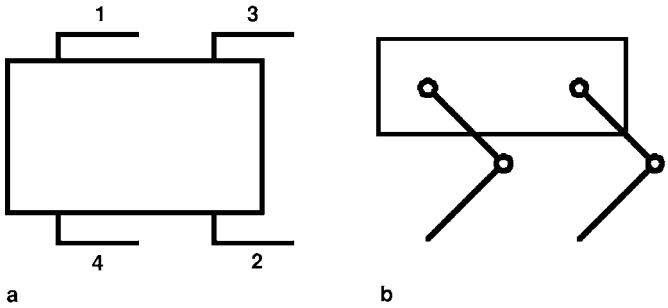


Fig. 1. A typical continuous walking robot. (a) Top view. (b) Front view.

To clarify this problem, a typical model of four-legged continuous walking robots with revolute joints is examined (Figure 1).

The robot can move forward in a straight line with a wave gait. At each time, one leg lifts up and does recovery motion, while the others support and propel the body forward. After the lifted leg finishes its recovery and touches the ground, one of the supporting legs lifts up and the remaining supporting legs continue their support and propulsion job with the new supporting leg.

There are two important properties in this process:

(i) The change of the state of one leg from touch-down to lift-up is in the middle of other legs' propulsion strokes.
(ii) In general, the instantaneous configuration of one leg is not parallel to those of the others (Figure 2).

These properties determine a strong coordination among all the supporting legs, which, in turn, requires a strong coordination among joints and a smooth trajectory for each joint. Continuously actuated robots are capable of this kind of job, but binary robots with revolute joints cannot. Smooth locomotion requires continuous and smooth trajectories for, if not all, at least some of the revolute joints. With only revolute joints, a binary robot has only several discrete states for each leg. It will inevitably cause sliding between

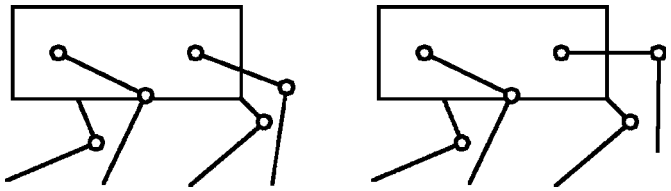


Fig. 2. Configurations in the propulsion process.

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the feet and the ground, which should be avoided from the point of view of position control.

To avoid this situation, some elastic and flexible parts can be introduced into binary robots. However, this will increase the complexity of analysis and control. Instead, a model of rigid-link binary walking robots is introduced in this paper, and the corresponding straight-line walking gait stability is studied in detail.

2. A MODEL OF RIGID-LINK BINARY WALKING ROBOT

Since the joint arrangement in Figure 1 is not applicable to rigid-link binary robots, a new model is introduced for studying straight-walking gait (Figure 3).

The leg arrangement of this binary robot is longitudinal axis-symmetric. We assume that the robot has n pairs of legs. The kth pair of legs is numbered as leg $2k-1$ and leg $2k$. In this simple model, each leg has only two linear degrees of freedom, one for back-and-forward locomotion, the other for up-and-down locomotion, implemented by a pair of binary prismatic joints. All the legs are assumed to have equal horizontal strokes and equal vertical strokes. In fact, the horizontal strokes must be the same for all the legs in order to move the body forward without causing interference. This kind of leg arrangement eliminates the possible sliding between the feet and the ground in straight-line walking. Furthermore, we assume that the mass of the legs can be ignored with respect to that of the body, and the center of mass is overlapped with the geometric center of the body.

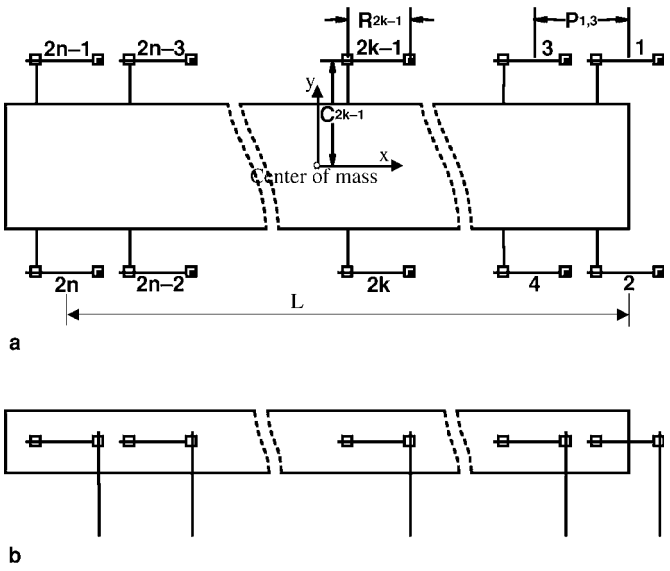


Fig. 3. A rigid-link binary walking robot with prismatic joints. (a) Top view. (b) Front view.

2.1. Related definitions

First we list out some definitions used in this paper:

(i) The cycle time T is the time for a complete cycle of leg locomotion of a periodic gait.[11]

(ii) The transfer phase ϕ_{it} of leg i is the period in which the foot i is not on the ground; the support phase ϕ_{is} of leg i is the period in which the foot i is on the ground.[11] $T=\phi_{is}+\phi_{it}$.

(iii) The phase ϕ_i is the fraction of a cycle period by which the placement of leg i on the ground lags behind the beginning of the cycle.

(iv) The propulsion phase ϕ_p of the body is the period in which the body moves forward.

(v) The stride λ is the distance that the center of mass of the body translates during one complete locomotion cycle.[11]

(vi) The leg stroke R_i is the distance through which the foot i is translated relative to the body during the support phase[11] (Figure 3). Because the leg strokes for all legs must be equal in order to keep the coordination, the common leg stroke is denoted as R. As a result, one obtains $\lambda=R$.

(vii) The pitch $P_{i,j}$ is the distance between the centers of strokes of the adjacent pairs of legs i and j (Figure 3), where $j=i-1$ or $j=i+1$.

(viii) The support pattern of a gait is a two-dimensional point set in the horizontal plane consisting of the convex hull of the vertical projection of all foot points in the support phase[11] (Figure 4). To keep a walking robot stable, its center of mass must stay inside the support pattern during the whole locomotion cycle.

(ix) The front stability margin S_f and the rear stability margin S_r are the distances from the vertical projection of center of mass to the front and rear boundaries

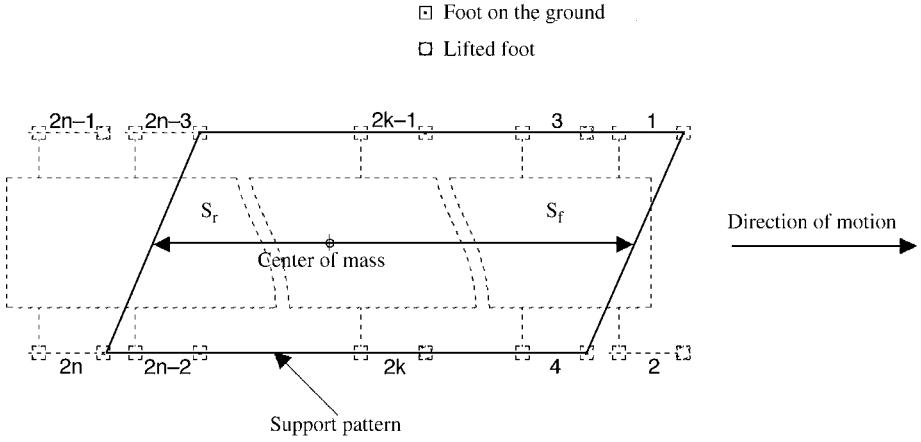


Fig. 4. Support pattern and stability margins.

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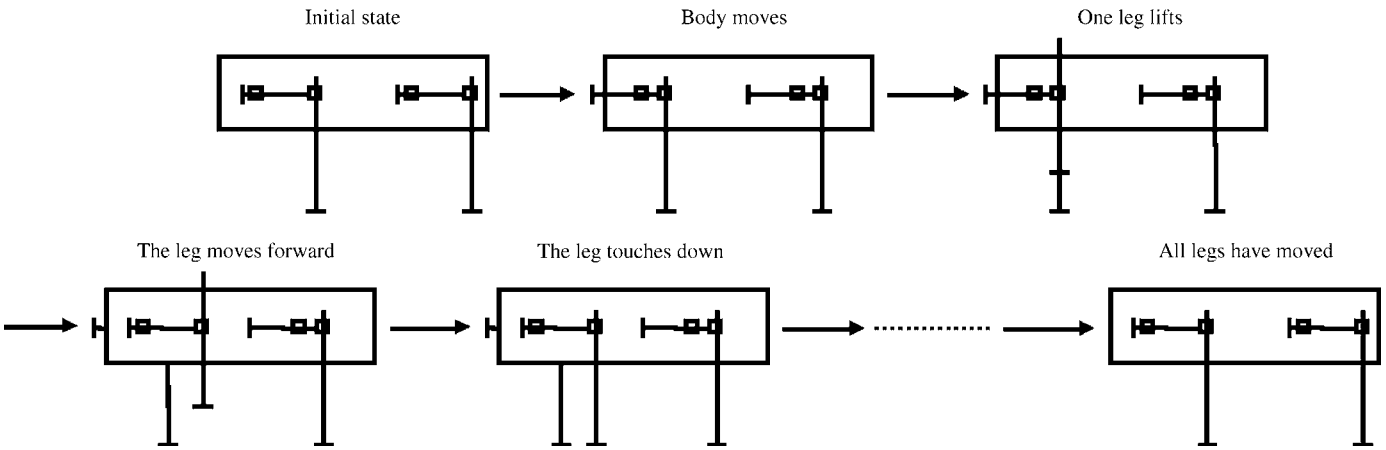


Fig. 5. Motion sequence with body moving first.

of the support polygon respectively, as measured in the direction of motion (Figure 4). The shorter of these two is the longitudinal stability margin S_L.[11]

(x) The longitudinal gait stability margin S for a periodic gait is the minimum S_L over an entire cycle of locomotion. A gait is statically stable if $S \geq 0$. Otherwise, it is statically unstable.[11]

To simplify the analysis, it is reasonable to normalize all the time data to the cycle time so that $T=1$, and normalize all the distance data to the stride so that $\lambda=1$.

2.2. General characteristics of periodic gaits for the binary walking robot

Without a loss of generality, non-singular regular periodic gaits are studied, which means that only one event, either the placement or the lifting of a foot occurs at any time. Therefore, with the binary nature of the robot, it is reasonable to consider the locomotion in a complete cycle as a sequence of leg and body motions.

Intuitively, there are three possible sequences of motion:

(a) The body moves forward at the beginning of the cycle before any leg moves forward (Figure 5). Here, the motion of a leg is also its recovery motion.

(b) The body moves forward at the end of the cycle after all the legs move forward. Here, the body motion is also the recovery motion of all the legs (Figure 6).

(c) The body moves in the middle of the legs' locomotion. In this case, some legs stay at the beginning of their strokes while the others have reached the end. This situation is well accepted by continuously-actuated robots, but makes it impossible for a rigid-link binary robot to move its body.

Therefore, only the body-move-first and body-move-last sequences can be realized in practice. Moreover, by simply observing these two locomotion sequences, it is clear that $\lambda=R$ must hold for the binary walking robot model.

2.3. Formulas

To ease the stability analysis, based on the definitions in Section 2.1 and the characteristics of binary walking robots, some formulas are derived in their standard forms as follows:

(i) The distance between the centers of stroke of the two end pairs of legs is (Figure 3)

$$L=\sum_{i=1}^{n-1} P_{i,j+1}. \tag{1}$$

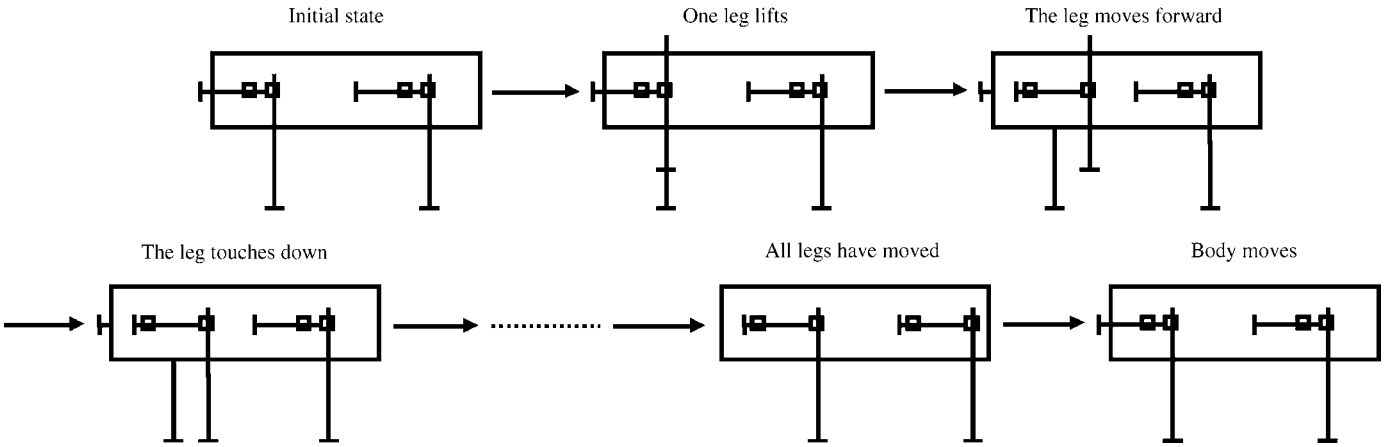


Fig. 6. Motion sequence with body moving last.

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(ii) Let the center of mass be the origin of the body-fixed local coordinate system. The x coordinate of the center of stroke of the kth pair of legs is

$$X_{Pk}=\frac{1}{2}L-\sum_{i=1}^{k}P_{i-1,i}, \tag{2}$$

where $P_{0,1}=0$.

(iii) The x coordinate of foot j with respect to the center of stroke of leg j is

$$X_{fj}=\begin{cases}-\dfrac{R}{2} & t\in[\phi_p,\ \phi_j-\phi_{jt}]\\ \dfrac{R}{2} & t\in[\phi_j,\ 1]\end{cases}, \tag{3}$$

when the body moves first; or

$$X_{fj}=\begin{cases}-\dfrac{R}{2} & t\in[0,\ \phi_j-\phi_{jt}]\\ \dfrac{R}{2} & t\in[\phi_j,\ 1-\phi_p]\end{cases}, \tag{4}$$

when the body moves last.

(iv) The position of foot j with respect to the center of mass is

$$X_j=X_{p\frac{j+1}{2}}+X_{fj},\ Y_j=C_j, \tag{5}$$

when j is odd; or

$$X_j=X_{p\frac{j}{2}}+X_{fj},\ Y_j=-C_j, \tag{6}$$

when j is even. Here C_j is the constant size of leg j in y direction (Figure 3).

If all the pitches are equal, (1) becomes

$$L=(n-1)P, \tag{7}$$

where P denoting the common pitch; and (2) becomes

$$X_{pk}=\frac{n-2k+1}{2}P. \tag{8}$$

3. STABILITY OF GAITS

Defined in Section 2.1, the longitudinal gait stability margin S is used as the measure of static stability. To simplify the analysis, all the legs are assumed to have common P. The cases of R<P, R>P and R=P are studied respectively.

3.1. R<P

In this case, the leg stroke is shorter than the pitch between two neighboring pairs of legs, which means that the (i+1)th pair of legs cannot move to the front of the ith pair of legs. As a result, only the two front pairs of legs and two rear pairs of legs determine S. Therefore, to calculate the static stability, one only needs to consider all the legs when n≤4; otherwise only four pairs of legs are involved. Moreover, the

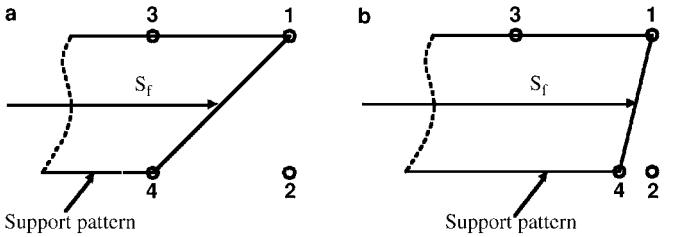


Fig. 7. Front stability margin when R<P (in Fig. 7, feet 1, 3 and 4 are on the ground while foot 2 is not). (a) Leg 4 has not moved forward before leg 2 lifts up. (b) Leg 4 has moved forward before leg 2 lifts up.

S can only occur when either the first of the front pair of legs or the second of the rear pair lifts up. Here, first and second means the lifting order among the front legs and the rear legs.

For the front-leg-lift-up case, without a loss of generality, let leg 2 lift up before leg 1 does. Then S_f is determined by the positions of leg 1 and 4 (Figure 7):

$$S_f=X_{Sf}=\frac{X_1+X_4}{2},\quad Y_{Sf}=\frac{Y_1+Y_4}{2}. \tag{9}$$

The greatest S_f occurs when leg 4 has moved forward before leg 2 lifts up.

For the rear-leg-lift-up case, without a loss of generality, let leg 2n–1 lift up after leg 2n does. Then S_r is determined by the positions of leg 2n–3 and 2n (Figure 8):

$$S_r=X_{Sr}=\frac{X_{2n-3}+X_{2n}}{2},\quad Y_{Sr}=\frac{Y_{2n-3}+Y_{2n}}{2}. \tag{10}$$

The smallest S_r occurs when 2n–3 has not moved forward before leg 2n–1 lifts up.

As a result, $S=\min(|S_f|, |S_r|)$.

(i) 4-leg gait stability

The greatest gait S_f occurs when leg 2 lifts up and leg 4 has moved forward, and the smallest gait S_r occurs when leg 3 lifts up and leg 1 has not moved forward. The same result is obtained in both cases $S_f=S_r=0$, therefore S=0. It is on the edge of stability and therefore is called weak stability. The wave gait with leg order 4–2–3–1 has this greatest S (Figure 9).

(ii) 6-leg gait stability

The greatest gait S_f occurs when leg 2 lifts up and leg 4 has moved forward: $S_f=P/2$. The smallest gait S_r

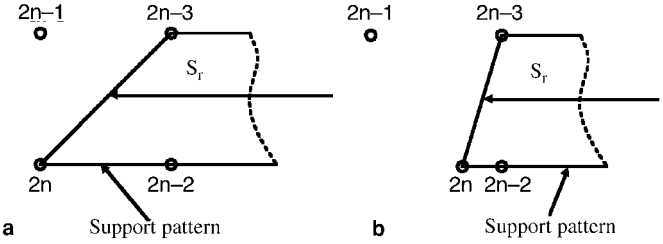


Fig. 8. Rear stability margin when R<P (in Fig. 8, feet 2n–3, 2n–2 and 2n are on the ground while foot 2n–1 is not). (a) Leg 2n has not moved forward before leg 2n–1 lifts up. (b) Leg 2n has moved forward before leg 2n–1 lifts up.

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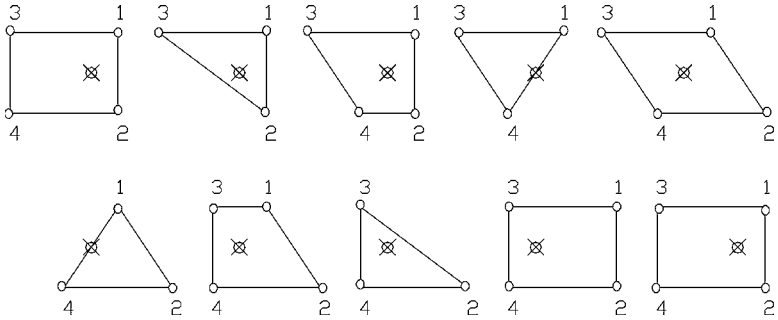


Fig. 9. Evolution of support pattern of 4-leg wave gait. (The small circles represent the feet on the ground. The circle with a cross represents the center of mass.)

occurs when leg 5 lifts up and leg 3 has not moved forward: $S_r = -P/2$. So $S = P/2$. The wave gait with leg order 6–4–2–5–3–1 has this greatest S (Figure 10). It is statically stable.

(iii) 6-leg tripod gait (Figure 11)

When leg 2, 3, 6 move forward simultaneously, S_f is determined by leg 1 and 4, both of which have not moved forward, so $S_f = ((P - R)/2)$. When leg 1, 4, 5 move forward simultaneously, S_r is determined by leg 3 and 6, both of which have moved forward, so $S_r = -((P - R)/2)$. Therefore, $S = ((P - R)/2)$, which means that the tripod gait is stable when $R < P$.

3.2. $R > P$

In this case, the (i+1)th pair of legs can move to the front of the ith pair. However, the S still occurs when either the first front leg or the second rear leg lifts up.

For the front-leg-lift-up case, without a loss of generality, let leg 2 lift up first, then

$$S_f = X_{Sf} = \frac{X_i + X_j}{2}, \quad Y_{Sf} = \frac{Y_i + Y_j}{2}, \tag{11}$$

where i denotes the farthest odd-numbered leg in front of leg 1, and j the farthest even-numbered leg in front of leg 4 (Figure 12). If no odd-numbered leg moves to the front of leg 1, then X_i and Y_i in the above formula are substituted with X_1 and Y_1. Similarly, if no even-numbered leg moves to the front of leg 4, then X_j and Y_j in the above formula are substituted with X_4 and Y_4.

For the rear-leg-lift-up case, without a loss of generality, let leg 2n–1 lift up second, then

$$S_r = X_{Sr} = \frac{X_i + X_j}{2}, \quad Y_{Sr} = \frac{Y_i + Y_j}{2}, \tag{12}$$

where i denotes the farthest odd-numbered leg behind leg 2n–3, and j the farthest even-numbered leg behind leg 2n (Figure 13). If no odd-numbered leg lags behind leg 2n–3, then X_i and Y_i in the above formula are substituted with X_{2n-3} and Y_{2n-3}. Similarly, if no even-numbered leg lags behind leg 2n, then X_i and Y_j in the above formula are substituted with X_{2n} and Y_{2n}.

As a result, $S = \min(|S_f|, |S_r|)$.

(i) 4-leg gait stability

It is impossible for both leg 3 and leg 4 to move to the front of leg 1 and leg 2, because the center of mass will move out of the support pattern. Therefore, the greatest gait S_f occurs when leg 2 lifts up and leg 4 has moved forward, and the smallest gait S_r occurs when leg 3 lifts up and leg 1 has not moved forward. Therefore, similar to the $R < P$ case, $S = 0$ (weak stability), and the wave gait with leg order 4–2–3–1 has this greatest S.

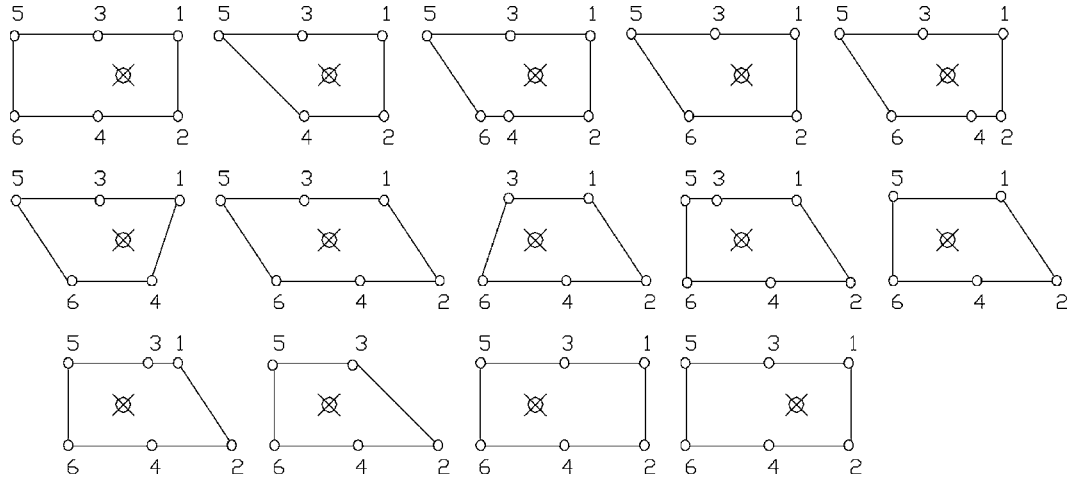


Fig. 10. Evolution of support pattern of 6-leg wave gait. (The small circles represent the feet on the ground. The circle with a cross represents the center of mass.)

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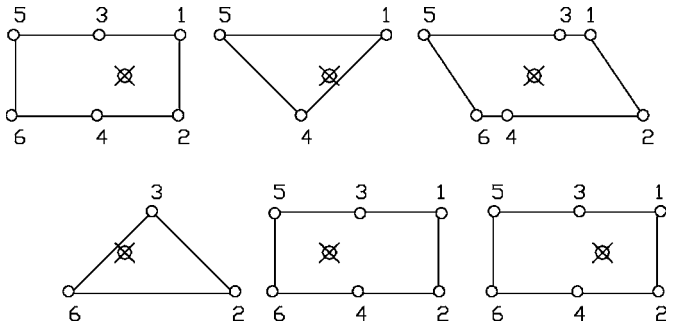


Fig. 11. Evolution of support pattern of 6-leg tripod gait. (The small circles represent the feet on the ground. The circle with a cross represents the center of mass.)

(ii) 6-leg gait stability

The greatest gait S_f occurs when leg 2 lifts up and leg 4 has moved forward, and the smallest gait S_r occurs when leg 5 lifts up and leg 3 has not moved forward. Therefore, similar to the R<P case, $S=(P/2)$, and the wave gait with leg order 6–4–2–5–3–1 has this greatest S. It is statically stable.

(iii) 6-leg tripod gait

Same as the R<P case, one obtains $S=((P-R)/2)$. However, this means that the tripod gait is not stable when R>P, because S<0 in this case.

3.3. R=P

The R=P case represents a boundary situation. It is clear that there is no change for the calculation of the longitudinal gait stability margin from those of R>P and R<P. However, the 6-leg tripod gait becomes weakly stable.

3.4. Comparison

An obvious result from the above analysis is that the tripod gait is stable when R≤P, but not stable when R>P. Here, "weakly stable" is considered as stable.

Meanwhile, although appearing the same in formulas, the stability of the 4-leg and 6-leg wave gaits when R>P is different from the situation when R≤P.

In the 4-leg case, if the body moves first, the instant stability margin S_f is determined by leg 1 and 2: $S_f=(P/2)-(R/2)$, and S_r is determined by leg 3 and 4: $S_r=-(P/2)-(R/2)$. If the body moves last, $S_f=(P/2)+(R/2)$ and $S_r=-(P/2)+(R/2)$. Because $S_f\geq 0$ and $S_r\leq 0$ are the necessary conditions for being stable, it is stable when R≤P but not stable when R>P.

In the 6-leg case, if the body moves first, S_f is determined by leg 1 and 2: $S_f=P-(R/2)$, and S_r is determined by leg 5 and 6: $S_r=-P-(R/2)$. If the body moves last, $S_f=P+(R/2)$ and $S_r=-P+(R/2)$. For the same reason as above, it is stable when R≤2P but not stable when R>2P.

Generally, in the 2n-leg case, if the body moves first, S_f is determined by leg 1 and 2: $S_f=((n-1)/2)\cdot P-(R/2)$, and S_r is determined by leg 2n–1 and 2n: $S_r=-((n-1)/2)\cdot P-(R/2)$. If the body moves last, $S_f=((n-1)/2)\cdot P+(R/2)$ and $S_r=-((n-1)/2)\cdot P+(R/2)$. For the same reason as above, it is stable when R≤(n–1)P but not stable when R>(n–1)P.

4. RESULTS, SIMULATION AND DISCUSSION

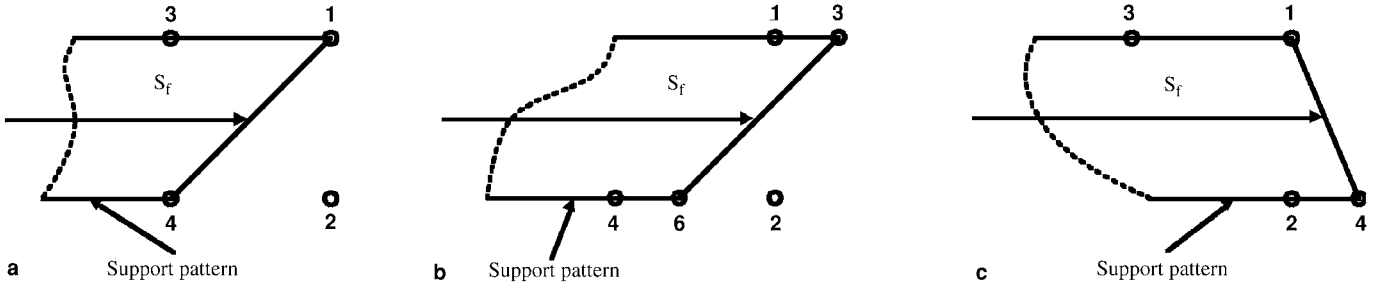


Fig. 12. Front stability margin when R>P. (In Fig. 12, foot 2 is not on the ground while other feet are. Here the small circles represent feet.) (a) Leg 4 has not moved forward before leg 2 lifts up, and no legs have moved to the front of leg 3 and leg 4. (b) Leg 4 has not moved forward before leg 2 lifts up, leg 3 has moved to the front of leg 1, and leg 6 has moved to the front of leg 4. (c) Leg 4 has moved forward before leg 2 lifts up.

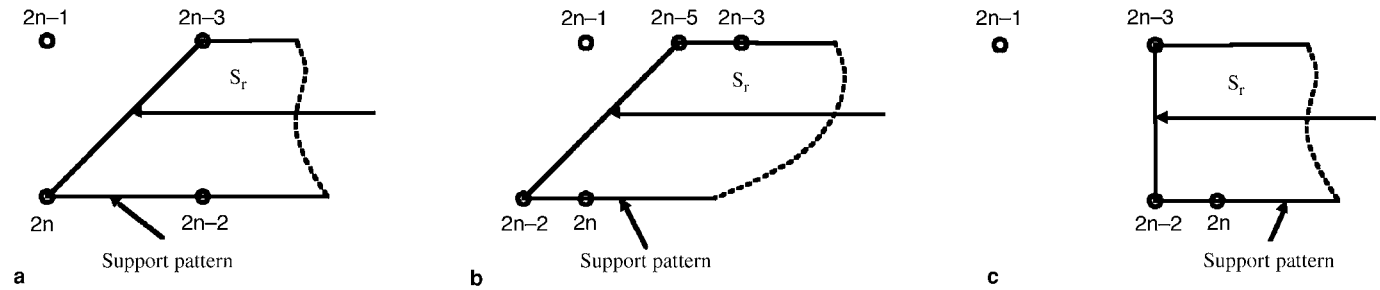


Fig. 13. Rear stability margin when R>P. (In Fig. 13, foot 2n–1 is not on the ground while other feet are. Here the small circles represent feet.) (a) Leg 2n has not moved forward before leg 2n–1 lifts up, and no legs lag behind leg 2n–3 and leg 2n. (b) Leg 2n has not moved forward before leg 2n–1 lifts up, leg 2n–5 lags behind leg 2n–3, and leg 2n–2 lags behind leg 2n. (c) Leg 2n has moved forward before leg 2n–1 lifts up.

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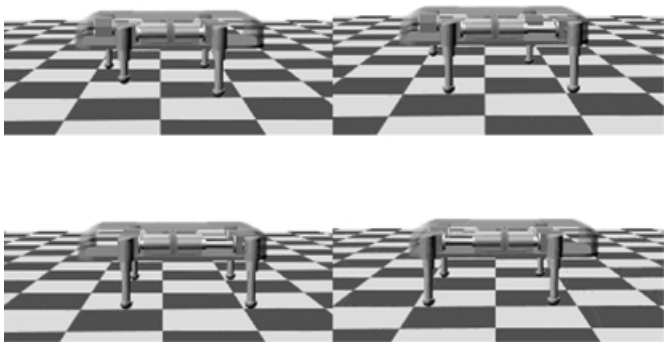


Fig. 14. Simulation of a 4-legged binary walking robot. Upper left: leg 4 lifts up and moves forward; upper right: leg 4 touches down; lower left: all legs have moved forward; lower right: body moves forward.

4.1. Results

Based on the stability analysis of the rigid-link binary walking robot model, some important results are obtained:

(i) A 2n-legged binary walking robot can maintain its stability only if $R \leq (n-1)P$.
(ii) A 4-legged binary walking robot can only obtain weak stability.
(iii) A 6-legged binary walking robot requires $R \leq P$ to use tripod gait, though the necessary condition for stability is $R \leq 2P$.
(iv) The more legs a binary robot has, the stronger stability it can obtain.

These results make up a set of general criteria which should be followed in the design of rigid-link binary walking robots. In addition, the cases of $R = P$ and $R > P$ require some special adjustment in the leg arrangement of the binary walking robot. For instance, the shoulders of any two neighboring legs must be at different altitudes, and any two neighboring legs must have different C sizes. In practice, a design with $R = P$ is not recommended because it usually can only offer a weak stability which is hard to implement.

4.2. Simulation

As an example, a 4-legged rigid-link binary walking robot with $R < P$ is simulated and visualized in 3DSMAX (Figure 14). In the simulation, the wave gait with leg order 4–2–3–1 is implemented.

4.3. Discussion on improving stability

In practice, being stable is highly desired. How to maintain and improve stability is very important for the design of walking machines. It becomes critical for such cases as 4-legged binary walking machines, where only weak stability can be obtained, because they are not guaranteed in real implementation.

Here we give some suggestions to increase the stability margin:

(a) Mount a movable weight on the body to maintain the stability (Figure 15). Using a 4-legged binary walking machine as an example, the locomotion of the weight is implemented by a prismatic joint. One can see that, when leg 2 lifts up, the support pattern is a triangle determined by feet 1, 3 and 4. As for the robot without the movable weight, $S_f = 0$, and the center of mass is located on the side connecting feet 1 and 4 (Figure 9). If the mounted weight moves backward before leg 2 lifts

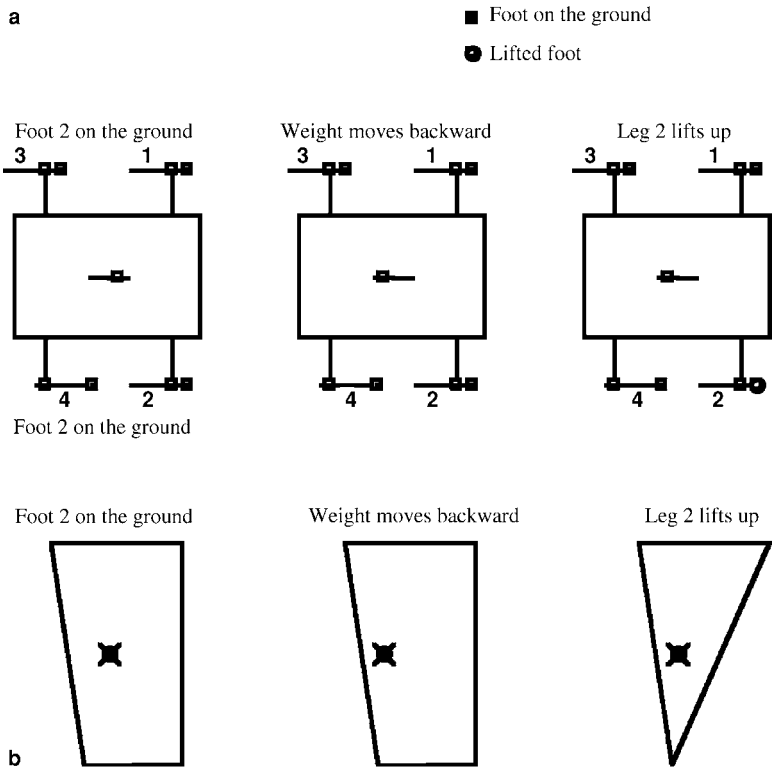


Fig. 15. Using movable weight to improve the stability. (a) Motion sequence. (b) Correspondingly evolving support pattern.

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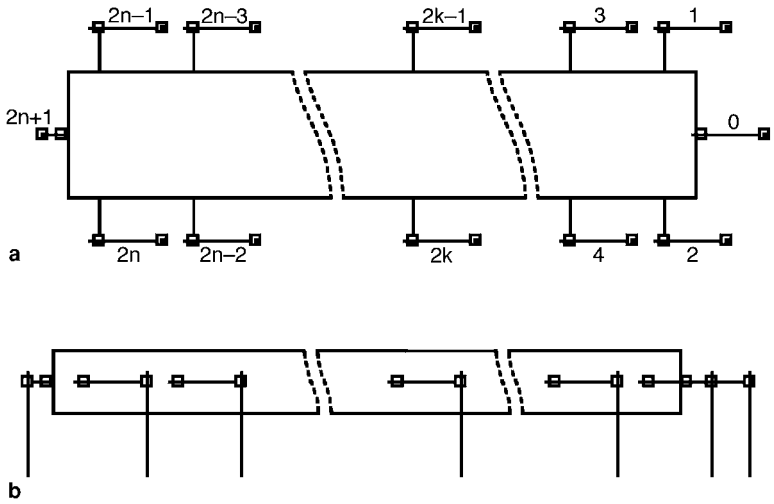


Fig. 16. A 2n+2 arrangement of the rigid-link binary walking robot. (a) Top view. (b) Front view.

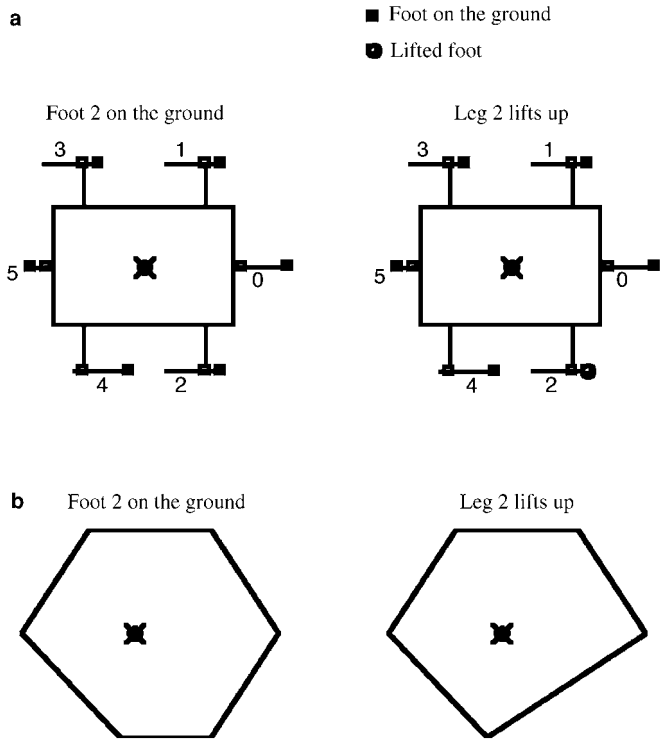


Fig. 17. Using 2n+2 arrangement to improve the stability. (a) Motion sequence. (b) Correspondingly evolving support pattern.

up, the center of mass is moved backward correspondingly. Then, when leg 2 lifts up, $S_f>0$, the center of mass is inside the triangle instead of on the boundary so that the robot obtains a stronger stability.

(b) Use a 2n+2 leg arrangement to provide generally stronger stability than 2n arrangement (Figure 16). Here the two additional legs are numbered as leg 0 and leg 2n+1. Using 4+2 arrangement as an example (Figure 17), when leg 2 lifts up, The support pattern is the polygon determined by feet 0, 1, 3, 4 and 5 instead of feet 1, 3 and 4 of the 4-leg arrangement. As a result, $S_f>0$, and the stability is not weak any more.

(c) Add another prismatic joint to each leg so that the leg can move outward (Figure 18). Using a 6-leg robot as an example, when leg 2 lifts up, the support pattern is determined by feet 1, 3, 4, 5 and 6. If leg 4 has moved outward before leg 2 lifts up, the support pattern covers bigger area and S_f is bigger. Thus, a stronger stability is obtained. This method only applies to binary robots with more than 4 legs. As for a 4-legged walking robot, if leg 4 moves outward, when it lifts up, the instantaneous stability is still weak.

Clearly the above-mentioned methods do improve the stability of binary walking robots. The negative impact is that the total number of DOFs of the robot increases, and a more complex gait is required to take care of the extra locomotion.

8. CONCLUSIONS

Binary joints make the locomotion of binary walking robots largely different from that of traditional continuously-actuated walking machines. From the perspective of position control, a new model of rigid-link binary walking robot is introduced. Correspondingly, a set of formulas are derived. These seemingly simple formulas make up a powerful enough tool of stability analysis for this category of binary walking robots. Conceptual robots with different number of legs and different gaits are employed to study both the capability and limitation of the model. Research shows that the current model can implement the straight-walking scheme of rigid-link binary walking robots effectively, though each leg has only two degrees of freedom. This paper sets a starting point of the stability research for binary walking robots. Because of their binary nature and resulting convenience of control and low cost, this category of robots provides a potential economic solution to discretizable environments and modular robotic systems. It is the author's belief that it is promising to extend the current model to implement the turning gait and rough terrain adaptation.

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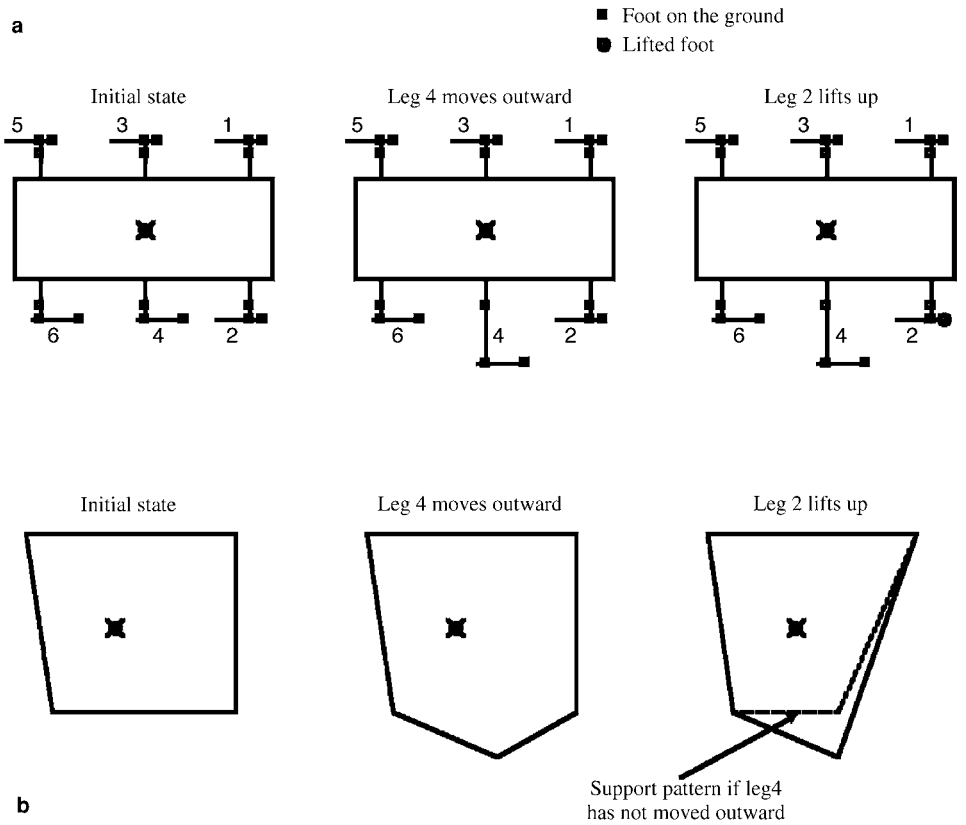


Fig. 18. Using additional prismatic joints to improve the stability. (a) Motion sequence. (b) Correspondingly evolving support pattern.

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