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Optimal Estimators for Weak Gravitational Lensing

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Abstract. Through the use of a new novel implementation of the quadratic maximum likelihood estimator, we tested the comparative performance of the Pseudo-C_ℓ estimator for upcoming Stage-IV cosmic shear surveys. We found a ∼20% and over an order of magnitude increase in the errors associated with the E- and B-mode spectra, respectively, when comparing the Pseudo-C_ℓ estimator to QML. This highlights that QML estimators should be applied to data from upcoming Stage-IV surveys.

Keywords. Gravitational lensing, Large-scale structure of universe, Methods: numerical.

1. Introduction

Cosmic shear is the coherent distortion in the shapes of background galaxies due to the matter distribution of the intervening large-scale structure (see Bartelmann & Schneider (2001) and Kilbinger (2015) for comprehensive reviews). One such method for extracting cosmological information from the observation of galaxy shapes is through the power spectrum. As many robust methods have been developed to produce estimates for the theoretical prediction for the power spectrum as a function of cosmological parameters, the extraction of the power spectrum from data has become a key aspect in any analysis using cosmic shear data.

There exists two broad categories of power spectrum estimators that have gained popular support (see Efstathiou (2004) for a comprehensive review of these estimators): the quadratic maximum likelihood (QML) estimator and the Pseudo-C_ℓ estimator. The QML estimator follows from a derivation of the pixel-level likelihood of the power spectrum coefficients given a map, and has been shown that the errors associated with this estimator are optimal (that is, its covariance matrix is given by the inverse Fisher matrix and thus satisfying the Cramér-Rao bound). The Pseudo-C_ℓ estimator is a derivative of the QML estimator where the pixel covariance matrix is assumed to be diagonal, which vastly simplifies calculations which dramatically increases the performance of the estimator at a cost of having non-minimal errors. Given that QML estimators have traditionally been very computationally expensive, the potential gain from smaller errors have been overlooked in favour of the more performant Pseudo-C_ℓ methods. Hence, the investigation into the development of an improved QML estimator, specifically one that performs much faster than existing implementations, is of vital importance to the cosmic shear community - and something that has been investigated here.

2. Method

Central to our new QML method is the use of the conjugate gradient algorithm to numerically invert the pixel covariance matrix. This is in direct comparison to previous

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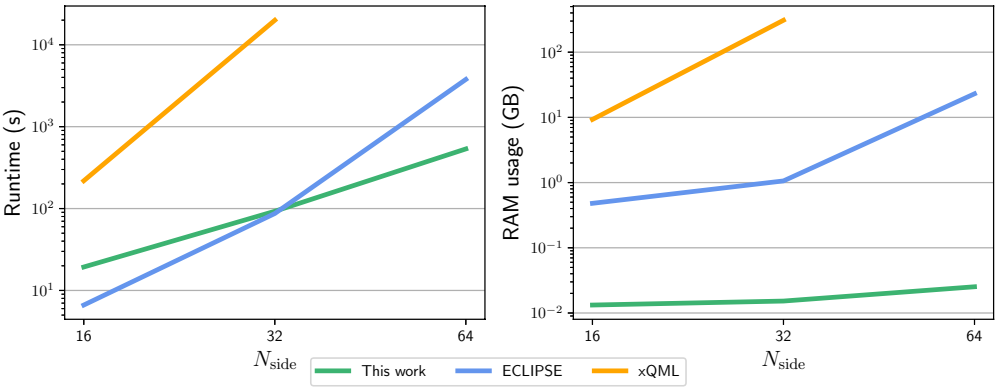


Figure 1. Comparison of runtime and RAM usage between different QML implementations as a function of input map resolution, with our code showing significantly better scaling.

methods where the computation of the full form of the covariance matrix is required, which is a slow and numerically complex problem. Since the covariance matrix size grows with the square of the number of pixels in the map, this has traditionally placed strict limits on the processing high-resolution maps using QML methods due to memory constraints. However, as our conjugate gradient approach only needs to compute the action of a trial vector on the covariance matrix, we subvert the memory challenges of existing methods and thus can produce estimates for the power spectrum for much higher resolution maps than traditional methods can process.

Figure 1 compares the runtime and memory usage of two existing QML implementations (`xQML` – presented in Vanneste *et al.* (2018), and `ECLIPSE` – presented in Bilbao-Ahedo *et al.* (2021)) with our new method. Map resolutions are presented as a function of the `HealPix` $N_{\rm side}$ parameter, where the number of pixels in a map is given by $N_{\rm pix} = 12N_{\rm side}^2$, and the maximum reliable multipole obtainable from a map is $\ell_{\rm max} = 2N_{\rm side}$. Figure 1 shows that the use of the conjugate gradient algorithm means that our method requires orders of magnitude less RAM than the existing methods over the range of map resolutions and runs significantly faster at higher-resolution maps. This allows our method to push to much higher resolution maps than is possible for existing methods before we run into computational bottlenecks, with our method able to produce estimates for resolutions of $N_{\rm side} = 256$ whereas the existing methods were limited to $N_{\rm side} = 64$ for the same hardware.

3. Results

With our new QML implementation able to produce power spectrum estimates for significantly higher resolution maps than were previously possible, we could now, for the first time, produce a direct comparison for the power spectrum errors between the Pseudo-C_ℓ and QML methods to much higher multipoles than was previously possible. The result of this comparison is shown in Figure 2. Here, we see that the sub-optimality of the Pseudo-C_ℓ estimator results in larger errors for the power spectrum estimates when compared to the QML estimator, with the increase in errors most pronounced for low multipoles. In addition, we see that there is an over order of magnitude improvement for the QML estimates in the ratios for the BB-spectra errors (which should be zero for parity-conserving gravitational lensing). Hence, applying QML methods to existing and forthcoming cosmic shear data sets will provide the maximal amount of constraining power possible and is thus something that should be considered for all future surveys.

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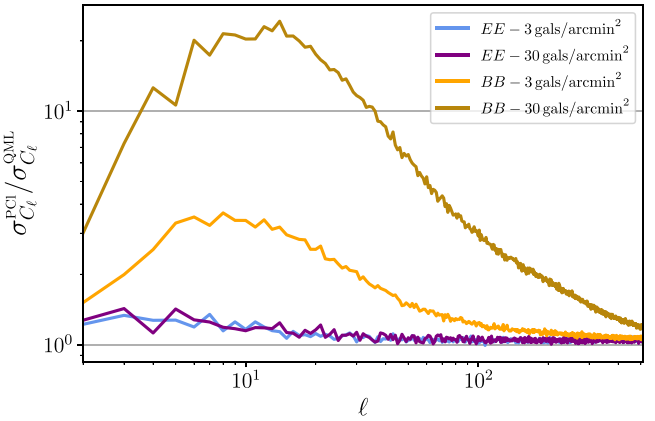


Figure 2. Ratio of the Pseudo-C_ℓ to QML errors for two different noise levels, showing a vast improvement for the QML estimator at low-ℓ multipoles, particularly for the BB-spectra.

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