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THE AMBIGUITY OF KNOWABILITY

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Abstract. In this paper it is shown that the Verification Thesis (all truths are knowable) is only susceptible to Fitch's Paradox if one conflates the *de re* and *de dicto* interpretation of knowability. A formalisation shows that if one treats knowability as a complex second-order predicate, then the paradox falls apart.

§1. Introduction. Fitch's paradox (1963) poses a serious challenge for the Verification Thesis (VT) that all truths are knowable. If one assumes that there is an unknown truth p, then *p is an unknown truth* is itself a truth. According to VT this implies that *p is an unknown truth* is knowable, but this leads to a contradiction, because if *p is an unknown truth* were known, then p would be both known and unknown. Therefore VT implies that there are no unknown truths. This consequence of VT is clearly absurd, but it is unclear how this absurdity can be avoided.

After its rediscovery in the 1970s, a great number of publications have discussed the paradox and proposed ways to solve it. Brogaard and Salerno (2012) distinguish two categories for approaches to the paradox: (i) those that avoid the absurdity by weakening the logic that led to it, (ii) those that avoid the absurdity by restricting the quantifier in such a way that VT does not apply to the so-called Moore sentence: *p is an unknown truth*. In this paper I argue that there is a third way of approaching the paradox, namely by maintaining that the paradox is based on a fallacy of equivocation. Sentences such as VT containing the word 'knowable' can be read in two ways: they can be given a *de re* and *de dicto* interpretation. I show that the *de re* interpretation of VT is not susceptable to the argument above, while the *de dicto* interpretation of VT is susceptable to it. Therefore, a verificationist can argue that the *de re* interpretation of VT is the right interpretation and in doing so ward off the paradox.

The approach presented here is in the same spirit—though different from—approaches that Brogaard and Salerno (2012) categorize under (ii), namely Kvanvig (1995; 2006) who concludes that the truths over which VT quantifies are *indexical* due to implicit quantifiers over individuals and times in VT, and Edgington (1985) who takes knowledge to depend on a situation and takes VT to contain implicit quantifiers over situations. Recently, Kennedy (2014) proposed a formal solution in the same spirit, that identifies the error in the argument as a substitution of non-rigid expressions into modal contexts. My analysis identifies the ambiguity of VT and the resulting equivocation as the source of the paradox, and provides a simple formal setting, based on the work of Van Benthem (2004), to support the analysis.

To be precise, my analysis locates the fallacious shift in interpretation when the argument proceeds from

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(1) *p is an unknown truth* is knowable,

to

(2) It is possible that *p is an unknown truth* is known.

Sentence 1 is true in its *de re* interpretation, but sentence 2 only follows from the *de dicto* interpretation of sentence 1. In other words, Moore sentences are knowable, while simultaneously it is impossible for them to be known. I will provide a formal logic in which this inference pattern is invalid.

§2. *De re* and *de dicto*. It is well known that ambiguity comes about when an expression may fall inside or outside the scope of a modality. In the context of first-order modal logic, Stalnaker and Thomason (1968) provide the following example of a natural language sentence, which has two different interpretations:

(3) The President of the U.S. is necessarily a citizen of the U.S.

This sentence can be read *de re* as:

(4) The President of the U.S. has the property of being necessarily a citizen of the U.S.

It can be read *de dicto* as:

(5) It is necessary that: the President of the U.S. is a citizen of the U.S.

Stalnaker and Thomason argue that the *de re* reading of the sentence is false given that one can easily describe a possible world where the person who is the President of the U.S. in the actual world, is not a U.S. citizen, whereas the *de dicto* reading is true (provided that the modality ranges over possible worlds consistent with U.S. law). The ambiguity in the original sentence depends on whether one interprets the singular term 'the President of the U.S.' outside or inside the scope of the modality of necessity. Another way of putting this point is to say that one can regard the modality as being part of the complex predicate of 'necessarily being a U.S. citizen' that applies to the term 'the President of the U.S.' and hence *de re* (concerning the thing). Or one can regard the modality as applying to the sentence 'the President of the U.S. is a citizen of the U.S.' an hence *de dicto* (concerning what is said). Generally, the term *de re* applies to interpretations of an expression outside the scope of a modality and *de dicto* applies to interpretations of an expression inside the scope of a modality.

The history of the distinction between *de re* and *de dicto* interpretations can be traced back to Aristotle's *Sophistical Refutations* where he treats fallacies of ambiguity and equivocation. When discussing fallacies arising from combination and division of words (166a23-25), he uses the examples of 'being able to walk while sitting' and 'being able to write while not writing' (see Hasper, 2009). Clearly, the ambiguity arises because the scope of the modality can be read in two ways. The ambiguity of 'being possibly known while unknown' is very similar.

The main difference between expressions such as 'being able to walk' and 'being possibly known' is that the first applies to objects such as human beings and animals, whereas the second applies to propositions. Consequently, the former can be rendered in some sort of first-order modal logic, and the latter in second-order modal logic, where propositions feature as terms to which second-order predicates apply. Propositions are usually not considered as terms, but there are many contexts in which we use terms to refer to propositions, e.g., John 1:1, the Brouwer fixed point theorem, proposition 4.1 of the Tractatus, Article 5 of the Universal Declaration of Human Rights, etc. In such cases properties such as

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believed, proved, true, and known can be applied to such terms and thereby modalities can be thought of as second-order predicates. The inference pattern underlying Fitch's paradox is not bound to second-order modal logic and can be interpreted in both first and second-order modal logic:

$$(6)\quad \frac{t \text{ is a possible } P.}{\text{Therefore it is possible that } t \text{ is a } P.}$$

If we substitute 'p is an unknown truth' for t, and 'known' for P, then we find part of the argument in Fitch's paradox. One can easily construct counterexamples to the validity of this pattern, when t is interpreted *de re* in the premiss, and *de dicto* in the conclusion:

t	P
the person closest to the north pole writing a letter	person who is not writing
the President of the U.S.	non-U.S. citizen
the winning lottery ticket of the next *Mega Millions* lottery	nonwinning lottery ticket
the largest country pope Francis will never visit	place pope Francis will visit
the last theorem proved by Fermat	not proved by Fermat

The elements are invariably some contingent property of some class of entities and a definite description of an entity within the relevant class that does not have the property by its description.

§3. A formal language with second-order predicate abstraction. In order to give a formal treatment of Fitch's paradox we need a language with which we can syntactically distinguish the *de re* and *de dicto* readings of VT. Besides the usual Boolean operators, the language has three operators: one for expressing knowledge (K), one for necessity ($\Box$), and one for second-order predicate abstraction ($\langle \lambda p.\cdot \rangle \cdot$). The operator for second-order predicate abstraction is instrumental to distinguishing *de re* readings from *de dicto* readings.

DEFINITION 3.1 (language). *The language L is given by the following Backus–Naur Form:*

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K\varphi \mid \Box\varphi \mid \langle \lambda p.\varphi \rangle \varphi,$$

where p ranges over the elements of some set P of atomic propositions. The sublanguage of L without any necessity or abstraction operators is the language of epistemic logic and denoted as L_{EL}. We use $(\varphi \rightarrow \psi)$ as an abbreviation of $\neg(\varphi \wedge \neg\psi)$, and $\Diamond\varphi$ as an abbrevation of $\neg\Box\neg\varphi$. Outer parentheses will often be omitted.

In first-order modal logic, one can syntactically distinguish *de re* from *de dicto* statements using first-order predicate abstraction (Stalnaker and Thomason, 1968; Fitting and Mendelsohn, 1998). Predicate abstraction allows one to treat complex predicates as a distinct syntactic category. An expression of the form $\langle \lambda x.\varphi \rangle$ behaves as a complex unary predicate that can be applied to terms. In this expression x is a variable ranging over individuals and φ a first-order modal formula with x occurring freely. This construction is very much like lambda abstraction from lambda calculus, but rather than yielding a function, the result is a predicate, hence the name "predicate abstraction". Let t be a

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(nonrigid) term for the 'President of the U.S.' and let P be the unary predicate of being a U.S. citizen, then

(7) $\langle \lambda x.\Box P(x)\rangle(t)$

(8) $\Box\langle \lambda x.P(x)\rangle(t)$

are formal renditions of (4) and (5) above. Note that in (7), the term t occurs outside the scope of $\Box$ whereas in (8) t occurs inside the scope of $\Box$.

Second-order predicate abstraction occurs in the work of Fitting (1998) and we have included it in the language defined above. An expression of the form $\langle \lambda q.\varphi\rangle$, where q is a dummy propositional variable, is a second-order predicate that can be applied to formulas. In the language of this logic one can express for instance that "Fermat's last theorem is known" by $\langle \lambda q.Kq\rangle(t)$ where t is the theorem. In this way one can faithfully formalize predicates such as 'known' as second-order predicates.

The formal rendition of VT in the literature is usually taken to be the following:

(9) $\varphi \rightarrow \Diamond K\varphi$

with implicit universal quantification over all sentences φ. Since the occurrence of φ in the consequent of VT is inside the scope of $\Diamond$, this formalization interprets the consequent of VT as a *de dicto* statement. However, when one takes 'knowable' as a complex second-order predicate $\langle \lambda q.\Diamond Kq\rangle$, one would end up with a *de re* reading of VT:

(10) $\varphi \rightarrow \langle \lambda q.\Diamond Kq\rangle(\varphi)$.

In contrast to (9), here the second occurrence of φ is outside the scope of $\Diamond$.[1] So, we now have a language with which we can syntactically distinguish the *de re* reading from the *de dicto* reading of VT. In natural language we could paraphrase these respectively as:

(11) If φ, then it is possible that φ is known.

(12) If φ, then φ has the property of being knowable.

It may seem implausible that these two sentences have different meanings. In the remainder we show that we can distinguish the two corresponding formulas semantically.

§4. Semantics. A key element of a semantics for this language must be that the meaning of a formula depends on whether it falls inside or outside the scope of the operator $\Diamond$. Otherwise *de re* and *de dicto* interpretations would merely differ syntactically. The proposal of Van Benthem (2004, 2009) and subsequently Balbiani et al. (2008) fits the bill. Van Benthem interprets $\Diamond$ as a quantifier over truthful public announcements (a particular kind of information changing event where all agents receive the same true information and it is common knowledge among them that this is so (Plaza, 1989)). One of the features of public announcements is that they may change the extension of a formula, i.e., the set of worlds where the formula φ holds may change due to an announcement. This may happen because in Van Benthem's approach the operator $\Diamond$ corresponds to a relation between *models*, rather than a relation between *worlds*. Hence the meaning of a formula depends on its modal nesting.

[1] This analysis of knowability is less radical than Fuhrmann (2014), who treats knowability as an operator which cannot be defined in terms of possibility and knowledge. Although knowability is syntactically a second-order predicate in my approach, it is still analyzed in terms of possibility and knowledge.

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Van Benthem paraphrases (9) as: if φ is true, then there is a true ψ such that φ becomes known after announcing ψ. The implication is taken to be a classical material implication and K is taken to be a normal epistemic modality that distributes over conjunction (if a conjunction is known, then so are its conjuncts) and satisfies at least the axiom T (all knowns are true: $K\varphi \rightarrow \varphi$). Yet, in his approach to Fitch's paradox, the formal rendition of VT is also (9), and hence a *de dicto* interpretation of VT. In the logic proposed by Van Benthem, the paradoxical argument goes through and so he concludes that VT cannot be saved. We will see that the *de re* interpretation of VT with the same interpretation for K and $\Diamond$ as Van Benthem is consistent.

Simple Kripke models for epistemic logic will be used to interpret the language. We will not consider multiple agents, but just one single knower, and so we can do without an accessibility relation and simply consider the set of all possible worlds to consist of the knower's epistemic alternatives.[2]

DEFINITION 4.1 (models). *Given a set of atoms P, a model M is a triple (W, V, w), where*

- *$W \neq \emptyset$ is a nonempty set of possible worlds,*
- *$V : P \rightarrow \wp(W)$ is a valuation that assigns a set of possible worlds to each atom,*
- *$w \in W$ is a world taken to be the actual world.*

We can interpret the language introduced in the previous section in these models.

DEFINITION 4.2 (semantics). *We define $\models$ as a relation between models and formulas of L recursively as follows. Let a model $M = (W, V, w)$ be given.*

$$
\begin{array}{ll}
M \models p & \text{iff } w \in V(p) \\
M \models \neg\varphi & \text{iff } M \not\models \varphi \\
M \models (\varphi \wedge \psi) & \text{iff } M \models \varphi \text{ and } M \models \psi \\
M \models K\varphi & \text{iff } (W, V, w') \models \varphi \text{ for all } w' \in W \\
M \models \Box\varphi & \text{iff } M_\psi \models \varphi \text{ for all } \psi \in L_{EL} \text{ such that } M \models \psi \\
M \models \langle\lambda p.\varphi\rangle\psi & \text{iff } M_{p:=\psi} \models \varphi
\end{array}
$$

In the clause for $\Box\varphi$, the model $M_\psi = (W_\psi, V_\psi, w)$ is such that

- $W_\psi = \{w' \in W \mid (W, V, w') \models \psi\}$,
- $V_\psi(p) = V(p) \cap W_\psi$.

In the clause for $\langle\lambda p.\varphi\rangle\psi$, the model $M_{p:=\psi} = (W, V_{p:=\psi}, w)$ is such that

- $V_{p:=\psi}(q) = \begin{cases} \{w' \in W \mid (W, V, w') \models \psi\} & \text{if } q = p \\ V(q) & \text{otherwise} \end{cases}$.

So, a formula of the form $\Box\varphi$ quantifies over models with a (possibly) smaller set of possible worlds (and hence more knowledge). The formula $\langle\lambda p.\varphi\rangle\psi$ is a statement about a model with a different valuation (and hence formulas may have a different extension).

§5. A formal analysis of the paradox. The *de dicto* formalization of VT indeed implies that all truths are known. The proof of the following theorem is just Fitch's paradox in a more formal setting.

[2] As it turns out, the analysis does not rely on this simplification. All one needs to assume is that the accessibility relation is reflexive. For the sake of elegance and clarity we have kept the presentation as simple as possible.

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THEOREM 5.1. *For every model M, if for all formulas $\varphi \in L$ it holds that $M \models \varphi \rightarrow \Diamond K\varphi$, then for all formulas $\psi \in L$ it holds that $M \models \psi \rightarrow K\psi$.*

Proof. Take an arbitrary model M and suppose that for all $\varphi \in L$ it holds that $M \models \varphi \rightarrow \Diamond K\varphi$. Take an arbitrary formula $\psi \in L$ and suppose that $M \models \psi$ Suppose that $M \not\models K\psi$. Then $M \models \psi \wedge \neg K\psi$. Therefore by the assumption above it follows that $M \models \Diamond K(\psi \wedge \neg K\psi)$. There is no model that satisfies $K(\psi \wedge \neg K\psi)$. This formula is equivalent to $K\psi \wedge K\neg K\psi)$. The left conjunct implies that ψ is true at all worlds, but the right conjunct implies that ψ is false at some world. Therefore there is also no submodel of M that satisfies this formula. Hence $M \not\models \Diamond K(\psi \wedge \neg K\psi)$. Therefore $M \models K\psi$ and so $M \models \psi \rightarrow K\psi$. □

It turns out that the *de re* formalization of VT is a tautology in this logic. This is somewhat surprising, because one might expect it to be merely consistent.[3]

THEOREM 5.2. *For every $\varphi \in L$, the formula $\varphi \rightarrow \langle \lambda q.\Diamond Kq\rangle\varphi$ is a tautology.*

Proof. Take an arbitrary formula φ and a model M^0 and suppose that $M^0 \models \varphi$. We now have to show that $M^0 \models \langle \lambda q.\Diamond Kq\rangle(\varphi)$. Take the model $M^1 = M^0_{q:=\varphi}$. By the semantics of predicate abstraction, we have to show that $M^1 \models \Diamond Kq$. Therefore, we have to find an appropriate formula $\psi \in L_{EL}$ such that $M^1 \models \psi$ and ${M^1}_\psi \models Kq$. We take this formula to be q. Note that $q \in L_{EL}$ and $M^1 \models q$, because $M^0 \models \varphi$. Now consider the model $M^2 = {M^1}_q$. Note that $W^2 = \{w' \mid (W^1, V^1, w') \models q\}$. Therefore $(W^2, V^2, w'') \models q$ for all $w'' \in W^2$. By the semantics of K it is now clear that $M^2 \models Kq$. Therefore $M^0 \models \langle \lambda q.\Diamond Kq\rangle(\varphi)$. Since M^0 was arbitrary, it follows that $\varphi \rightarrow \langle \lambda q.\Diamond Kq\rangle(\varphi)$ is a tautology. □

Since this theorem holds for all formulas it should also hold for the Moore formula $p \wedge \neg Kp$, stating that p is an unknown truth. This formula is indeed consistent and we can provide a model in which $p \wedge \neg Kp$ is true. The simplest model would be $M^0 = (W, V, w)$, where $W = \{w, w'\}$ and $V(p) = \{w\}$. In order to evaluate the formula that states that $p \wedge \neg Kp$ is knowable, we also need a dummy propositional variable q where $V(q)$ can be any subset of W. Let us see how the argument continues using this model and evaluate the formula $\langle \lambda q.\Diamond Kq\rangle(p \wedge \neg Kp))$, which states that $p \wedge \neg Kp$ is knowable. The formula $p \wedge \neg Kp$ is true at (W, V, w), but false at (W, V, w'). Now when we move to $M^1 = M^0_{q:=p\wedge\neg Kp}$, it will be the case that $V^1(q) = \{w_1\}$. Updating this model with q will result in model $M^2 = M^1_q$ where $W^2 = \{w\}$ and $V^2(q) = W^2$. Hence $M^2 \models Kq$. So $M^0 \models \langle \lambda q.\Diamond Kq\rangle(p \wedge \neg Kp))$, and so indeed the Moore formula $p \wedge \neg Kp$ is knowable, even though, as we saw above, the formula $K(p \wedge \neg Kp)$, which states that $p \wedge \neg Kp$ is known, is a contradiction.

As far as the paradox goes, this analysis suggests that when we interpret VT as a *de dicto* statement, the paradox's argument is valid, and the conclusion that all truths are known follows. However, the premise VT is not acceptable. When we interpret VT as a *de re* statement, then it is certainly acceptable, but then the argument is not valid, and accepting VT would not force one to accept that all truths are known.

In particular, the inference where one concludes that *p is an unknown truth* is *de dicto* possibly known from *p is an unknown truth* being *de re* knowable is invalid in this logic.

[3] One can adapt the logic such that it is merely consistent and not a tautology, but this would unnecessarily complicate the presentation here.

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Formally:

$$(13)\ \frac{\langle \lambda q.\Diamond Kq\rangle(p \wedge \neg Kp)}{\Diamond\langle \lambda q.Kq\rangle(p \wedge \neg Kp)}$$

is invalid. Yet, because of the equivocation of *de re* knowability and *de dicto* possible knowledge it may seem an unexceptional step in the argument.

§6. Conclusion. An adherent of VT will likely say that knowable should now obviously be taken as a complex second-order predicate, and the analysis shows that she can argue that this position does not entail that all truths are known. Regarding some unknown truth p (say, the twin prime conjecture), she can maintain that *the twin prime conjecture is an unknown truth* is knowable if true, whereas it is not possible that *the twin prime conjecture is an unknown truth* is known. This is completely analogous to maintaining that the President of the U.S. is a possible non-U.S. citizen, whereas it is not possible that the President of the U.S. is not a U.S. citizen.

By regarding knowledge as a propositional attitude and propositions as sets of possible worlds, we can take the *de re* reading of VT to say that an agent can take on the attitude of knowledge towards any set of possible worlds containing the actual world, provided that this set is the extension of some sentence.[4] If 'the twin prime conjecture is an unknown truth' expresses a proposition and this proposition is some set of possible worlds t, such that the actual world is in it, then VT states that some truthful announcement can be made, such that this announcement will exclude all possible worlds which fall outside t. Therefore t becomes known, yet the sentence 'the twin prime conjecture is an unknown truth' now expresses some other proposition different from t. Analogously, in a possible world where Obama is not a U.S. citizen, the President of the U.S. will describe someone else. In this way the paradox is kept at bay.

One may object that the position of the adherent of VT is closely tied to Van Benthem's reading of the diamond. But one can even see a similar pattern in Gödel's incompleteness theorem. Gödel's incompleteness theorem implies that there is a sentence in the language of first-order arithmetic which is true when this language is interpreted over the natural numbers, but which is unprovable in Peano Arithmetic (*PA*). Gödel provides such a sentence G which states that G is unprovable in *PA*. G is therefore very much like an unknowable truth. One can add G to *PA*, thereby obtaining a new formal system $PA + G$ in which G is provable (see Smith, 2007, p. 149). Yet in that system G does not state that G is not provable in $PA + G$, just as 'the twin prime conjecture is an unknown truth' will no longer state an unknown truth once we learn that the twin prime conjecture is true.

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[4] This provision is crucial. There may be sets of worlds that include the actual world which are not the extension of a sentence. Such sets can be unknowable in the sense that an agent may be unable to take on the attitude of knowledge towards such a set by any means. However, such sets are not truths in a linguistic sense. This suggests that, in line with verificationism, VT is best understood as a statement about truths expressible in language.

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