Canad. Math. Bull. 2026, pp. 1–11
http://dx.doi.org/10.4153/S0008439526102781
© The Author(s), 2026. Published by Cambridge University Press on behalf of Canadian Mathematical Society

Zelevinsky segment and strongly positive representations

Ivan Matić

Abstract. Let G_n denote either the group $SO(2n+1,F)$ or $Sp(2n,F)$ over a non-Archimedean local field of characteristic zero. We determine all composition factors of a parabolically induced representation of the form $\langle[\nu^a\rho,\nu^b\rho]\rangle\rtimes\sigma$, where $\langle[\nu^a\rho,\nu^b\rho]\rangle$ denotes the Zelevinsky segment representation of the general linear group attached to the segment $[\nu^a\rho,\nu^b\rho]$, and σ denotes a strongly positive discrete series of G_n.

1 Introduction

In this note, we present a further step in our investigation of induced representations of the form $\langle[\nu^a\rho,\nu^b\rho]\rangle\rtimes\sigma$, where $\langle[\nu^a\rho,\nu^b\rho]\rangle$ stands for the Zelevinsky segment representation of the general linear group attached to the segment $[\nu^a\rho,\nu^b\rho]$, and σ denotes a discrete series of either symplectic or special odd orthogonal group over a non-Archimedean field of characteristic zero. A key step in the determination of the composition factors of $\langle[\nu^a\rho,\nu^b\rho]\rangle\rtimes\sigma$, obtained for half-integral a in [11], is to understand the irreducible tempered subquotients. This task appears to be more complicated in the integral case than in the half-integral one, and we show how this difficulty can be overcome under the assumption that σ is strongly positive. This enables us to complete an explicit description of the composition factors of $\langle[\nu^a\rho,\nu^b\rho]\rangle\rtimes\sigma$, for a strongly positive discrete series σ. Note that the strongly positive discrete series serves as a cornerstone in discrete series classification [7, 13, 15], while the Zelevinsky segment representation plays an important role in the classification of unitary representations [17].

Reducibility criterion, provided in [10], reduces the determination of the composition factors for $\langle[\nu^a\rho,\nu^b\rho]\rangle\rtimes\sigma$ to the case $2a\in\mathbb{Z}$ and self-contragredient ρ. The case of half-integral a is a particular instance of the results of [11], while the case $a>0$ follows from [2]. This reduces our determination to case $a\in\mathbb{Z}$, $a<0$, $a+b\geq 0$, and $\rho\cong\widetilde{\rho}$.

Received by the editors August 7, 2026 UTC; revised September 23, 2026 UTC; accepted September 25, 2026 UTC.

This work is supported in part by the Croatian Science Foundation under the project number HRZZ-IP-2022-10-4615. This work is also funded by the EU NextGeneration under the Juraj Dobrila University of Pula institutional research project number IIP_UNIPU_010159 and by the National Recovery and Resilience Plan 2021–2026 under the project 581-UNIOS-54.

AMS subject classification: **11F70, 22E50.**

Keywords: Composition factors, Zelevinsky segment, strongly positive discrete series, classical p-adic groups.

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

We use several advantages of the strongly positive case. First is the knowledge of the Jacquet modules and embeddings of such representations [9, 12], which enables one to use the structural formula and reduce to the cuspidal case, in certain situations. Second one is a complete description of the composition factors for the generalized principal series in the strongly positive case [16], enabling one to obtain useful embeddings of potential subquotients.

In Section 2, we introduce the notations and summarize the required preliminaries. In Section 3, we provide a complete description of the composition factors for the induced representation of the studied type.

2 Preliminaries

Let F stand for a non-Archimedean local field of characteristic zero. By $J_n = (\delta_{i,n+1-j})_{1\leq i,j\leq n}$, we denote the $n \times n$ matrix, where $\delta_{i,n+1-j}$ stands for the Kronecker symbol. For a square matrix g, we denote by g^t its transposed matrix, and by g^τ its transposed matrix with respect to the second diagonal. In what follows, we fix one of the series of classical groups

$$Sp(n,F) = \left\{ g \in GL(2n,F) : \begin{pmatrix} 0 & -J_n \\ J_n & 0 \end{pmatrix} g^t \begin{pmatrix} 0 & -J_n \\ J_n & 0 \end{pmatrix} = g^{-1} \right\},$$

or

$$SO(2n+1,F) = \left\{ g \in GL(2n+1,F) : g^\tau = g^{-1} \right\},$$

and denote by G_n the rank n group belonging to the series which we fixed.

The set of standard parabolic subgroups will be fixed in a usual way, that is, we fix a minimal F-parabolic subgroup in G_n consisting of upper triangular matrices in the usual matrix realization. Then, the Levi factors of standard parabolic subgroups have the form $M \cong GL(m_1,F) \times \cdots \times GL(m_k,F) \times G_{m'}$, where $GL(m,F)$ stands for the general linear group of rank m over F. If δ_i is a representation of $GL(m_i,F)$ and τ is a representation of $G_{m'}$, the normalized parabolically induced representation $\mathrm{Ind}_M^{G_n}(\delta_1 \otimes \cdots \otimes \delta_k \otimes \tau)$ will be denoted by $\delta_1 \times \cdots \times \delta_k \rtimes \tau$. We use a similar notation to denote a parabolically induced representation of $GL(m,F)$.

By $\mathrm{Irr}(G_n)$, we denote the set of all irreducible admissible representations of G_n. Let $R(G_n)$ denote the Grothendieck group of admissible representations of finite length of G_n and define $R(G) = \oplus_{n\geq 0} R(G_n)$. In a similar way, we define $\mathrm{Irr}(GL(n,F))$, $R(GL(n,F))$, and $R(GL) = \oplus_{n\geq 0} R(GL(n,F))$.

For $\sigma \in \mathrm{Irr}(G_n)$ and $0 \leq k \leq n$, we denote by $r_{(k)}(\sigma)$ the normalized Jacquet module of σ with respect to the parabolic subgroup $P_{(k)}$ having the Levi subgroup equal to $GL(k,F) \times G_{n-k}$. We identify $r_{(k)}(\sigma)$ with its semisimplification in $R(GL(k,F)) \otimes R(G_{n-k})$ and consider

$$\mu^*(\sigma) = 1 \otimes \sigma + \sum_{k=1}^{n} r_{(k)}(\sigma) \in R(GL) \otimes R(G).$$

Let ν stand for a composition of the determinant mapping with the normalized absolute value on F. Let $\rho \in R(GL)$ denote an irreducible supercuspidal

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

representation. By a segment, we mean a set of the form $[\rho, \nu^k\rho] := \{\rho, \nu\rho, \dots, \nu^k\rho\}$, where k is a nonnegative integer. The induced representation $\nu^k\rho \times \nu^{k-1}\rho \times \cdots \times \rho$ has a unique irreducible subrepresentation, denoted by $\delta([\rho, \nu^k\rho])$, which is essentially square-integrable. By [20], every irreducible essentially square-integrable representation in $R(GL)$ can be obtained in this way.

The induced representation $\rho \times \nu\rho \times \cdots \times \nu^k\rho$ has a unique irreducible subrepresentation, which is denoted by $\langle[\rho, \nu^k\rho]\rangle$, and called the Zelevinsky segment representation attached to segment $[\rho, \nu^k\rho]$.

Let us recall the structural formulas, obtained in [5, Theorem 1.4] and in [18, Theorems 5.4 and 6.5].

Theorem 2.1 *Let $\rho \in R(GL)$ denote an irreducible cuspidal representation and $k, l \in \mathbb{R}$ such that $k + l$ is a nonnegative integer. Let $\sigma \in R(G)$ denote an irreducible admissible representation of finite length. Write $\mu^*(\sigma) = \sum_{\pi,\sigma'} \pi \otimes \sigma'$. Then, we have*

$$\begin{aligned}\mu^*(\langle[\nu^{-k}\rho, \nu^l\rho]\rangle \rtimes \sigma) = &\sum_{i=0}^{k+l+1} \sum_{j=0}^{i} \sum_{\pi,\sigma'} \langle[\nu^{-l}\widetilde{\rho}, \nu^{-i+k}\widetilde{\rho}]\rangle \times \langle[\nu^{-k}\rho, \nu^{j-k-1}\rho]\rangle \times \pi \\ &\otimes \langle[\nu^{j-k}\rho, \nu^{i-k-1}\rho]\rangle \rtimes \sigma', \\ \mu^*(\delta([\nu^{-k}\rho, \nu^l\rho]) \rtimes \sigma) = &\sum_{i=-k-1}^{l} \sum_{j=i}^{l} \sum_{\pi,\sigma'} \delta([\nu^{-i}\widetilde{\rho}, \nu^k\widetilde{\rho}]) \times \delta([\nu^{j+1}\rho, \nu^l\rho]) \times \pi \\ &\otimes \delta([\nu^{i+1}\rho, \nu^j\rho]) \rtimes \sigma'.\end{aligned}$$

We omit $\langle[\nu^x\rho, \nu^y\rho]\rangle$ and $\delta([\nu^x\rho, \nu^y\rho])$ if $x > y$.

Let $\delta \in \mathrm{Irr}(R(GL))$ denote an irreducible essentially square-integrable representation. Then, there is a unique $e(\delta) \in \mathbb{R}$ such that $\nu^{-e(\delta)}\delta$ is unitarizable [20]. We use the subrepresentation version of the Langlands classification for classical groups and realize a non-tempered irreducible representation π of G_n as the unique irreducible (Langlands) subrepresentation of an induced representation of the form $\delta_1 \times \delta_2 \times \cdots \times \delta_k \rtimes \tau$, where τ is an irreducible tempered representation of some G_t, and $\delta_1, \delta_2, \dots, \delta_k \in R(GL)$ are irreducible essentially square-integrable representations such that $e(\delta_1) \le e(\delta_2) \le \cdots \le e(\delta_k) < 0$. We write $\pi = L(\delta_1, \delta_2, \dots, \delta_k; \tau)$.

According to the Mœglin–Tadić classification of discrete series [13, 15] (which is now known to hold unconditionally, due to [1], [3, Theorem 7.8], and [14, Théorème 3.1.1]), a discrete series $\sigma \in G_n$ corresponds to an admissible triple consisting of the Jordan block, the partial cuspidal support, and the ε-function. For more details on these invariants, we refer the reader to [15], or to [7], where a slightly different approach has been used. For a discrete series σ, we denote the corresponding admissible triple by $(\mathrm{Jord}(\sigma), \sigma_c, \varepsilon_\sigma)$. For an irreducible self-contragredient cuspidal representation ρ_1 of $GL(n_1, F)$, let $\mathrm{Jord}_{\rho_1}(\sigma) = \{x : (x, \rho_1) \in \mathrm{Jord}(\sigma)\}$. If $\mathrm{Jord}_{\rho_1}(\sigma) \neq \emptyset$ and $x \in \mathrm{Jord}_{\rho_1}(\sigma)$, denote $x_ = \max\{y \in \mathrm{Jord}_{\rho_1}(\sigma) : y < x\}$, if it exists.

Discrete series $\sigma \in R(G)$ is called strongly positive if for every embedding

$$\sigma \hookrightarrow \nu^{s_1}\rho_1 \times \nu^{s_2}\rho_2 \times \cdots \times \nu^{s_k}\rho_k \rtimes \sigma_c,$$

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

where $\rho_i \in R$, $i = 1, 2, \dots, k$, are irreducible cuspidal unitary representations, we have $s_i > 0$ for $i = 1, 2, \dots, k$.

Strongly positive discrete series σ corresponds to an alternated triple, that is, we have $\varepsilon_\sigma((x_-, \rho_1), (x, \rho_1)) = -1$, for every $(x, \rho_1) \in \mathrm{Jord}(\sigma)$ such that x_- is defined. We also provide an inductive description of strongly positive discrete series, which follows from [8, Theorem 1.2].

Proposition 2.2 *Suppose that σ is a strongly positive discrete series and let ρ denote an irreducible cuspidal unitary representation such that some twist of ρ appears in the cuspidal support of σ. Then, there exist unique $c, d \in \mathbb{R}$ such that $c > 0$, $d - c \in \mathbb{Z}_{\geq 0}$, and the unique strongly positive discrete series σ_1 without $\nu^c\rho$ in the cuspidal support, with the property that σ is the unique irreducible subrepresentation of $\delta([\nu^c\rho, \nu^d\rho]) \rtimes \sigma_1$. Furthermore, there is a unique nonnegative integer l such that $c + l = \alpha$, for $\alpha > 0$ such that $\nu^\alpha\rho \rtimes \sigma_c$ reduces. If $l = 0$, there are no twists of ρ appearing in the cuspidal support of σ_1, and if $l > 0$, there exist unique $d' > d$ and a unique strongly positive discrete series σ_2, which contains neither $\nu^c\rho$ nor $\nu^{c+1}\rho$ in its cuspidal support, such that σ_1 is a unique irreducible subrepresentation of $\delta([\nu^{c+1}\rho, \nu^{d'}\rho]) \rtimes \sigma_2$.*

Throughout the rest of the article, we fix a strongly positive discrete series σ and an irreducible supercuspidal self-contragredient representation ρ. Let $\alpha \geq 0$ be such that $\nu^\alpha\rho \rtimes \sigma_c$ reduces. We assume that α is an integer and fix integers a and b such that $a \leq 0$ and $a + b \geq 0$ (note that, in $R(G)$, we have $\langle[\nu^a\rho, \nu^b\rho]\rangle \rtimes \sigma = \langle[\nu^{-b}\rho, \nu^{-a}\rho]\rangle \rtimes \sigma$).

3 Composition factors

In this section, we describe the composition factors of $\langle[\nu^a\rho, \nu^b\rho]\rangle \rtimes \sigma$, after a certain amount of the preparation work.

Lemma 3.1 *If $a \leq -1$, then the induced representation $\langle[\nu^a\rho, \nu^b\rho]\rangle \rtimes \sigma$ does not contain an irreducible tempered subquotient.*

Proof Suppose, on the contrary, that there is an irreducible tempered subquotient τ of $\langle[\nu^a\rho, \nu^b\rho]\rangle \rtimes \sigma$. Inspecting its cuspidal support, we deduce that τ is not strongly positive, and it is easy to see that τ can be written as a subrepresentation of an induced representation of the form $\delta([\nu^x\rho, \nu^y\rho]) \rtimes \tau'$, where $x \leq 0$, $y \geq 0$, $x + y \geq 0$, and τ' is irreducible tempered. Using the Frobenius reciprocity, we obtain $\mu^*(\langle[\nu^a\rho, \nu^b\rho]\rangle \rtimes \sigma) \geq \delta([\nu^x\rho, \nu^y\rho]) \otimes \tau'$, and Theorem 2.1 implies that there are $0 \leq j \leq i \leq b - a + 1$ and an irreducible constituent $\pi \otimes \sigma'$ of $\mu^*(\sigma)$ such that

$$\delta([\nu^x\rho, \nu^y\rho]) \leq \langle[\nu^{-b}\rho, \nu^{-i-a}\rho]\rangle \times \langle[\nu^a\rho, \nu^{j+a-1}\rho]\rangle \times \pi.$$

Using the same reasoning as in the proof of [11, Theorem 3.9], we conclude that ρ appears in the cuspidal support of π, which contradicts [9, Theorem 5.3] (or [12, Section 7]). ∎

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

The following proposition can be proved in the same way as [11, Proposition 4.1].

Proposition 3.2 *Suppose that $L(\delta_1, \delta_2, \dots, \delta_k; \tau)$ is an irreducible non-tempered subquotient of $\langle [\nu^a\rho, \nu^b\rho] \rangle \rtimes \sigma$, and let $\delta_i \cong \delta([\nu^{a_i}\rho_i, \nu^{b_i}\rho_i])$ for $i = 1, 2, \dots, k$. Then, for all $i = 1, 2, \dots, k$, we have $\rho_i \cong \rho$ and $b_i - a_i \in \{0, 1\}$.*

Proposition 3.3 *Suppose that $L(\delta_1, \delta_2, \dots, \delta_k; \tau)$ is an irreducible non-tempered subquotient of $\langle [\nu^a\rho, \nu^b\rho] \rangle \rtimes \sigma$, and let $\delta_i \cong \delta([\nu^{a_i}\rho_i, \nu^{b_i}\rho_i])$ for $i = 1, 2, \dots, k$. Suppose that there is a $j \in \{1, 2, \dots, k\}$ such that $b_j - a_j = 1$, and let $l = a + b$. Then, the following holds:*

(1) *$-a < b$, $a_l = b_l - 1$ and for $i < l$, we have $a_i = b_i = -b + i - 1$;*
(2) *if there is a $j' \in \{j+1, j+2, \dots, k\}$ such that $b_{j'} - a_{j'} = 1$, then $b_i - a_i = 1$ for all $i \in \{j+1, j+2, \dots, j'-1\}$;*
(3) *either $\tau \cong \sigma$ or there is an integer x, $1 \le x \le -a$, such that $\langle [\rho, \nu^x\rho] \rangle \rtimes \sigma$ contains an irreducible tempered subquotient;*
(4) *if $\tau \cong \sigma$, then $a_k = -1$, $b_k = 0$, and $\langle [\rho, \nu\rho] \rangle \rtimes \sigma$ contains $L(\delta([\nu^{-1}\rho, \rho]); \sigma)$.*

Proof The proof closely follows that of [11, Proposition 4.2]. We restrict our attention to the main differences.

Let $i \in \{1, 2, \dots, k\}$ be such that $b_i - a_i = 1$. Following the same lines as in the proof of [11, Proposition 4.2], we deduce that there is a c such that $1 \le c \le -a + 1$, $a_i = -c$, $L(\delta_i, \dots, \delta_k; \tau)$ is an irreducible subquotient of $\langle [\nu^{-c+1}\rho, \nu^c\rho] \rangle \rtimes \sigma$, and $L(\delta_{i+1}, \dots, \delta_k; \tau)$ is an irreducible subquotient of $\langle [\nu^{-c+2}\rho, \nu^{c-1}\rho] \rangle \rtimes \sigma$.

If for all $j \in \{i+1, \dots, k\}$, we have $b_j - a_j = 1$, a repeated application of [10, Lemma 2.6] shows that either $\tau \cong \sigma$ or there is a $d \ge 1$ such that τ is an irreducible subquotient of $\langle [\nu^{-d+1}\rho, \nu^d\rho] \rangle \rtimes \sigma$. If $\tau \cong \sigma$, it follows that $a_k = -1, b_k = 0$, and $L(\delta([\nu^{-1}\rho, \rho]); \sigma)$ is an irreducible subquotient of $\langle [\rho, \nu\rho] \rangle \rtimes \sigma$. If τ is an irreducible subquotient of $\langle [\nu^{-d+1}\rho, \nu^d\rho] \rangle \rtimes \sigma$, with $d \ge 1$, Lemma 3.1 implies $d = 1$.

Suppose that there is a $j \in \{i+1, \dots, k\}$ such that $a_j = b_j$. In a completely analogous way as in the proof of [11, Proposition 4.2], we deduce that $a_t = b_t$ holds for all $t \in \{j, \dots, k\}$, and there are x and y, $0 \le -y < x \le -a$, such that τ is an irreducible subquotient of $\langle [\nu^y\rho, \nu^x\rho] \rangle \rtimes \sigma$. Lemma 3.1 gives $y = 0$ and ends the proof. ∎

Lemma 3.4 *Suppose that σ is a subrepresentation of an induced representation of the form $\delta([\nu^c\rho', \nu^d\rho']) \rtimes \sigma'$, where $(c, \rho') \ne (1, \rho)$ and σ' is irreducible. If $L(\delta([\nu^{-1}\rho, \rho]); \sigma)$ is an irreducible subquotient of $\langle [\rho, \nu\rho] \rangle \rtimes \sigma$, then $L(\delta([\nu^{-1}\rho, \rho]); \sigma')$ is an irreducible subquotient of $\langle [\rho, \nu\rho] \rangle \rtimes \sigma'$.*

Proof By [9, Theorem 5.3] or [12, Section 7], we have $c > 0$, σ' is strongly positive, and $\delta([\nu^c\rho', \nu^d\rho']) \otimes \sigma'$ is a unique irreducible constituent of $\mu^*(\sigma)$ of the form $\delta([\nu^c\rho', \nu^d\rho']) \otimes \pi$.

If $\rho' \cong \rho$, then $c \ge 2$. This leads to

$$\delta([\nu^{-1}\rho, \rho]) \times \delta([\nu^c\rho', \nu^d\rho']) \cong \delta([\nu^c\rho', \nu^d\rho']) \times \delta([\nu^{-1}\rho, \rho]).$$

Thus, $L(\delta([\nu^{-1}\rho, \rho]); \sigma)$ is a subrepresentation of

$$\delta([\nu^c\rho', \nu^d\rho']) \times \delta([\nu^{-1}\rho, \rho]) \rtimes \sigma'.$$

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

By [6, Lemma 5.5], there is an irreducible subquotient π of $\delta([\nu^{-1}\rho,\rho])\rtimes\sigma'$ such that $L(\delta([\nu^{-1}\rho,\rho]);\sigma)$ is a subrepresentation of $\delta([\nu^{c}\rho',\nu^{d}\rho'])\rtimes\pi$. By the Frobenius reciprocity, $\mu^*(L(\delta([\nu^{-1}\rho,\rho]);\sigma))$ contains $\delta([\nu^{-1}\rho,\rho])\otimes\sigma$, and it follows that $\mu^*(\delta([\nu^{c}\rho',\nu^{d}\rho'])\rtimes\pi)$ also contains $\delta([\nu^{-1}\rho,\rho])\otimes\sigma$. Now, Theorem 2.1 implies that $\mu^*(\pi)$ contains an irreducible constituent of the form $\delta([\nu^{-1}\rho,\rho])\otimes\pi'$. Strong positivity of σ' implies $\pi\cong L(\delta([\nu^{-1}\rho,\rho]);\sigma')$, so

$$\mu^*(L(\delta([\nu^{-1}\rho,\rho]);\sigma))\geq\delta([\nu^{c}\rho',\nu^{d}\rho'])\otimes L(\delta([\nu^{-1}\rho,\rho]);\sigma').$$

From

$$L(\delta([\nu^{-1}\rho,\rho]);\sigma)\leq\langle[\rho,\nu\rho]\rangle\rtimes\sigma$$

follows that $\mu^*(\langle[\rho,\nu\rho]\rangle\rtimes\sigma)$ contains $\delta([\nu^{c}\rho',\nu^{d}\rho'])\otimes L(\delta([\nu^{-1}\rho,\rho]);\sigma')$.

It is now a direct consequence of Theorem 2.1 that $L(\delta([\nu^{-1}\rho,\rho]);\sigma')$ is an irreducible subquotient of $\langle[\rho,\nu\rho]\rangle\rtimes\sigma'$. ∎

Proposition 3.5 *The induced representation $\langle[\rho,\nu\rho]\rangle\rtimes\sigma$ contains the irreducible representation $L(\delta([\nu^{-1}\rho,\rho]);\sigma)$ if and only if $1\notin Jord_\rho(\sigma)$.*

Proof We use a case-by-case consideration. Let us first consider the case $1\notin\mathrm{Jord}_\rho(\sigma)$ and $3\notin\mathrm{Jord}_\rho(\sigma)$. By [16, Theorem 2.1], in $R(G)$, we have

$$\delta[\rho,\nu\rho]\rangle\rtimes\sigma=L(\delta([\nu^{-1}\rho,\rho]);\sigma)+\sigma_{ds}^{(1)}+\sigma_{ds}^{(2)},$$

where $\sigma_{ds}^{(1)}$ and $\sigma_{ds}^{(2)}$ are discrete series subrepresentations of $\delta[\rho,\nu\rho]\rangle\rtimes\sigma$.

Since $\mu^*(\sigma)$ does not contain an irreducible constituent of the form $\nu\rho\otimes\pi$, and $\rho\rtimes\sigma$ is a length-two representation, Theorem 2.1 can be used to show that $\mu^*(L(\delta([\nu^{-1}\rho,\rho]);\sigma))$ does not contain an irreducible constituent of the form $\nu\rho\otimes\pi$.

By [16, Proposition 3.1], the induced representation $\nu\rho\rtimes\sigma$ is irreducible, so we have

$$L(\delta([\nu^{-1}\rho,\rho]);\sigma)\hookrightarrow\rho\times\nu^{-1}\rho\rtimes\sigma\cong\rho\times\nu\rho\rtimes\sigma.$$

By [6, Lemma 5.5], there is an irreducible subquotient π of $\rho\times\nu\rho$ such that $L(\delta([\nu^{-1}\rho,\rho]);\sigma)$ is a subrepresentation of $\pi\rtimes\sigma$. Since, in $R(GL)$, we have

$$\rho\times\nu\rho=\delta([\rho,\nu\rho])+\langle[\rho,\nu\rho]\rangle,$$

and $L(\delta([\nu^{-1}\rho,\rho]);\sigma)$ is not a subrepresentation of $\nu\rho\times\rho\rtimes\sigma$, we get $\pi\cong\langle[\rho,\nu\rho]\rangle$.

Now, we consider the case $1\notin\mathrm{Jord}_\rho(\sigma)$ and $3\in\mathrm{Jord}_\rho(\sigma)$. We denote by σ' a strongly positive discrete series such that σ is a subrepresentation of $\nu\rho\rtimes\sigma'$. It follows from [16, Theorem 4.1(iii)] that, in $R(G)$, we have

$$\delta([\rho,\nu\rho])\rtimes\sigma=L(\delta([\nu^{-1}\rho,\rho]);\sigma)+\tau,$$

where τ is an irreducible tempered representation, which is a subrepresentation of both $\delta([\nu^{-1}\rho,\nu\rho])\rtimes\sigma'$ and $\nu\rho\times\nu\rho\times\rho\rtimes\sigma'$. We note that $\rho\rtimes\sigma'$ is irreducible. It can be seen using Theorem 2.1 and the description of the Jacquet modules of strongly positive discrete series ([9, Theorem 5.3] or [12, Section 7]) that $\nu\rho\times\nu\rho\otimes\rho\rtimes\sigma'$ is a

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

unique irreducible constituent of the form $\nu\rho \times \nu\rho \otimes \pi$ appearing in $\mu^*(\delta[\rho, \nu\rho]\rangle \rtimes \sigma)$. Consequently, $\mu^*(L(\delta([\nu^{-1}\rho, \rho]); \sigma))$ does not contain an irreducible constituent of the form $\nu\rho \times \nu\rho \otimes \pi$.

Since $\nu\rho \rtimes \sigma$ is irreducible, we have

$$L(\delta([\nu^{-1}\rho, \rho]); \sigma) \hookrightarrow \rho \times \nu\rho \rtimes \sigma.$$

We denote by π an irreducible subquotient of $\rho \times \nu\rho$ such that $L(\delta([\nu^{-1}\rho, \rho]); \sigma)$ is a subrepresentation of $\pi \rtimes \sigma$.

If $\pi \cong \delta([\rho, \nu\rho])$, we have

$$\begin{aligned} L(\delta([\nu^{-1}\rho, \rho]); \sigma) &\hookrightarrow \delta([\rho, \nu\rho]) \rtimes \sigma \\ &\hookrightarrow \delta([\rho, \nu\rho]) \times \nu\rho \rtimes \sigma' \\ &\cong \nu\rho \times \delta([\rho, \nu\rho]) \rtimes \sigma' \\ &\hookrightarrow \nu\rho \times \nu\rho \times \rho \rtimes \sigma', \end{aligned}$$

which is impossible. Thus, $\pi \cong \langle[\rho, \nu\rho]\rangle$.

Let us now assume $1 \in \mathrm{Jord}_\rho(\sigma)$ and $3 \in \mathrm{Jord}_\rho(\sigma)$. Then, $\mu^*(\sigma)$ does not contain an irreducible constituent of the form $\nu\rho \otimes \pi$. Also, in this case, $\delta([\rho, \nu\rho]) \rtimes \sigma$ is irreducible, so

$$L(\delta([\nu^{-1}\rho, \rho]); \sigma) \cong \delta([\rho, \nu\rho]) \rtimes \sigma.$$

Thus, $\mu^*(L(\delta([\nu^{-1}\rho, \rho]); \sigma))$ contains an irreducible constituent of the form $\nu\rho \otimes \pi'$, and it follows that $L(\delta([\nu^{-1}\rho, \rho]); \sigma)$ is not an irreducible subquotient of $\langle[\rho, \nu\rho]\rangle \rtimes \sigma$.

It remains to consider the case $1 \in \mathrm{Jord}_\rho(\sigma)$ and $3 \notin \mathrm{Jord}_\rho(\sigma)$. Note that, in this case, $\nu\rho$ does not appear in the cuspidal support of σ, and there is a positive integer d, $d \geq 2$, such that σ is a subrepresentation of $\delta([\nu^2\rho, \nu^d\rho]) \rtimes \sigma'$, for a strongly positive discrete series σ'.

Suppose that $L(\delta([\nu^{-1}\rho, \rho]); \sigma)$ is an irreducible subquotient of $\langle[\rho, \nu\rho]\rangle \rtimes \sigma$. Using a repeated application of Proposition 2.2 and Lemma 3.4, we obtain that then $L(\delta([\nu^{-1}\rho, \rho]); \sigma_c)$ is an irreducible subquotient of $\langle[\rho, \nu\rho]\rangle \rtimes \sigma_c$. Thus, it is enough to prove that in this case $\langle[\rho, \nu\rho]\rangle \rtimes \sigma_c$ does not contain $L(\delta([\nu^{-1}\rho, \rho]); \sigma_c)$.

Note that $\alpha \geq 1$. The case $\alpha = 1$ follows directly from [19, Proposition 3.8], so we comment only case $\alpha > 1$.

For $\alpha > 1$, $\delta([\rho, \nu\rho]) \rtimes \sigma_c$ is irreducible by [16, Theorem 4.1(i)], so we have $L(\delta([\nu^{-1}\rho, \rho]); \sigma_c) \cong \delta([\rho, \nu\rho]) \rtimes \sigma_c$ and $\mu^*(L(\delta([\nu^{-1}\rho, \rho]); \sigma_c)) \geq \nu\rho \otimes \rho \rtimes \sigma_c$. Since, by Theorem 2.1, $\mu^*(\langle[\rho, \nu\rho]\rangle \rtimes \sigma_c)$ does not contain $\nu\rho \otimes \rho \rtimes \sigma_c$, we obtain that $\langle[\rho, \nu\rho]\rangle \rtimes \sigma_c$ does not contain $L(\delta([\nu^{-1}\rho, \rho]); \sigma_c)$. ∎

Now, we complete a determination of irreducible tempered subquotients.

Theorem 3.6 *Suppose that $d \geq 1$. The induced representation $\langle[\rho, \nu^d\rho]\rangle \rtimes \sigma$ contains an irreducible tempered subquotient if and only if $x \in \mathrm{Jord}_\rho(\sigma)$ for all $x \in \{1, 3, \ldots, 2d - 1\}$ and $2d + 1 \notin \mathrm{Jord}_\rho(\sigma)$. If $\langle[\rho, \nu^d\rho]\rangle \rtimes \sigma$ contains an irreducible tempered subquotient, then it contains a unique irreducible tempered subquotient, which is a subrepresentation.*

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

Proof Let us first show that there are no irreducible tempered subquotients in the case $1 \notin \mathrm{Jord}_\rho(\sigma)$. Suppose, on the contrary, that $1 \notin \mathrm{Jord}_\rho(\sigma)$ and there is an irreducible tempered subquotient τ of $\langle [\rho, \nu^d\rho]\rangle \rtimes \sigma$. If $2d+1 \in \mathrm{Jord}_\rho(\sigma)$, using cuspidal support considerations, similarly as in [4, Section 4.2.1], we deduce that τ is a subrepresentation of an induced representation of the form $\delta([\nu^{-d}\rho, \nu^d\rho]) \rtimes \sigma_1$, where σ_1 is a strongly positive discrete series.

The Frobenius reciprocity implies $\mu^*(\langle [\rho, \nu^d\rho]\rangle \rtimes \sigma) \geq \delta([\nu^{-d}\rho, \nu^d\rho]) \otimes \sigma_1$. Using Theorem 2.1 and [9, Theorem 5.3], we easily get $d = 1$ and $\mu^*(\sigma) \geq \nu\rho \otimes \sigma_1$. Consequently, $1 \in \mathrm{Jord}_\rho(\sigma_1)$.

Thus,

$$\tau \hookrightarrow \nu\rho \times \delta([\nu^{-1}\rho, \rho]) \rtimes \sigma_1,$$

and [6, Lemma 5.5] implies that there is an irreducible subquotient π of $\delta([\rho, \nu\rho]) \rtimes \sigma_1 = \delta([\nu^{-1}\rho, \rho]) \rtimes \sigma_1$ such that τ is a subrepresentation of $\nu\rho \rtimes \pi$.

By [16, Theorem 4.1(ii)], in $R(G)$, we have

$$\delta([\rho, \nu\rho]) \rtimes \sigma_1 = \tau' + L(\delta([\nu^{-1}\rho, \rho]); \sigma_1),$$

where τ' is an irreducible tempered subrepresentation of $\delta([\rho, \nu\rho]) \rtimes \sigma_1$. If $\pi \cong \tau'$, we have

$$\begin{aligned}\tau &\hookrightarrow \nu\rho \rtimes \tau' \hookrightarrow \nu\rho \times \delta([\rho, \nu\rho]) \rtimes \sigma_1 \\ &\hookrightarrow \nu\rho \times \nu\rho \times \rho \rtimes \sigma_1,\end{aligned}$$

which is impossible since $\mu^*(\langle [\rho, \nu\rho]\rangle \rtimes \sigma)$ does not contain an irreducible constituent of the form $\nu\rho \times \nu\rho \otimes \pi'$. Thus, $\pi \cong L(\delta([\nu^{-1}\rho, \rho]); \sigma_1)$, and $\mu^*(\langle [\rho, \nu\rho]\rangle \rtimes \sigma)$ contains $\nu\rho \otimes L(\delta([\nu^{-1}\rho, \rho]); \sigma_1)$. Using Theorem 2.1 and [9, Theorem 5.3] again, we obtain $L(\delta([\nu^{-1}\rho, \rho]); \sigma_1) \leq \langle [\rho, \nu\rho]\rangle \rtimes \sigma_1$, contradicting Proposition 3.5.

If $2d+1 \notin \mathrm{Jord}_\rho(\sigma)$, using cuspidal support considerations, we obtain that τ is a discrete series representation, such that $\mathrm{Jord}_\rho(\tau) = \mathrm{Jord}_\rho(\sigma) \cup \{1, 2d+1\}$. Also, τ is a subrepresentation of an induced representation of the form $\delta([\nu^x\rho, \nu^y\rho]) \rtimes \sigma_1$, where σ_1 is a strongly positive discrete series, $x \leq 0$, $x + y > 0$, and in $\mathrm{Jord}_\rho(\tau)$, we have $(2y+1)_- = -2x+1$.

Using the Frobenius reciprocity, together with the transitivity of the Jacquet modules, we deduce $\mu^*(\langle [\rho, \nu^d\rho]\rangle \rtimes \sigma) \geq \delta([\nu^x\rho, \nu^y\rho]) \otimes \sigma_1$. The calculation of Jacquet modules gives $x \in \{0, -1\}$, $d = 1$ if $x = -1$, and $\mu^*(\sigma) \geq \delta([\nu\rho, \nu^y\rho]) \otimes \sigma'$. Consequently, $2y+1 = \min(\mathrm{Jord}_\rho(\sigma))$, and σ' is a strongly positive discrete series such that $1 \in \mathrm{Jord}_\rho(\sigma')$ and $2y+1 \notin \mathrm{Jord}_\rho(\sigma')$.

If $y < d$, we have $x = 0$, and the calculation of Jacquet modules leads to $\sigma_1 \leq \langle [\nu\rho, \nu^d\rho]\rangle \rtimes \sigma'$. This contradicts [11, Proposition 3.3], the proof of which covers the integral case without any modification.

If $d < y$, we have $x = -d$, so $d = 1$. From $\mu^*(\langle [\rho, \nu\rho]\rangle \rtimes \sigma) \geq \delta([\nu^{-1}\rho, \nu^y\rho]) \otimes \sigma_1$, we directly get $\sigma_1 \cong \sigma'$.

Note that

$$\tau \hookrightarrow \delta([\nu^{-1}\rho, \nu^y\rho]) \rtimes \sigma_1 \hookrightarrow \delta([\nu\rho, \nu^y\rho]) \times \delta([\nu^{-1}\rho, \rho]) \rtimes \sigma_1,$$

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

so there is an irreducible subquotient π of $\delta([\nu^{-1}\rho,\rho])\rtimes\sigma_1$ such that τ is a subrepresentation of $\delta([\nu\rho,\nu^y\rho])\rtimes\pi$.

By [16, Theorem 4.1(ii)], in $R(G)$, we have

$$\delta([\rho,\nu\rho])\rtimes\sigma_1=\tau''+L(\delta([\nu^{-1}\rho,\rho]);\sigma_1),$$

where τ'' is an irreducible tempered subrepresentation of $\delta([\rho,\nu\rho])\rtimes\sigma_1$. If $\pi\cong\tau''$, we have

$$\begin{aligned}\tau&\hookrightarrow\delta([\nu\rho,\nu^y\rho])\times\delta([\rho,\nu\rho])\rtimes\sigma_1\\&\hookrightarrow\delta([\nu\rho,\nu^y\rho])\times\nu\rho\times\rho\rtimes\sigma_1\\&\cong\nu\rho\times\delta([\nu\rho,\nu^y\rho])\times\rho\rtimes\sigma_1.\end{aligned}$$

Since $y>1$ and $2y+1=\min(\mathrm{Jord}_\rho(\sigma))$, it follows that $\mu^*(\langle[\rho,\nu\rho]\rangle\rtimes\sigma)$ does not contain an irreducible constituent of the form $\nu\rho\otimes\pi'$. Thus, $\pi\cong L(\delta([\nu^{-1}\rho,\rho]);\sigma_1)$ so $\mu^*(\langle[\rho,\nu\rho]\rangle\rtimes\sigma)$ contains $\delta([\nu\rho,\nu^y\rho])\otimes L(\delta([\nu^{-1}\rho,\rho]);\sigma_1)$. The calculation of the Jacquet modules implies $\langle[\rho,\nu\rho]\rangle\rtimes\sigma_1\geq L(\delta([\nu^{-1}\rho,\rho]);\sigma_1)$, which contradicts Proposition 3.5, since $\sigma_1\cong\sigma'$ and $1\in\mathrm{Jord}_\rho(\sigma')$.

Let us now assume $1\in\mathrm{Jord}_\rho(\sigma)$. Using cuspidal support considerations, we obtain that an irreducible tempered subquotient τ of $\langle[\rho,\nu^d\rho]\rangle\rtimes\sigma$ is a subrepresentation of an induced representation of the form $\rho\rtimes\sigma'$, for a strongly positive discrete series σ'. Thus, $\mu^*(\langle[\rho,\nu^d\rho]\rangle\rtimes\sigma)$ contains $\rho\otimes\sigma'$. Theorem 2.1 and [9, Theorem 5.3] imply $\sigma'\leq\langle[\nu\rho,\nu^d\rho]\rangle\rtimes\sigma$. It can now be seen, following the same lines as in the proof of [11, Proposition 3.3], that $x\in\mathrm{Jord}_\rho(\sigma)$ for all $x\in\{1,3,\dots,2d-1\}$ and $2d+1\notin\mathrm{Jord}_\rho(\sigma)$.

If $x\in\mathrm{Jord}_\rho(\sigma)$ for all $x\in\{1,3,\dots,2d-1\}$ and $2d+1\notin\mathrm{Jord}_\rho(\sigma)$, [10, Lemma 5.3] shows that the induced representation $\langle[\rho,\nu^d\rho]\rangle\rtimes\sigma$ contains an irreducible tempered subrepresentation. Also, by [11, Proposition 3.3], the induced representation $\langle[\nu\rho,\nu^d\rho]\rangle\rtimes\sigma$ contains a unique discrete series subquotient, which we denote by σ_{ds}. Using Theorem 2.1, together with [9, Theorem 5.3], we deduce that $\rho\otimes\sigma_{ds}$ is a unique irreducible constituent of $\mu^*(\langle[\rho,\nu^d\rho]\rangle\rtimes\sigma)$ of the form $\rho\otimes\sigma'$, for a discrete series σ', and appears with multiplicity one.

Consequently, if $\langle[\rho,\nu^d\rho]\rangle\rtimes\sigma$ contains an irreducible tempered subquotient, then it contains a unique irreducible tempered subquotient, which is a subrepresentation. ∎

Propositions 3.3 and 3.5, together with Theorem 3.6, enable us to describe the composition factors of $\langle[\nu^a\rho,\nu^b\rho]\rangle\rtimes\sigma$. Complete description is given by the following result, whose proof we omit, since it can be established by a very easy modification of the proof of [11, Theorem 4.6].

Theorem 3.7 *Let us denote by x_1 the minimal positive odd integer such that $x_1\notin Jord_\rho(\sigma)$, and let $x_{\min}=\frac{x_1-1}{2}$.*

(1) *If $x_{\min}>b$, then the induced representation $\langle[\nu^a\rho,\nu^b\rho]\rangle\rtimes\sigma$ is irreducible, and we have*

$$\langle[\nu^a\rho,\nu^b\rho]\rangle\rtimes\sigma=L(\nu^{-b}\rho,\dots,\nu^{a-1}\rho,\nu^a\rho,\nu^a\rho,\dots,\nu^{-1}\rho,\nu^{-1}\rho;\rho\rtimes\sigma).$$

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

(2) *Suppose that* $x_{\min} = 0$. *We denote by* τ_1 *and* τ_2 *mutually non-isomorphic irreducible tempered subrepresentations of* $\rho \rtimes \sigma$. *If* $-a = b$, *in* $R(G)$, *we have*

$$\langle [\nu^a\rho, \nu^b\rho] \rangle \rtimes \sigma = L(\nu^a\rho, \nu^a\rho, \dots, \nu^{-1}\rho, \nu^{-1}\rho; \tau_1) + L(\nu^a\rho, \nu^a\rho, \dots, \nu^{-1}\rho, \nu^{-1}\rho; \tau_2).$$

If $-a < b$, *in* $R(G)$, *we have*

$$\begin{aligned}\langle [\nu^a\rho, \nu^b\rho] \rangle \rtimes \sigma = {} & L(\nu^{-b}\rho, \dots, \nu^{a-1}\rho, \nu^a\rho, \nu^a\rho, \dots, \nu^{-1}\rho, \nu^{-1}\rho; \tau_1) \\ & + L(\nu^{-b}\rho, \dots, \nu^{a-1}\rho, \nu^a\rho, \nu^a\rho, \dots, \nu^{-1}\rho, \nu^{-1}\rho; \tau_2) \\ & + L(\nu^{-b}\rho, \dots, \nu^{a-2}\rho, \delta([\nu^{a-1}\rho, \nu^a\rho]), \dots, \delta([\nu^{-1}\rho, \rho]); \sigma).\end{aligned}$$

(3) *Suppose that* $0 < x_{\min} \le b$, *and let us denote by* τ *a unique irreducible tempered subquotient of* $\langle [\rho, \nu^{x_{\min}}\rho] \rangle \rtimes \sigma$. *If* $-a < x_{\min}$, *in* $R(G)$, *we have*

$$\begin{aligned}\langle [\nu^a\rho, \nu^b\rho] \rangle \rtimes \sigma = {} & L(\nu^{-b}\rho, \dots, \nu^{a-1}\rho, \nu^a\rho, \nu^a\rho, \dots, \nu^{-1}\rho, \nu^{-1}\rho; \rho \rtimes \sigma) \\ & + L(\nu^{-b}\rho, \dots, \nu^{-x_{\min}-1}\rho, \nu^a\rho, \dots, \nu^{-1}\rho; \tau).\end{aligned}$$

If $-a = b$, *in* $R(G)$, *we have*

$$\begin{aligned}\langle [\nu^a\rho, \nu^b\rho] \rangle \rtimes \sigma = {} & L(\nu^a\rho, \nu^a\rho, \dots, \nu^{-1}\rho, \nu^{-1}\rho; \rho \rtimes \sigma) \\ & + L(\nu^a\rho, \nu^a\rho, \dots, \nu^{-x_{\min}-1}\rho, \nu^{-x_{\min}-1}\rho, \nu^{-x_{\min}}\rho, \dots, \nu^{-1}\rho; \tau).\end{aligned}$$

If $x_{\min} \le -a < b$, *in* $R(G)$, *we have*

$$\begin{aligned}\langle [\nu^a\rho, \nu^b\rho] \rangle \rtimes \sigma = {} & L(\nu^{-b}\rho, \dots, \nu^{a-1}\rho, \nu^a\rho, \nu^a\rho, \dots, \nu^{-1}\rho, \nu^{-1}\rho; \rho \rtimes \sigma) \\ & + L(\nu^{-b}\rho, \dots, \nu^{a-1}\rho, \nu^a\rho, \nu^a\rho, \dots, \nu^{-x_{\min}-1}\rho, \nu^{-x_{\min}-1}\rho, \nu^{-x_{\min}}\rho, \dots, \nu^{-1}\rho; \tau) \\ & + L(\nu^{-b}\rho, \dots, \nu^{a-2}\rho, \delta([\nu^{a-1}\rho, \nu^a\rho]), \dots, \delta([\nu^{-x_{\min}-1}\rho, \nu^{-x_{\min}}\rho]), \\ & \nu^{-x_{\min}+1}\rho, \dots, \nu^{-1}\rho; \tau).\end{aligned}$$

Acknowledgments The author would like to thank the referee for his or her careful reading, useful comments, and suggestions.

References

[1] J. Arthur, *The endoscopic classification of representations. Orthogonal and symplectic groups*, volume 61 of American Mathematical Society Colloquium Publications, American Mathematical Society, Providence, RI, 2013.
[2] B. Bošnjak, *Representations induced from essentially Speh and strongly positive discrete series*. Manuscripta Math. **173**(2024), 591–617.
[3] W. T. Gan and L. Lomelí, *Globalization of supercuspidal representations over function fields and applications*. J. Eur. Math. Soc. **20**(2018), 2813–2858.
[4] M. Hanzer, *The generalized injectivity conjecture for classical p-adic groups*. Int. Math. Res. Not. IMRN **2010**(2010), 195–237.
[5] M. Hanzer and G. Muić, *On an algebraic approach to the Zelevinsky classification for classical p-adic groups*. J. Algebra **320**(2008), 3206–3231.
[6] C. Jantzen, *On supports of induced representations for symplectic and odd-orthogonal groups*. Amer. J. Math. **119**(1997), 1213–1262.

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press

[7] Y. Kim and I. Matić, *Discrete series of odd general spin groups*. Monatsh. Math. **203**(2024), 419–473.
[8] I. Matić, *Strongly positive representations of metaplectic groups*. J. Algebra **334**(2011), 255–274.
[9] I. Matić, *Jacquet modules of strongly positive representations of the metaplectic group Sp(n)* Trans. Amer. Math. Soc. **365**(2013), 2755–2778.
[10] I. Matić, *Reducibility of representations induced from the Zelevinsky segment and discrete series*. Manuscripta Math. **164**(2021), 349–374.
[11] I. Matić, *Representations induced from the Zelevinsky segment and discrete series in the half-integral case*. Forum Math. **33**(2021), 193–212.
[12] I. Matić and M. Tadić, *On Jacquet modules of representations of segment type*. Manuscripta Math. **147**(2015), 437–476.
[13] C. Mœglin, *Sur la classification des séries discrètes des groupes classiques p-adiques: Paramètres de Langlands et exhaustivité*. J. Eur. Math. Soc. **4**(2002), 143–200.
[14] C. Mœglin, *Paquets stables des séries discrètes accessibles par endoscopie tordue; leur paramètre de Langlands*. In: J. W. Cogdell, F. Shahidi, and D. Soudry (eds.), Automorphic forms and related geometry: Assessing the legacy of I.I. Piatetski-Shapiro, volume 614 of Contemporary Mathematics, American Mathematical Society, Providence, RI, 2014, pp. 295–336.
[15] C. Mœglin and M. Tadić, *Construction of discrete series for classical p-adic groups*. J. Amer. Math. Soc. **15**(2002), 715–786.
[16] G. Muić, *Composition series of generalized principal series; the case of strongly positive discrete series*. Israel J. Math. **140**(2004), 157–202.
[17] M. Tadić, *Classification of unitary representations in irreducible representations of general linear group (non-Archimedean case)*. Ann. Sci. École Norm. Sup. (4) **19**(1986), 335–382.
[18] M. Tadić, *Structure arising from induction and Jacquet modules of representations of classical p-adic groups*. J. Algebra **177**(1995), 1–33.
[19] M. Tadić, *Unitarizability in corank three for classical p-adic groups*. Mem. Amer. Math. Soc. **286**(2023), vii+120.
[20] A. V. Zelevinsky, *Induced representations of reductive p-adic groups. II. On irreducible representations of $GL(n)$*. Ann. Sci. École Norm. Sup. (4) **13**(1980), 165–210.

School of Applied Mathematics and Informatics, University of Osijek, Osijek, Croatia
e-mail: imatic@mathos.hr

https://doi.org/10.4153/S0008439526102781 Published online by Cambridge University Press