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DIRECTED TECHNOLOGICAL CHANGE: A KNOWLEDGE-BASED MODEL

TAILONG LI
Zhejiang Sci-Tech University

SHIYUAN PAN
Zhejiang University

HENG-FU ZOU
Central University of Finance and Economics

In a knowledge-based growth model where skilled workers are used in innovation and production, skill-biased technological change may lower average R&D productivity via an *innovation possibilities frontier* effect that eliminates scale effects. We show that skill-biased technological change increases the skill premium even if the elasticity of substitution between skilled and unskilled workers is less than two. Trade between developed countries promotes skill-biased technological change, thus raising wage inequality. Trade between developed and developing countries has differing effects: it induces relatively skill-replacing technological change and lowers wage inequality in the developed country but has the opposite effects in the developing country. Finally, we show that trade can stimulate or hurt economic growth.

Keywords: Directed Technological Change, Scale Effect, Wage Inequality, Trade

1. INTRODUCTION

This paper studies the effect of trade on the direction of technological change, wage inequality, and economic growth. It does so by involving skilled workers in both goods and knowledge production and by allowing skill-biased technological change to lower average R&D productivity.

Many knowledge-based growth models imply that there exist scale effects; that is, economic growth rate rises with the total population or the number of scientists. The data, however, are against this theoretical implication. For example, despite a

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substantial increase in the amount of labor engaged in R&D sectors, there has been no acceleration in productivity trends in many OECD countries for the postwar period [see Figure 1 in Jones (1995a, p. 763)]. Moreover, the U.S. productivity growth rate has remained roughly unchanged even though the total personnel and R&D personnel have grown since World War II [see Figures 2 and 3 in Laincz and Peretto (2006, p. 267)]. During the same time period, the relative supply of skilled workers has increased sharply and technological change has been skill-biased in the United States [e.g., Acemoglu (1998, 2002b, 2003b)]. Autor et al. (1998) report that on the average the supply of skills has grown 3.06% a year during 1970 and 1995, while it has increased 2.63% a year between 1940 and 1970. Assuming the elasticity of substitution between skilled and unskilled workers to be 1.4, Acemoglu (2002b) computes that the degree of skill bias of technology was approximately 0.030 in 1960, became 0.069 and 0.157 in 1970 and 1980, respectively, and increased to 0.470 in 1990. Acemoglu (2009) proposes a framework of directed technological change to explain many problems in macroeconomics, labor economics, and international trade. In his model, the number of scientists engaged in R&D sectors is given exogenously [see also Acemoglu (2002a, 2003a)]. This is a weakness in the sense that the growth rate of the economy depends on scientists' input, which is not optimally determined by the allocation of skilled workers between innovation and production. Therefore, it is necessary to develop a knowledge-based model of directed technological change, where skilled workers are allocated across sectors endogenously. We can use this model to study the relationship between directed technological change and economic growth and to suggest a possible explanation for the two facts mentioned, the absence of scale effects and the acceleration in the skill bias of technological change.

Acemoglu (2003b) provides an explanation for patterns of skill premia, relying on the precondition that the elasticity of substitution between skilled and unskilled workers is "greater than 2, which is on the higher side of the estimates, but still plausible, and consistent with a number of studies." The relatively widespread consensus on the estimates of the elasticity, however, is "greater than 1, most likely, greater than 1.4, and perhaps as large as 2" [Acemoglu (2002a)]. So there is a gap between the precondition and the value of 1.4, which is "the typical elasticity used in the literature" [Acemoglu (2002b, 2003b)]. Using our knowledge-based model, we are able to show that skill-biased technological change increases the skill premium even if the elasticity is smaller than 2. Therefore, results obtained by Acemoglu can be improved upon to match the empirical consensus.

Existing directed-technological-change models mainly focus on the effect of trade between developed and developing countries on wage inequality, but disregard trade between developed countries (for a short summary of several different frameworks treating trade between developed countries, see Section 2). The international trade literature, however, documents that trade in intermediate goods between developed countries or regions has grown substantially and played a dominant role in recent decades [e.g., Krugman (1995, 2000); Feenstra and

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Hanson (1996); Feenstra (1998)]. Moreover, in most situations, the impact of international trade on economic growth is not clear [for details, see Alesina et al. (2005)]. We show that the potential impact of international trade on the skill bias of technology, wage inequality, and economic growth differs depending on whether the participating country is developed or developing.

The rest of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, we introduce a knowledge-based growth model founded on Rivera-Batiz and Romer (1991) and Acemoglu (2009). In the model, technological innovations respond to profit incentives; knowledge and skilled workers are the only R&D inputs; and the production and R&D sectors compete for skilled workers. Section 4 applies the model to address issues of directed technological change, scale effects, and skill premia. Section 5 extends the model to investigate the impact of international trade on wage inequality and economic growth. Section 6 concludes.

2. RELATED LITERATURE

In the past twenty years there has been a flourishing controversy about scale effects. Does a large economy grow faster? The answer from the first-generation knowledge-based growth models is yes [Romer (1990); Grossman and Helpman (1991); Aghion and Howitt (1992)], but the empirical analysis has questioned the validity of scale effects. The second-generation models have been built to remove scale effects [Jones (1995a, 1995b); Kortum (1997); Segerstrom (1998)]. In these models, technological change is endogenous in the sense that it requires real resources, but the long-run growth rate of the economy is exogenous and proportional to the population growth rate. The exogeneity of economic growth is sufficiently at odds with the spirit of the endogenous growth literature so that third-generation models have been constructed to eliminate scale effects while preserving the endogeneity of the steady-state growth rate [Aghion and Howitt (1998); Dinopoulos and Thompson (1998); Peretto (1998); Young (1998); Jones (1999, 2005)].

During the past decade, Acemoglu and his co-authors (1998, 2001, 2002a, 2003b, 2007, 2009) have made a serious effort to study issues related to the bias of technological change. Allowing skilled workers to be simultaneously allocated across production and innovation, this paper extends Acemoglu's model to endogenize the economic growth rate and to explore the relationship between skill-biased technological change and scale effects. Creating spillovers in two R&D sectors asymmetrically, knowledge (or technology) accumulation in different directions influences R&D productivity differently. For example, an increase in skill-complementary knowledge reduces its own average productivity in the skill-complementary R&D sector, but it improves the average productivity of labor-complementary knowledge in the labor-complementary R&D sector. This implies that skill-biased technological change has an *innovation-possibilities frontier* effect, which is introduced to describe such double-faced influences as

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in the preceding example, on the overall innovations: it positively affects labor-complementary innovations but negatively affects skill-complementary inventions. The economic growth rate, therefore, remains unchanged if the two competing strengths exactly offset each other and declines if the negative strength is dominant. This suggests that skill-biased technological change caused by an increase in the supply of skilled workers may remove scale effects. To the best of our knowledge, this paper is the first to discuss skill-biased technological change and scale effects in a unified framework.

A number of empirical studies have reported a substantial increase in skill premia in most developed countries. In response to these studies, the theoretical literature has stressed two main explanations, increased international trade and skill-biased technological change [see Aghion et al. (1999); Hornstein et al. (2005)]. Dinopoulos and Segerstrom (1999) suggest that international trade leads to more skill-intensive innovations, thus increasing skill premia. Neary (2002) and Thoenig and Verdier (2003) show that trade between developed countries encourages imitation and induces firms to adopt defensive innovation strategies to reinforce nonreplication measures. These strategies lead to an increase in the relative demand for skills, thus raising skill premia. Maurin et al. (2002), Matsuyama (2007), and Unel (2010) argue that international trade inherently requires more intensive use of skills, and thereby increases skill premia. Epifani and Gancia (2006, 2008) report that, if the elasticity of substitution between outputs of two sectors is higher than one and the skill-intensive sector has greater returns to scale, trade between developed countries increases skill premia. Dinopoulos et al. (2010) highlight that intraindustry trade causes an expansion of firm size, which is assumed to be skill-biased, and hence raises skill premia. Some economists emphasize the important role of skill-biased technological change. They argue that if the introduction of new technology increases the demand for skills, wage inequality rises during the period of economic development [Katz and Murphy (1992); Acemoglu (1998, 2002a, 2002b, 2003b); Krusell et al. (2000)]. Our paper revisits this territory and differs from the existing papers in considering the effect of international trade on the direction of technological change.[1] Acemoglu (2003b) considers the impact of trade between developed and developing countries on the bias of technological change, whereas we enrich the discussion with trade between developed countries. Moreover, exposing a rather different mechanism, we provide results quite distinct from those of Acemoglu (see Section 5). Especially, we show that trade between developed countries could be a major cause of rising wage inequality because it induces skill-biased technological change. The mechanism behind this story is that trade between developed countries makes more advanced technologies available and raises R&D productivity sharply, thus increasing expected future outputs greatly. Therefore, to balance the economy, skilled workers transfer across the production and R&D sectors. In some circumstances, due to-because of smoothing of consumption (or the trade-off between current and future consumption), more skilled workers are employed in the skill-intensive sector after trade opening. As a result, the relocation of skilled workers leads to skill-biased

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technological change and a rise in wage inequality. A similar mechanism in the other situation of developed countries trading with developing countries implies that skill-replacing technological change happens in the developed countries and their wage inequality falls. The degree of skill bias of technology in the developing countries is greater after trade opening, because their trade partners, the developed countries, are skill-abundant. Therefore, trade leads to an increase in wage inequality in the developing countries. We call such mechanisms the *skilled workers reallocation* effect of international trade on the direction of technological change.

Trade between developed countries could hurt economic growth if the negative effect of trade-induced skill-biased technological change on growth were powerful. Similarly, trade between developed and developing countries might slow down economic growth when the negative impact of trade-induced skill-replacing (or skill-biased) technological change was dominant. Both imply that international trade does not always stimulate economic growth even if it enhances the opportunities of countries to access more advanced technologies. This result is different from the existing one that trade and the flow of ideas can permanently accelerate economic growth [Rivera-Batiz and Romer (1991)].

In fact, there are three forces determining the direction of technological change: (i) the price effect, which creates incentives to develop labor-complementary technologies when the number of skilled workers is large; (ii) the market size effect, which induces skill-biased technological change when the relative supply of skills is large; and (iii) the *innovation-possibilities frontier* effect, which encourages further development of skill-complementary technologies when current technological change is skill-replacing. What determines the relative strength of the first two competing forces is the elasticity of substitution between skilled and unskilled workers. The larger the elasticity of substitution, the more powerful the market size effect, and vice versa. The *innovation-possibilities frontier* effect increases with the degree of state dependence, which measures the spillover effect of one type of knowledge (or technologies) on the production of the other [for the definition of the degree of state dependence, see Acemoglu (2002a, 2003a)].

Intuitively, skilled workers are not bound to be favored by skill-biased technological change, because the price effect implies that an increase in the supply of skills raises the relative wages of unskilled workers. However, the price effect is dominated by the market size effect if the elasticity of substitution between skilled and unskilled workers is large and the degree of state dependence is medium. In this case, technological change is skill-biased and the skill premium increases. This statement looks similar to that in Acemoglu (2002a), but the state-dependence condition and the R&D process in this paper are characterized differently: skilled workers and skill- and labor-complementary knowledge are used as the only inputs into innovation; the production and R&D sectors are allowed to compete for skilled workers; and the innovation possibilities frontier is set to be knowledge-based [for the knowledge-based specification of R&D, see River-Batiz and Romer (1991)]. Acemoglu (2002a) also introduces a knowledge-based model, but he assumes

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a constant supply of scientists. When studying labor- and capital-augmenting technological change, Acemoglu (2003a) has modified his baseline model, where skilled workers are replaced by physical capital, to endogenize the amount of labor engaged in R&D sectors, but he considers only an extreme state-dependence case.[2] Our model can identify the *innovation-possibilities frontier* effect and present the *skilled workers reallocation* effect of international trade. Because of the first effect, a rise in the supply of skilled workers affects the bias of technological change and changes average R&D productivity. Therefore, scale effects and the skill premium have to change. The second effect implies that international trade could influence the direction of technological change, wage inequality, and economic growth via transferring skilled workers between production and innovation.

3. THE MODEL

In this section, we outline the basic model. The key assumption is that skilled workers are used in both goods and knowledge production.

3.1. The Environment

Consider a closed economy populated by H skilled and L unskilled workers. All consumers have identical constant relative risk aversion preferences,

$$\int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt, \tag{1}$$

where $C(t)$ is consumption at time t, θ is the coefficient of relative risk aversion (or the intertemporal elasticity of substitution), and ρ is the discount rate. The time index will be suppressed for simplicity. Subject to a standard budget constraint, the utility maximization yields

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r-\rho), \tag{2}$$

where the L.H.S. is the growth rate of consumption and r is the rental price of capital (or the interest rate). Note that, on the balanced growth path (BGP), this growth rate is equal to that of output. The aggregate output is given by

$$Y = \left[(Y_l)^{\frac{\epsilon-1}{\epsilon}} + (Y_h)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

where Y_l is the total output of the labor-intensive good, Y_h is the total output of the skill-intensive good, and $\epsilon \in [0, \infty)$ is the elasticity of substitution between them. Prices of goods are denoted as p_l and p_h, respectively. In a competitive market,

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market clearing implies that the relative price satisfies

$$p = \frac{p_h}{p_l} = \left(\frac{Y_h}{Y_l}\right)^{-1/\epsilon}. \tag{4}$$

Assume that the skill-intensive good is produced using skilled workers and a set of differentiated intermediate goods ("machines," technologies, or knowledge), and that the labor-intensive is produced using unskilled workers and a different set of machines. Some machines are skill-complementary, whereas others are labor-complementary, so technological change could be biased toward skilled or unskilled workers. Specifically, production functions of the two goods are

$$Y_h = \int_0^{A_h} [k_h(i)]^{1-\alpha} di \cdot H_Y^{\alpha} \tag{5}$$

and

$$Y_l = \int_0^{A_l} [k_l(i)]^{1-\alpha} di \cdot L^{\alpha}, \tag{6}$$

where $0 < \alpha < 1$, A_z measures the set of machines complementary to workers of skill level z, $k_z(i)$ is the quantity of z-complementary machines of variety i, $z = h$ or l, and H_Y is the number of skilled workers employed to produce the skill-intensive good. Given A_z, production functions (5) and (6) exhibit constant returns to scale. However, there will be aggregate increasing returns when technological change is endogenized.

The innovation possibilities frontier takes the form of knowledge-based R&D specification. With skill- and labor-complementary R&D sectors, there are

$$\dot{A}_h = (A_h)^{\frac{1+\delta}{2}} (A_l)^{\frac{1-\delta}{2}} H_h \quad \text{and} \quad \dot{A}_l = (A_h)^{\frac{1-\delta}{2}} (A_l)^{\frac{1+\delta}{2}} H_l, \tag{7}$$

respectively, where H_z is the number of skilled workers inventing z -complementary machines,[3] and $0 \leq \delta \leq 1$ measures the degree of state dependence. When $\delta = 0$, there is no state dependence because A_z creates identical spillovers in both R&D sectors. In contrast, if $\delta = 1$, there is an extremely high value of state dependence and A_z only spills over into z -complementary R&D. Clearly, $H = H_Y + H_h + H_l$; that is, the skill-intensive good and two R&D sectors compete for skilled workers.

3.2. Equilibrium

Taking wages, prices of machines, and the product price as given, production firms solve their profit maximization problems to decide the demand for machines:

$$k_h(i) = [(1-\alpha) \cdot p_h / \chi_h(i)]^{1/\alpha} \cdot H_Y \tag{8}$$

and

$$k_l(i) = [(1-\alpha) \cdot p_l / \chi_l(i)]^{1/\alpha} \cdot L, \tag{9}$$

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where $\chi_z(i)$ is the price of machine i. Without loss of generality, we normalize the marginal cost of producing any machine to $\eta = (1-\alpha)^2$. The producer of machine i, as a monopolist, chooses a level of output, $k_z(i)$, to maximize profits:

$$\pi_z(i) = \max_{k_z(i)} \chi_z(i)k_z(i) - \eta k_z(i). \tag{10}$$

From (8), (9), and (10), the monopoly (profit-maximizing) price is

$$\chi_z(i) = \chi_z = 1-\alpha, \tag{11}$$

which is the same at all points in time and for all varieties of machines. Substitution of (11) into (8) and (9) yields

$$k_h = k_h(i) = (p_h)^{1/\alpha} \cdot H_Y \tag{12}$$

and

$$k_l = k_l(i) = (p_l)^{1/\alpha} \cdot L. \tag{13}$$

At time τ, the monopoly profits are

$$\pi_h(\tau) = \alpha(1-\alpha)[p_h(\tau)]^{1/\alpha} \cdot H_Y \tag{14}$$

and

$$\pi_l(\tau) = \alpha(1-\alpha)[p_l(\tau)]^{1/\alpha} \cdot L. \tag{15}$$

Substituting (12) and (13) into (5) and (6), respectively, we obtain

$$Y_h = (p_h)^{(1-\alpha)/\alpha} \cdot A_h H_Y \quad \text{and} \quad Y_l = (p_l)^{(1-\alpha)/\alpha} \cdot A_l L. \tag{16}$$

From (4) and (16), the relative price of skill-intensive goods is

$$p = \frac{p_h}{p_l} = \left(\frac{A_h}{A_l} \cdot \frac{H_Y}{L}\right)^{-\frac{\alpha}{1+\alpha(\epsilon-1)}}. \tag{17}$$

Intuitively, when technological change is highly skill-biased (high A_h/A_l) or the number of skilled workers in the skill-intensive sector is large (high H_Y/L), the relative supply of skill-intensive goods is large, and so the relative price is low.

From (3), (16), and (17), the elasticity of substitution between skilled and unskilled workers is $1+\alpha(\epsilon-1)$. It is difficult to estimate the elasticity, but there is a relatively widespread consensus: it is greater than 1, most likely greater than 1.4, and perhaps as high as 2.[4] Hence, we take $\epsilon > 1$ in the following discussion. For R&D, the price for a new machine variety, $P_z(t)$, is equal to the present value of profits seized by the monopolist:

$$P_z(t) = \int_t^\infty e^{-\int_t^\tau r(s)ds} \pi_z(\tau)\, d\tau. \tag{18}$$

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Free entry into R&D sectors implies that, at equilibrium, $P_z(t)$ is also the cost of discovering a new variety. Along the BGP, $P_z(t)$ is constant for all t. Thus, differentiating (18) with respect to t results in

$$\pi_z(t) = r(t) \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} \pi_z(\tau)\, d\tau. \tag{19}$$

Combination of (18) and (19) leads to

$$r(t) = \frac{\pi_z(t)}{P_z(t)}. \tag{20}$$

That is, on the BGP, the interest rate is identical to the ratio of the profit flow to the lump-sum cost of discovery. Equations (14), (15), and (20) give

$$P_h = \alpha(1-\alpha)(p_h)^{1/\alpha} \cdot H_Y/r \quad \text{and} \quad P_l = \alpha(1-\alpha)(p_l)^{1/\alpha} \cdot L/r. \tag{21}$$

These equations imply that the present value of monopoly profits increases with the product price, p_h or p_l and with production firms' employment, H_Y or L, and decreases with the rental price of capital, r. Intuitively, a higher price for the product increases the value of the marginal product of all factors, including that of machines, thus encouraging firms to rent more machines and raising the instantaneous profits of the monopolist. On the other hand, a higher level of employment implies more workers to use machines, increasing demand, and thereby raises the profits. Finally, a higher rental price of capital raises the discount rate for the future profit flow, and so reduces the present value.

From (16), wages paid for skilled workers in the skill-intensive sector are

$$w_h = \alpha(p_h)^{1/\alpha} \cdot A_h. \tag{22}$$

From (7), wages in the R&D sectors are

$$w_h = P_h \cdot (A_h)^{\frac{1+\delta}{2}} (A_l)^{\frac{1-\delta}{2}} = P_l \cdot (A_h)^{\frac{1-\delta}{2}} (A_l)^{\frac{1+\delta}{2}}. \tag{23}$$

Thus, the equated equilibrium wages yield

$$H_Y = \frac{r(A_h/A_l)^{\frac{1-\delta}{2}}}{1-\alpha}. \tag{24}$$

This implies that if the rental price of capital is higher, production firms employ more labor and less capital. That is, labor is substituted for capital in (5). Equation (24) also indicates that if the relative productivity of skilled workers, A_h/A_l, is higher, the skill-intensive sector is more profitable and hires more skilled workers.

Along the BGP, A_h and A_l must grow at the same rate. So (7) implies that

$$(A_h/A_l)^{\delta-1} H_h = H_l. \tag{25}$$

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In addition, on the BGP, output, consumption, and knowledge grow at the same rate. Using (2), (7), (24), and (25), we obtain

$$H_Y = \frac{\theta H + \rho \left(A_h/A_l\right)^{\frac{1-\delta}{2}} \left[1 + \left(A_h/A_l\right)^{\delta-1}\right]}{\theta + (1-\alpha)\left[1 + \left(A_h/A_l\right)^{\delta-1}\right]}. \tag{26}$$

Combination of (17), (21), and (23) gives

$$A_h/A_l = \left(H_Y/L\right)^{1/\lambda}, \tag{27}$$

where $\lambda = \frac{1-\delta[1+\alpha(\epsilon-1)]}{\alpha(\epsilon-1)}$. Obviously, (26) and (27) are two consequences of wage equalization of skilled workers in the BGP. From them, we obtain

$$f\left(\frac{A_h}{A_l}\right) \triangleq L\left(\frac{A_h}{A_l}\right)^{\lambda}\left\{\theta + (1-\alpha)\left[1 + \left(\frac{A_h}{A_l}\right)^{\delta-1}\right]\right\}$$
$$- \rho\left(\frac{A_h}{A_l}\right)^{\frac{1-\delta}{2}}\left[1 + \left(\frac{A_h}{A_l}\right)^{\delta-1}\right] - \theta H = 0. \tag{28}$$

The existence of solutions to (28) is easily proven. More than one solution may exist, all of them are locally stable, and in some cases there is a unique optimal BGP outcome (see Appendix B).[5] Denote the optimal solution by $(A_h/A_l)^*$. Using (2), (24), and (27), we obtain the optimal BGP growth rate:

$$g^* = \frac{(1-\alpha)\, L[(A_h/A_l)^*]^{\frac{2\lambda+\delta-1}{2}} - \rho}{\theta}. \tag{29}$$

From (17), (21), and (27), we can show that

$$\frac{\partial\left\{\left[P_h \cdot (A_h)^{\frac{1+\delta}{2}}(A_l)^{\frac{1-\delta}{2}}\right] \Big/ \left[P_l \cdot (A_h)^{\frac{1-\delta}{2}}(A_l)^{\frac{1+\delta}{2}}\right]\right\}}{\partial(A_h/A_l)} < 0$$

for $\lambda > 0$, and then prove the stability of BGP equilibria (see Appendix B): when $\lambda > 0$, any equilibrium state converges to the BGP. In contrast, when $\lambda < 0$, equilibrium dynamics is unstable and takes us to a corner solution, meaning that only one type of R&D is undertaken. In the rest of the paper, we restrict our attention to the stable case: $\lambda > 0$, or equivalently, $\delta < \frac{1}{1+\alpha(\epsilon-1)}$.

4. APPLICATIONS

In this section, we apply the model to address issues of directed technological change, scale effects, and skill premia.

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4.1. Directed Technological Change

Using (21), we obtain the profitability of innovating a new skill-complementary machine relative to a new labor-complementary one:

$$\frac{P_h}{P_l} = \underbrace{p^{\frac{1}{\alpha}}}_{\substack{\text{price}\\ \text{effect}}} \cdot \underbrace{\frac{H_Y}{L}}_{\substack{\text{market}\\ \text{size effect}}}. \tag{30}$$

On the BGP, the relative profitability of two different types of innovations is determined by the price effect and the market size effect. From (12) and (13), higher employment of skilled workers in production increases the relative demand for skill-complementary machines, and thus raises the relative profitability through the market size effect.

On the side of the innovation-possibilities frontier, using (7) or (23), we find

$$\frac{P_h}{P_l} = \underbrace{\left(\frac{A_h}{A_l}\right)^{-\delta}}_{\substack{\text{effect of innovation-}\\ \text{possibilities frontier}}}. \tag{31}$$

At equilibrium, if technological change today is biased toward one factor, innovations complementing this factor will be cheaper tomorrow. In fact, more machine varieties complementing skilled workers increase the output of skill-intensive goods substantially, thus decreasing the relative price. As a result, other things being equal, the expected profitability of skill-complementary innovations falls.

It is proved in Appendix C that the relationship between H_Y and H satisfies

$$\frac{\partial\left(H_Y/L\right)}{\partial\left(H/L\right)} > 0, \quad \text{if } 0 < \lambda \leq \frac{1-\delta}{2}. \tag{32}$$

Therefore, combining (27) and (32), we obtain

PROPOSITION 1. *Suppose that* $\epsilon > 1$ *and* $0 < \lambda \leq \frac{1-\delta}{2}$. *Technological change is skill-biased.*

Proof. See Appendix E. ■

Like Acemoglu's (2002b) model, ours provides a possible interpretation for why technological change over the past decades was skill-biased, and for why the skill bias may have accelerated during the past 25 years, beginning in the 1970s. Indeed, an increase in the supply of skilled workers increases the number of skilled workers used in the skill-intensive sector, thus encouraging skill-biased technological change via the market size effect and pushing technological change

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to be skill-replacing through the price effect. This proposition concludes that, when $\epsilon > 1$ and $0 < \lambda \leq \frac{1-\delta}{2}$ (equivalently, $\frac{2-\alpha(\epsilon-1)}{2+\alpha(\epsilon-1)} \leq \delta < \frac{1}{1+\alpha(\epsilon-1)}$) hold, the market size effect is dominant and technological change is biased toward skills.

4.2. Scale Effects

Many empirical studies have shown that economic growth exhibits no large persistent change even when large increases in skilled workers in R&D. Many economists try to model the absence of scale effects [e.g., Jones (1995a, 1995b); Kortum (1997); Aghion and Howitt (1998); Peretto (1998); Young (1998); Peretto and Smulders (2002)]. There are two major mechanisms: the diminishing returns to knowledge, and the entry of new innovation firms, which splits the large market and dilutes the rent value of invention. We provide a new insight here by focusing on the effect of an increase in H on the BGP growth rate g^*.

PROPOSITION 2. *Suppose that* $\epsilon > 1$ *and* $\lambda > 0$. *When* $\lambda = \frac{1-\delta}{2}$, *the economic growth rate is unrelated to the number of skilled workers; i.e.,* $\partial g^*/\partial H = 0$. *When* $\lambda < \frac{1-\delta}{2}$, *the growth rate decreases with the number of skilled workers; i.e.,* $\partial g^*/\partial H < 0$.

Proof. See Appendix E. ■

Let us recall (7). Skill-biased technological change reduces the average productivity of A_h in the A_h-R&D sector, thus slowing economic growth. This is the negative side of the *innovation possibilities frontier* effect. On the other hand, it increases the average productivity of A_l in the A_l -R&D sector, thus speeding up growth. This is the positive side. Therefore, even if the supply of skilled workers goes up, the growth rate may remain unchanged or decrease because skill-biased technological change may reduce average R&D productivity. Grossman and Helpman (1991) show that an expansion in the factor used most intensively in the R&D sector speeds up innovation and growth necessarily, but an expansion of the least intensive factor in R&D slows down growth. They claim that an increase in the stock of human capital raises the economic growth rate, whereas the growth rate declines with the number of unskilled workers. In our mechanism, skill-replacing technological change increases the average productivity of A_h in the A_h-R&D sector, whereas it decreases that of A_l in the A_l-R&D sector. Therefore, a large economy with skill-replacing technological change may also exhibit no scale effects. Obviously, our interpretation is different from the existing ones.

The empirical ranges for parameters allow the knife-edge assumption, $\lambda = \frac{1-\delta}{2}$, to hold. For instance, $1 + \alpha(\epsilon - 1) = 1.4$ and $\delta = 2/3$, or $1 + \alpha(\epsilon - 1) = 2$ and $\delta = 1/3$. Indeed, Jones (1999) has pointed out that in most of the third-generation models, exogenous knife-edge restrictions, such as the proportionality of the number of varieties to the size of the economy, are acceptable in some sense to eliminate scale effects. Besides, the models might exhibit a negative scale effect if knife-edge restrictions were relaxed [e.g., Young (1998)]. Without empirical

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work designed to estimate the parameter values, it is impossible to say which provides the best characterization. Obviously, the relaxed condition, $\lambda < \frac{1-\delta}{2}$, is not very restricted, but holds for a slightly wider set of cases.

In (29), the BGP growth rate increases in the number of unskilled workers even if the parameter restriction holds. Strictly speaking, scale effects with respect to unskilled workers still exist. But this may not have a significant impact in practice, provided that technological change is skill-biased. Indeed, $[(A_h/A_l)^*]^{\frac{2\lambda+\delta-1}{2}}$ declines with $(A_h/A_l)^*$ when $0 < \lambda < \frac{1-\delta}{2}$. Hence, the growth rate may fall or slightly rise if both L and H/L increase. In Acemoglu (2002a), the elasticity of substitution between skilled and unskilled workers is assumed to be 1.4, and the degree of skill bias of technology was approximately 0.069 in 1970 and increased to 0.470 in 1990. In this circumstance, for instance, supposing that $2\lambda + \delta - 1 = -0.2$ where $\delta = 0.7$, and the number of unskilled workers has increased by 20%, 25%, or 30% between 1970 and 1990, the value of $L \cdot [(A_h/A_l)^*]^{\frac{2\lambda+\delta-1}{2}}$ decreases by 1%, or increases by 3.1% or 7.3%, respectively.[6] Now, the change in g^* is extremely limited, and in brief terms, economic growth rate in 1990 roughly equals that in 1970.

4.3. Skill Premia

Inspection of (16) yields the ratio of wages paid for skilled to unskilled workers:

$$\frac{w_h}{w_l} = \left(\frac{p_h}{p_l}\right)^{1/\alpha} \cdot \left(\frac{A_h}{A_l}\right). \tag{33}$$

So the skill premium is greater when the relative price of skill-intensive goods is higher or technological change is skill-biased. From (17), (27), and (33),

$$\frac{w_h}{w_l} = \left(\frac{A_h}{A_l}\right)^{\frac{\alpha(\epsilon-1)+\delta-1}{\alpha(\epsilon-1)}}. \tag{34}$$

Obviously, $\frac{\alpha(\epsilon-1)+\delta-1}{\alpha(\epsilon-1)}$ is positive when $\epsilon > 1$ and $0 < \lambda \leq \frac{1-\delta}{2}$. Therefore, Proposition 1 and (34) yield

PROPOSITION 3. *Suppose that* $\epsilon > 1$ *and* $0 < \lambda \leq \frac{1-\delta}{2}$. *The skill premium increases with the number of skilled workers.*

Proof. See Appendix E. ■

More skilled workers decrease the skill premium by reducing the relative price of skill-intensive goods, and at the same time, they increase the skill premium by improving skill-biased technological change. In Acemoglu (2003b), $1 + \alpha(\epsilon - 1)$ must be assumed to be greater than 2 to guarantee that skill-biased technological change increases wage inequality. Acemoglu (2002a) suggests that, if some high values are set for of the state dependence in the R&D process, skill-biased technological change raises the skill premium when the elasticity of substitution is less

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than 2. The mechanism proposed in this paper does not need such a condition of the state dependence. Our proposition directly improves Acemoglu's conclusions by making them consistent with the findings in empirical studies, because we have

Remark 1. Skill-biased technological change leads to a rise in wage inequality even if the elasticity of substitution between skilled and unskilled workers, $1 + \alpha(\epsilon - 1)$, is less than 2.

5. TRADE, WAGE INEQUALITY, AND GROWTH

This section investigates the impact of international trade on wage inequality and economic growth. We first focus on trade between two identical developed countries, and then consider trade between developed and developing countries. Knowledge flow with trade in intermediate goods is permitted.

5.1. Trade between Identical Countries

Machines are exported only if neither invented nor produced in the foreign country, and are imported only if unavailable domestically (this is likely to occur if there is a high trade cost or strong patent protection). To simplify the discussion, assume that two identical countries develop different sets of machines in autarky, and that in the open world there is balanced trade between them. In the home country, taking wages, prices of machines, and the product price as given, producers of skill-intensive goods maximize profits:

$$\max_{k_h(i),\, k_h(\widehat{i})} p_h \left\{ \int_0^{A_h} [k_h(i)]^{1-\alpha} di + \int_0^{\widehat{A}_h} [k_h(\widehat{i})]^{1-\alpha} d\widehat{i} \right\} \cdot H_Y^{\alpha}$$

$$- \int_0^{A_h} k_h(i)\chi_h(i) di - \int_0^{\widehat{A}_h} k_h(\widehat{i})\chi_h(\widehat{i}) d\widehat{i} - w_h H_Y, \tag{35}$$

where machines produced at home and abroad are denoted by i and $\widehat{i}$, respectively. The quantities of domestic and imported machines are, respectively,

$$k_h(i) = [(1-\alpha) \cdot p_h/\chi_h(i)]^{1/\alpha} \cdot H_Y \text{ and } k_h(\widehat{i}) = [(1-\alpha) \cdot p_h/\chi_h(\widehat{i})]^{1/\alpha} \cdot H_Y. \tag{36}$$

Similar arguments give

$$k_l(i) = [(1-\alpha) \cdot p_l/\chi_l(i)]^{1/\alpha} L \text{ and } k_l(\widehat{i}) = [(1-\alpha) \cdot p_l/\chi_l(\widehat{i})]^{1/\alpha} L. \tag{37}$$

As in (11), the monopoly price of any machine is $1-\alpha$. Substituting (36) and (37) into (5) and (6), respectively, we get

$$Y_h = 2(p_h)^{(1-\alpha)/\alpha} \cdot A_h H_Y \text{ and } Y_l = 2(p_l)^{(1-\alpha)/\alpha} \cdot A_l L. \tag{38}$$

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Doubling the varieties of machines doubles outputs of skill- and labor-intensive goods by making the productivity of workers twice what it was. From (4), doubling of outputs has no effect on the relative price, so (17) holds. For types of machines complementing skilled or unskilled workers, at time τ, the total monopoly profits in home and abroad markets are, respectively,

$$\pi_h(\tau) = 2\alpha(1-\alpha)[p_h(\tau)]^{1/\alpha} \cdot H_Y \tag{39}$$

and

$$\pi_l(\tau) = 2\alpha(1-\alpha)[p_l(\tau)]^{1/\alpha} \cdot L. \tag{40}$$

Thus, (20), (39), and (40) imply that, on the BGP,

$$P_h = 2\alpha(1-\alpha)(p_h)^{1/\alpha} \cdot H_Y/r \text{ and } P_l = 2\alpha(1-\alpha)(p_l)^{1/\alpha} \cdot L/r. \tag{41}$$

Knowledge flows between countries. In pursuit of profits, R&D firms specialize in creating different types of machines. Hence, the effective knowledge in the open world is twice the amount in autarky. Research depends on the total worldwide stock of knowledge, so R&D equations now are

$$\dot{A}_h = 2(A_h)^{\frac{1+\delta}{2}}(A_l)^{\frac{1-\delta}{2}} H_h \text{ and } \dot{A}_l = 2(A_h)^{\frac{1-\delta}{2}}(A_l)^{\frac{1+\delta}{2}} H_l, \tag{42}$$

which means that doubling the stock of ideas doubles the R&D productivity. Taking advantage of tedious algebra like that in Section 2, we obtain

$$H_Y = \frac{2\theta H + \rho(A_h/A_l)^{\frac{1-\delta}{2}}\left[1+(A_h/A_l)^{\delta-1}\right]}{2\theta + (1-\alpha)\left[1+(A_h/A_l)^{\delta-1}\right]}, \tag{43}$$

(27), and[7]

$$L\left(\frac{A_h}{A_l}\right)^{\lambda}\left\{2\theta + (1-\alpha)\left[1+\left(\frac{A_h}{A_l}\right)^{\delta-1}\right]\right\} - \rho\left(\frac{A_h}{A_l}\right)^{\frac{1-\delta}{2}}\left[1+\left(\frac{A_h}{A_l}\right)^{\delta-1}\right] - 2\theta H = 0. \tag{44}$$

When comparing (43) with (26), we can prove that trade between developed countries results in more skilled workers being used in the skill-intensive sector for $0 < \lambda \leq \frac{1-\delta}{2}$ (see Appendix D). Moreover, we find

PROPOSITION 4. *Suppose that* $\epsilon > 1$ *and* $0 < \lambda \leq \frac{1-\delta}{2}$. *Trade between two identical countries increases the degree of skill bias of technology and wage inequality. If* $\sigma < 2$, *trade speeds up growth; if* $\sigma = 2$, *it has no effect on growth; otherwise, for* $\sigma > 2$, *it slows down growth, where* $\sigma = [(A_h/A_l)^*_o/(A_h/A_l)^*]^{(1-2\lambda-\delta)/2}$ *has a positive correlation with the shift of the degree of skill bias, and* $(A_h/A_l)^*_o$ *is the degree of skill bias of technology in the open economy.*[8]

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Proof. See Appendix E. ■

Because knowledge and technology are endogenized, doubling the effective stock of ideas increases the efficiency of innovation and the expected future output greatly. Trade-off between current and future consumption leads to more skilled workers being engaged in production after trade opening. When the market size effect is dominant, trade results in skill-biased technological change and increases wage inequality. This is referred to as the *skilled workers reallocation* effect of international trade.

Most knowledge-based models [e.g., Rivera-Batiz and Romer (1991)] argue that trade with flows of ideas can permanently speed up growth. They do not consider the effect of international trade on the direction of technological change. Proposition 2 claims that skill-biased technological change may have a negative effect on economic growth. Thus, international trade is not always helpful to economic growth because of its impact on the bias of technological change, even though it definitely enhances the opportunities to access more advanced technologies.

5.2. Trade between Different Countries

When trade between a developed and a developing country is considered, suppose that the fraction of skilled workers in the developed country is higher than that in the developing country by far, namely, $H^{\mathrm{N}}/L^{\mathrm{N}} \gg H^{\mathrm{S}}/L^{\mathrm{S}}$, where N and S denote the developed and the developing country, respectively.

Following Acemoglu (2003b), assume that the developing country is incompetent to research and develop new machines and that producers in the developing country adopt machines invented in the developed country. The machine productivity is assumed to be identical in the two countries.[9] Demand for skill- and labor-complementary machines is, respectively,

$$k_h(i) = [(1-\alpha)\cdot p_h/\chi_h(i)]^{1/\alpha}(H_Y^{\mathrm{N}}+H^{\mathrm{S}}) \text{ and}$$
$$k_l(i) = [(1-\alpha)\cdot p_l/\chi_l(i)]^{1/\alpha}(L^{\mathrm{N}}+L^{\mathrm{S}}). \quad (45)$$

As before, the monopoly price is $1-\alpha$. At time τ, the total monopoly profits of selling skill- and labor-complementary machines are, respectively,[10]

$$\pi_h(\tau) = \alpha(1-\alpha)[p_h(\tau)]^{1/\alpha}\cdot\left(H_Y^{\mathrm{N}}+H^{\mathrm{S}}\right), \quad (46)$$

and

$$\pi_l(\tau) = \alpha(1-\alpha)[p_l(\tau)]^{1/\alpha}\cdot(L^{\mathrm{N}}+L^{\mathrm{S}}). \quad (47)$$

Combination of (20), (46), and (47) yields

$$P_h = \alpha(1-\alpha)(p_h)^{1/\alpha}\cdot\left(H_Y^{\mathrm{N}}+H^{\mathrm{S}}\right)/r \text{ and}$$
$$P_l = \alpha(1-\alpha)(p_l)^{1/\alpha}\cdot(L^{\mathrm{N}}+L^{\mathrm{S}})/r. \quad (48)$$

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Outputs of consumer goods in the developed country are, respectively,

$$Y_h^{\mathrm{N}} = (p_h)^{(1-\alpha)/\alpha} \cdot A_h H_Y^{\mathrm{N}} \text{ and } Y_l^{\mathrm{N}} = (p_l)^{(1-\alpha)/\alpha} \cdot A_l L^{\mathrm{N}}. \tag{49}$$

Similarly,

$$Y_h^{\mathrm{S}} = (p_h)^{(1-\alpha)/\alpha} \cdot A_h H^{\mathrm{S}} \text{ and } Y_l^{\mathrm{S}} = (p_l)^{(1-\alpha)/\alpha} \cdot A_l L^{\mathrm{S}}. \tag{50}$$

Equations (4), (49), and (50) imply

$$p = \frac{p_h}{p_l} = \left(\frac{A_h}{A_l} \cdot \frac{H_Y^{\mathrm{N}} + H^{\mathrm{S}}}{L^{\mathrm{N}} + L^{\mathrm{S}}} \right)^{-\frac{\alpha}{1+\alpha(\epsilon-1)}}. \tag{51}$$

Taking advantage of reasoning similar to that preceding, we get

$$H_Y^{\mathrm{N}} + H^{\mathrm{S}} = \frac{\theta(H^{\mathrm{N}} + H^{\mathrm{S}}) + \rho(A_h/A_l)^{\frac{1-\delta}{2}}[1 + (A_h/A_l)^{\delta-1}]}{\theta + (1-\alpha)[1 + (A_h/A_l)^{\delta-1}]}, \tag{52}$$

$$\frac{A_h}{A_l} = \left(\frac{H_Y^{\mathrm{N}} + H^{\mathrm{S}}}{L^{\mathrm{N}} + L^{\mathrm{S}}} \right)^{\frac{1}{\lambda}}, \tag{53}$$

and

$$\begin{aligned} &(L^{\mathrm{N}} + L^{\mathrm{S}}) \left(\frac{A_h}{A_l} \right)^{\lambda} \left\{ \theta + (1-\alpha) \left[1 + \left(\frac{A_h}{A_l} \right)^{\delta-1} \right] \right\} \\ &- \rho \left(\frac{A_h}{A_l} \right)^{\frac{1-\delta}{2}} \left[1 + \left(\frac{A_h}{A_l} \right)^{\delta-1} \right] - \theta(H^{\mathrm{N}} + H^{\mathrm{S}}) = 0. \end{aligned} \tag{54}$$

Using (52) and (53), we obtain

PROPOSITION 5. *Trade between a developed and a developing country reduces the degree of skill bias of technology in the developed country. For the developed country, trade increases the economic growth rate when* $\phi > \psi$*; the growth rate remains unchanged when* $\phi = \psi$*; trade decreases the growth rate when* $\phi < \psi$*, where* $\phi = [(H_Y^{\mathrm{N}})_o^* + H^{\mathrm{S}}][(A_h/A_l)_o^*]^{\frac{\delta-1}{2}}$ *and* $\psi = (H_Y^{\mathrm{N}})^*[(A_h/A_l)_{\mathrm{N}}^*]^{\frac{\delta-1}{2}}$*. For the developing country, trade speeds up growth when* $\phi > \varphi$*; the growth rate is invariant when* $\phi = \varphi$*; trade slows down growth when* $\phi < \varphi$*, where* $\varphi = (H_Y^{\mathrm{S}})^*[(A_h/A_l)_{\mathrm{S}}^*]^{(\delta-1)/2}$.

Proof. See Appendix E. ■

Due to the smaller market size for skill-complementary machines in the developing country, trade between developed and developing countries reduces the relative profits of skill-complementary innovations. Therefore, the *skilled workers reallocation* effect induces skill-replacing technological change in the developed country. This result is different from those in Acemoglu (2003b) and Theonig and Verdier (2003), where trade only causes skill-biased technological change.

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Trade between developed countries doubles the effective stock of ideas via knowledge flows. However, trade between developed and developing countries does not necessarily enrich this stock, because developing countries usually create fewer technologies. Consequently, it does not stimulate the growth of the world economy by promoting R&D productivity. Using some algebra, we have

$$\left(\frac{w_h}{w_l}\right)^*_o = \left(\frac{H_Y^{\mathrm{N}} + H^{\mathrm{S}}}{L^{\mathrm{N}} + L^{\mathrm{S}}}\right)^{\frac{\alpha(\epsilon-1)-\lambda}{\lambda(1+\alpha(\epsilon-1))}} = \left[\left(\frac{A_h}{A_l}\right)^*\right]^{\frac{\alpha(\epsilon-1)+\delta-1}{\alpha(\epsilon-1)}}. \tag{55}$$

Therefore,

PROPOSITION 6. *Suppose that* $\epsilon > 1$ *and* $0 < \lambda \leq \frac{1-\delta}{2}$. *Trade between a developed and a developing country increases wage inequality in the developing country, whereas it decreases wage inequality in the developed country.*

Proof. See Appendix E. ■

Because the degree of skill bias of technology in the developing country is larger after trade opening, its wage inequality increases. However, owing to the relatively skill-replacing technological change, wage inequality in the developed country declines. This result is different from those of Acemoglu (2003) and Theonig and Verdier (2003), who argue that trade between developed and developing countries contributes to an increase in skill premia.[11]

It is worth emphasizing that (17) and Propositions 4 and 5 imply that trade between developed countries reduces the relative price of the skill-intensive good, whereas trade between developed and developing countries raises the relative price. Because the volume of trade between developed countries is substantially greater than that between developed and developing countries, international trade may cause a decrease in the relative price of skill-intensive goods in the real world.

6. CONCLUSION

We have built a knowledge-based growth model to explore how directed technological change affects scale effects and skill premia and how international trade influences wage inequality and economic growth when participating countries are developed or developing. The key assumption is that R&D sectors compete with the production sector for skilled workers.

When technologies that are complementary to skilled workers are being innovated, the average productivity of skill-complementary knowledge decreases with the degree of skill bias of technology. This implies that skill-biased technological change could result in a fall in economic growth. Thist is referred to as the negative *innovation-possibilities frontier* effect. On the other hand, the average productivity of labor-complementary knowledge in the labor-complementary R&D sector increases with the degree of skill bias of technology, and so skill-biased technological change has a positive impact on growth. This is the positive *innovation-possibilities frontier* effect. Therefore, scale effects may be absent if the negative effect is more powerful.

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In this paper, the two facts of accelerated skilled-biased technological change and the absence of scale effects have been jointly accounted for in a unified framework. As far as we know, we are the first to connect directed technological change and scale effects. We also show that, by endogenous allocation of skilled workers across production and innovation, skill-biased technological change increases skill premia even if the elasticity of substitution between skilled and unskilled workers is smaller than 2. This is quite an improvement on the existing studies.

Trade between developed countries increases the demand for skilled workers used in the skill-intensive sector through the *skilled workers reallocation* effect, thus inducing skill-biased technological change and enlarging skill premia. As a result, the observed growth in trade between developed countries could be a major cause of rising wage inequality. In contrast, the *skilled workers reallocation* effect of trade between developed and developing countries encourages skill-replacing technological change in the developed country and lowers its wage inequality. At the same time, for the developing country, the degree of skill bias of technology in the open regime is greater, and therefore international trade causes an increase in wage inequality.

Because directed technological change has two opposite effects on economic growth, international trade promotes growth in some circumstances but hinders it in other circumstances. This implies that trade opening does not always promote economic growth in developed (or developing) countries even though it increases their opportunities to access more advanced technologies.

NOTES

1. We appreciate one referee's comment in this territory.
2. We benefit from the editor's comment on this point.
3. Existing studies give attention to the relationship between the number of scientists and economic growth. Therefore, we assume that R&D sectors only employ skilled workers. Of course, R&D sectors can alternatively hire both skilled and unskilled workers. Because of space limits, we leave out the discussion.
4. See Acemoglu (2002a) for related references. He also discusses the magnitude of δ.
5. The reader should be aware of the multiplicity result. We are not able to give a global analysis of the dynamics because it is difficult to identify the system of differential equations relevant to this problem. In Appendix B, we present an analytical framework making clear several local situations. Fortunately, the main results are locally independent of the uniqueness. The appendices, especially Appendix C, imply that, as long as (28) is continuously differentiable and the BGP outcome is chosen in some way or selected like ad hoc dynamics, the main results absolutely hold. For example, if utilities are equal at P_1 and P_2 (see Figure B.1 in Appendix B), solving the uniqueness problem is difficult. Assuming that P_1 is selected as the BGP outcome for some unknown reasons, the multiplicity has no essential influence on the following discussion.
6. If $2\lambda + \delta - 1 = -0.26$, where $\delta = 0.71$, then $L \cdot [(A_h/A_l)^*]^{\frac{2\lambda+\delta-1}{2}}$ decreases by 6.5% or 2.6%, or increases by 1.3%, respectively. We benefit from one referee's comment that, according to the data, the supply of unskilled labor increased by around 25% from 1970 to 1990, and if quality-adjusted unskilled labor is considered, this is around 30%.
7. Solutions to (44) are similar to those to (28). We do not discuss the case of symmetric countries with identical initial data being in different steady states. Such a discussion is quite difficult.
8. Labor immobility is the main difference between exposure to trade with an identical country and doubling the size of the economy.

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9. The main results will not be altered under the more realistic assumption that the productivity of machines in the developing country is proportional to that in the developed country.

10. We assume that R&D firms in the developed country can capture all revenues generated by machine sales in the developing country. In the real world, because of poor property rights protection, R&D firms can only capture a proportion of these revenues. However, the main results are unchanged if the property rights are considered.

11. Although some empirical studies document that trade between identical countries increases skill premia [e.g., Epifani and Gancia (2008)], it is very difficult to separate the effect of trade between identical countries from that of trade between different countries by empirical efforts.

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APPENDIX A: SOLUTIONS TO (28)

A.1. EXISTENCE

Equation (28) implies that $f(+\infty) = +\infty$, $f(0) = -\infty$ as $\lambda > \frac{1-\delta}{2}$, and $f(+\infty) = -\infty$, $f(0) = +\infty$ as $\lambda < \frac{1-\delta}{2}$. In the meantime, $f(\cdot)$ is continuous. Therefore, there exists at least one solution to (28) when $\lambda \neq \frac{1-\delta}{2}$. In addition, when $\lambda = \frac{1-\delta}{2}$, (28) can be rewritten as $(A_h/A_l)^{2\lambda}[(1+\theta-\alpha)L-\rho] - \theta H(A_h/A_l)^{\lambda} + (1-\alpha)L - \rho = 0$. Hence, when $(1+\theta-\alpha)L-\rho \neq 0$ and $\theta^2H^2 - 4[(1+\theta-\alpha)L-\rho]\cdot[(1-\alpha)L-\rho] \geq 0$, there exists at least one solution to (28).

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A.2. THE NUMBER OF SOLUTIONS

When $\delta = 1$, (28) is

$$f\left(\frac{A_h}{A_l}\right) = [\theta + 2(1-\alpha)]L\left(\frac{A_h}{A_l}\right)^{\lambda} - 2\rho - \theta H = 0.$$

It has the unique solution $\frac{A_h}{A_l} = [\frac{2\rho+\theta H}{\theta L+2(1-\alpha)L}]^{1/\lambda}$. Assume $\delta \neq 1$. Letting $x = (A_h/A_l)^{(1-\delta)/2}$, (28) is changed to be

$$F(x) \triangleq f\left(x^{\frac{2}{1-\delta}}\right) = (\theta L + a)x^m + ax^{m-2} - \rho x - \rho x^{-1} - \theta H = 0, \qquad \textbf{(A.1)}$$

where $0 \leq x < +\infty$, $a = (1-\alpha)L$, and $m = \frac{2\lambda}{1-\delta}$. Simple calculus gives

$$F_1(x) \triangleq x^2 F'(x) = m\,(\theta L + \alpha)\,x^{m+1} + (m-2)ax^{m-1} - \rho x^2 + \rho,$$

$$F_2(x) \triangleq F_1'(x) = m\,(m+1)\,(\theta L + \alpha)\,x^m + (m-2)(m-1)ax^{m-2} - 2\rho x,$$

$$F_3(x) \triangleq F_2'(x) = m^2(m+1)(\theta L + a)x^{m-1} + (m-2)^2(m-1)ax^{m-3} - 2\rho,$$

and

$$F_4(x) \triangleq F_3'(x) = (m-1)\,m^2(m+1)(\theta L + a)x^{m-2} + (m-3)(m-2)^2(m-1)ax^{m-4},$$

which clarify completely the properties of solutions to (A.1). First, for $m = 1$,

$$F(x) = (\theta L + a - \rho)x + (a - \rho)x^{-1} - \theta H.$$

- If $a > \rho$, $F(0) = F(+\infty) = +\infty$ and $F(x)$ arrives at its minimum at $x = (\frac{a-\rho}{\theta L+a-\rho})^{\frac{1}{2}}$. Hence
  - (A.1) has two solutions when the minimum is negative;
  - (A.1) has only one solution when the minimum is zero;
  - (A.1) has no solution when the minimum is positive.
- If $\theta L + a < \rho$, $F(0) = F(+\infty) = -\infty$ and $F(x)$ arrives at its maximum at $x = (\frac{a-\rho}{\theta L+a-\rho})^{\frac{1}{2}}$. Hence
  - (A.1) has two solutions when the maximum is positive;
  - (A.1) has only one solution when the maximum is zero;
  - (A.1) has no solution when the maximum is negative.
- If $\theta L + a > \rho > a$, $F(0) = -\infty$, $F(+\infty) = +\infty$ and $F'(x) > 0$. Hence (A.1) is uniquely solved.
- If $\theta L + a = \rho > a$, there is no solution.
- If $\theta L + a > \rho = a$, the unique solution is $x = H/L$.

Because $F(0) = -\infty$ and $F(+\infty) = +\infty$ if $m > 1$, and $F(0) = +\infty$ and $F(+\infty) = -\infty$ if $m < 1$, there exists at least one solution to (A.1) for $m \neq 1$. By analyzing functions F_1, F_2, F_3, and F_4, we can show clearly the number of solutions. We provide some qualitative conclusions as follows:

- $m \geq 3 : F_4(x) > 0$, and therefore $F_3(x)$ strictly increases. Because $F_3(0) = -2\rho$ and $F_3(+\infty) = +\infty$, $F_3(x) = 0$ is uniquely solved. Moreover, $F_2(x)$ decreases before increasing. Now, that $F_2(0) = 0$ implies that $F_2(x) = 0$ has two solutions (one is

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zero and the other is assumed to be $x_1 > 0$). Consequently, $F_1(x)$ also decreases before increasing, and $F_1(0) = \rho$ and $F_1(+\infty) = +\infty$ imply that
  - (A.1) has only one solution because $F(x)$ is strictly increasing, if $F_1(x_1) \geq 0$;
  - The number of solutions to (A.1) may be 1, 2, or 3 if $F_1(x_1) < 0$, which means that $F(x)$ is a strictly increasing–decreasing–increasing function.
- $2 < m < 3 : F_4(x)$ is strictly increasing and has only one zero point (assumed to be $x_2 > 0$). Thus $F_3(x)$ is strictly decreasing–increasing, and from $F_3(0) = +\infty$ and $F_3(+\infty) = +\infty$, we get that the solutions of $F_3(x) = 0$ depend on its minimum, i.e., $F_3(x_2)$. Generally, $F_3(x)$ may have 0, 1, or 2 zero points. Therefore, $F_2(x)$ maybe strictly increasing or increasing–decreasing–increasing. Now, that $F_2(0) = 0$ and $F_2(+\infty) = +\infty$ imply that $F_2(x) = 0$ may have 1 (precisely $x = 0$), 2 (one is zero and the other is positive), or 3 (the last supposed to be $x_3 > 0$) solutions. The former two cases suggest that $F_1(x)$ is strictly increasing, and the last suggests that $F_1(x)$ is increasing–decreasing–increasing. Hence $F_1(0) = \rho$ and $F_1(+\infty) = +\infty$ imply that
  - (A.1) has only one solution because $F(x)$ is strictly increasing, if $F_1(x_3) \geq 0$;
  - The number of solutions to (A.1) may be 1, 2, or 3, if $F_1(x_3) < 0$, which means that $F(x)$ is an increasing–decreasing–increasing function.
- $m = 2 : F_4(x) > 0$, and thus $F_3(x)$ strictly increases. Because $F_3(0) = -2\rho$ and $F_3(+\infty) = +\infty$, $F_3(x) = 0$ is uniquely solved, and thus $F_2(x)$ is strictly decreasing–increasing. Now, that $F_2(0) = 0$ implies that $F_2(x) = 0$ has two solutions (one is zero and the other is assumed to be $x_4 > 0$). As a result, $F_1(x)$ is also strictly decreasing–increasing, and $F_1(0) = \rho$ and $F_1(+\infty) = +\infty$ imply that
  - (A.1) has only one solution because $F(x)$ is strictly increasing, if $F_1(x_4) \geq 0$;
  - The number of solutions to (A.1) may be 1, 2, or 3, if $F_1(x_4) < 0$, which means that $F(x)$ is a strictly increasing–decreasing–increasing function.
- $1 < m < 2 : F_4(x) = 0$ is uniquely solved at $x_5 > 0$, $F_4(x) < 0$ for $x < x_5$, and $F_4(x) > 0$ for $x > x_5$. In fact, $F_4(x)$ increases from $-\infty$ to its positive maximum and then decreases to 0. Now $F_3(x)$ is strictly decreasing–increasing from $F_3(0) = +\infty$ to $F_3(+\infty) = +\infty$. Thus the solutions of $F_3(x) = 0$ depend on its minimum, i.e., $F_3(x_5)$. Generally, $F_3(x)$ may have 0, 1, or 2 zero points. Therefore, $F_2(x)$ maybe strictly increasing or increasing–decreasing–increasing. Because $F_2(0) = -\infty$ and $F_2(+\infty) = +\infty$, we have that $F_2(x) = 0$ may have 1, 2, or 3 positive solutions. The former two cases ensure that $F_1(x)$ is strictly decreasing–increasing, and the last ensures that $F_1(x)$ is strictly decreasing–increasing–decreasing–increasing. Therefore $F_1(0) = \rho$ and $F_1(+\infty) = +\infty$ imply that
  - (A.1) has only one solution, if the minimum of $F_1(x)$ is nonnegative;
  - The number of solutions to (A.1) may be 1, 2, or 3, if the negative minimum of $F_1(x)$ is reached at only one minimal point;
  - The number of solutions to (A.1) may be 1, 2, or 3, if the local maximum of $F_1(x)$ is nonpositive;
  - The number of solutions to (A.1) may be 1, 2, 3, 4, or 5, if the negative local minima of $F_1(x)$ are reached at two minimal points and the local maximum is positive.
- $m < 1 : F_4(x) = 0$ is uniquely solved at $x_6 > 0$, $F_4(x) > 0$ for $x < x_6$, and $F_4(x) < 0$ for $x > x_6$. In fact, $F_4(x)$ decreases from $+\infty$ to its negative minimum and then increases to 0. Now $F_3(x)$ is strictly increasing–decreasing from $F_3(0) = -\infty$ to $F_3(+\infty) = -2\rho$. Thus the solutions of $F_3(x) = 0$ depend on its maximum, i.e.,

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$F_3(x_6)$. Generally, $F_3(x)$ may have 0, 1, or 2 zero points. Therefore, $F_2(x)$ may be strictly decreasing or decreasing–increasing–decreasing. Because $F_2(0) = +\infty$ and $F_2(+\infty) = -\infty$, we have $F_2(x) = 0$ may have 1, 2, or 3 positive solutions. The former two cases ensure that $F_1(x)$ is strictly increasing–decreasing, and the last ensures that $F_1(x)$ is increasing–decreasing–increasing–decreasing. Therefore, $F_1(0) = -\infty$ and $F_1(+\infty) = -\infty$ imply that

- (A.1) has only one solution, if the maximum of $F_1(x)$ is nonpositive;
- The number of solutions to (A.1) may be 1, 2, or 3 if the positive maximum of $F_1(x)$ is reached at only one maximal point;
- The number of solutions to (A.1) may be 1, 2, or 3 if the local minimum of $F_1(x)$ is nonnegative;
- The number of solutions to (A.1) may be 1, 2, 3, 4, or 5 if the positive local maxima of $F_1(x)$ are reached at two maximal points and the local minimum is negative.

APPENDIX B: BGP EQUILIBRIA

B.1. EXISTENCE OF THE OPTIMAL BGP

Without loss of generality, we suppose the economy in its BGP at time 0. From (1), the representative agent's utility is

$$U = \frac{C(0)^{1-\theta}}{(1-\theta)[\rho-(1-\theta)g]} - \frac{1}{\rho(1-\theta)},$$

where $C(0)$ is the consumption at time 0; formally,

$$C(0) = p_h(0)Y_h(0) + p_l(0)Y_l(0) - (1-\alpha)\cdot A_h(0)k_h(0) - (1-\alpha)\cdot A_l(0)k_l(0),$$

and g is the economic growth rate. From (2), (24), and (27),

$$g = \frac{(1-\alpha)L\cdot(A_h/A_l)^{\frac{2\lambda+\delta-1}{2}} - \rho}{\theta}.$$

Using (16) and (27), we can compute

$$C(0) = \left[(p_h)^{1/\alpha}\cdot(A_h/A_l)^{\lambda+1} + (p_l)^{1/\alpha}\right]L\cdot A_l(0) - (1-\alpha)\cdot[A_h(0)k_h(0) + A_l(0)k_l(0)].$$

From (3), we normalize $(p_l)^{1-\epsilon} + (p_h)^{1-\epsilon} = 1$. Furthermore, (17) and (27) imply

$$p_h = \left[(A_h/A_l)^{\delta-1} + 1\right]^{\frac{1}{\epsilon-1}} \text{ and } p_l = \left[(A_h/A_l)^{1-\delta} + 1\right]^{\frac{1}{\epsilon-1}}.$$

As the total amount of initial capital $A_h(0)k_h(0) + A_l(0)k_l(0)$, the number of unskilled workers L, and the initial technology level $A_l(0)$ are given, the utility is a continuous function of the variable A_h/A_l (or equivalently, g). Therefore, among the BGPs, there is an optimal one maximizing the social welfare.

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B.2. LOCAL STABILITY

Suppose that $\delta < \frac{1}{1+\alpha(\epsilon-1)}$. Consider an economy starting out of but near the BGP. Let the economy remain in equilibrium. Initial $A_h(0)$, $A_l(0)$, and $H_Y(0)$ are given. If

$$A_h(0)/A_l(0) < [H_Y(0)/L]^{1/\lambda}, \tag{B.1}$$

we can prove that, over an interval of time $t \in [0, s]$, the growth rate of A_h is higher than that of A_l, or equivalently, A_h/A_l is increasing in t. Rewrite (18) as

$$r(t)P_z(t) = \pi_z(t) + \dot{P}_z(t).$$

Because P_z satisfies (23),

$$P_h \cdot (A_h/A_l)^{\delta} = P_l.$$

Differentiating it with respect to time, we obtain

$$\dot{P}_h \cdot (A_h/A_l)^{\delta} + P_h \cdot \left[\left(\dot{A}_h/A_l\right)^{\delta}\right] = \dot{P}_l.$$

The preceding three equations imply the dynamic equation

$$P_h \cdot \left[\left(\dot{A}_h/A_l\right)^{\delta}\right] = \pi_h \cdot (A_h/A_l)^{\delta} - \pi_l.$$

Using (14), (15), and (17) on the R.H.S., we obtain

$$P_h\left[\left(\frac{\dot{A}_h}{A_l}\right)^{\delta}\right] = \alpha(1-\alpha)\cdot(p_l)^{1/\alpha}\cdot L\left[\left(\frac{A_h}{A_l}\cdot\frac{H_Y}{L}\right)^{-\frac{1}{1+\alpha(\epsilon-1)}}\cdot\left(\frac{H_Y}{L}\right)\cdot\left(\frac{A_h}{A_l}\right)^{\delta} - 1\right].$$

From (B.1) and $\delta < \frac{1}{1+\alpha(\epsilon-1)}$, the R.H.S. is positive, and so A_h/A_l is increasing in t.

Similarly, if $A_h(0)/A_l(0) > (H_Y(0)/L)^{1/\lambda}$, then over an interval of time, the growth rate of A_h is lower than that of A_l, or equivalently, A_h/A_l is decreasing in time. This analysis shows that the economy converges to satisfy $A_h/A_l = (H_Y/L)^{1/\lambda}$ and $(\dot{A}_h/A_l) = 0$. Moreover, p_z , π_z, and P_z converge to be constant. That is, the economic equilibrium approaches the BGP.

B.3. DYNAMICS AND UNIQUENESS

Suppose that the economy remains in equilibrium and that there exist exactly two solutions to (28), denoted by $(A_h/A_l)_1$ and $(A_h/A_l)_2$. In Figure B.1, the first quadrant is divided into six zones designated $I, II, \ldots, VI$; there are two BGP (equilibrium) points P_i corresponding to $(A_h/A_l)_i$, $i = 1, 2$, respectively; and the economy starts from an equilibrium point P_0.

First, because A_h/A_l is increasing in time if (B.1) holds, P_0 will not be located in zone II. Similarly, P_0 must not stand in zone V. In other words, all the points in zones II and V are out of equilibrium.

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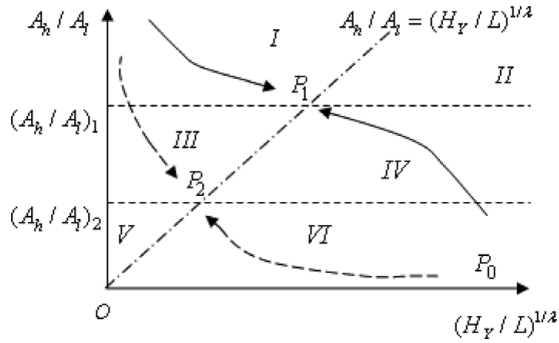


FIGURE B.1. Dynamics of the economy.

Second, because A_h/A_l is increasing if (B.1) holds, the economy has to approach P_1 when P_0 is located in zone IV. Likewise, the economy converges to P_2 when P_0 stands in zone III. Thus, the unique BGP is obtained from the initial location.

Finally, if P_0 is located in zone I or VI, the economy converges to P_1 or P_2. The BGP outcome may be multiple, and to solve it through conventional dynamic analysis is difficult. Simply, assuming the representative agent has greater utility at P_1 than P_2, we select P_1 as the unique BGP outcome.

APPENDIX C: $\frac{\partial(H_Y/L)}{\partial(H/L)} > 0$ IF $0 < \lambda \leq \frac{1-\delta}{2}$

Let $y = H_Y/L$ and $x = H/L$. Obviously, $0 \leq y \leq x$. From (26) and (27),

$$y = \frac{\theta x + \rho L^{-1} y^{\frac{1-\delta}{2\lambda}} \left(1 + y^{\frac{\delta-1}{\lambda}}\right)}{\theta + (1-\alpha)\left(1 + y^{\frac{\delta-1}{\lambda}}\right)}.$$

Differentiating it with respect to x and using simple computations, we get

$$f(x, y) \cdot \frac{\partial y}{\partial x} = \theta \left[\theta + (1-\alpha)\left(1 + y^{\frac{\delta-1}{\lambda}}\right)\right],$$

where

$$f(x, y) = \left[\theta + (1-\alpha)\left(1 + y^{\frac{\delta-1}{\lambda}}\right)\right]^2 + \frac{\theta\rho}{2\lambda L}(1-\delta) y^{\frac{1-\delta}{2\lambda}-1} \cdot \left(y^{\frac{\delta-1}{\lambda}} - 1\right)$$
$$- \frac{\rho}{2\lambda L}(1-\alpha)(1-\delta) y^{\frac{1-\delta}{2\lambda}-1} \cdot \left(1 + y^{\frac{\delta-1}{\lambda}}\right)^2 - \frac{\theta}{\lambda}(1-\alpha)(1-\delta) x \cdot y^{\frac{\delta-1}{\lambda}-1}.$$

First, $f(x, y) \neq 0$, and its sign will never reverse, by the continuity. Suppose x and y small. On the R.H.S. of the last expression, the second term is positive because $\frac{\delta-1}{\lambda} < 0$, and the last two negative terms are dominated by the first positive one because $\frac{1-\delta}{2\lambda} - 1 \geq 0$ and $2\frac{\delta-1}{\lambda} < \frac{\delta-1}{\lambda} - 1 < 0$ hold. Now, $f(x, y)$ is positive for x small, and moreover, for all $x > 0$. Thus, $\partial y/\partial x > 0$.

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APPENDIX D: LARGER H_Y IN (43) THAN IN (26)

Comparing (43) with (26), it is sufficient to show that doubling θ induces bigger H_Y. For y in Appendix C, differentiate it w.r.t. θ and use computations to get

$$f(x,y)\cdot\frac{\partial y}{\partial\theta}=\left[(1-\alpha)x-\rho L^{-1}y^{\frac{1-\delta}{2\lambda}}\right]\left(1+y^{\frac{\delta-1}{\lambda}}\right),$$

where

$$f(x,y)=\left[\theta+(1-\alpha)\left(1+y^{\frac{\delta-1}{\lambda}}\right)\right]^2+\frac{\theta\rho}{2\lambda L}(1-\delta)y^{\frac{1-\delta}{2\lambda}-1}\cdot\left(y^{\frac{\delta-1}{\lambda}}-1\right)$$
$$-\frac{\rho}{2\lambda L}(1-\alpha)(1-\delta)y^{\frac{1-\delta}{2\lambda}-1}\cdot\left(1+y^{\frac{\delta-1}{\lambda}}\right)^2-\frac{\theta}{\lambda}(1-\alpha)(1-\delta)x\cdot y^{\frac{\delta-1}{\lambda}-1}.$$

Because $\frac{1-\delta}{2\lambda}\geq 1$, $0\leq y\leq x$ and the magnitude of L is much greater than that of ρ, arguments similar to those in Appendix C can show that the R.H.S. of the two preceding equations above must be positive, and thus $\partial y/\partial\theta>0$.

APPENDIX E: PROOFS OF PROPOSITIONS

E.1. PROOF OF PROPOSITION 1

By (27), we have

$$\frac{\partial\left(A_h/A_l\right)}{\partial\left(H/L\right)}=\frac{1}{\lambda}\left(\frac{A_h}{A_l}\right)^{1-\lambda}\cdot\frac{\partial\left(H_Y/L\right)}{\partial\left(H/L\right)}.$$

From Appendix C, $\partial(A_h/A_l)/\partial(H/L)>0$; that is, technological change is skill-biased.

E.2. PROOF OF PROPOSITION 2

This is obvious from Proposition 1 and (29).

E.3. PROOF OF PROPOSITION 3

This is clear from Proposition 1 and (34).

E.4. PROOF OF PROPOSITION 4

Results about the degree of skill bias of technology and wage inequality are based on (27) and (34). In the open economy,

$$g_o^*=\frac{2\left(1-\alpha\right)L\left[\left(A_h/A_l\right)_o^*\right]^{\frac{2\lambda+\delta-1}{2}}-\rho}{\theta}.$$

This gives the result on the growth rate straightforwardly.

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E.5. PROOF OF PROPOSITION 5

Because $H^{\mathrm{N}}/L^{\mathrm{N}} \gg H^{\mathrm{S}}/L^{\mathrm{S}}$, the combination of (52), (53), and Appendix C implies that international trade leads to $(A_h/A_l)_o^* < (A_h/A_l)^*$. Now the growth rate in the open economy is

$$g_o^* = \frac{(1-\alpha)\left[\left(H_Y^{\mathrm{N}}\right)_o^* + H^{\mathrm{S}}\right]\left[(A_h/A_l)_o^*\right]^{\frac{\delta-1}{2}} - \rho}{\theta}.$$

Therefore, the growth rate varies depending on sizes of ϕ, ψ, and φ.

E.6. PROOF OF PROPOSITION 6

Before trade opening, skill premia in the developed and the developing country are, respectively,

$$\left(\frac{w_h}{w_l}\right)^{\mathrm{N}} = \left[\left(\frac{A_h}{A_l}\right)_N^*\right]^{\frac{\alpha(\epsilon-1)+\delta-1}{\alpha(\epsilon-1)}} \text{ and } \left(\frac{w_h}{w_l}\right)^{\mathrm{S}} = \left[\left(\frac{A_h}{A_l}\right)_S^*\right]^{\frac{\alpha(\epsilon-1)+\delta-1}{\alpha(\epsilon-1)}}.$$

Proposition 5 suggests that $(A_h/A_l)_N^* > (A_h/A_l)_o^*$; therefore wage inequality declines in the developed country. In the meantime, because of $H^{\mathrm{N}}/L^{\mathrm{N}} \gg H^{\mathrm{S}}/L^{\mathrm{S}}$; (52), (53), and arguments similar to those in Appendix C, $(A_h/A_l)_S^* < (A_h/A_l)_o^*$ holds, and consequently wage inequality increases in the developing country.

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