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Delay-dependent robust H_∞ control for neutral time-delay systems

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We derive a delay-dependent H_∞ performance criterion with a decoupled structure for systems with neutral time delay. We then extend it to an H_∞ controller synthesis for systems with polytopic uncertainty. All conditions are given in terms of linear matrix inequalities (LMIs). In some previous descriptor system methods, the products of the controller and Lyapunov matrices are completely separated for the performance analysis, but not for controller synthesis – the method developed in the current paper eliminates this weakness. We present a numerical example to illustrate the effectiveness of the solution.

1. Introduction

Neutral delay systems have attracted much interest recently because they are both theoretically important and are often encountered in various practical fields such as chemical processes, aircraft, biology and economics (Mahmoud 2000; Xu *et al.* 2001; Tarbouriech and Garcia 1999). Stability conditions for neutral delay system based on LMIs or Riccati equations have been obtained for both the delay-independent and delay-dependent cases (Ma *et al.* 1995; Fan *et al.* 2002; Yue *et al.* 2003; Park 2004). Although the method for solving the delay-independent case is simpler, it is known that the delay-dependent case is generally less conservative, especially when the size of the delay is small. The uncertainties model, including both norm-bounded and polytopic cases, has received considerable attention, though more has been directed to the analysis and synthesis of neutral delay systems based on the norm-bounded model. For polytopic uncertain systems, the difficulties appear to be in determining how to separate the structure between the Lyapunov and controller matrices. Many researchers have obtained useful results in solving both discrete (de Oliveira *et al.* 1999; de Oliveira *et al.* 1999) and continuous (Apkarian *et al.* 2001; Ebihara 2001) time systems. However, they did not consider time delay. Recently, some researchers have investigated neutral delay systems with polytopic uncertainty (Fridman and Shaked 2002; Fridman and Shaked 2003), though the products

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of the Lyapunov and controller matrices are not completely separated in the controller design, which is the motivation for the current paper.

In the current paper, we solve the robust H_∞ control problems for systems with neutral time delay and polytopic uncertainty. We begin by presenting a new delay-dependent H_∞ performance condition. Then we derive an H_∞ performance criterion that separates the products of the controller and Lyapunov matrices. This can reduce the conservatism inherent in the classical quadratic method by providing a parameter-dependent Lyapunov function. We then design robust H_∞ state-feedback controllers for time-delay systems with polytopic uncertainty. The advantages of our results over previous methods are shown by a numerical example.

To simplify the notation, we define T_{zw} to be the transfer function from w to z and use the symbol $Sym\{\cdot\}$ to denote $Sym\{X\} \stackrel{\text{def}}{=} X + X^T$ and the symbol $*$ to denote the symmetric part.

2. Problem formulation

Consider the following system with neutral time delay

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h) + F\dot{x}(t-d) + B_1 w(t) + B_2 u(t) \\ z(t) &= Cx(t) + D_1 w(t) + D_2 u(t) \\ x(t) &= 0, \forall t \in [-\tau, 0] \end{aligned} \tag{1}$$

where:

— $x(t) \in R^n$ is the state;
— $w(t) \in R^l$ is the disturbance of finite energy in the space $L_2[0, \infty)$;
— $z(t) \in R^p$ is the exogenous output;
— $u(t) \in R^q$ is the control input;
— A, B_1, B_2, A_d, F, C, D_1, D_2 are known constant matrices of appropriate dimensions.

The time delays $d > 0$ and $h > 0$ are assumed to be known, and we let $\tau = \max\{d, h\}$.

Lemma 2.1 (Saadni and Mehdi 2004). Given matrices ϕ, a and b with appropriate dimension, the following two statements are equivalent:

(i) ϕ, a and b satisfy

$$\begin{aligned} \phi &< 0 \\ \phi + ab^T + ba^T &< 0. \end{aligned}$$

(ii) ϕ, a and b are such that the following LMI

$$\begin{bmatrix} \phi & a + bG^T \\ * & -G - G^T \end{bmatrix} < 0$$

is feasible in the variable G.

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Lemma 2.2 (Park 1999). For any $2n \times 2n$ symmetric and positive-definite matrix $R > 0$ and matrix M, the inequality

$$-2\int_{t-h}^{t} b^T(s)a(s)ds \leqslant \int_{t-h}^{t} \delta^T(s)\begin{bmatrix} R & RM \\ * & (2,2) \end{bmatrix}\delta(s)ds$$

holds for $a(s) \in R^{2n}$, $b(s) \in R^{2n}$ given for $s \in [t-h, t]$ where

$$\delta(s) = \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}$$
$$(2,2) = (M^T R + I)R^{-1}(RM + I).$$

3. Main results

Let $u(t) = 0$. We begin by providing a delay-dependent H_∞ performance condition for the system (1).

Theorem 3.1. Given positive scalars $\gamma > 0$ and $h > 0$, and a scalar λ, system (1) is stable and satisfies $\|T_{zw}\|_\infty < \gamma$ if there exist symmetric and positive-definite matrices $Q > 0$, $\bar{S} > 0$, $\bar{R} > 0$ and $\bar{U} > 0$ such that the following LMI holds:

$$\begin{bmatrix} (1,1) & -\lambda A_d\bar{S} & (1,3) & F\bar{U} & B_1 & QC^T & (1,7) & QA^T & Q \\ * & -\bar{S} & (2,3) & 0 & 0 & 0 & 0 & \bar{S}A_d^T & 0 \\ * & * & -h\bar{R} & hA_dF\bar{U} & hA_dB_1 & 0 & (3,7) & 0 & 0 \\ * & * & * & -\bar{U} & 0 & 0 & 0 & \bar{U}F^T & 0 \\ * & * & * & * & -\gamma^2 I & D_1^T & 0 & B_1^T & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -h\bar{R} & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{U} & 0 \\ * & * & * & * & * & * & * & * & -\bar{S} \end{bmatrix} < 0 \qquad (2)$$

where

$$(1,1) = Sym\{(A + A_d + \lambda A_d)Q\}$$
$$(1,3) = hQ(A + (1+\lambda)A_d)^T A_d^T$$
$$(1,7) = h(\lambda + 1)\bar{R}$$
$$(2,3) = -h\lambda\bar{S}A_d^T A_d^T$$
$$(3,7) = h^2(\lambda + 1)A_d\bar{R}.$$

Proof. Consider the Lyapunov function

$$V(t) = \sum_{i=1}^{4} V_i(t)$$

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where

$$V_1(t) = x^T(t)Px(t)$$
$$V_2(t) = \int_{t-h}^{t} x^T(\alpha)Sx(\alpha)d\alpha$$
$$V_3(t) = \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(\alpha)A_d^T RA_d\dot{x}(\alpha)d\alpha d\theta$$
$$V_4(t) = \int_{t-d}^{t} \dot{x}^T(\alpha)U\dot{x}(\alpha)d\alpha$$

The time derivative of $V(t)$ along the solution of (1) is given by

$$\dot{V}(t) = \sum_{i=1}^{4} \dot{V}_i(t),$$

where

$$\begin{aligned}\dot{V}_1(t) &= 2x^T(t)P(A+A_d)x(t) + 2x^T(t)PB_1w(t)\\ &\quad - 2\int_{t-h}^{t} x^T(t)PA_d\dot{x}(\alpha)d\alpha + 2x^T(t)PF\dot{x}(t-d)\\ \dot{V}_2(t) &= x^T(t)Sx(t) - x^T(t-h)Sx(t-h)\\ \dot{V}_3(t) &= -\int_{t-h}^{t} \dot{x}^T(\alpha)A_d^T RA_d\dot{x}(\alpha)d\alpha\\ &\quad + h\dot{x}^T(t)A_d^T RA_d\dot{x}(t)\\ \dot{V}_4(t) &= \dot{x}^T(t)U\dot{x}(t) - \dot{x}^T(t-d)U\dot{x}(t-d).\end{aligned}$$

Let

$$\begin{aligned}\eta &= RA_d\dot{x}(t)\\ \zeta^T &= \begin{bmatrix} P & 0 & hA_d^T & 0\end{bmatrix}\\ \xi^T &= \begin{bmatrix} x^T(t) & x^T(t-h) & \eta^T & \dot{x}^T(t-d)\end{bmatrix}.\end{aligned}$$

Note that we have the equation

$$-2\int_{t-h}^{t} x^T(t)PA_d\dot{x}(\alpha)d\alpha = -2\int_{t-h}^{t} \xi^T\zeta A_d\dot{x}(\alpha)d\alpha + 2h\int_{t-h}^{t} \eta^T A_dA_d\dot{x}(\alpha)d\alpha.$$

By Lemma 2.2, we can get

$$\begin{aligned}-2\int_{t-h}^{t} \xi^T\zeta A_d\dot{x}(\alpha)d\alpha &\leqslant \int_{t-h}^{t} \dot{x}^T(\alpha)A_d^T RA_d\dot{x}(\alpha)d\alpha + 2\lambda\int_{t-h}^{t} \xi^T\zeta A_d\dot{x}(\alpha)d\alpha\\ &\quad + (\lambda+1)^2\int_{t-h}^{t} \xi^T\zeta R^{-1}\zeta^T\xi d\alpha\\ &= \int_{t-h}^{t} \dot{x}^T(\alpha)A_d^T RA_d\dot{x}(\alpha)d\alpha + 2\lambda\int_{t-h}^{t} x^T(t)PA_d\dot{x}(\alpha)d\alpha\\ &\quad + 2\lambda h\int_{t-h}^{t} \eta^T A_dA_d\dot{x}(\alpha)d\alpha + (\lambda+1)^2\int_{t-h}^{t} \xi^T\zeta R^{-1}\zeta^T\xi d\alpha.\end{aligned}$$

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Note that

$$2\lambda h\int_{t-h}^{t}\eta^T A_d A_d\dot{x}(\alpha)d\alpha = 2h\eta^T A_d(A+(1+\lambda)A_d)x(t)+2h\eta^T A_d B_1 w(t) - 2\lambda h\eta^T A_d A_d x(t-h)+2h\eta^T F\dot{x}(t-d) - 2h\eta^T R^{-1}\eta - 2h\int_{t-h}^{t}\eta^T A_d A_d\dot{x}(\alpha)d\alpha.$$

Hence we have

$$\frac{dV(t)}{dt}+z^T(t)z(t)-\gamma^2 w^T(t)w(t)=\delta^T\Xi\delta+z^T(t)z(t) \tag{3}$$

where

$$\delta^T=\begin{bmatrix}\xi^T & w^T(t)\end{bmatrix}$$

$$\Xi=\begin{bmatrix}(1,1) & \begin{bmatrix}PB_1\\0\\hA_dB_1\\0\end{bmatrix}\\ * & -\gamma^2 I\end{bmatrix}+\Xi_4$$

$$(1,1)=\begin{bmatrix}\Xi_1 & -\lambda PA_d & \Xi_2 & PF\\ * & -S & -h\lambda A_d^T A_d^T & 0\\ * & * & -hR^{-1} & hA_dF\\ * & * & * & -U\end{bmatrix}+\Xi_3$$

$$\Xi_1=Sym\{P(A+A_d+\lambda A_d)\}+S$$

$$\Xi_2=h(A+A_d+\lambda A_d)^T A_d^T$$

$$\Xi_3=h(\lambda+1)^2\zeta R^{-1}\zeta^T$$

$$\Xi_4=\begin{bmatrix}A^T\\A_d^T\\0\\F^T\\B_1^T\end{bmatrix}U\begin{bmatrix}A & A_d & 0 & F & B_1\end{bmatrix}.$$

We now integrate equation (3) with respect to t from 0 to ∞ using $V(0)=0$, $V(\infty)\geqslant 0$ and

$$\int_0^\infty z^T(t)z(t)=\int_0^\infty\left(x^T(t)C^T+w^T(t)D_1^T\right)(Cx(t)+D_1w(t)).$$

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From the Schur complement Lemma, we can get $\dot{V} < 0$ if the following LMI holds:

$$\begin{bmatrix} (1,1) & -\lambda P A_d & (1,3) & PF & PB_1 & C^T & h(\lambda+1)P & A^T \\ * & -S & -h\lambda A_d^T A_d^T & 0 & 0 & 0 & 0 & A_d^T \\ * & * & -hR^{-1} & hA_dF & hA_dB_1 & 0 & (3,7) & 0 \\ * & * & * & -U & 0 & 0 & 0 & F^T \\ * & * & * & * & -\gamma^2 I & D_1^T & 0 & B_1^T \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -hR & 0 \\ * & * & * & * & * & * & * & -U^{-1} \end{bmatrix} < 0 \quad (4)$$

where

$$\begin{aligned} (1,1) &= Sym\{P(A + A_d + \lambda A_d)\} + S \\ (1,3) &= h(A + A_d + \lambda A_d)^T A_d^T \\ (3,7) &= h^2(\lambda+1)A_d. \end{aligned}$$

Pre- and post-multiplying both sides of equation (4) by

$$diag\{Q, \bar{S}, \bar{R}, \bar{U}, I, I, \bar{R}, I\},$$

where

$$\begin{aligned} Q &= P^{-1} \\ \bar{S} &= S^{-1} \\ \bar{U} &= U^{-1} \\ \bar{R} &= R^{-1}, \end{aligned}$$

then making a series of congruence transformations and using the Schur complement Lemma, we can get LMI (2), which completes the proof. □

Our goal now is to find a state-feedback control law

$$u(t) = Kx(t) \quad (5)$$

that will asymptotically stabilise the system and make it satisfy

$$\| T_{zw}(s) \|_\infty < \gamma.$$

Applying (5) to the system (1), we get the closed-loop system

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + A_d x_c(t-h) + F\dot{x}(t-d) + B_1 w(t) \\ z_c(t) &= C_c x_c(t) + D_1 w(t) \end{aligned} \quad (6)$$

where

$$\begin{aligned} A_c &= A + B_2 K \\ C_c &= C + D_2 K. \end{aligned} \quad (7)$$

We will now design a state-feedback H_∞ controller for system (6).

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Theorem 3.2. Given positive scalars $\gamma > 0$, $h > 0$ and $\varepsilon > 0$, and a scalar λ, there exists a state-feedback controller such that the closed-loop system (6) is asymptotically stable and satisfies

$$\| T_{zw} \|_\infty < \gamma$$

if there exist symmetric and positive-definite matrices $Q > 0$, $\bar{S} > 0$, $\bar{R} > 0$, $\bar{U} > 0$ and matrices V and W such that the following LMI holds:

$$\begin{bmatrix} -\varepsilon Q & (1,2) & \begin{bmatrix} B_1 & 0 \end{bmatrix} & (1,4) & (1,5) \\ * & (2,2) & \begin{bmatrix} 0 & 0 \\ hA_dB_1 & 0 \\ 0 & 0 \end{bmatrix} & (2,4) & (2,5) \\ * & * & \begin{bmatrix} -\gamma^2 I & D_1^T \\ * & -I \end{bmatrix} & \begin{bmatrix} 0 & B_1^T & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & CV + D_2W \end{bmatrix} \\ * & * & * & (4,4) & \begin{bmatrix} 0 \\ AV + B_2W \\ V \end{bmatrix} \\ * & * & * & * & -V - V^T \end{bmatrix} < 0 \qquad (8)$$

where

$$\begin{aligned}
(1,2) &= \begin{bmatrix} -\lambda A_d \bar{S} & 0 & F\bar{U} \end{bmatrix} \\
(1,4) &= \begin{bmatrix} h(\lambda+1)\bar{R} & 0 & 0 \end{bmatrix} \\
(1,5) &= Q + \left(A + A_d + \lambda A_d + \frac{1}{2}\varepsilon I\right) V + B_2 W \\
(2,2) &= \begin{bmatrix} -\bar{S} & -h\lambda \bar{S} A_d^T A_d^T & 0 \\ * & -h\bar{R} & hA_dF\bar{U} \\ * & * & -\bar{U} \end{bmatrix} \\
(2,4) &= \begin{bmatrix} 0 & \bar{S}A_d^T & 0 \\ h^2(\lambda+1)A_d\bar{R} & 0 & 0 \\ 0 & \bar{U}F^T & 0 \end{bmatrix} \\
(2,5) &= \begin{bmatrix} 0 \\ hA_d(A + (1+\lambda)A_d)V + hA_dB_2W \\ 0 \end{bmatrix} \\
(4,4) &= \begin{bmatrix} -h\bar{R} & 0 & 0 \\ * & -\bar{U} & 0 \\ * & * & -\bar{S} \end{bmatrix}
\end{aligned}$$

If this condition holds, the desired state-feedback controller is given by

$$u(t) = WV^{-1}x(t).$$

Proof. Applying Theorem 3.1 to the closed-loop system (6), we can get the LMI (2), where A is substituted by A_c and C is substituted by C_c. We then substitute (7) into the

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LMI (2) and note that the LMI (2) can be rewritten to give the following LMI:

$$\Pi + ab^T + ba^T < 0 \tag{9}$$

where

$$\Pi = \begin{bmatrix} -\varepsilon Q & (1,2) & \begin{bmatrix} B_1 & 0 \end{bmatrix} & \begin{bmatrix} h(\lambda+1)\bar{R} & 0 & 0 \end{bmatrix} \\ * & (2,2) & \begin{bmatrix} 0 & 0 \\ hA_dB_1 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & \bar{S}A_d^T & 0 \\ h^2(\lambda+1)A_d\bar{R} & 0 & 0 \\ 0 & \bar{U}F^T & 0 \end{bmatrix} \\ * & * & \begin{bmatrix} -\gamma^2 I & D_1^T \\ * & -I \end{bmatrix} & \begin{bmatrix} 0 & B_1^T & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ * & * & * & \begin{bmatrix} -h\bar{R} & 0 & 0 \\ * & -\bar{U} & 0 \\ * & * & -\bar{S} \end{bmatrix} \end{bmatrix}$$

$$(1,2) = \begin{bmatrix} -\lambda A_d\bar{S} & 0 & F\bar{U} \end{bmatrix}$$

$$(2,2) = \begin{bmatrix} -\bar{S} & -h\lambda\bar{S}A_d^TA_d^T & 0 \\ * & -h\bar{R} & hA_dF\bar{U} \\ * & * & -\bar{U} \end{bmatrix}$$

$$a = \begin{bmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$b = \begin{bmatrix} b_1 & 0 & b_3 & 0 & 0 & b_6 & 0 & b_8 & I \end{bmatrix}^T$$

$$b_1 = (A + B_2K + (1+\lambda)A_d)^T + \frac{1}{2}\varepsilon I$$

$$b_3 = h(A + B_2K + (1+\lambda)A_d)^TA_d^T$$

$$b_6 = C^T + K^TD_2^T$$

$$b_8 = A^T + K^TB_2^T.$$

It is obvious that there exists a positive scalar ε such that $\Pi < 0$ holds.

We define $W = KV$. From Lemma 2.1, we can get that equations (9) and $\Pi < 0$ are equivalent to equation (8), which completes the proof. □

Remark 3.3. If we compare our results with the descriptor system method applied in Fridman and Shaked (2002), we can see that the products of the system matrices and Lyapunov matrices are completely separated in Fridman and Shaked (2002, Theorem 2.1), but the controller matrix is coupled with a symmetric and positive-definite matrix Q_1 in Fridman and Shaked (2002, Theorem 3.1). Therefore, the products of the controller and Lyapunov matrices are completely separated in the performance analysis for the method in Fridman and Shaked (2002), but not in the controller synthesis. However, this weakness is eliminated with our method, and thus it is expected to give less conservative results.

The LMIs in our Theorem 3.2 are affine in the system matrices, which means it can also be applied to the case where these matrices are uncertain but known to reside within

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a given polytope. So we now consider the system (1) and denote

$$\Omega = \begin{bmatrix} A & A_d & F & B_1 & B_2 \\ C & D_1 & D_2 & & \end{bmatrix}$$

where we assume that

$$\Omega \in C_o\{\Omega_i, i = 1, \cdots, N\},$$

where the N vertices of the polytope are described by

$$\Omega_i = \begin{bmatrix} A_i & A_{di} & F_i & B_{1i} & B_{2i} \\ C_i & D_{1i} & D_{2i} & & \end{bmatrix}$$

We can then obtain the following corollary.

Corollary 3.4. Consider the system (1), where the system matrices reside within the polytope Ω. Given positive scalars $\gamma > 0$ and $\varepsilon > 0$, and a scalar λ, there exists a state-feedback controller such that the closed-loop system (6) is asymptotically stable and satisfies

$$\| T_{zw} \|_\infty < \gamma$$

for all the matrices in Ω if there exist symmetric and positive-definite matrices $Q_i > 0$, $\bar{S}_i > 0$ and $\bar{R}_i > 0$, and matrices V and U that satisfy LMIs (8) for $i = 1, \ldots, N$, where the matrices

$$A_i, A_{di}, B_{1i}, F_i, B_{2i}, C_i, D_{1i}, D_{2i}, Q_i, \bar{S}_i, \bar{R}_i$$

are taken with the index i. The state-feedback gain is then given by $K = UV^{-1}$.

4. Example

Consider the system (1) with the following parameters:

$$A = \begin{bmatrix} 0 & e \\ f & 0.8 \end{bmatrix}$$

$$A_d = \begin{bmatrix} 0.1 & 0.02 \\ 0.03 & 0.1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.3 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D_1 = 0$$

$$D_2 = 1$$

$$\rho = 0$$

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where

$$0 \leqslant e \leqslant 0.1$$
$$-0.5 \leqslant f \leqslant -0.4.$$

Hence, the system can be described as a polytope with four vertices. Our goal is to find a state-feedback gain K minimising the worst case H_∞ cost for all possible values of the parameters e and f.

From Fridman and Shaked (2002, Theorem 3.1), for $h = 0.5$, a minimum value of $\gamma = 2.0357$ is obtained with a corresponding state-feedback gain of

$$K = \begin{bmatrix} 8.8175 & -10.2655 \end{bmatrix} \times 10^4.$$

For $\gamma = 3$, the gain of

$$K = \begin{bmatrix} 7.9316 & -9.2434 \end{bmatrix} \times 10^3$$

is obtained. Applying Theorem 3.2 in the current paper, we obtain, for the same h and for $\lambda = -0.1$, $\varepsilon = 150$, a minimum γ of 1.9438 with corresponding state-feedback gain of

$$K = \begin{bmatrix} 502.3887 & -599.8634 \end{bmatrix}$$

and for $\gamma = 3$, the gain of

$$K = \begin{bmatrix} 156.5060 & -171.2643 \end{bmatrix}$$

is obtained.

It can be seen that the method in the current paper achieves an improvement in the H_∞ upper bounds and much lower gain than produced by the method in Fridman and Shaked (2002), so the method in the current paper is less conservative.

5. Conclusions

We have proposed a new delay-dependent H_∞ performance criterion based on an LMI approach for a class of neutral time-delay systems with parameter-dependent Lyapunov variables. Our method possesses a separated structure, which is then used to synthesise the controller for polytopic uncertain systems. A numerical example shows that the proposed method provides an improvement by reducing the conservativeness for time-delay systems with polytopic uncertainty.

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