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FAT-TAIL DISTRIBUTIONS AND BUSINESS-CYCLE MODELS

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Recent empirical findings suggest that macroeconomic variables are seldom normally distributed. For example, the distributions of aggregate output growth-rate time series of many OECD countries are well approximated by symmetric exponential-power (EP) densities with Laplace fat tails. In this work, we assess whether real business cycle (RBC) and standard medium-scale New Keynesian (NK) models are able to replicate this statistical regularity. We simulate both models, drawing Gaussian- vs Laplace-distributed shocks, and we explore the statistical properties of simulated time series. Our results cast doubts on whether RBC and NK models are able to provide a satisfactory representation of the transmission mechanisms linking exogenous shocks to macroeconomic dynamics.

Keywords: Growth-Rate Distributions, Normality, Fat Tails, Time Series, Exponential-Power Distributions, Laplace Distributions, DSGE Models, RBC Models

1. INTRODUCTION

Gaussian assumptions are extensively employed in theoretical and applied macroeconomics. An increasing number of recent studies, however, have started to question this practice. For example, Christiano (2007) criticizes using normal likelihood functions. He shows that the distributions of the residuals of standard VAR analyses display marked excess kurtosis, concluding that "the evidence against the normality assumption is substantial" [Christiano (2007, p. 145)].

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TABLE 1. U.S. growth-rate time series: Summary statistics and estimated EP parameters

	Moments			Normality tests			EP parameters			
Series	Std. dev.	Skewness	Kurtosis	JB	Lill	AD	$\widehat{b}$	S.E.	$\widehat{a}$	S.E.
Y	0.010	−0.083	4.240	16.376**	0.071**	1.923**	1.050	0.007	0.008	0.124
C	0.008	−0.425	8.337	305.418**	0.095**	3.037**	1.200	0.042	0.010	0.146
I	0.053	−0.389	4.897	43.986**	0.081**	1.868**	1.090	0.006	0.008	0.131
E	0.007	−0.403	3.903	15.322**	0.073**	1.852**	1.360	0.006	0.004	0.171
W	0.008	0.751	4.060	35.359**	0.064*	1.813**	1.510	0.007	0.004	0.197
P	0.008	−0.572	10.088	539.075**	0.102**	4.724**	0.954	0.005	−0.000	0.111

Notes: Y, real GDP; C, consumption; I, investment; E, employment; P, inflation; W, real wage; JB, Jarque–Bera test; Lill, Lilliefors test; AD, Anderson–Darling test; S.E., standard error. **Significant at 5% level; *Significant at 1% level.

Similarly, Mishkin (2011) warns against the use of Gaussian shocks in quantitative studies of optimal monetary policy, arguing that in reality "the distribution of shocks hitting the economy is more complex" and "may exhibit excess kurtosis, that is, tail risk" (p. 24). Empirical findings strongly support Mishkin's argument. For instance, growth rates of macroeconomic variables are seldom normally distributed, no matter whether they are observed over a cross section or along a time series. More specifically, Fagiolo et al. (2008) show that, in the majority of OECD countries, the distribution of within-country time-series output growth rates—properly purified of autocorrelation, outliers, and heteroskedasticity—is well approximated by symmetric exponential-power (EP) densities with Laplace tails, persistently fatter than Gaussian ones. In other words, extreme output changes typically occur more frequently than implied by a normal distribution.[1]

Non-normality and fat tails do not characterize only output growth rates. In Table 1, we summarize the statistical properties of growth-rate distributions for some important macroeconomic U.S. time series such as real GDP (Y), consumption (C), investment (I), employment (E), inflation (P), and real wage (W). We report their second, third, and fourth moments, together with a battery of normality tests for the null hypothesis that data are normally distributed. Both the large kurtosis figures and normality tests suggest that growth-rate distributions of U.S. time series are not normal and exhibit fat tails. Note also that skewness is quite mild in all series.

In this work, we ask whether existing standard macroeconomic models are able to replicate this statistical regularity. In particular, we focus on two standard workhorse models: the real business cycle (RBC) model and the standard medium-scale New Keynesian (NK) model [e.g., Smets and Wouters (2003); Christiano et al. (2005); Schmitt-Grohé and Uribe (2006)]. There exist two basic sources allowing fat-tail emergence in RBC or NK models. First, a given model can

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endogenously generate fat tails because of its structural assumptions, even if it is hit by purely Gaussian uncorrelated shocks. In this case, fat-tail distributed time series arise via the *endogenous* transmission mechanism embodied in the model. Second, fat tails could just be the result of *exogenous* stochastic disturbances, whose distribution is non-Gaussian. Obviously, such an approach would make it necessary to find appropriate distributions for the exogenous shocks. Following Fagiolo et al. (2008), we suggest in the following that EP densities with Laplace tails are a very good candidate. This is justified by the observation that Laplace distributions have been shown to proxy the properties of U.S. output growth-rate time series distributions well. Furthermore, as we discuss in the following, growth-rate distributions of other U.S. macroeconomic variables are close to Laplacian.

This paper explores both sources of fat-tail emergence in macroeconomics by simulating RBC and NK models, using shocks drawn from either a Gaussian or an EP/Laplace distribution. Note that, because the model has to be nonlinear to endogenously produce non-Gaussian distributions from Gaussian shocks, we shall simulate both RBC and NK models using a second-order approximation.

We try to answer a simple research question: can the two workhorse macroeconomic models replicate the statistical regularity regarding higher moments of growth-rate macroeconomic time-series distributions? In particular, can such models generate fat tails? Our results suggest a *negative* answer. We show that, in the RBC case, simulated time-series distributions simply mirror those of the shocks hitting the model. This is because the RBC model lacks an internal shock propagation mechanism [see Cogley and Nason (1995)]. In other words, it does not add structure to the nature of the shocks, thus acting like a neutral filter. This implies that the RBC model is able to replicate fat tails *exogenously*, but not *endogenously*. The NK model, instead, never generates fat-tail growth-rate distributions, either endogenously or exogenously. What is more, the NK structure points in the wrong direction: even if the model is hit by fat-tail shocks, it delivers quasi-normal growth-rate distributions for simulated macroeconomic time series.

The idea that the economy can be hit by positive-probability big shocks is not new in macroeconomics [see, e.g., Rietz (1988); Barro (2006)]. The literature on fat tails and macroeconomic modeling, however, is still in its infancy, even if it may gain more attention as a consequence of the global financial crisis. Of course, there may be alternative sources of non-normality in the data that we do not investigate in this work. Non-normality in DSGE models may arise from stochastic volatility, as in Fernández-Villaverde and Rubio-Ramírez (2007) and Justiniano and Primiceri (2008), who introduce low-frequency movements in volatility to explain the source of the Great Moderation period. Moreover, Posch (2009) shows that non-normality can also result from the presence of jumps in aggregate productivity in a standard neoclassical growth model.

The rest of the paper is organized as follows. Section 2 discusses the methodology that we employ in our exercises. Results are presented in Section 3. Finally, Section 4 concludes.

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2. METHODOLOGY

We begin by comparing growth-rate distributions of quarterly U.S. time series of real GDP (Y), consumption (C), investment (I), employment (E), inflation (P), and real wage (W) with simulated distributions for the same variables, as generated by RBC and NK models under different assumptions about the underlying shock distributions. Observed U.S. series range from 1948Q1 to 2010Q4 (251 observations) and are drawn from the St. Louis Federal Reserve Economic Data (FRED) database.[2] We take second-order approximations[3] of RBC and NK models and we perform a Monte Carlo analysis, generating 1,000 series of 1,000 observations for each macroeconomic variable.

We use the most standard RBC and NK models. The former is a plain-vanilla RBC model: the basic textbook neoclassical growth model with a technology shock.[4] The NK model is a version of the standard medium-scale model used in various papers [cf., for example, Smets and Wouters (2003); Christiano et al. (2005); Schmitt-Grohé and Uribe (2006)]. It features both real and nominal frictions. The real frictions are monopolistic competition, habit persistence in consumption, and fixed cost in an otherwise standard Cobb–Douglas production function that generates increasing returns to scale and guarantees zero profit at equilibrium, variable capacity utilization, and adjustment costs in investment. The nominal frictions are a cash-in-advance constraint on wage payments of firms, sticky wages and prices à la Calvo and Yun, and past inflation indexation of nonreset prices. Christiano et al. (2005) argue that all these frictions appear to be crucial in replicating the dynamics of macroeconomic variables over the business cycle. Monetary policy is described by a standard Taylor rule. Shocks and calibration are as in Smets and Wouters (2003).

We play with two scenarios as far as shocks are concerned. In the first scenario, we draw i.i.d. shocks from a standard Gaussian distribution. In the second, shocks are i.i.d. and Laplace distributed, with zero mean and unit standard deviation.[5] As mentioned previously, Laplace distributions are good first-pass proxies for empirically observed growth-rate time series in the United States and in OECD countries. More precisely, Fagiolo et al. (2008) find that the EP density is the family that best fits the data vis-à-vis other fat-tailed distributions such as the Lévy stable and the Student t.

In our analysis, we compute growth rates $g(t)$ for any variable as the first difference of the log of any series $X(t)$. Given the time interval $T_n = \{t_1, \ldots, t_n\}$, over which we observe our data, we define the time-series distribution of growth rates as

$$G_{T_n} = \{g(t), t \in T_n\}. \tag{1}$$

To compare simulated and observed time-series distributions, we employ a parametric approach. More precisely, following Fagiolo et al. (2008, 2009), we fit growth-rate distributions with exponential-power (EP) densities [for details see Agrò (1995); Bottazzi and Secchi (2003)]. The EP probability density function

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reads as follows:

$$f(x;b,a,m)=\frac{1}{2ab^{\frac{1}{b}}\Gamma\left(1+\frac{1}{b}\right)}e^{-\frac{1}{b}\left|\frac{x-m}{a}\right|^{b}}, \tag{2}$$

where $a>0$, $b>0$, and $\Gamma(\cdot)$ is the Gamma function.

The EP is a generalization of a Gaussian random variable, and it is fully characterized by three parameters: a *location* parameter m, a *scale* parameter a, and a *shape* parameter b. The location parameter controls for the mean of the distribution, whereas the scale parameter is proportional to the absolute deviation. The parameter b determines the fatness of the tails: the larger b, the thinner the tails. In particular, if $b=2$, the EP distribution reduces to a Gaussian, whereas if $b=1$, one recovers a Laplace (with unit standard deviation if $a=1/\sqrt{2}$).

The EP distribution allows one to precisely measure how far the empirical distribution is from the normal and Laplace benchmarks. Note also that the EP density is characterized by exponentially shaped tails, and thus it has finite moments of any order. This is important, as in many financial applications one typically deals with heavy-tailed distributions whose higher moments (and sometimes even whose mean) do not converge [Embrechts et al. (1997)]. Macroeconomic growth-rate time-series distributions, conversely, almost always possess finite moments. Therefore, we do not expect, at least in principle, RBC and NK macroeconomic models to behave badly with such smooth shock distributions.

We fit empirical and simulated distributions with the EP density in equation (2) by jointly estimating the three parameters via maximum likelihood (ML).[6] Next, we compare EP density parameter estimates obtained from the data with the mean of the ones obtained by fitting the EP to each Monte Carlo simulated time series. To do so, we calibrate the RBC and NK as in the standard literature [see, e.g., Smets and Wouters (2003)], properly adjusting the standard deviations of simulated shocks in either set of scenarios. Furthermore, before carrying out ML estimation, we subtract the mean from all time series. Because we are mainly interested in the tail parameter b, this does not affect our estimates, but improves overall estimation efficiency. Note also that Monte Carlo parameter-estimate distributions are unimodal and quite symmetric. Therefore, the mean is a good proxy for the aggregate behavior of our models. No dramatic difference is detected if instead of taking the mean of estimates, one simply pools all zero-mean time-series together and estimates a unique b parameter by fitting the EP to the pooled sample.

3. RESULTS

Table 1 suggests that empirical growth-rate distributions of U.S. series are characterized by fat tails. This is confirmed by our EP fits. In line with previous empirical evidence [Fagiolo et al. (2008, 2009)], the GDP growth-rate distribution is well approximated by a Laplace density (cf. Table 1). Moreover, growth-rate distributions of all other U.S. time series are markedly non-Gaussian: estimates of

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TABLE 2. Simulated growth-rate distributions in the real business cycle model (second-order approximation): Summary statistics and estimated EP parameters

		Moments			Normality tests			EP parameters			
Series	Shocks	Std. dev.	Skewness	Kurtosis	JB	Lill	AD	$\widehat{b}$	S.E.	$\widehat{a}$	S.E.
Y	Gauss	0.018	−0.007	2.996	0.046	0.061	0.057	2.012	0.142	0.018	0.0006
C	Gauss	0.004	0.028	2.995	0.057	0.051	0.058	2.018	0.142	0.004	0.0001
I	Gauss	0.058	−0.025	3.097	0.137	0.058	0.069	1.934	0.134	0.057	0.0020
E	Gauss	0.014	−0.006	2.997	0.050	0.051	0.048	2.010	0.141	0.014	0.0005
Y	Lapl.	0.018	−0.022	5.780	1	1	0.994	1.059	0.063	0.013	0.0005
C	Lapl.	0.004	0.047	5.454	1	1	0.995	1.158	0.070	0.003	0.0001
I	Lapl.	0.058	−0.202	6.038	1	1	0.988	1.078	0.064	0.043	0.0017
E	Lapl.	0.014	−0.031	5.582	1	1	0.996	1.118	0.067	0.010	0.0004

Notes: Normality tests report the percentage of series for which the normality hypothesis cannot be accepted at a 5% significance level. Y, real GDP; C, consumption; I, investment; E, employment; P, inflation; W, real wage; JB, Jarque–Bera test; Lill, Lilliefors test; AD, Anderson–Darling test; S.E., standard error.

the shape parameter ($\hat{b}$) range from a maximum of 1.51 for real wage to a minimum of 0.95 for inflation. These results indicate that the presence of quasi-Laplacian fat tails in growth-rate distributions is a pervasive statistical regularity for U.S. macroeconomic time series.

Fernández-Villaverde and Rubio-Ramírez (2007) and Justiniano and Primiceri (2008) show that stochastic volatility can be the source of the switch from the Great Inflation to the Great Moderation period. Thus, one may think that stochastic volatility may be as well the source of the fat tails observed in the EP density we estimated. However, we did a simple exercise that provides some evidence against this conjecture. We estimated output growth-rate distribution in subsamples such as the Great Inflation (1960–1983) and the Great Moderation (1984–2007), in which there should not be breaks in fundamentals or changes in the shock volatilities. We find that the estimated output growth-rate distributions still exhibit fat tails, as described by EP densities, and are far from the Gaussian benchmark.[7]

We now turn to assessing whether RBC and NK models are able to account for this empirical evidence.

3.1. The RBC Model

Our main results for the RBC model are summarized in Table 2. To begin, note that the second and third moments of RBC simulated time-series distributions do not change substantially if shocks are drawn from a Gaussian or Laplace distribution.[8] This is not the case for the fourth moment: in the presence of Laplace shocks, growth-rate series strongly exhibit excess kurtosis, whereas Gaussian shocks imply fourth moments that lie very close to 3, i.e., the normal benchmark. This result is confirmed by Jarque–Bera, Lilliefors, and Anderson–Darling normality

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TABLE 3. Simulated growth-rate distributions in the New Keynesian model (second-order approximation): Summary statistics and estimated EP parameters

		Moments			Normality tests			EP parameters			
Series	Shocks	Std. dev.	Skewness	Kurtosis	JB	Lill	AD	$\widehat{b}$	S.E.	$\widehat{a}$	S.E.
Y	Gauss	0.004	−0.028	2.998	0.076	0.064	0.069	2.014	0.142	0.004	0.0002
C	Gauss	0.002	0.004	2.996	0.078	0.067	0.075	2.014	0.142	0.002	0.0001
I	Gauss	0.004	−0.041	2.977	0.184	0.115	0.168	2.042	0.144	0.004	0.0002
E	Gauss	0.009	−0.002	2.997	0.054	0.052	0.049	2.013	0.142	0.009	0.0003
W	Gauss	0.002	−0.026	2.994	0.092	0.060	0.074	2.016	0.142	0.002	0.0001
P	Gauss	0.001	−0.023	3.009	0.069	0.053	0.064	2.002	0.141	0.001	0.0001
Y	Lapl.	0.004	−0.013	3.743	0.888	0.395	0.655	1.625	0.107	0.004	0.0001
C	Lapl.	0.002	0.016	4.388	0.999	0.925	0.991	1.424	0.091	0.001	0.0001
I	Lapl.	0.004	−0.047	3.196	0.382	0.193	0.271	1.878	0.129	0.004	0.0002
E	Lapl.	0.009	0.008	4.990	1	0.999	0.996	1.266	0.078	0.008	0.0003
W	Lapl.	0.002	−0.028	3.257	0.377	0.127	0.200	1.837	0.126	0.002	0.0001
P	Lapl.	0.001	−0.052	4.053	0.983	0.736	0.919	1.517	0.098	0.001	0.0001

Notes: Normality tests report the percentage of series for which the normality hypothesis cannot be accepted at a 5% significance level. Y, real GDP; C, consumption; I, investment; E, employment; P, inflation; W, real wage; JB, Jarque–Bera test; Lill, Lilliefors test; AD, Anderson–Darling test; S.E., standard error.

tests. In the presence of Gaussian shocks, the null hypothesis of normality is not rejected at the 5% significance level in at least 86% of the series generated by the model. In contrast, with Laplace shocks, the null hypothesis of normality is always rejected.

Fitting EP densities to the series reveals that the assumptions concerning the nature of shocks have a very strong impact on the growth-rate distributions of the series generated by the RBC model (see Table 2). If shocks are Gaussian, estimates of the shape parameter $\hat{b}$ oscillate around 2 for all the simulated time series, hinting at Gaussian tails. On the other hand, if shocks are Laplacian, the $\hat{b}$'s are slightly larger than 1, the hallmark of Laplacian tails. These findings, well in line with the results in Cogley and Nason (1995), suggest that the propagation mechanism is almost neutral in RBC models: distributional properties of growth-rate time series completely reflect those of the underlying shocks [see also Rotemberg and Woodford (1996)].

3.2. The NK Model

The first interesting moments of simulated time series generated by the NK model are reported in Table 3. As for the RBC model, second and third moments—as well as relative standard deviation, autocorrelations, and cross correlations with output, not reported here—are not dramatically affected by shock distributions. Conversely, kurtosis changes when we move from Gaussian to Laplace shocks.

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However, changes in kurtosis are less pronounced in the NK than in the RBC model. In the presence of Gaussian shocks, time-series distributions still exhibit quasi-Gaussian kurtosis, whereas if shocks are Laplace distributed, kurtosis ranges from 3.2 (investment) to 5 (employment). As it happens in the RBC model, normality-test outcomes are also influenced by the distribution of the shocks. However, the percentage of cases for which normality is rejected is lower than in the RBC case.

All that is reflected in the estimation of the $\hat{b}$'s; see Table 3. In the presence of Gaussian shocks, growth-rate time-series are normally distributed, as $\hat{b}$'s lie very close to 2. This is in line with the preceding RBC results. If shocks become Laplace distributed, simulated growth-rate distributions display fatter tails. However, in contrast with the RBC model, tails are still far from the Laplace benchmark. The minimum estimated $\hat{b}$ is 1.27 for employment, whereas $\hat{b}$ is very close to 2 for investment and real wage.

The NK model is therefore unable to reproduce the statistical regularity concerning fat-tailed growth-rate time-series distributions, even in the presence of Laplace-distributed (fat-tail) shocks. Rather than inducing stronger nonlinearities, it seems that the ensemble of real and nominal rigidities, added in the NK model upon the bare-bones structure of the RBC model, tends to smooth time series of exogenous, fat-tail Laplacian shocks.

To shed some light on the latter conjecture, we simulate two variations of the NK model. In the first, we jettison from the NK model habit persistence in consumption, investment adjustment costs, sticky wages, and past indexation of prices and wages. The resulting model is very close to a plain-vanilla three-equation NK model with capital. In the second case, we allow perfect price and wage flexibility, thus obtaining a kind of RBC model with real frictions. Simulation results show that when both models are fed with Laplace shocks, the distributions of the generated time series become more fat-tailed than in the case of a complete medium-scale New Keynesian model, even if they are still quite far from the Laplace benchmark (see Table 4).

The intuition for this result rests on the role of adjustment cost frictions.[9] To generate kurtosis in simulated economic-model time series, one needs features that amplify the effects of the shocks. Note that both real and nominal frictions are in essence adjustment costs that do the opposite: they smooth out the effects of shocks. The standard RBC model does not have any adjustment costs; hence fat-tailed shocks are simply transferred to fat-tailed simulated time series. The medium-scale NK model, instead, exhibits many real and nominal adjustment cost features, so fat tails are filtered out. The RBC model with real frictions and the stripped-down NK model with capital both have fewer adjustment costs than the medium-scale NK model, but more than the standard RBC model. This explains why the results for these two "middle way" models fall somewhat between our two extreme models. Moreover, the foregoing results suggest that higher price flexibility is not always desirable, as it can magnify the impact of shocks.[10]

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TABLE 4. Simulated growth-rate distributions in different New Keynesian models with Laplace shocks (second-order approximation): Estimated EP parameters

	EP parameters			
Series	$\widehat{b}$	S.E.	$\widehat{a}$	S.E.
	Plain vanilla			
Y	1.432	0.099	0.017	0.0007
C	1.537	0.109	0.004	0.0001
I	1.408	0.097	0.062	0.0024
E	1.522	0.111	0.025	0.0009
W	1.581	0.116	0.024	0.0009
P	1.613	0.117	0.004	0.0001
	Fully flexible price			
Y	1.418	0.093	0.007	0.0003
C	1.358	0.085	0.003	0.0001
I	1.482	0.104	0.012	0.0006
E	1.510	0.106	0.005	0.0002
W	1.254	0.074	0.008	0.0003
P	1.457	0.107	0.025	0.0010

Notes: Y, real GDP; C, consumption; I, investment; E, employment; P, inflation; W, real wage; S.E., standard error.

3.3. Robustness Analysis

The foregoing results are robust to a number of additional tweaks. First, we modify the approximation method employed in the models, switching from second-order linear approximations to first-order ones. We find that our results do not substantially change. Moreover, we test whether the *endogenous* transmission mechanisms embodied in both RBC and NK models are sufficient to generate fat-tailed distributed time series in the case of *fourth-order* approximation, as kurtosis may affect the solutions of the models.[11] We find that in the presence of Gaussian shocks, growth-rate distributions of macroeconomic variables produced by both models are still normal, even when fourth-order approximation is taken into account.

Second, we squeeze the econometric length of Monte Carlo samples to that of empirical time series (251 observations). In this case, also, the robustness of our results is confirmed.

Finally, we compute growth rates as $g(t) = \Delta X(t)/X(t-1) - 1$, to check whether the log-difference approximations could have affected our results. We find again that the main results of our study are not altered.

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4. CONCLUDING REMARKS

Observed growth-rate distributions of U.S. macroeconomic time series (output, consumption, investment, inflation, employment, and real wage) strongly depart from the Gaussian benchmark and are well proxied by EP densities with Laplacian tails.

This paper asks whether standard workhorse macroeconomic models of the business cycle, such as the basic RBC and NK models, are able to replicate this statistical regularity. The answer is far from encouraging.

We find that neither the RBC nor the NK model can generate fat-tailed time-series distributions if they are hit by i.i.d. Gaussian shocks. This implies that *endogenous* transmission mechanisms embedded in both RBC and NK models are not able to reproduce fat tails from normal disturbances. Conversely, when the RBC model is subject to i.i.d. Laplace-distributed shocks, it tends to generate Laplace-distributed growth-rate distributions for simulated time series. This results from quasi-linearity of the RBC model and from neutrality of its endogenous transmission mechanisms to shocks. In other words, the growth-rate distributions of the main series generated by the model have the same shape as the shock distributions that are fed into the model. In contrast, the NK structure points in the wrong direction. When shocks are Laplacian, the NK model tends to deliver growth-rate distributions whose tails are slightly fatter than Gaussian ones. This suggests that the endogenous transmission mechanism of the NK model induced by the several real and nominal rigidities hardwired into the model is actually smoothing extremal events generated by the fat tails of Laplace disturbances.

Ideally, one would like a model that can endogenously reproduce fat tails from Gaussian shocks, thanks to transmission mechanisms built into the model. This might allow a better understanding of how alternative mechanisms lead to the endogenous emergence of high-probability large events. Our results seem to suggest that the two workhorse models employed in current macroeconomic business-cycle analysis are not able to fulfill this expectations. Therefore, further research in this field is required to bridge business-cycle models more firmly to empirical statistical regularities concerning growth-rate time-series distributions of macroeconomic variables. For instance, one should study whether the new generation of DSGE models [see Gertler and Kiyotaki (2010) for a survey], which embody some features of credit and financial market dynamics, possess the appropriate *endogenous* transmission mechanisms to transform Gaussian shocks into fat-tailed distributed macroeconomic time series. Another possible route is to relax the rational-expectation assumption as in De Grauwe (2012).

NOTES

1. Interestingly, fat-tail output growth-rate distributions emerge also for cross-section data, at the levels of countries [Canning et al. (1998); Lee et al. (1998); Castaldi and Dosi (2009)], industries [Castaldi and Sapio (2008)], and firms [Bottazzi and Secchi (2003); Fu et al. (2005)].

2. Freely available online at http://research.stlouisfed.org/fred2/.

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3. Our choice is in line with the literature on RBC and NK models [see, e.g., the very recent Fernandez-Villaverde et al. (2012)]. In the robustness section we control whether our results are sensitive to the approximation order employed.

4. To be more precise, we use the model in example 1 of the DYNARE examples, as described in "Stochastic Simulations with DYNARE: A Practical Guide" (www.dynare.org/documentation-and-support/tutorial/guide.pdf/.../file), with only the technology shocks.

5. Whenever shocks follow an AR(1) process, we assume that the innovation terms are Laplace distributed.

6. Data, codes, and our full results are available from the authors upon request.

7. The same result emerges when we consider the period from 1919 to 1952. Note that given the short length of the subsamples, we had to switch to a monthly series, that is, industrial production. Given the strong correlations between GDP and industrial production, the latter is a good proxy of output dynamics [see, e.g., Fagiolo et al. (2008)].

8. Relative standard deviations, autocorrelations, and cross correlations with output are not affected by the source of the shocks either.

9. We thank an anonymous referee for suggesting this interpretation to us.

10. Although this is expected for real frictions, it may seem surprising for nominal frictions, which may intuitively appear to amplify the effects of shocks on real variables. However, this is not always the case in microfounded models. For example, Ascari et al. (2012) show that price stickiness reduces the variability of output in response to supply shocks. See also Greenwald and Stiglitz (1993).

11. We perform fourth-order approximation of both RBC and NK models employing Dynare++, developed by Ondra Kamenik. More on that at http://www.dynare.org/documentation-and-support/dynarepp.

REFERENCES

Agrò, G. (1995) Maximum likelihood estimation for the exponential power function parameters. *Communications in Statistics* 24, 523–536.

Ascari, G., A. Flamini, and L. Rossi (2012) Nominal Rigidities, Supply Shocks and Economic Stability. DEM working paper 24, Department of Economics and Management, University of Pavia.

Barro, R.J. (2006) Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics* 1213, 823–866.

Bottazzi, G. and A. Secchi (2003) Common properties and sectoral specificities in the dynamics of U.S. manufacturing firms. *Review of Industrial Organization* 23, 217–232.

Canning, D., L.A.N. Amaral, Y. Lee, M. Meyer, and H.E. Stanley (1998) Scaling the volatility of GDP growth rates. *Economic Letters* 60, 335–341.

Castaldi, C. and G. Dosi (2009) The patterns of output growth of firms and countries: Scale invariances and scale specificities. *Empirical Economics* 373, 475–495.

Castaldi, C. and S. Sapio (2008) Growing like mushrooms? Sectoral evidence from four large European economies. *Journal of Evolutionary Economics* 18, 509–527.

Christiano, L.J. (2007) Comment on "On the Fit of New Keynesian Models" by Del Negro, Schorfheide, Smets and Wouters. *Journal of Business and Economic Statistics* 252, 143–151.

Christiano, L.J., M. Eichenbaum, and C.L. Evans (2005) Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113, 1–45.

Cogley, T. and J.M. Nason (1995) Output dynamics in real-business-cycle models. *American Economic Review* 85, 492–511.

De Grauwe, Paul (2012) Booms and busts in economic activity: A behavioral explanation. *Journal of Economic Behavior & Organization* 83(3), 484–501.

Embrechts, P., C. Küppelberg, and T. Mikosch (1997) *Modelling Extremal Events for Insurance and Finance*. Berlin: Springer.

Fagiolo, G., M. Napoletano, M. Piazza, and A. Roventini (2009) Detrending and the distributional properties of U.S. output time series. *Economics Bulletin* 29, 3155–3161.

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Fagiolo, G., M. Napoletano, and A. Roventini (2008) Are output growth-rate distributions fat-tailed? Some evidence from OECD countries. *Journal of Applied Econometrics* 23, 639–669.

Fernandez-Villaverde, P. Guerròn-Quintana, and J. Rubio-Ramìrez (2012) Estimating Dynamic Equilibrium Models with Stochastic Volatility. NBER Working Paper 18399, National Bureau of Economic Research.

Fernández-Villaverde, J. and J. F. Rubio-Ramírez (2007) Estimating macroeconomic models: A likelihood approach. *Review of Economic Studies* 744, 1059–1087.

Fu, D., F. Pammolli, S. Buldyrev, M. Riccaboni, K. Matia, K. Yamasaki, and H. Stanley (2005) The growth of business firms: Theoretical framework and empirical evidence. *Proceedings of the National Academy of Science* 102, 18,801–18,806.

Gertler, M. and N. Kiyotaki (2010) Financial intermediation and credit policy in business cycle analysis. In B.M. Friedman and M. Woodford (eds.), *Handbook of Monetary Economics*, pp. 547–599. Amsterdam: North Holland Elsevier.

Greenwald, B. and J. Stiglitz (1993) Financial market imperfections and business cycles. *Quarterly Journal of Economics* 108, 77–114.

Justiniano, A. and G. Primiceri (2008) The time-varying volatility of macroeconomic fluctuations. *American Economic Review* 983, 604–641.

Lee, Y., L.A.N. Amaral, D. Canning, M. Meyer, and H.E. Stanley (1998) Universal features in the growth dynamics of complex organizations. *Physical Review Letters* 81, 3275–3278.

Mishkin, F.S. (2011) Monetary Policy Strategy: Lessons from the Crisis. NBER working paper 16755, National Bureau of Economic Research.

Posch, O. (2009) Structural estimation of jump-diffusion processes in macroeconomics. *Journal of Econometrics* 153, 196–210.

Rietz, T.A. (1988) The equity risk premium: A solution. *Journal of Monetary Economics* 22, 1002–1037.

Rotemberg, J. and M. Woodford (1996) Real-business-cycle models and forecastable movements in output, hours, and consumption. *American Economic Review* 86, 71–89.

Schmitt-Grohé, S. and M. Uribe (2006) Optimal fiscal and monetary policy in a medium-scale macroeconomic model. In M. Gertler and K. Rogoff (eds.), *NBER Macroeconomics Annual*, pp. 383–425. Cambridge, MA: MIT Press.

Smets, F. and R. Wouters (2003) An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association* 1, 1123–1175.

https://doi.org/10.1017/S1365100513000473 Published online by Cambridge University Press