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FIELD'S *SAVING TRUTH FROM PARADOX*: SOME THINGS IT DOESN'T DO

DONALD A. MARTIN

Departments of Mathematics and Philosophy,
University of California, Los Angeles

Abstract. I will discuss Field's book mainly by comparison with Saul Kripke's *Outline of a Theory of Truth*. I will point out important properties of Kripke's "least fixed point" that are absent from Field's constructions and theory. I do this not to demean Field's superb work on truth but rather to suggest that there may be no really satisfactory conditional connective for languages containing their own truth predicates.

I will do my assigned job and be a critic, and this might lead to a wrong impression of my overall opinion. It is therefore important to say that I have a high regard for Hartry Field's work on the semantic paradoxes and for the book, *Saving Truth From Paradox*.[1] I regard his as the best work I know of in the subject since Kripke's *Outline of a Theory of Truth*.[2] I believe he is right in rejecting the kinds of accounts of truth he rejects. I think the goals that he has aimed for in developing his own account are laudable and that he has succeeded to a remarkable degree in achieving these goals. If I don't believe that he has achieved a fully satisfying account of truth and its logic, I also doubt that there is such a thing.

My remarks will be centered around the relation between Field's account and the one he (rightly, I believe) associates with Kripke's least fixed point construction. Doing so will make it possible to highlight some things Field has done and some things he has not. But there will be many important issues that I won't touch on.

The first half of Kripke's paper is devoted to an informal discussion of truth. The rest of the paper is technical. By the time the formal construction of a hierarchy of languages begins, Kripke has pretty much given—informally, piecemeal, and in a heterogeneous fashion—a set of rules a subset of which is sufficient to generate the least fixed point. He has indicated, via an example, that an existential quantification follows from any of its instances, regardless of the status of other instances. He has indicated that the truth of a sentence and the sentence itself follow from one another and that the falsity of a sentence and the negation of the sentence follow from one another. Though his discussion is not couched in terms of valid rules of implication, it is natural to take it in this way, and doing so is suggested by his sketch[3] of how to explain the word "true" to someone who does not know it.

In discussing Kripke's model construction and its significance, I will mostly use Field's terminology rather than Kripke's.

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1 Field (2008).

2 Kripke (1975).

3 Kripke (1975, p. 701).

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The construction starts with an interpreted first-order language whose quantifiers range over a set U. (The word "interpreted" is mine, not Field's.) I will take this language to consist of a first-order language and and a model for that language.[4] I will call the language $\mathcal{L}$ to bring the notation in line with the notation Field uses for his own construction. The connectives of $\mathcal{L}$ are $\neg$, $\vee$, and $\wedge$. It is assumed that $\mathcal{L}$ can express its own syntax, either directly or via coding by Gödel numbers or sets. For this to be possible, the model associated with $\mathcal{L}$ needs to be an ω-model in a natural sense that Field defines, and this is assumed. It is also assumed that every sentence A of $\mathcal{L}$ (or its code) gets a name $\langle A \rangle$ in this way.

Let $\mathcal{L}^+$ be the semi-interpreted language gotten by adding a predicate "True" to $\mathcal{L}$. Kripke's construction defines what Field calls a *quasi-extension* for "True." One possible reason for the prefix "quasi" is that there are sentences, for example, Liar sentences, such that neither they nor their negations belong to the quasi-extension. Another possible reason is Field's view that it wrong to identify membership in the quasi-extension with being true in $\mathcal{L}^+$.

The quasi-extension is defined using transfinite recursion. Field's version of the construction defines a transfinite increasing sequence $\langle X_\alpha \mid \alpha \leq \beta \rangle$ of sets of sentences of $\mathcal{L}^+$ for a certain ordinal β. Let $Y = X_\beta$. Y is the quasi-extension of "True." The members of Y are assigned *semantic value* 1. Sentences whose negations belong to Y get *semantic value* 0. The remaining sentences get *semantic value* 1/2 (or, more in accordance with Kripke, have undefined semantic value). The set Y is a *fixed point* in the following sense: Suppose we expand the model associated with $\mathcal{L}$ to a three-valued model for $\mathcal{L}^+$ by giving to True($\langle A \rangle$) the semantic value of A for each sentence A and by making True(x) have value 0 when a nonsentence is assigned to x. If we then define satisfaction using Kleene strong truth tables, each sentence gets its semantic value as its truth value. In other words, Y is mapped to itself by a certain operation $\mathcal{O}$ mapping sets of sentences of $\mathcal{L}^+$ to sets of sentences of $\mathcal{L}^+$.

The set Y is the least fixed point of the operation $\mathcal{O}$. There are other fixed points. Any fixed point would work for accomplishing everything Field says Kripke's construction accomplishes. As far as I know, Field has no preference for one or another. Much of what I will do in this paper will be to point out virtues of the least fixed point that are lacking in the fixed point Field constructs for an operation he defines for a richer language than Kripke's. The least fixed point is the only fixed point of Kripke's $\mathcal{O}$ that has all the virtues I will mention.

Assume, to avoid dealing with satisfaction, that every member u of the domain U has a name c_u in $\mathcal{L}$. Then another way to define the quasi-extension is to say that it is the set of sentences that are consequences of the set of true sentences of $\mathcal{L}$—indeed, of the true atomic sentences and negations of atomic sentences of $\mathcal{L}$—under certain of the rules of inference.[5]

[4] To interpret a first-order language could be, for example, not just to give it a model but also to give meaning to its symbols.

[5] Essentially a set of such rules can be extracted from Kripke's paper—even from the informal first half of the paper. The "essentially" is because Kripke works in terms of truth and falsity and what Field could call quasi-extensions and quasi-anti-extensions. This is more complicated in one way, but it is simpler in that it would allow the rules I list involving negation to be omitted in favor of two simple ones.

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The rules in question are the ones Field calls $\wedge$-Introd, $\neg\wedge$-Introd, $\vee$-Introd, $\neg\vee$-Introd, $\neg\neg$-Introd, T-Introd, and $\neg$T-Introd, plus

$$\frac{A(x/c_u)}{\exists x\, A}, \qquad \frac{\neg A(x/c_u)}{\neg\forall x\, A},$$

and two infinitary rules,

$$\frac{\{A(x/c_u) \mid u \in U\}}{\forall x\, A} \quad \text{and} \quad \frac{\{\neg A(x/c_u) \mid u \in U\}}{\neg\exists x\, A}.$$

When the assumption that everything in U has a name fails, we can modify our rules into implication rules for satisfaction. Often we can instead expand $\mathcal{L}$ by adding elements of U to be the needed names and do this in such a way that the added naming relation is defined by a formula of $\mathcal{L}$. Then we can let the quasi-extension be the set of those consequences of the expanded truth set that belong to the unexpanded $\mathcal{L}^+$.

The alternative definition I have given of Kripke's least fixed point is hardly something Field would have been unaware of. He would certainly point out that it becomes a definition only when one defines "consequence," and one of the ways to do this is using ordinals and transfinite recursion. The reason I bring up the alternative definition is because it will be helpful in discussing both Field's assessment of Kripkes construction and some differences between Field's own model construction and Kripke's.

One point that Field makes is that Kripke's construction cannot be carried out for unrestricted languages: it needs U to be a set. I think he is correct.[6] And the problem is with defining a quasi-extension, not just with the construction itself. I doubt that there is a satisfactory way in the nonset case to give an explicit definition or a definition that is inductive in the sense Field has in mind. I doubt that there is a really satisfactory inductive definition in any sense, but I do think there is an interesting attempt, a variant of the alternative definition.

The version of the alternative definition that I gave involved sets unnecessarily. For one thing, what we care about defining is just being a sentence with semantic value 1. It doesn't really matter whether there is a set of all the things with this property. Also we do not care whether the premises form a set. We can thus declare that a sentence of $\mathcal{L}^+$ has semantic value 1 if and only if it is a consequence of true atomic sentences and negations of atomic sentences (or satisfaction facts) of $\mathcal{L}$. Even in unrestricted cases, we can say in $\mathcal{L}$ which sentences these are. Finally, the two infinitary rules can be replaced by infinite-premise rules. Even in unrestricted cases, we can specify, essentially in $\mathcal{L}$, what the rules are. In those cases we seem to have no explicit way to say what it is to be a consequence of our premises under our rules, but perhaps we have a kind of intuitive grasp of the idea.

Another way to present the definition is as an *inductive* one, in the sense that the definition of "formula" is inductive. *Some* of the clauses are the following (in the case we have names for everything).

- Every true atomic or negation-of-atomic sentence of $\mathcal{L}$ has semantic value 1.
- If A has semantic value 1, so does True($\langle A \rangle$).
- If $A(x/c_u)$ has semantic value 1 for all $u \in U$, then $\forall x\, A$ has semantic value 1.

[6] At the end of this paper, I will say something about the possibility of giving a bivalent definition of the construction in a nonbivalent language that contains a truth predicate.

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When U is a set we can make this into an explicit set-theoretic definition—*á la* Dedekind, by transfinite recursion, or in other ways. But even in such cases a common informal way to finish off inductive definitions of this sort would be with a clause saying that nothing has semantic value 1 unless the other clauses require it to.

Consider, for example, the case when the quantifiers of $\mathcal{L}$ range over all sets and when is the $\mathcal{L}$ is the language of set theory. For either the added names version or the satisfaction version of the attempted inductive definition, all the positive clauses are expressible in $\mathcal{L}$ with a predicate "has semantic value 1" added. The last clause, "Nothing has semantic value 1 unless…," cannot be stated in the same way. Second-order quantifiers would do the job, but I doubt their legitimacy. In short, I do not think that the attempted inductive definition counts as a genuine definition. But, once again, we perhaps get a kind of grasp of the idea. We may feel that the definition gives us something conceptual.

There is another way to define the least fixed point that does not extend to the unrestricted case but nevertheless seems to provide something conceptual even in that case. This is the definition via *games*. If A is a sentence of $\mathcal{L}^+$, then there is game naturally connected with A. Play of the game proceeds by two players listing a sequence of sentences of $\mathcal{L}^+$. (I am presenting the version of the game for the case that every object has a name.) When a sentence is listed, one of the players is *responsible* for that occurrence of the sentence. Play of the game begins with Player I's listing A and being responsible for it. If a disjunction is listed, the player who is is responsible for it and must list one of the disjuncts. If a conjunction is listed, the player not responsible for it must list a conjunct. The player responsible for the conjunction is responsible for the conjunct. The cases when quantified sentences are listed are analogous to those for disjunctions and conjunctions, with one of the players listing a instance. When a negation is listed, the player not responsible for it lists the negated sentence and is responsible for it. If an atomic sentence of $\mathcal{L}$ is listed, the player responsible for it wins if it is true and loses if it is false. Note that, though play of the game may go on forever, winning happens in finite time if it happens at all. It is not hard to show that the sentence A has value 1 in the least fixed point if and only if Player I has a winning strategy and it has value 0 if and only if Player II has a winning strategy. No explicit definition of "winning strategy" can be given in $\mathcal{L}$, but perhaps speakers of $\mathcal{L}$ could have, in some weak sense, an intuitive grasp of the concept.

Field has a second reason for holding that Kripke's quasi-extension construction does not define truth. Even in the case that U is a set, Field denies that having semantic value 1 should be identified with truth. He holds that the sentences that have semantic value 1 are true and that the sentences that have semantic value 0 are false. For any one of the the remaining sentences of $\mathcal{L}^+$, he holds that we cannot say (on the basis of what we should extract from Kripke's construction) that it is true, that it is false, that it is not true, or that it is not false. I fully agree with him about this.

Kripke's comments about his sketch of how to explain the word "true" to someone seem in accordance with this "internal" view that Field favors. Kripke says, of a sentence (a Liar sentence) that is not assigned value 1 or 0,

> It will never be called "true." But the subject cannot express this fact by saying "(3) is not true." Such an assertion would conflict directly with the stipulation that he should deny that a sentence is true precisely under the circumstances under which he would deny the sentence itself.[7]

[7] Kripke (1975, p. 701).

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But Kripke thinks that the language for which he is explaining the use of "true" is only a prephilosophical stage of our language, a stage that "continues in the daily life of non-philosophical speakers."[8]

Let us take to heart Field's view about the difference between truth and having semantic value 1 and reconsider in its light the alternative definition of the latter. Let us think of it as an attempted definition of truth for $\mathcal{L}^+$.

Consider first the rules-of-inference version. All the rules of seem clearly valid. The "only if" part of "A sentence of $\mathcal{L}^+$ is true if and only if it is a consequence of the true atomic or negation-of-atomic sentences of $\mathcal{L}$" should not be accepted. But the "if" part should be. Indeed it seems a tenable position that the truth of every sentence of $\mathcal{L}^+$ that we should accept is such a consequence. This seems a tenable position even in unrestricted cases, even though we then have no explicit definition of "consequence."

Kripke says something that might suggest that he holds this position:

> In this manner, the subject will eventually be able to attribute truth to more and more statements involving the notion of truth itself. There is no reason to suppose that *all* statements involving 'true' will be decided in this way, but most will. Indeed, our suggestion is that the "grounded" sentences can be characterized as those which eventually get a truth value in this process.[9]

Admittedly Kripke is only talking about "pre-philosophical" truth but there is nothing to suggest that he is talking about *restricted truth* (though my guess is that, if pressed, he would say that he is).

Now consider the inductive definition version of the alternative definition. Since the clauses involve the word "True," we should not state them in a bivalent language. The language $\mathcal{L}$ with an added predicate "True" won't do, because the clauses contain a conditional. Because there is no nesting of conditionals, any reasonable semantics for the conditional (e.g., Field's), makes all the positive clauses true. The negative clause, "Nothing is true unless the other clauses require it to be," should clearly not be accepted.

Field is concerned not with defining truth but with the *logic* of truth, *theories* of truth, and *model theories* for languages containing a truth predicate. The main thing Field thinks Kripke's construction accomplishes is to give a three-valued model theory that shows the consistency and "conservativeness over ω-models" of a certain theory of truth. This theory, which Field calls KFS, is based on what he calls *the Kleene logic* or K_3. Field leaves some room for play with the exact content of KFS. A part of it, consisting of basic axioms about syntax or coded syntax, varies with the interpreted ground language. At the heart of the theory is a rule of inference, *the Intersubstitutivity Principle*, that allows substitution in any nonintensional context of either of True($\langle A \rangle$) or A for the other, for any sentence A. (Field favors theories of truth where intersubstitutivity holds, and I think he is right in doing so.)

As Field remarks, any fixed point of the operation Kripke's construction defines would do for the consistency and conservancy proof for KFS, and, as far as I know, Field has no preference for one or another. I do not know whether he thinks there might be valid extensions of KFS that rule out the least fixed point.

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[8] Kripke (1975, p. 714).

[9] Kripke (1975, p. 701).

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KFS and the other theories of truth Field considers are effectively given: they have recursive sets of axioms and recursive inference rules. Considering effectively given theories has obvious virtues. One seeming virtue in the Kripke case is that KFS has completeness properties with respect to conservativeness. The conservativeness theorem for KFS says that any interpreted language $\mathcal{L}$ of the kind we are considering can be conservatively extended to a three-valued interpretation of $\mathcal{L}^+$ in which the axioms of KFS get value 1 and both the rules of the logic K_3 and the Intersubstitutivity Principle hold in the sense that they preserve value 1. By "conservatively extended" I mean that the model associated with $\mathcal{L}$ does not change—in particular that the domain U does not change. The proof of the conservativeness theorem is that every Kripke fixed point witnesses the conservativeness of KFS over its ground language and that such fixed points exist. A converse to part of this is that any witness to conservativeness of KFS is a Kripke fixed point. This is a kind of completeness result for KFS. Another one is that every sentence having semantic value 1 in the least fixed point over $\mathcal{L}$ has value 1 in every witness to the conservativeness of KFS over $\mathcal{L}$. In other words, KFS plus the truths of $\mathcal{L}$ provide an *axiomatization* of the least fixed point over $\mathcal{L}$ relative to conservative extensions.

A more direct way to express the same facts would be to put conservativeness into the logic. What we are interested in is conservative extensions of interpreted languages to languages containing extra symbols. Thus it makes sense to consider logics which are the logics of such extensions. For any set U, we can get such a logic K_U by replacing some of the quantifier rules of K_3 by infinite-premise rules. The axioms of KFS and the Intersubsitutivity Principle of KFS yield a theory based on the logic K_U that I will call KFS_U. The three-valued models in which KFS_U holds are precisely the Kripke fixed points over interpreted languages with domain U. For a language $\mathcal{L}$ with domain U, KFS_U plus the truths of $\mathcal{L}$ provide an axiomatization of the least fixed point over $\mathcal{L}$.

The weaknesses Field rightly finds in Kripke's construction are connected with the basic language's lack of expressive power. One weakness is the lack of a "decent" conditional or biconditional. In particular, there is no way to state in the languages $\mathcal{L}^+$ a valid version of the Tarski truth schema, $T(\langle A\rangle) \leftrightarrow A$. A second weakness is that there is nothing one can say in $\mathcal{L}^+$ about what is wrong with, for example, Liar sentences.

Field deals with these weaknesses by adding to the ground language $\mathcal{L}$ both a "True" predicate and a conditional $\rightarrow$. For the resulting language $\mathcal{L}^+$ he produces a three-valued model by a combination of Kripkes construction with a construction related to Herzberger's "revisionist" construction. He also has a lattice-valued version of the models, in which the connectives correspond to algebraic operations, the usual ones in the case of $\vee$, $\wedge$, and the quantifiers.

The Tarski truth schema comes out valid: all its instances have semantic value 1. Some of the laws that the classical conditional obeys hold, but of course some of those laws fail. Field has much to say in the book about why laws that hold should hold and why laws that fail should have to fail.

The problem of how to say what is wrong with a Liar sentence Q is handled via a definable *determinateness operator*: DA is defined to be $A \wedge \neg(A \rightarrow \neg A)$. The sentence DQ gets value 0 in the models, and DA gets value 1 if A gets value 1. The obvious revenge problem is solved: If Q_1 is constructed so that Q_1 is equivalent to $\neg D\mathrm{True}(\langle Q_1\rangle)$, then Q_1 does not get value 0 but DQ_1 does. Using this D and conjunction at limits, one gets a transfinite sequence of stronger and stronger determinateness operators.

Without disagreeing with Field's assertion that Kripke's construction does not define truth, I have tried to suggest that the construction does something in that direction. The

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revisionists think that their transfinite constructions get at the heart of the notion of truth. Field's construction is related to theirs, but happily he makes no claim analogous to theirs. He seems to regard his construction's role as being a tool for producing a model theory that yields a conservativeness theorem for a theory of truth (and perhaps provides an intuitive aid in discovering the correctness and incorrectness of various classical implication principles).

There is a question of *what theory* the construction yields a conservativeness proof for. In terms of axiom schemas and rules of inference, Field has given only a very partial theory. With Kripke's construction, there are easily described and comprehended theories directly connected with the construction: the effective theory KFS or the infinitary theories I have called KFS_U. It would not be ridiculous to hold that these theories provide *complete* axiomatizations for their limited languages. In the Field case, there are no such theories. As Welch (2008) shows, the logic of Field's construction is not axiomatizable in an analogous sense. Field considers some aspects of his construction to be arbitrary, and he thinks that not all the inference rules it yields are genuinely valid. He says:

> It might be better to adopt the view that what is validated by the given notion of the formal semantics outruns "real validity": that the genuine logical validities are some effectively generable subset of those inferences that preserve value 1.[10]

He admits that there would be "some arbitrariness in which effectively generable subset to choose."[11] This seems likely to be a very large understatement. Also it suggests that there isn't a genuine *concept* of "real validity" in play. And it raises the question: Why assume that real validity is effectively axiomatizable?

Field remarks that some have said that his account does not really yield the Tarski schema because his conditional is different from the classical conditional Tarski is using. Field points out that it agrees with the classical conditional where the law of the excluded middle holds. He says that it is a *generalization* of the classical conditional. This reply is fine, so far as it goes. What bothers me is that I don't see how it is *a* generalization; that is, I don't see *what* generalization it is supposed to be. In the end, all we are given is the model-theoretic construction and a (necessarily very partial) list of the laws and nonlaws. Contrast this with the connectives in the Kripke case. I would go so far as saying that Kripke's disjunction and conjunction *are* the classical disjunction and conjunction. I would not go this far with his negation, but axiomatization does give us a some sense of what generalization of classical negation is in play.

I mentioned earlier two ways in which one can "come close" to defining a Kripke quasi-extension in the unrestricted case: inductive definition and games. Nothing coming nearly that close is possible for Field's construction. For example, Welch (2009) gives a game definition of the set of Herzberger stable truths. Welch can modify this definition to give a game definition of the Field quasi-extension. Whereas Kripke case games, if won, are won in finite time, these games are only won after playing forever. In the case of an arithmetic ground language, the games are Σ^0_3, and Welch shows that no simpler games can work.

The comparisons I've been making are only intended to show that what Field has done for languages with a truth predicate and a conditional is not as analogous to what Kripke did for languages for truth without a conditional as Field seems to suggest. This is not

[10] Field (2008, p. 277).
[11] Field (2008, p. 277).

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intended as a criticism of what Field has achieved, which is impressive and important. The comparisons are also not intended to suggest that there is something analogous to what Kripke did and Field failed to find it. I doubt that there is any really natural notion of a conditional that will work in the context of truth.

One topic that Field doesn't pay a lot of attention to and which I think interesting is the resources of the languages $\mathcal{L}^+$, in particular, the resources of their bivalent parts. Some of the important issues about these languages live at the boundaries between their bivalent parts and the rest.

Adding a truth predicate greatly increases the expressive power of the languages $\mathcal{L}$ under consideration. Consider, for example, the Kripke case when $\mathcal{L}$ is the language of arithmetic. Every Δ^1_1 set of natural numbers is definable by a formula of $\mathcal{L}^+$ that satisfies excluded middle. Thus in essence any proper initial segment of Kripkes construction can be bivalently defined in $\mathcal{L}^+$. Moreover, this can be done uniformly. Ordinals less than the β of the fixed point—namely, ω_1^{CK}—can be represented bivalently in various ways. For example, there is a recursive linear ordering whose well-ordered initial part has order type ω_1^{CK}. Using such substitutes for ordinals, one can give a definition of the construction in $\mathcal{L}^+$ that is bivalent and correct for all the stages before the ω_1^{CK}th stage. In particular, one can uniformly bivalently characterize the initial segments of the prewellordings "becomes true before" and "becomes true or false before." There is a formula $A(x, y)$ of $\mathcal{L}^+$ with the following property. If C has semantic value 1, then for any sentence B, the sentence $A(\langle B\rangle, \langle C\rangle)$ has semantic value 0 or 1 and has value 1 if and only if the least α such that $B \in X_\alpha$ is $<$ the least α such that $C \in X_\alpha$ (i.e., iff B becomes true before C). Thus the sentences with value one can themselves more or less substitute for ordinal numbers.

What I said in the last paragraph is surely well known, though I don't know a reference for it. Philip Welch has proved similar results for the three-valued Field construction (and for more complex variants of it). The order of "becoming true" is replaced by the order of "becoming stably true." The ordinal Field calls Δ_0 replaces ω_1^{CK}. Also Welch is able, for each ordinal $\alpha < \Delta_0$, to define bivalently in $\mathcal{L}^+$ a determinateness hierarchy of length α.[12]

I have been talking about the special case when $\mathcal{L}$ is the language of arithmetic. But I am pretty confident that what I've said applies generally to the languages that Field considers.

The ordinals β and Δ_0 are at the boundaries of the bivalent parts of the Kripke and Field languages—just outside those parts.[13] One could, of course, try to get a kind of internal semantics for $\mathcal{L}^+$ by taking some definitions that work bivalently below these ordinals and treating them as full internal definitions. I do not know whether this would lead to anything of value.

I will end with a small remark. Earlier I discussed the question of whether Kripke's construction and the associated quasi-extension of truth could be defined in unrestricted (nonset domain) cases. If Field's language $\mathcal{L}^+$, with both a truth predicate and a conditional, behaves in unrestricted cases anything like the way it does in restricted cases, then, by Welch's work, it provides a witness that the answer is yes. For example, the Kripke quasi-extension can be defined by a bivalent formula of Field's language.

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[12] Welch's determinateness hierarchy theorem assumes that there are constants for elements of the domain U or else that $\mathcal{L}^+$ gotten by adding a satisfaction predicate rather than just a truth predicate. For some sets U, this seems not to have been what Field had in mind, but I think that Welch's theorem fully settles the the natural question about lengths of determinateness hierarchies for set domains U, and only for such U is there a clear mathematical question.

[13] There are differences in the way this is true for the two languages.

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DEPARTMENT OF MATHEMATICS
UNIVERSITY OF CALIFORNIA,
LOS ANGELES
520 PORTOLA PLAZA
BOX 951555
LOS ANGELES, CA 90095–1555

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