Hence in both cases we have the Mascheroni series

$$\gamma = \int_0^1 y(x)\,dx = \frac{1}{2} + \frac{1}{24} + \frac{1}{72} + \frac{19}{2880} + \frac{3}{800} + \ldots . \qquad (3)$$

The series of diagonal sums gleaned from the double series (1) is slower to converge to γ than the series (3) for comparable labour.

Reference

1. J. A. Scott, Evaluating the Euler constant, *Math. Gaz.* **94** (July 2010) pp. 304-306.

10.1017/mag.2017.133

J. A. SCOTT

1 Shiptons Lane, Great Somerford, Chippenham SN15 5EJ

101.32 Yet another Cantor series

With the product of the first n prime numbers aptly termed the nth primorial, the series of reciprocal primorials may be converted to a Cantor series with irrational sum ([1]).

In this *Gazette* Note we will discover a natural Cantor series with irrational sum by replacing the primes by the corresponding Fibonacci numbers.

Fibonacci prerequisites

The well-known Fibonacci sequence of positive integers

$$1, 1, 2, 3, 5, 8, \ldots$$

may be denoted (u_n) where

$$u_1 = u_2 = 1, \qquad u_n = u_{n-1} + u_{n-2}\ (n > 2).$$

We also need to define $u_0 = u_2 - u_1 = 0$ to be consistent with the recurrence relation in order to establish the following lemma ([2]).

Each $k \in N$ divides u_n for some n.

Now for $k \geqslant 2$ we consider the $k^2 + 1$ pairs $(u_i, u_{i+1}) \pmod k$ for $0 \leqslant i \leqslant k^2$. Then since only k^2 pairs are possible modulo k, two pairs must coincide whence there exist $i \geqslant 0$ and $j \geqslant 1$ such that $u_i \equiv u_{i+j}$ and $u_{i+1} \equiv u_{i+j+1} \pmod k$.

The use of the recurrence relation (both forwards and backwards) leads easily to $u_{n+j} \equiv u_n \pmod k$ for all n, that is, the Fibonacci sequence (u_n) is periodic modulo k.

Furthermore, since $u_j \equiv u_0 \pmod k$ in particular, u_j is a multiple of k.

The Cantor theorem

For the Cantor series

$$a_0 + \frac{a_1}{b_1} + \frac{a_2}{b_1 b_2} + \frac{a_3}{b_1 b_2 b_3} + \ldots$$

https://doi.org/10.1017/mag.2017.134 Published online by Cambridge University Press

the sequences (a_n) and (b_n) comprise integers such that $b_n \geqslant 2$ and $0 \leqslant a_n \leqslant b_n - 1$ $(n \geqslant 1)$. Suppose further that for each $k \in N$ there exists some $m \in N$ such that $k \mid b_1 b_2 \ldots b_m$.

Then the Cantor series is irrational if, and only if, both $a_n > 0$ and $a_n < b_n - 1$ infinitely often ([3]).

Hence, by virtue of our lemma, the theorem applies directly with the obvious choice $a_n = 1$ and $b_n = u_n$ (omitting the first two terms).

Note finally that $\sum_{n=1}^{\infty} \frac{1}{u_n}$ is also irrational ([4]).

Acknowledgement

Thanks to the anonymous referee for expert guidance with this Note.

References

1. M. Griffiths, On the sum of the reciprocals of the primorials, *Math. Gaz*. **99** (November 2015) pp. 522-523.
2. P. Erdős and J. Surányi, *Topics in the theory of numbers*, Springer (2003) p. 84.
3. I. Niven, Irrational numbers, CM11 *MAA* (1956) pp. 6-11.
4. R. André-Jeannin, Irrationalité de la somme des inverses de certaines suites récurrentes, *C. R. Acad. Sci. Paris Sér*. I **308** (1989) pp. 539-541.

10.1017/mag.2017.134

J. A. SCOTT

1 Shiptons Lane, Great Somerford, Chippenham SN15 5EJ

101.33 The application of the Weierstrass approximation theorem in the Riemann-Lebesgue lemma

Introduction

A widely-used result in Fourier Analysis is the Riemann-Lebesgue lemma, named after two giants of mathematics: George Friedrich Bernhard Riemann and Henri Léon Lebesgue. The lemma states that the Fourier transform of any L^1 function tends to zero at infinity. Its power lies in its generality, and therefore a worthy lemma to study.

This Note provides an interesting proof of a slightly weaker form of the lemma, namely by using the Weierstrass approximation theorem. The objective is to show the potential of such approximation theorems, so that a similar approach can be taken to solve more complex problems. We have attempted to reduce mathematical jargon and other complexities so that even an undergraduate can appreciate the proof.

Let us begin by stating the lemma in its weaker form.

Lemma 1: (Weak form of the Riemann-Lebesgue lemma)

$$\text{For all } f \in C^0[0,\ 2\pi],\ \lim_{n \to \infty} \int_0^{2\pi} f(t)\, e^{-int} dt = 0.$$

https://doi.org/10.1017/mag.2017.134 Published online by Cambridge University Press