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Simple anti-swing feedback control for a gantry crane

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SUMMARY

We propose a simple quasi time optimal control law for a gantry crane with a payload. The force applied to the trolley is a controlling parameter. The control law consists of two parts: a feedforward term and a trolley position and velocity feedback term.

Initially, we synthesize the feedforward term and the corresponding reference trajectory by computing the time optimal control for the system mass center. The computed optimal control is a discontinuous function of time with several switching time instants. Undesirable large vibrations due to the payload sway appear under this control. Therefore, we transform this control, replacing its jumps by the piecewise linear continuous functions. The computed feedforward term and the reference trajectory are used as input signals of the PD-controller.

KEYWORDS: Gantry crane; Trolley; Payload (or hanging load); Feedforward term; Reference trajectory; Feedback control.

1. INTRODUCTION

A planar motion of a gantry crane with a payload is considered in this paper. A payload is hanging at the extremity of a cable the length of which is fixed. Hence we study the crane as a two-degree-of-freedom mechanical system. In our system, only the trolley is actuated. The force applied to the trolley as a controlling parameter is studied here. Thus a crane with a payload is an under-actuated system.

The load is at rest at the start and must be at rest at the end of the transportation process. At the start and at the stop of the crane motion, undesirable oscillations of the cable with a hanging load are induced. The main problem of the load transportation is to suppress the load swing and the oscillations as quickly as possible or to avoid them if possible. If we try to avoid large load oscillations, the duration of the transportation increases. However, the requirement for the control system to obtain fast motion is important.

Much research[1–18] has been devoted to the control problem of a crane with a payload. Often aiming to synthesize a control law for this mechanical system, this work has also produced several techniques to solve different problems of an optimality, a time-optimality.[1–4,6] Most problems concern the synthesis of an optimal control as a function of time. Pontryagin's maximum principle[19] is usually applied to solve these problems. The problem of the optimal feedback synthesis is much harder and to the best of our knowledge has not yet been solved. Paper [7] is devoted to the modeling of an overhead crane space motion and to the synthesis of an anti-swing linear feedback control. The authors of papers[10,18] consider cranes with a flexible cable.

The first papers devoted to the modeling and to the control of the cranes were published a long time ago (see, for example references [1–3]). However in the last years there were publications in this topic too (see references [12–18]). This means that the problem of controller synthesis for a crane is still under consideration.

For designing a control law for our system, we take into account constraints both in the controlling force and the crane velocity. In the presence of the constraints, a time optimal control function has usually discontinuities.[19] The time optimal control for the crane also has discontinuities.[1–4,6] The planar motion of a gantry crane with a payload is described by a fourth order system of nonlinear differential equations. The linearised motion equations in an open loop have two real and two complex conjugate eigenvalues with both real parts which are equal to zero, because a crane with a payload is an oscillatory system. The number of discontinuity points in the time optimal control for the oscillatory systems is not limited as for the systems with real eigenvalues.[19] That is the reason why the problem of the time optimal feedback synthesis for the oscillatory systems and in particular for the crane with a payload is difficult. Therefore, we develop here another approach and we synthesize a feedback control, which is "quasi" time optimal. First, we synthesize the time optimal control for the system mass center under indicated above constraints in the controlling force and the crane velocity. The form of this control is well known and very simple, because the motion equation of the mass center is of a second order and of the simplest form. After adjusting this control to the complete system of the crane with a payload simultaneously to ensure fast motion and small vibrations due to the payload sway. The time optimal control for the mass center has discontinuities, of course. They are placed at the start, in the middle and at the end of the transportation process. If we apply this control to the crane with a payload, unwanted vibrations with a large amplitude of this payload appear, because the switching time instants for the mass center are not suitable for the complete two-degree-of-freedom system. To damp these vibrations takes time. Therefore, we modify the designed discontinuous time optimal control around the switching instants to decrease the influence of the discontinuities. The discontinuous control function is

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replaced near the switching instants by a piecelike linear continuous function. In this manner we compute a new control function – fluent control.[20] The inclination of the linear parts depends on the frequency of the system vibrations. If the frequency is "small", this inclination has to be "small" as well. Thus, we are able to adapt the inclination to any payload and even to a collection of payloads. Under a chosen fluent control, small oscillations of the payload appear and the dynamics of the complete system is close to that of its mass center. Particularly, the motion of the trolley is not far from the mass center motion. We employ the fluent controlling force as a feedforward term. As a reference, the corresponding trajectory of the mass center is used. The control law for the crane with a payload contains a feedforward term and a linear trolley position and velocity feedback control. We use in the PD-controller the current position and the current velocity of the trolley, but not those of the mass center, because the trolley position can be easily measured and it is close to the mass center position, if the oscillations amplitude is small enough. The designed feedback control with feedforward term does not excite undesirable large oscillations due to the payload sway. Let us note that the motion of the system is sensitive with respect to the switching time instants of the time optimal control, and this is bad from a practical point of view. The control designed here is more robust.

If the transportation distance is large, the crane moves a "long" time with a maximal possible velocity. The initial time intervals when the velocity increases from zero to its maximum, and the terminal time interval when it decreases from its maximum to zero, are relatively small. In our approach, both intervals are increased in order to decrease the swinging amplitude, therefore, we deviate from a time optimality. However, for a large distance the complete duration of the transportation increases not so much, and a deviation from an optimality is relatively small. So, the larger the distance of the transportation, the smaller the deviation from a time optimality. In this manner, we synthesize an anti-swing feedback control law (simple), but instead of an exact time optimality, we obtain a quasi time optimality.

The approach, described above, was applied to controller synthesis for an arm with one and two flexible links in authors' works.[21,22] Here this approach is developed for a gantry crane, because it is possible to consider this subject as a system with a flexible mechanical structure. We consider the angle of a payload swing as a "flexible" variable and the mass center position as a "rigid" variable. To our knowledge, the feedback control designed here for a gantry crane is new.

The paper is organized as follows: The dynamical model of a gantry crane is presented in Section 2. In Subsection 2.1, the motion equations are written. The dimensionless motion equations are obtained in Subsection 2.2. Subsection 2.3 is devoted to the motion equations analysis. The problem is stated in Section 3. The well known time optimal control for the mass center under the controlling force and the velocity constraints is considered in Section 4. Section 5 is devoted to the analysis and the comparison of the time optimal and a fluent control. Numerical investigations of a discontinuous and a fluent control in an open loop are presented in Section 6. Section 7 is devoted to numerical experiments with a fluent control in a closed loop. Section 8 contains our conclusion.

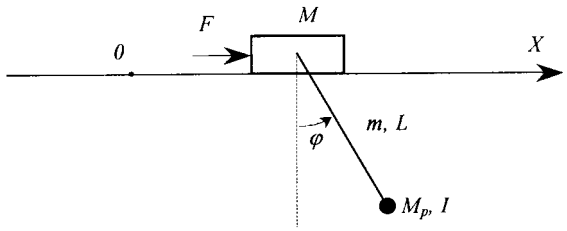


Fig. 1. Scheme of a gantry crane.

2. DYNAMICAL MODEL OF GANTRY CRANE

The scheme of a gantry crane is shown in Figure 1.

2.1. Motion equations

Let M be the mass of the trolley, m be the mass of the cable with the fixed length L. We denote by M_p and I, respectively, the load mass and the load inertia moment about its mass center. The cable is modeled as a homogeneous rigid straight beam with the mass center in the middle. It is clamped to the trolley at one extremity and to the mass center of a hanging load at the other extremity. A motor actuates the motion of the trolley. Let F be the controlling force produced by this motor (Figure 1).

Let us denote by x a linear displacement of the trolley with respect to some origin O, by φ an angle of the cable deviation from the vertical axis (the load swing angle). This angle is defined as positive for counter-clockwise cable rotation.

The expressions for kinetic energy K and potential energy V of our two degree of freedom system are the following (g is the gravity acceleration):

$$2K=(M+m+M_p)\dot{x}^2+(m+2M_p)L\cos\varphi\dot{x}\dot{\varphi}$$
$$+[(m/3+M_p)L^2+I]\dot{\varphi}^2 \qquad (1)$$
$$V=-(m/2+M_p)Lg\cos\varphi$$

The equations of a crane motion can be derived using Lagrange's method and expressions (1):[2–4]

$$(M+m+M_p)\ddot{x}+(m/2+M_p)L\ddot{\varphi}\cos\varphi$$
$$-(m/2+M_p)L\dot{\varphi}^2\sin\varphi=F \qquad (2)$$
$$[(m/3+M_p)L^2+I]\ddot{\varphi}+(m/2+M_p)L\ddot{x}\cos\varphi$$
$$+(m/2+M_p)gL\sin\varphi=0 \qquad (3)$$

2.2. Dimensionless motion equations

Let us introduce the dimensionless variables τ, χ and u:

$$\tau=t/T\ (T^2=L/g),\ \chi=x/L,\ u=F/(M+m+M_p)g \qquad (4)$$

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Substituting the dimensionless quantities in Equations (2), (3), we rewrite them in simple form (with only two parameters j and μ):

$$\ddot{\chi}+\mu\ddot{\varphi}\cos\varphi-\mu\dot{\varphi}^2\sin\varphi=u \tag{5}$$

$$(j-\mu\cos^2\varphi)\ddot{\varphi}+\frac{\mu}{2}\dot{\varphi}^2\sin 2\varphi+\sin\varphi+u\cos\varphi=0 \tag{6}$$

Here dots denote the differentiation with respect to dimensionless time τ; the dimensionless mass μ of the cable with a payload and the payload moment of inertia j are given by

$$\mu=\frac{m/2+M_p}{M+m+M_p},\quad j=\frac{(m/3+M_p)L^2+I}{(m/2+M_p)L^2} \tag{7}$$

Equations (2) and (5) express the linear momentum theorem. Let x_G denote the abscissa of the mass center G of the gantry crane with a payload. Introducing the dimensionless variable $\chi_G=x_G/L$, we can rewrite Equation (5) in a simple form

$$\ddot{\chi}_G=u \tag{8}$$

The controlling force F produced by the motor is limited by constant F_{max}:

$$|F|\le F_{max}\quad (|u|\le u_{max},\quad u_{max}=F_{max}/(M+m+M_p)g) \tag{9}$$

The crane velocity is usually limited by some constant V_{max} as well. We assume here that this constraint relates to the mass center velocity:

$$\left|\frac{dx_G}{dt}\right|\ge V_{max}\Rightarrow(|\dot{\chi}_G|\le v_{max},\quad v_{max}=V_{max}/\sqrt{Lg}) \tag{10}$$

2.3. Analysis of motion equations

Equations (6) and (8) are connected only due to the controlling parameter u. Under condition

$$u=0, \tag{11}$$

these equations are independent. In this case, they have two integrals: the linear momentum and energy integrals:

$$\dot{\chi}_G=const,\quad \frac{1}{2}(j-\mu\cos^2\varphi)\dot{\varphi}^2-\cos\varphi=const \tag{12}$$

Differentiating the second expression (12) with respect to time, we obtain Equation (6). The energy integral (see the second expression (12)) exists because the system is conservative under condition (11). Existence of this energy integral means that the oscillations of the crane cannot be damped without an active control u.

In reality, a frictional torque may exist in torque-transfer mechanism. But it is not well known and we assume that it can be neglected. The damping force will be presented later in the control law.

Linearising the Equation (6) near state

$$\varphi=\dot{\varphi}=0 \tag{13}$$

we obtain

$$(j-\mu)\ddot{\varphi}+\varphi=-u \tag{14}$$

The frequencies of our system, Ω_1, and of a usual pendulum, Ω_2 are defined by the following expressions

$$\Omega_1^2=\frac{1}{(j-\mu)T^2}=\frac{g}{L(j-\mu)},\quad \Omega_2^2=\frac{g}{Lj}$$

and the first frequency is larger than the second one.

The value μ increases strictly monotonically, if the payload mass M_p increases, and $\mu\to 1$, if $M_p\to\infty$ (see (7)). If the cable mass m and payload inertia moment I are negligibly small, then $j=1$. In this case, the frequency Ω_1 of the crane (with a payload) and mass M_p increase strictly monotonically together, and $\Omega_1\to\infty$, if $M_p\to\infty$.

3. STATEMENT OF THE PROBLEM

The problem is to find an admissible (satisfying inequalities (9) and (10)) feedback control u, which transfers a crane from its initial state

$$x(0)=\dot{x}(0)=0\quad (\chi(0)=\dot{\chi}(0)=0) \tag{15}$$

to an arbitrary final state

$$x(T_f)=D,\ \dot{x}(T_f)=0\ (\chi(\tau_f)=d,\ \dot{\chi}(\tau_f)=0,\ \tau_f=T_f/T,\ d=D/L) \tag{16}$$

and keeps it in this state. Here $D>0$ is the desired displacement of the trolley (and of the payload of course). The initial values of angle φ ("flexible" variable) and its time derivative are equal to zero; the final values are desired to be zero as well:

$$\varphi(0)=\dot{\varphi}(0)=0,\quad \varphi(T_f)=\dot{\varphi}(T_f)=0 \tag{17}$$

The final time T_f (or τ_f) is not given, but it is required that this time should be as short as possible.

From (15)–(17) we obtain the following boundary conditions for the mass center position x_G:

$$x_G(0)=\dot{x}_G(0)=0,\ x_G(T_f)=D,\ \dot{x}_G(T_f)=0$$
$$(\chi_G(\tau_f)=d,\ \dot{\chi}_G(\tau_f)=0) \tag{18}$$

Designing the time optimal feedback control $F=F(x,\dot{x},\varphi,\dot{\varphi})$ $(u=u(\chi,\dot{\chi},\varphi,\dot{\varphi}))$ is a very difficult problem. Besides to realize this feedback we need to measure all state variables $x, \dot{x}, \varphi, \dot{\varphi}$; but the load swing angle φ is not easily measured. Therefore, we will synthesize a quasi time optimal feedback control.

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4. TIME OPTIMAL CONTROL FOR MASS CENTER

Let us consider the problem of time optimal control synthesis for the mass center motion

$$\ddot{\chi}_G = u, \quad |u| \le u_{max}, \quad |\dot{\chi}_G| \le v_{max} \tag{19}$$

under boundary conditions (18)

$$\chi_G(0) = \dot{\chi}_G(0) = 0, \quad \chi_G(\tau_f) = d, \quad \dot{\chi}_G(\tau_f) = 0 \tag{20}$$

The solution of this time optimal problem is well known.[3,19,23] The behavior of the optimal control function $u(\tau)$ is described by the following expression:

$$u(\tau) = \begin{cases} u_{max}, & \text{if } 0 \le \tau < \tau_v \\ 0, & \text{if } \tau_v \le \tau \le \tau_f - \tau_v,\ \tau < 0,\ \tau > \tau_f \\ -u_{max}, & \text{if } \tau_f - \tau_v < \tau < \tau_f \end{cases} \tag{21}$$

The velocity of the mass center reaches its maximal admissible value v_{max} at instant τ_v, and τ_f is the minimal possible time

$$\tau_v = v_{max}/u_{max}, \quad \tau_f = d/v_{max} + v_{max}/u_{max}. \tag{22}$$

Control (21) is time optimal for the boundary value problem (19), (20), if and only if $2\tau_v \le \tau_f$, i.e. if $v_{max}^2 < du_{max}$. In the opposite case, the optimal control is the bang-bang function and we must in expression (21) delete the middle line and put

$$\tau_f = 2\tau_v = 2\left(\frac{d}{u_{max}}\right)^{1/2} \tag{23}$$

The profiles of the optimal acceleration (21) and the corresponding "trapezoidal" velocity are presented in Figure 2. Each of these profiles is symmetrical with respect to the middle time instant $\tau_f/2$

$$u(\tau) = -u(\tau_f - \tau), \quad \dot{\chi}_G(\tau) = \dot{\chi}_G(\tau_f - \tau) \tag{24}$$

Let $\Delta\tau_f$ denote the difference between the minimal possible time for the boundary value problem (6), (8), (17), (18) and the boundary value problem (19), (20). It is obvious that $\Delta\tau_f \ge 0$. This means that minimal duration (22) for the mass center transportation between boundary states (20) is a low estimation of the minimal possible duration for the complete two-degree-of-freedom system. However, there is countable set of values d, when control (21) is time optimal for a complete system (6), (8), i.e. when $\Delta\tau_f = 0$. Let us find several such distances d numerically. There is a similar consideration in reference [3], but for another term.

We use the following numerical parameters of the gantry crane:[12]

$$M = 600\ kg,\ L = 4\ m,\ M_p = 500\ kg,\ m = 0,\ I = 0,$$

$$V_{max} = 1.1\ m/s,\ F_{max} = 4400\ N,\ D = 20\ m \tag{25}$$

$$(j = 1,\ \mu = 0.5455,\ u_{max} = 0.4082,\ v_{max} = 0.1757,\ d = 5)$$

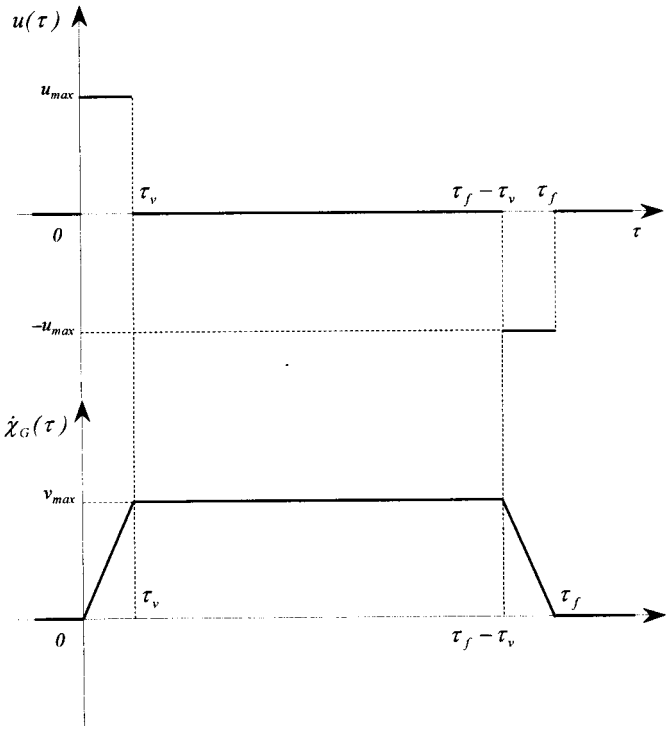


Fig. 2. Time optimal control (21) for the mass center and corresponding "trapeziodal" velocity.

The cable mass and the moment of the payload inertia around its mass center are neglected here. In other words, the load is regarded as a material point.

We will search for the solution of the boundary value problem (6), (8), (17), (18) in a symmetric form

$$\chi_G(\tau) - d/2 = -[\chi_G(\tau_f - \tau) - d/2],\ \varphi(\tau) = -\varphi(\tau_f - \tau) \tag{26}$$

because Equations (6), (8), and the boundary conditions given by (17), (18) admit a solution in such a form. It is derived from (26)

$$\chi_G(\tau_f/2) = d/2, \quad \varphi(\tau_f/2) = 0 \tag{27}$$

Let us assume that we have found distance d to satisfy the complete system (6), (8), under conditions (17), (18) at instant $\tau = 0$ and conditions (27) at instant $\tau_f/2$. After, using expressions (26), we are able to prolong the designed solution symmetrically to time interval $(\tau_f/2.\ \tau_f)$ and obtain in this manner the solution of the complete boundary value problem (6), (8), (17), (18). The boundary value problem has been solved numerically at interval $(0,\ \tau_f/2)$ with several distance values d. These values d and corresponding times τ_f (22) are shown (indicated by the stars) in Figure 3. These points $(d,\ \tau_f)$ belong, of course, to the curve (straight line) (22). The graph, which consists of two curves (22) and (23) is also shown in Figure 3. Expression (23) applies for "small" distances contrary to (22), which appplies for a "large" distance. The difference in time τ_f between two neighboring points, indicated by the stars, is equal to the oscillations period of system (6) with $u = 0$. Strictly speaking, this period depends on the amplitude of oscillations, because Equation (6) is nonlinear. But we are able to estimate this period approximately, using a linearised

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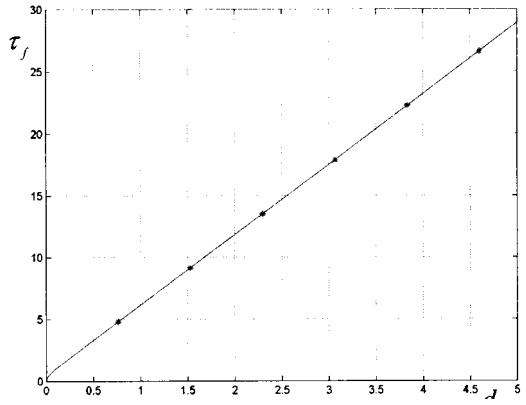


Fig. 3. Curves (22), (23) for minimal time.

Equation (14). The difference in distance d between neighboring points is constant too.

If the minimal possible time for the complete system depends on the transportation distance strictly monotonically, then, using results plotted in Figure 3, we are able to estimate the minimal duration of the payload transportation without solving the complete boundary value problem (6), (8), (17), (18).

In general, the minimal possible time for the complete boundary value problem (6), (8), (17), (18) is larger than for the "short" boundary value problem (19), (20). Pontryagin's maximum principle[19] can be used to design the optimal control as a function of time. However, to the authors' knowledge, the problem of optimal feedback synthesis has not been solved. Besides the optimal control is sensitive with respect to the switching instants and, consequently, to the parameter values. This is bad from a practical point of view. At the same time, only the sum of the trolley and the payload masses play a part in the design of the optimal control (21). For these reasons, we try to adjust the time optimal control (21) to the complete system. Function (21) contains jumps. In Section 5, we show that these jumps are not acceptable for the complete system because large vibrations of the hanging load appear. We modify this function and use "continuous"[20–22] control.

5. ANALYSIS OF TIME OPTIMAL AND CONTINUOUS CONTROL

Time optimal control (21) is a function with two switching time instants. At the start, $t=0$, of the controlling process and at its end, this optimal control has discontinuities as well. Let us consider the initial part of this control with one switching point (see Figure 4):

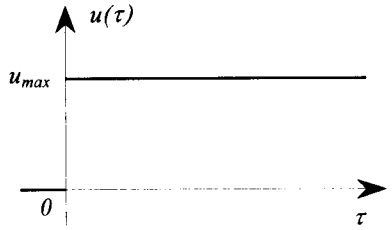


Fig. 4. Discontinuous control (28).

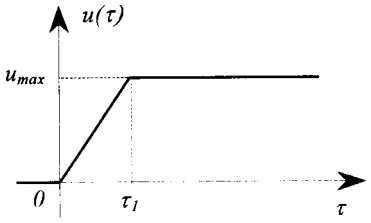


Fig. 5. Piecewise linear control function (31).

$$u(\tau)=\begin{cases}0, & \text{if } \tau<0,\\ u_{max}, & \text{if } \tau\geq 0.\end{cases} \tag{28}$$

Under control force (28), the linear Equation (14) has the following stationary solution:

$$\varphi_s=-u_{max} \tag{29}$$

With initial conditions (17), solution of Equation (14) under control (28) is:

$$\varphi(\tau)=\varphi_s(1-cos\ \omega\tau) \quad (\omega=(j-\mu)^{-1/2}) \tag{30}$$

Equation (30) describes under the linear approximation the oscillations of the system around a stationary solution (29).

Let us consider now, instead of (28), another control which changes continuously due to the piecemeal linear approximation of the control (28) near the point $\tau=0$ (see Figure 5):

$$u(\tau)=\begin{cases}0, & \text{if } \tau<0\\ \dfrac{u_{max}}{\tau_1}\tau, & \text{if } 0\leq\tau<\tau_1,\\ u_{max}, & \text{if } \tau\geq\tau_1.\end{cases} \tag{31}$$

With initial conditions (17), (at initial instant $\tau=0$), under the "fluent" control (31), we obtain the following solution of Equation (14):

$$\varphi(\tau)=\begin{cases}\varphi_s\left(\dfrac{\tau}{\tau_1}-\dfrac{sin\ \omega\tau}{\omega\tau_1}\right), & \text{if } 0\leq\tau\leq\tau_1\\ \varphi_s\left[1-\dfrac{2}{\omega\tau_1}sin\ \dfrac{\omega\tau_1}{2}cos\left(\omega\left(\tau-\dfrac{1}{2}\right)\right)\right], & \text{if } \tau_1\leq\tau\end{cases} \tag{32}$$

Equations (32) describe the angle φ vibrations around the piecemeal linear function of time

$$\varphi(\tau)=\begin{cases}\varphi_s\dfrac{\tau}{\tau_1}, & \text{if } 0\leq\tau\leq\tau_1,\\ \varphi_s, & \text{if } \tau_1\leq\tau\end{cases} \tag{33}$$

If time τ_1, increases, then the amplitude of these vibrations decreases. Hence, the larger the time τ_1, the smaller the

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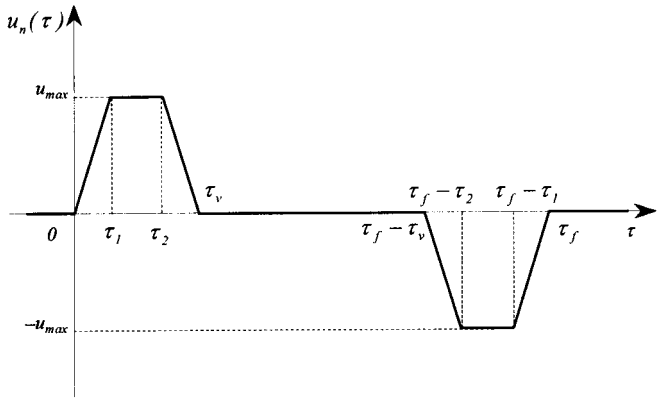


Fig. 6. Fluent control function (34).

vibration amplitude. The vibrations for time $\tau>\tau_1$, can be eliminated, if we choose $\tau_1=2\pi/\omega$ or $\tau_1=4\pi/\omega$, etc. (see the second line in equations (32)). However, if the parameters of the trolley and/or the hanging load are not well known, frequency ω is not well known too and we are not able to choose time τ_1 in this manner. It is possible, by the choice of time τ_1, to decrease the vibrations around the motion given by (33), as desired (this is clear from the physical point of view as well). This is especially relevant for time $\tau>\tau_1$. Therefore, for the synthesis of the control, we use the piecemeal linear approximation of the discontinuous function (21). Thus we design the desired nominal force $u_n(\tau)$ in the following form with two trapeziums (see Figure 6):

$$u_n(\tau)=\begin{cases}\dfrac{u_{max}}{\tau_1}\,\tau, & \text{if } 0\le\tau<\tau_1,\\ u_{max}, & \text{if } \tau_1\le\tau\le\tau_2,\\ \dfrac{u_{max}}{\tau_1}\,(\tau_v-\tau), & \text{if } \tau_2\le\tau<\tau_v,\\ 0, & \text{if } \tau_v\le\tau<\tau_f/2, \text{ or } \tau<0, \text{ or } \tau>\tau_f\\ -u_n(\tau_f-\tau), & \text{if } \tau_f/2\le\tau\le\tau_f.\end{cases} \tag{34}$$

At instant τ_v, the velocity of the mass center reaches its maximal admissible value v_{max}, and τ_f is the final time

$$\tau_v=\tau_1+\frac{v_{max}}{u_{max}},\quad \tau_f=\frac{d}{v_{max}}+\frac{v_{max}}{u_{max}}+3\tau_1-\frac{2u_{max}}{3v_{max}}\tau_1^2,$$

$$\tau_2=\tau_v-\tau_1 \tag{35}$$

Expressions (34), (35) are correct, if and only if $2\tau_1\le\tau_v$, i.e. $\tau_1\le v_{max}/u_{max}$. The function $u_n(\tau)$ is symmetric with respect to the instant $\tau_f/2$: $u_n(\tau)=-u_n(\tau_f-\tau)$.

Let us note that function (34) is the time optimal control for mass center motion with three constraints: force constraint (9), velocity constraint (10) and force derivative constraint, $|\dot u|\le u_{max}/\tau_1$.

If we adopt time τ_1 then, using the expressions (35), (34), (8), we are able to calculate the final time τ_f, the corresponding control and the trajectory (nominal) as functions of time.

If the inclination of the linear parts in function (34) is small (time τ_1 is large), the velocity reaches its maximal value after a long time and perhaps without using the acceleration potentiality of the trolley motor. Such a control and the corresponding motion is not acceptable, if the deviation from a time optimality is large. Let us note however, that the oscillation amplitude is reduced, if product $\omega\tau_1$ is large (see (32)). Hence, the deviation from a time optimality will be small, if the product $\omega\tau_1$ is large. We can choose a time τ_1 not large, if the frequency ω is large enough. The frequency ω_1 increases, if the payload mass increases. Thus, if time τ_1, is acceptable for some payload mass, it is acceptable for larger masses too and consequently for a collection of payloads.

It is important to note that to decrease the amplitude of payload oscillations we change not only the crane velocity continuously, but the crane acceleration as well.

Let us consider now the case when $\tau_v>2v_{max}/u_{max}$ ($\tau_1>v_{max}/u_{max}$). In this case, time τ_v is so large that the acceleration potentiality of the trolley drive motor cannot be used. The maximal amplitude u_0 of the desired nominal force $u_n(\tau)$ is inferior to u_{max} (see Figure 7), and instead of expression (34) we obtain the following:

$$u_n(\tau)=\begin{cases}\dfrac{2u_0}{\tau_v}\,\tau, & \text{if } 0\le\tau<\tau_v/2,\\ -\dfrac{2u_0}{\tau_v}\,\tau+2u_0, & \text{if } \tau_v/2\le\tau\le\tau_v,\\ 0, & \text{if } \tau_v\le\tau<\tau_f/2, \text{ or } \tau<0, \text{ or } \tau>\tau_f\\ -u_n(\tau_f-\tau), & \text{if } \tau_f/2\le\tau\le\tau_f.\end{cases} \tag{36}$$

At instant τ_v, the velocity of the mass center reaches its maximal admissible value v_{max}, the final time is τ_f

$$\tau_v=2\frac{v_{max}}{u_0},\quad \tau_f=\frac{d}{v_{max}}+\frac{5}{3}\tau_v \tag{37}$$

The function (36) is symmetric with respect to time instant $\tau_f/2$, as functions (21) and (34).

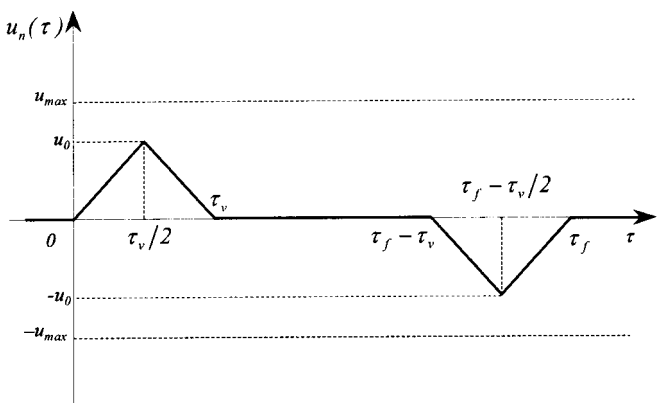


Fig. 7. Fluent control function (36).

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If we adopt time τ_v, then, using the expressions (37), (36), (8), we are able to calculate the final time τ_f, the corresponding control and the trajectory (nominal) as functions of time. During an experiment (numerical), these computed functions are used as the command signals. We denote them by $u_n(\tau)$, $\chi_n(\tau)$, $\dot{\chi}_n(\tau)$ (see Section 7).

6. PROCESSES WITH DISCONTINUOUS AND CONTINUOUS CONTROLS IN OPEN LOOP

Here numerical experiments with a gantry crane under open-loop controls are presented.

6.1. Process with discontinuous control

Using equations for the discontinuous control (21), (22) and the numerical parameter values (25), we obtain the controlling force $F(t)$ which is shown in Figure 8 (upper). The time to reach the maximal mass center velocity is $T_v = 0.275$ *s* ($\tau_v = 0.430$). Final time is $T_f = 18.46$ *s* ($\tau_f = 28.89$).

The next three graphs in Figure 8 show the behavior of the complete two-degree-of-freedom system under obtained controlling force $F(t)$ in an open-loop. There are oscillations of the trolley and the payload during a motion with a constant velocity of the mass center and after final time T_f. The amplitude of the oscillations of angle φ, during the motion with a constant mass center velocity, is equal to 0.25 approximately; the amplitude of these oscillations after instant T_f is 0.51. The mass center does not move in the horizontal direction after time T_f, but the trolley and the hanging load oscillate around it in the opposite phase. The amplitude of the trolley and the payload oscillations after time T_f are near 1.7 *m*. These oscillations are not damped in an open-loop of course.

6.2. Process with continuous control

Now let us calculate force $F(t)$, using equations (36), (37) for the "continuous" control. This force is shown in Figure 9 (upper). Ratio $2u_0/\tau_v$ in these calculations is chosen to be equal to 0.139 ($F_0/T_1 = 50.62$ *N/s*). Value u_0 is 0.06 ($F_0 = 648.9$ *N*). The time to reach the maximal mass center velocity (see equations (37)) is $T_v = 3.749$ *s* ($\tau_v = 5.868$). Final time is $T_f = 21.93$ *s* ($\tau_f = 34.33$).

The next three graphs in Figure 9 show the behavior of the complete two-degree-of-freedom system under obtained controlling the force $F(t)$ in an open-loop. The amplitude of the oscillations in the angle φ during the motion with a constant mass center velocity is smaller than 0.04; the amplitude of these oscillations after instant T_f is smaller than 0.07. The size of these amplitudes is seven times less approximately than under the discontinuous control (21). After the instant T_f, there are also oscillations of the trolley and the hanging load in the opposite phase. The amplitude of these oscillations is 0.13 *m* approximately. This amplitude is considerably smaller than under the discontinuous control (21). The oscillations are not damped in an open-loop.

The results of previous Subsection 6.1 and this Subsection 6.2 show that, using a "continuous" control, it is possible to decrease the oscillations of the payload at the end of the mass center transportation significantly. The final time T_f of the mass center transportation under "continuous" control is larger, of course, than under a discontinuous one. However, the final time 18.46 *s* under the discontinuous control is smaller than corresponding time 21.93 *s* under the "fluent" control by only 16%. To damp the payload vibrations the use of a feedback control is necessary. We investigate a feedback control in Section 7.

7. PROCESS WITH CONTINUOUS CONTROL IN CLOSED LOOP

Our strategy to control a gantry crane is as follows. At each sampling period, during the motion, the computer calculates the nominal command force $u_n(\tau)$, using equations (36), (37). The reference (nominal) motion $\chi_n(\tau)$ and $\dot{\chi}_n(\tau)$ can be calculated on-line, integrating Equation (8), which describes the behavior of the crane mass center. Functions $\chi_n(\tau)$ and $\dot{\chi}_n(\tau)$ can be designed analytically as well. The movements of the trolley and the crane mass center under "continuous" control (36) are close (see Subsection 6.2 and Figure 9). Besides, unlike the mass center position, the trolley position is easily measured. By these reasons, we combine the trolley position and velocity feedback control, but as a reference we use the motion of the mass center. Thus in our consideration, the controlling force $u(\tau)$ consists of the feedforward term $u_n(\tau)$ combined with a linear trolley position and velocity feedback (β, $\gamma = const$)[21,22]

$$u(\tau) = u_n(\tau) + \beta[\chi_n(\tau) - \chi] + \gamma[\dot{\chi}_n(\tau) - \dot{\chi}] \qquad (38)$$

After the final instant τ_f, the following relations obtain:

$$u_n(\tau) \equiv 0,\ \chi_n(\tau) \equiv d,\ \dot{\chi}_n(\tau) \equiv 0 \qquad (39)$$

A trivial solution

$$\chi(\tau) \equiv d,\ \dot{\chi}(\tau) \equiv 0,\ \varphi(\tau) \equiv 0,\ \dot{\varphi}(\tau) \equiv 0$$

of system (5), (6), (38) under conditions (39) is asymptotically stable, if and only if $\beta > 0$, $\gamma > 0$. We are able to prove this assertion, using the Hurwitz's criteria.

The force $F(t)$ applied to the trolley, the angle $\varphi(t)$, the trolley position and the hanging load position of the crane under the feedback control (38) are plotted in Figure 10. The nominal regime $u_n(\tau)$, $\chi_n(\tau)$, $\dot{\chi}_n(\tau)$ is calculated, using Equations (36), (8) with ratio $2u_0/\tau_v = 0.139$. This regime is described in Subsection 6.2 and shown in Figure 9. The feedback gains β and γ are chosen to be equal to 0.1 and 1.0, respectively.

In Figure 10, we see that after time T_f all the variables converge to their desired values asymptotically. Force $F(t)$ and angle $\varphi(t)$ change in the opposite phase. After time 22.5 *s*, the amplitude of the hanging load vibrations is smaller than 0.03 *m* and the vibrations vanish. If, for a concrete technological operation, such vibrations are admissible, we can say that the transition process has finished time 22.5 *s*. Otherwise, it is necessary to expect vibrations until they become admissible, or to decrease ratio $2u_0/\tau_v$. In both cases, the time of the transition process will be larger. Optimal duration 18.46 *s* for the mass center transportation is smaller than that of 22.5 *s* by 18%. Let us remember that

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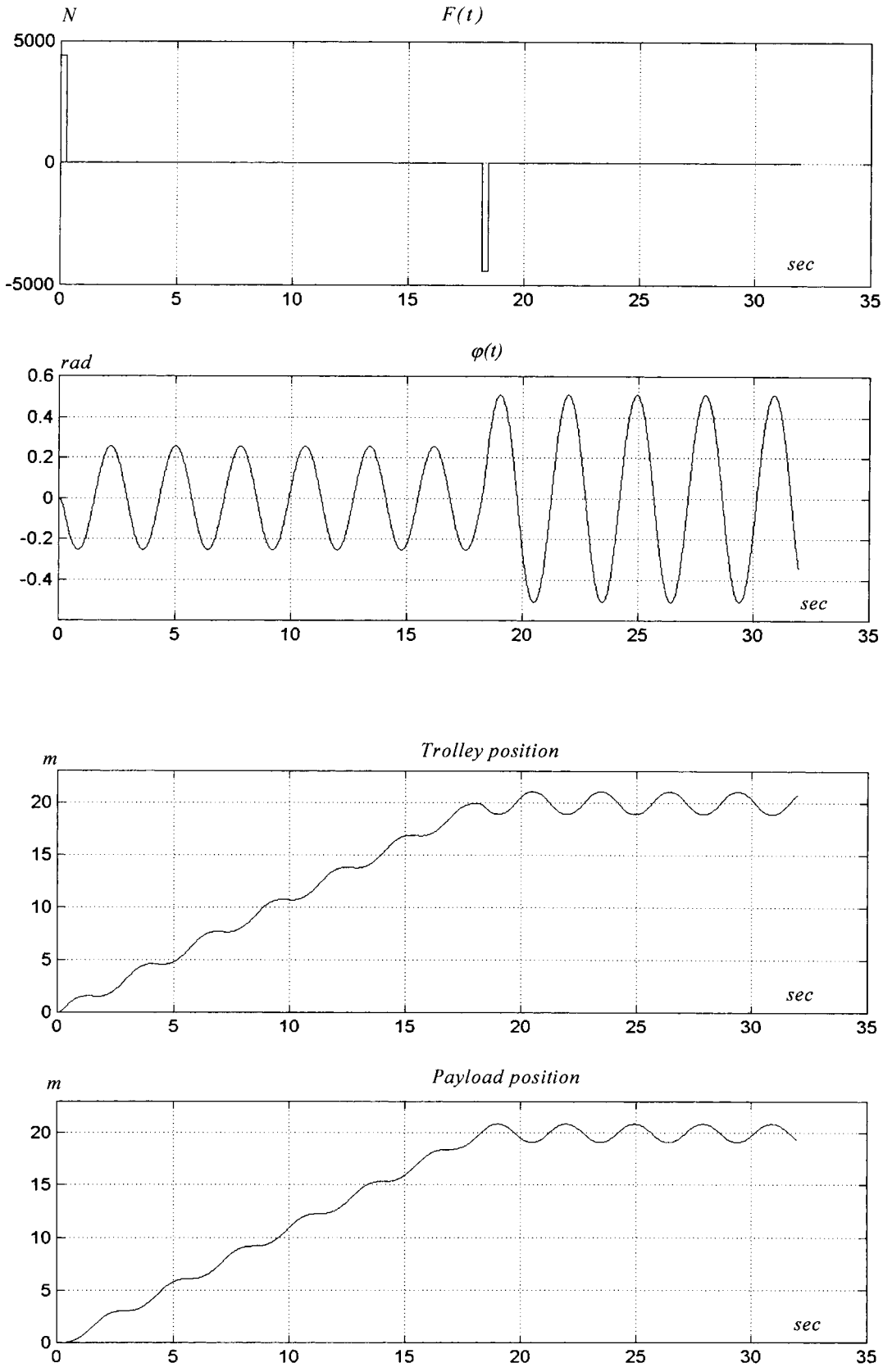


Fig. 8. Force $F(t)$ applied to the trolley, angle $\varphi(t)$, the trolley and payload positions under control (21).

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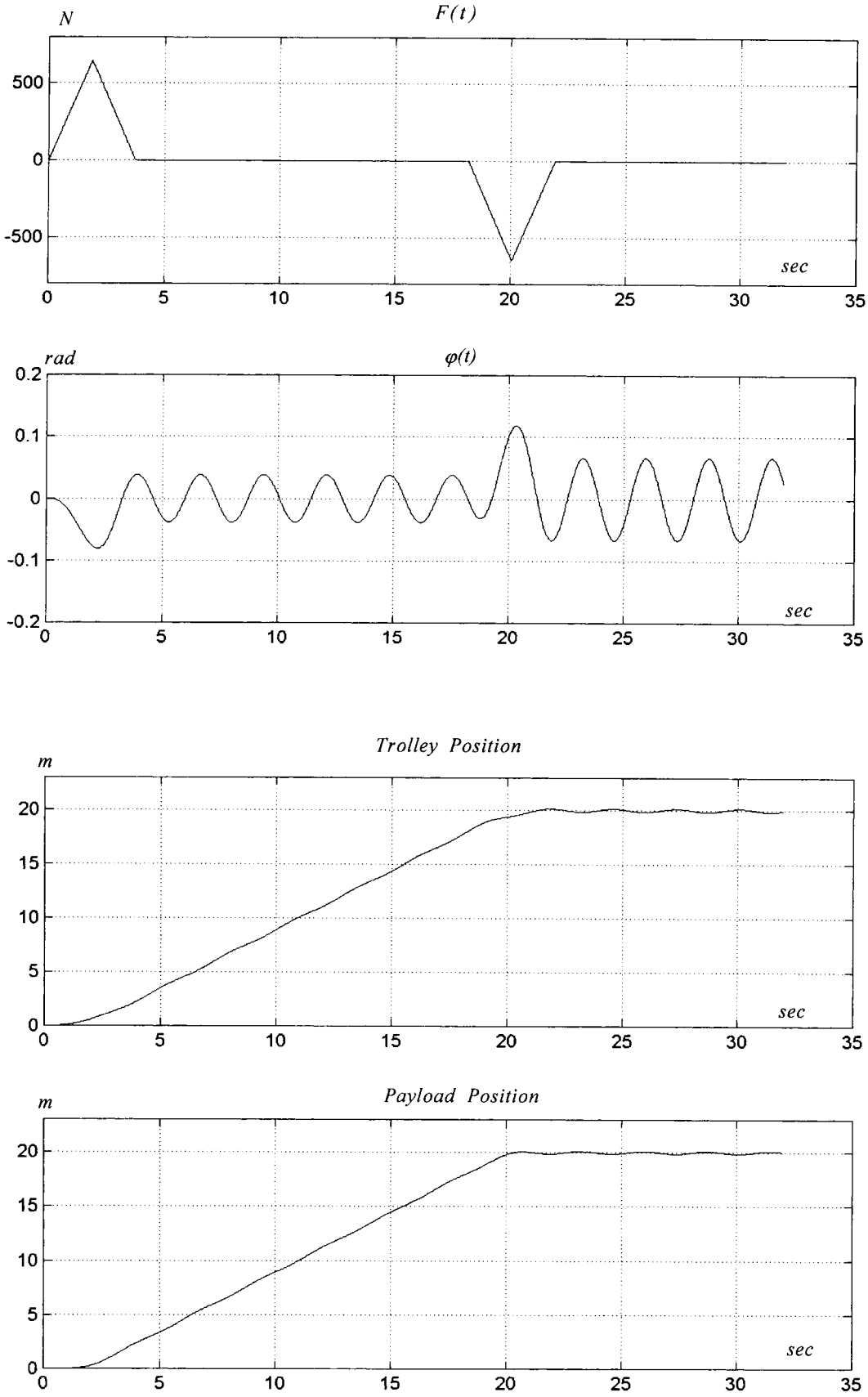


Fig. 9. Force $F(t)$ applied to the trolley, angle $\varphi(t)$, the trolley and payload positions under control (36).

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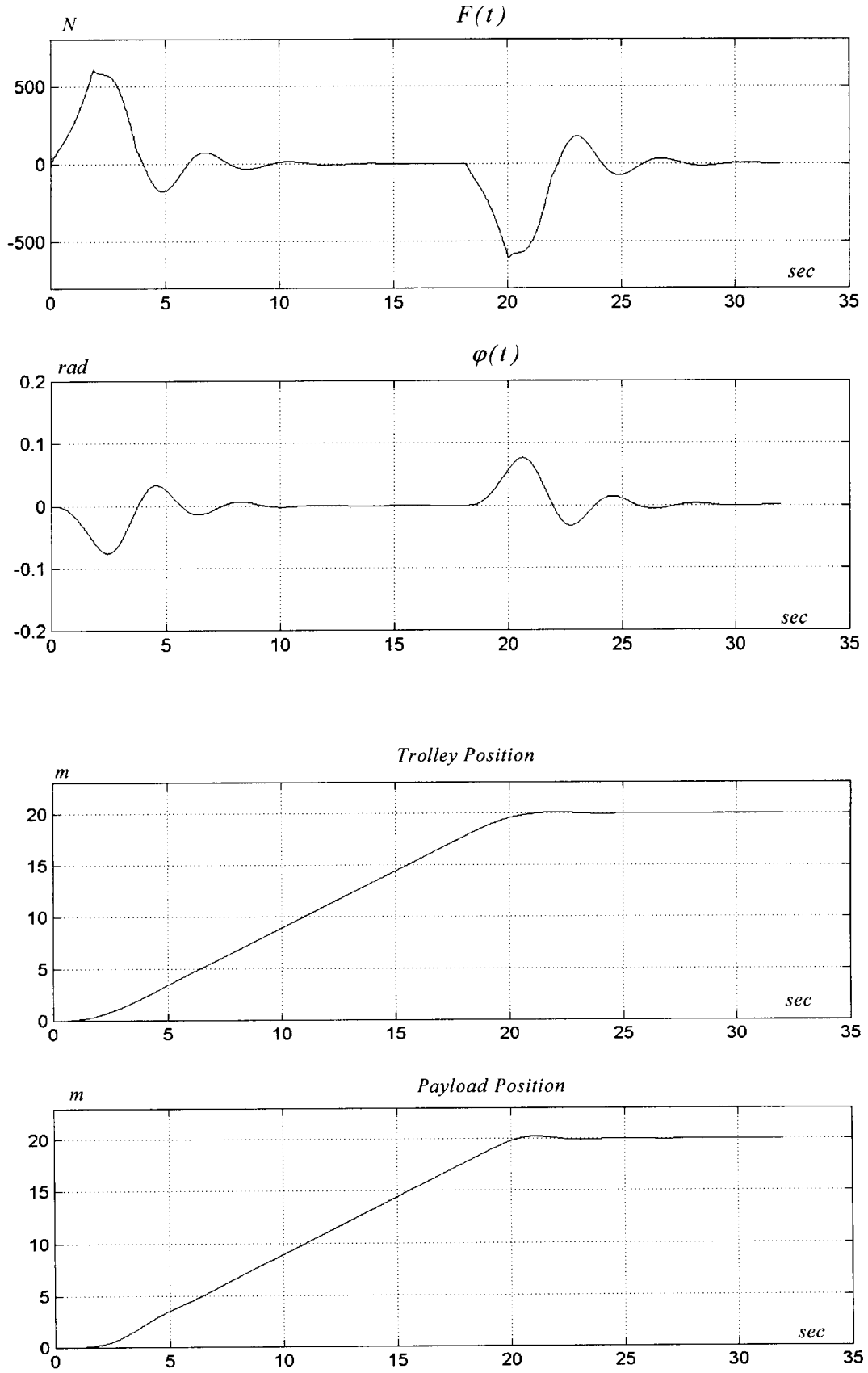


Fig. 10. Force $F(t)$ applied to the trolley, angle $\varphi(t)$, the trolley and payload positions under feedback control (38).

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the optimal time for the mass center transportation is a low estimation of the minimal possible duration for the complete system. This means that, in general, the difference between the optimal time for the complete system and duration 22.5 *s* is even smaller than 18%.

If we use the linear feedback (38) with the discontinuous function (21) as a feedforward term and with the corresponding reference motion, then the amplitudes of the oscillations in the system are much larger and the transition process is longer.

8. CONCLUSION

In this paper, the analytical and numerical study of the control problem for a gantry crane is presented. The control law for the actuator contains a feedforward term, combined with a linear trolley position and velocity feedback term. We propose a method to design both the feedforward term and the reference trajectory.

A high-quality controlled system has to be fast. However, time optimal feedback designing is a difficult problem for a gantry crane and this problem is not yet solved. Besides, to realize the optimal feedback control we have to know all the state variables. But the payload swing angle and the payload position are not easily measured. At the same time, the trolley position and its velocity are well known during the crane motion and can be used in the control law.

A gantry crane with a payload in our modeling is a two-degree-of-freedom mechanical system. We consider the position of the mass center of the crane with the cable and the payload as a "rigid" variable and the angle of the cable swing as a "flexible" variable. First, we compute separately for the rigid variable the time optimal control function, taking into account constraints both in the controlling force and the crane velocity. This function contains discontinuities. This is standard in the case of a bounded control. If we employ this discontinuous control function for a complete two degree of freedom system, large oscillations appear in the load swing angle ("elastic" oscillations), because the switching instants computed for the associated rigid variable are not appropriate for the complete system. Damping the resulting swinging takes a long time. To avoid a large swinging of the payload, we modify the obtained discontinuous control force by a continuous function, which is close to the discontinuous controlling force. The possibility to render the elastic vibrations as small as desired is proved analytically for a gantry crane with a payload. Under the modified continuous control the amplitude of the load swing angle oscillations is limited and the behaviors of the trolley and the mass center positions are close. We stress the point that it is not sufficient to change the crane velocity fluently in order to avoid large payload vibrations. The amplitude of the payload vibrations becomes small, if the crane *acceleration* changes fluently. By our modification of the discontinuous control we deviate, of course, from a time optimality. But if a required crane displacement is large enough, the trolley moves with the maximal possible velocity the most part of the movement duration. Then this deviation from a time optimality is small. In the numerical experiments with the complete system, we use the continuous control function as a feedforward term successfully. The corresponding mass center position and velocity can be used as a reference motion for the trolley PD-controller, because positions of the mass center and of the trolley are close under our control, and the trolley position is easily measured. The motion of a gantry crane under the designed feedback control is not far from the time optimal motion.

Some authors try to design a control, which quickly damps the excited vibrations of the payload. But this damping problem is not easy and damping takes time. Therefore, in our opinion, the problem of the controller design is for a gantry crane not closed. We propose here another approach. A control, which does not excite undesirable large vibrations is designed here. We hope that our method of the anti-swing feedback control design is interesting for practical applications.

It looks to us that our approach can be recommended not only for a gantry crane, but also for other systems with a flexible structure, and for some under-actuated systems.

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