A SUPERCONDUCTING GALVANOMETER

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Received 21 June 1951

ABSTRACT. A superconducting tangent galvanometer suitable for the measurement of thermoelectric voltages in circuits of resistance about 10^{-7} ohm is described. The inductance has been kept low by the use of only a single turn in each of the deflecting coils, but a high sensitivity has been attained by reducing the controlling field to about 0·01 gauss. The theory of operation of such a galvanometer in circuits of low resistance is discussed; the actual behaviour of the instrument is in satisfactory agreement with the theory. The galvanometer will detect a current of 10^{-5} A. corresponding to an e.m.f. of 10^{-12} V.

1. Introduction

In the course of developing apparatus for the investigation of the thermoelectric powers of metals at temperatures below 4·2° K., a superconducting galvanometer has been constructed which, in view of its improved sensitivity compared with other designs, seems worthy of separate description without reference to any particular application. The original aim of producing an instrument which could detect a thermoelectric voltage as small as 10^{-12} V. has been more than achieved.

At low temperatures the high conductivity of pure metals, and the use of superconductors wherever possible, makes it feasible to perform the measurements in a circuit whose resistance may be only 10^{-7} ohm; thus the galvanometer need only be capable of detecting a current of 10^{-5} A. On the other hand, this current must be measured without introducing an inductance of more than a few microhenries, or the time-constant of the circuit will become prohibitively long. In the present instrument, a tangent galvanometer, the inductance has been kept low by the use of very few turns in the deflecting coils. At the same time the required sensitivity has been attained by reducing the controlling field to a value much less than the horizontal component of the earth's field. The galvanometer contributes nothing to the resistance of the circuit, since the deflecting coils are of superconducting lead wire.

Details have been published of two superconducting galvanometers constructed by Grayson Smith and his co-workers. The first of these (1) was of the moving-coil type and was soon superseded by a more reliable instrument (2), which was essentially a moving-magnet galvanometer with a torsional control. This instrument had a sensitivity, under typical operating conditions, of 1·78 cm./mA. with an 'effective inductance' (see § 2) estimated from data given in the paper to be about 500 μH. The present galvanometer represents a very substantial improvement on this instrument, for it can be used conveniently at a sensitivity of 15 cm./mA. and then has an 'effective inductance' of 1·2 μH.

2. Theory

Consider a moving-magnet galvanometer as in Fig. 1 consisting of two coils in series, each of n turns and with their axes normal to a magnetic field H, having a small magnet of moment M suspended between them. The behaviour of the galvanometer

https://doi.org/10.1017/S0305004100027481 Published online by Cambridge University Press

is governed by two differential equations, one for the motion of the magnet and the other for the growth of current in the circuit. It is not, however, permissible to regard them as independent, since the current passing through the coils will exert a force on the magnet, and the changing magnetic flux through the coils due to the motion of the magnet will excite a back-e.m.f. in the circuit. It is easy to show that the field at the centre of the coils due to a current x is Jx, and that for small values of θ the total magnetic flux through the coils due to the magnet is equal to $JM\theta$, where J is the geometrical constant $4\pi na^2/r^3$. For small deflexions, therefore,

$$I\ddot{\theta}+k\dot{\theta}+(MH+\mu)\,\theta = JMx,$$

and

$$(L+L_c)\,\dot{x}+Rx+JM\dot{\theta} = E,$$

where I is the moment of inertia of the magnet and suspension, k is a damping constant, μ is the torsion constant of the suspension fibre, E is the e.m.f. acting in the circuit of resistance R, and L and L_c are the self-inductances of the galvanometer itself and the rest of the circuit. For simplicity it has been assumed, though in practice it is not usually realized, that when $\theta=0$ the suspension fibre is untwisted.

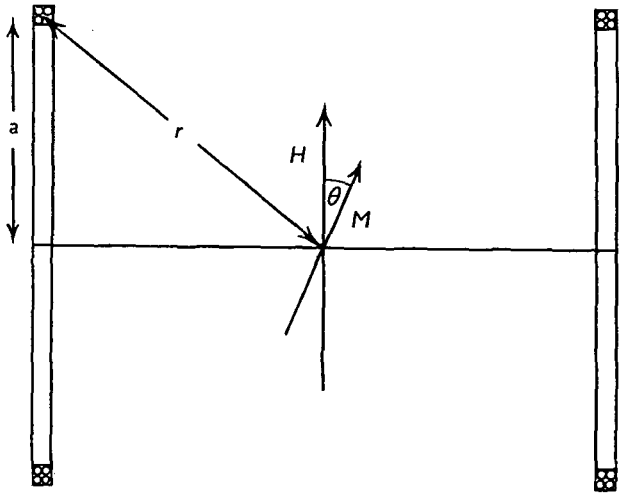


Fig. 1. Schematic diagram of galvanometer.

The above equations lead to the third-order equation for the motion of the magnet:

$$(L+L_c)\,I\ddot{\theta}+[(L+L_c)k+RI]\,\ddot{\theta}+[(L+L_c)\,(MH+\mu)+Rk+J^2M^2]\,\dot{\theta} + R(MH+\mu)\,\theta = JEM. \qquad (1)$$

If now we neglect μ, and imagine that I and k are made very small so that the motion of the magnet always keeps in step with the current, the equation simplifies to

$$\left(L+\frac{J^2M}{H}+L_c\right)\dot{\theta}+R\theta = \frac{JE}{H};$$

or, since now $\theta \doteqdot Jx/H$,

$$\left(L+\frac{J^2M}{H}+L_c\right)\dot{x}+Rx = E.$$

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This is similar to the equation $L\dot{x}+Rx = E$ for a circuit containing resistance and inductance only, and thus the effect of the motion of the magnet is just to increase the inductance of the galvanometer to an 'effective' value $L_{\text{eff.}}$ $(=L+J^2M/H)$, and correspondingly to increase the time-constant of the circuit to the value $(L_c+L_{\text{eff.}})/R$. It will be shown in § 6 that this concept of 'effective inductance' is still of value in estimating the time-constant of the circuit even when this is comparable with the period of free oscillation of the magnet and the above approximations are unjustified.

From equation (1) it may be seen that under steady conditions the voltage sensitivity, θ/E, is given by $JM/R\,(MH+\mu)$.

3. DESIGN OF THE GALVANOMETER

Substitution of appropriate numerical values in the above equations shows that it is impossible to attain the required sensitivity if the controlling field H is the horizontal component of the earth's field. Two methods were tried to increase the sensitivity. First a galvanometer was constructed with two magnets of nearly equal moment but opposite polarity constituting a nearly astatic pair. The restoring torque due to the field H is thus reduced, while the deflecting coils, acting on one magnet only, still produce the same couple per unit current.

Now in order to protect the instrument from external magnetic disturbances it is necessary to surround it by a magnetic shield. A superconductor is ideal for this purpose, for when it is cooled below its transition point the lines of magnetic flux passing through it are 'frozen' into small regions of the metal which remain non-superconducting; the field inside the shield subsequently remains constant and no externally applied field, unless it exceeds the critical field of the superconductor, can penetrate into the interior of the shield. It was found that, when the shield was of tin and was slowly cooled through its superconducting transition point (3·72° K.) by reducing the pressure of the vapour above the liquid helium, the frozen-in field was reasonably uniform and the 'astatic' galvanometer worked satisfactorily with a sensitivity about ten times greater than that of a single-magnet galvanometer working in the earth's field. A shield of lead (transition point 7·2° K.) was then tried so that the instrument could be used throughout the entire available temperature range below the boiling-point of liquid helium (4·2° K.). It was now found that when the shield was cooled by pouring liquid helium on it, the irregular growth of the superconducting regions of the shield destroyed the uniformity of the frozen-in field. The two magnets were therefore suspended in quite different fields and the instrument was completely unworkable.

For this reason it was decided to construct a single-magnet galvanometer working in a field much less than that of the earth. Inspection of the equations of § 2 shows that in order to obtain the greatest possible voltage sensitivity for a given time-constant it is advisable to make this field H and also the moment M of the magnet as small as possible. Experience has shown that it is quite possible to work in steady fields as small as 0·01 gauss, and it has been found that in this field a small Alnico magnet 2 mm. long and having a moment of about 0·2 e.m.u. is satisfactory. If the moment of the magnet is much smaller than this the torsion of the suspension fibre

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causes the equilibrium position of the magnet to lie at an angle to the field and brings about marked non-linearity of the instrument.

It is advantageous also to choose R as small as possible (in this case about 10^{-7} ohm) and to make the ratio a/r as near unity as is convenient. Some space must be left between the coils so that there is room for the suspension, and a value $2/\sqrt{5}$ was chosen for a/r; this figure corresponds to the Helmholtz arrangement of coils and has the added virtue of giving a high degree of uniformity of the deflecting field.

When H, M, R and a/r have been determined by the practical considerations listed above it remains only to select values of a and n giving a suitable sensitivity. It was decided to construct the galvanometer with only one turn of radius 1·2 cm. on each coil.

4. Construction

The galvanometer magnet is attached to the bottom of a long fused silica suspension described in §5 which hangs inside a vertical evacuated tube passing through the top-plate of the cryostat. The upper part of the tube is of brass and is fitted with a lens to admit light to the suspension mirror, while the lower section of the tube leading down into the liquid helium is of thin german silver 1 cm. in diameter along most of its length but narrowing to 8 mm. near its end. This tube is held rigidly in place by three taut stays of 2 mm. german silver tubing.

The coil assembly attached to the bottom of the galvanometer tube is shown in detail in Fig. 2. A represents the superconducting shield of lead 0·4 mm. thick. When the apparatus is filled with liquid helium a non-uniform field approximately equal to the earth's field is frozen within this shield. The shield is now turned by means of the shaft B and reduction gearing C until the frozen-in field at the magnet is perpendicular to the axis of the deflecting coils D. The circles E represent a Helmholtz pair of coils each consisting of 55 turns of 42 s.w.g. copper wire wound on a paxolin former. The current passing through these coils is adjusted until the frozen-in field at the magnet is very nearly compensated. By this procedure the very weak field needed to secure high sensitivity can be obtained with little difficulty; attempts to proceed by the obvious method of freezing-in the already nearly compensated earth's field met with little success.

The former F on which the superconducting coils D are wound is made of copper and is soldered to the galvanometer tube G. The motion of the magnet is damped by eddy currents induced in this former. A copper block I is held in position inside F by three weak springs and serves the double purpose of defining a small region in the centre of the coils in which the magnet can oscillate and of providing additional damping. A threaded hole in I enables it to be readily extracted from the galvanometer so that the degree of damping can be varied without reconstructing the entire instrument.

A special lead-cored copper wire is used for the deflecting coils D. The wire was made by filling a clean copper tube with molten lead and then drawing down the composite rod until the external diameter was 0·5 mm. When the copper is dissolved away in nitric acid the remaining lead is seen to form a quite uniform wire; one sample about 50 cm. long was found to have a resistance certainly less than 10^{-10} ohm at 4·2°K.,

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and currents of at least 0·75 A. could be passed through the wire without the appearance of resistance. The wire is wound directly on the copper former F; the copper is found to provide perfect insulation so long as the lead remains superconducting. Contact is made to the ends of the wire by soldering with Wood's metal

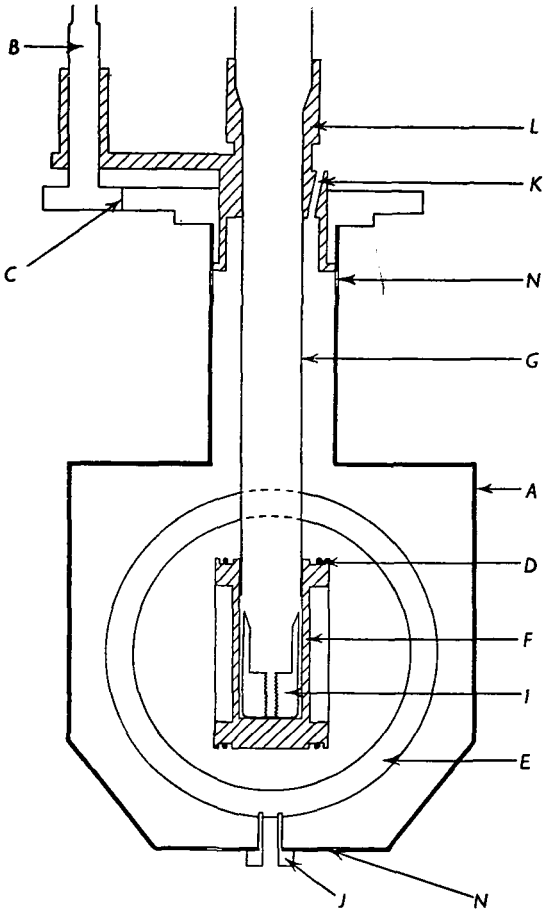


Fig. 2. The galvanometer coil assembly.

after the copper insulation has been removed with nitric acid. The superconducting wires leave the galvanometer through the distrene bush J in the bottom of the shield while the leads to the compensating coils are brought in through the hole K in the brass bush L. Small holes N in the shield allow liquid helium to flow freely round the coils.

5. THE SUSPENSION

The length of the galvanometer tube necessitates a long suspension which is very sensitive to vibration and which makes accurate levelling essential. The cryostat is supported on three screws which rest in a geometrical clamp on the top of a light alloy framework. It may be readily detached and carried away to the liquefier to be filled with liquid helium. When it is returned to its normal mounting it is a simple

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matter to adjust the three screws until a vertical beam of light, which is reflected by a horizontal mirror rigidly attached to the cryostat, retraces its original path. The light alloy framework rests on sponge rubber on a 'vibrationless support' consisting of a pillar of concrete blocks interleaved with felt. There are no rigid connexions between the cryostat and the rest of the building. With this support the galvanometer is unaffected by any disturbances within the building, but the suspension still vibrates irregularly, probably as a result of microseismic activity.

In order to reduce the amplitude of this motion as far as possible the suspension has been carefully constructed so as to have a high degree of symmetry, and a hollow brass cylinder has been added to increase the moment of inertia. The suspension consists of a straight silica rod 50 cm. long and about 1 mm. in diameter. The hollow cylinder, 5·4 mm. in external diameter and having a moment of inertia of about 0·01 g.cm.^{2}, is fastened to the rod 12 cm. from its upper end. The top of the rod passes centrally through a carefully machined duralumin block 3 mm. square and 1·4 mm. thick, to which two similar mirrors are fastened, one to each side. The magnet is cemented in a small duralumin holder at the bottom of the suspension. Each of these three components was first carefully balanced and then attached to the silica rod with a little Durofix.

A very fine hole passes down the centre of the silica, and a piece of thin copper wire is inserted in this hole and held in place with shellac. About a centimetre of wire projects from the top of the rod, and a 5μ quartz fibre is laid along this and fastened with shellac; this arrangement enables the rod to hang symmetrically from the fibre. The other end of the fibre, which is 6 cm. long, is attached to a heavy brass block which can be raised and lowered by a simple winch at the top of the galvanometer tube. This enables the suspension to rest on the bottom of the tube while the apparatus is carried about, thus preventing breakage of the fine fibre. The fibre easily supports the weight of the suspension (about 0·8 g.) and exerts a torsional couple on the magnet of 0·0006 dyne cm./radian; at sensitivities of 15 cm./mA. this is comparable with the magnetic restoring torque (0·0014 dyne cm./radian) and causes a certain amount of non-linearity in the instrument.

With this suspension the mean amplitude of the vibration is only about 1 mm. at room temperature, but this was found to rise to about 3 mm. under working conditions because of vibration due to the boiling of the liquid nitrogen surrounding the helium dewar. However, a device similar to that suggested by Preston-Thomas (3) has recently been incorporated to prevent bubbling of the nitrogen, and the galvanometer can now be read under actual working conditions to an accuracy of about 0·5 mm. at a scale distance of one metre.

6. Performance of the galvanometer

The sensitivity of the instrument used in the earth's field is about 0·7 cm./mA. This can be increased, by decreasing the controlling field H, to 5 cm./mA. without the instrument becoming appreciably non-linear, but for higher sensitivities some slight non-linearity appears due to the increasing importance of the suspension fibre.

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Fig. 3 shows that, as expected, the reciprocal of the sensitivity is a linear function of the current in the compensating coils. The galvanometer seems to work most satisfactorily at a sensitivity of about 15 cm./mA. when it is slightly less than critically damped and has a period of about 12 sec. Higher sensitivities are easily attained, but under these conditions the instrument becomes extremely sluggish, and the zero drift is rapid. At a sensitivity of 15 cm./mA., however, the zero drift is very slow, about 2 cm./h.; this indicates the very high degree of constancy of the field frozen within the shield. The zero is very sensitive to the levelling of the instrument—another indication of the non-uniformity of the frozen-in field already mentioned in § 3—and is affected slightly by changes in the pressure of the helium vapour in the dewar, presumably because of distortion of the galvanometer tube.

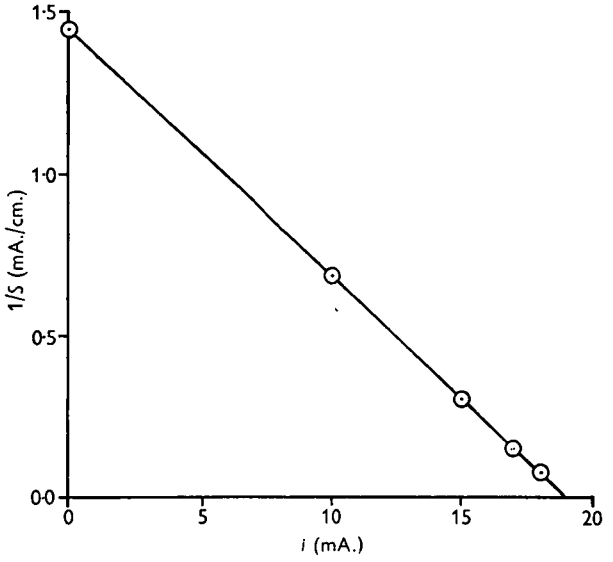


Fig. 3. The relation between the current sensitivity S of the galvanometer and the current i in the compensating coils.

Calculations based on the equations of § 2 show that the instrument has an ordinary self-inductance of about $0{\cdot}15\mu$H. and a total 'effective inductance' of about $1{\cdot}2\mu$H. Thus, in a circuit of resistance 10^{-7} ohm, we should expect a voltage sensitivity of 1·5 mm./$\mu\mu$V. to be attained with a time-constant of about 12 sec. (assuming $L_c = 0$). However, in developing the concept of 'effective inductance', we used the condition that this time-constant should be much larger than the period of swing of the galvanometer. Since the period is about 15 sec. at a sensitivity of 15 cm./mA. we must solve the accurate equation (1) in order to be sure that our estimated time-constant is not seriously in error. The solution consists, for the particular conditions we have chosen, of a damped sinusoidal oscillation superimposed on an exponential decay of time-constant 15 sec., so that even under these conditions the 'effective inductance' provides a useful estimate of the time-constant of the circuit. A detailed comparison between the experimentally observed transient oscillations and the predictions of

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equation (1) is not possible, since so many of the parameters appearing in the equation, particularly those which mainly determine the oscillatory part of the solution, are known only very approximately. However, the experimentally observed transient oscillations of Fig. 4 show clearly the combination of an exponential decay with a damped sinusoidal motion.

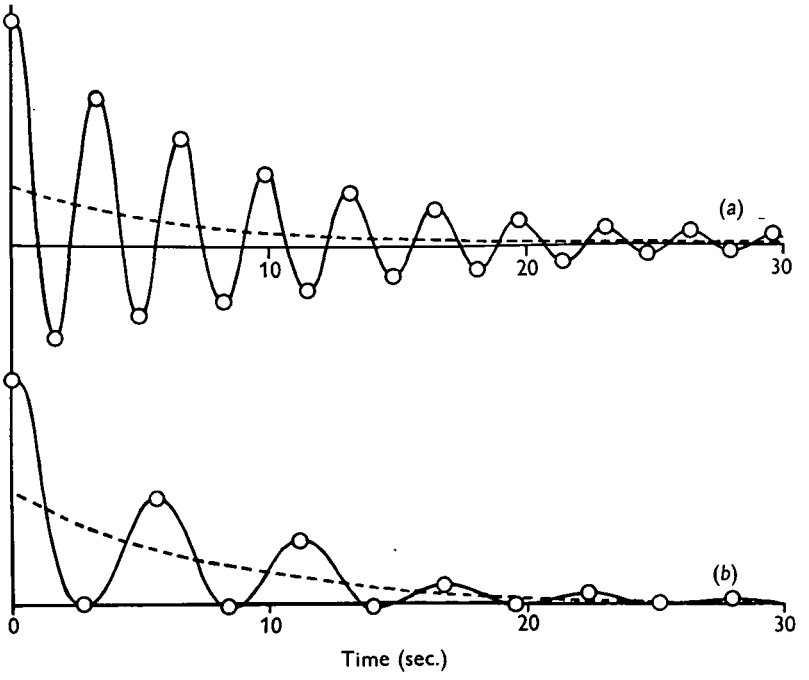


Fig. 4. Transient oscillations observed with the galvanometer in a circuit of resistance $1{\cdot}74 \times 10^{-7}$ ohm. Curve (*a*) at a sensitivity of 0·65 cm./mA.; curve (*b*) at a sensitivity of 2·75 cm./mA. At a sensitivity of 15 cm./mA. the oscillations are only just observable.

It has been found that the galvanometer can be used reliably for the determination of resistances of the order of 10^{-7} ohm. The circuit used for this purpose is shown in Fig. 5. G represents the galvanometer, R_1 and R_2 the low resistance to be determined and a comparison resistance of about 2×10^{-5} ohm and S represents a 'superconducting switch'. This consists of a lead film evaporated on to a german silver strip which can be heated above the transition point of lead by passing current through a suitable heating coil. A resistance of about 5×10^{-3} ohm can thus be introduced into the circuit at will. With S superconducting a current is passed through the leads AA'; most of this flows through R_1, and only a small fraction $R_1/(R_1+R_2)$ passes through the galvanometer. The apparent sensitivity of the galvanometer is measured, and this is compared with the true sensitivity observed when S is resistive and practically all the current passes through G. R_2 is determined at the same time by the usual potentiometric methods using the potential leads BB'. By this method the two resistances can be compared to an accuracy of 2 % or better, so that R_1 may be determined absolutely with similar precision.

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The experience already gained with the galvanometer indicates that it should prove a very reliable instrument in the investigation of thermoelectric effects.

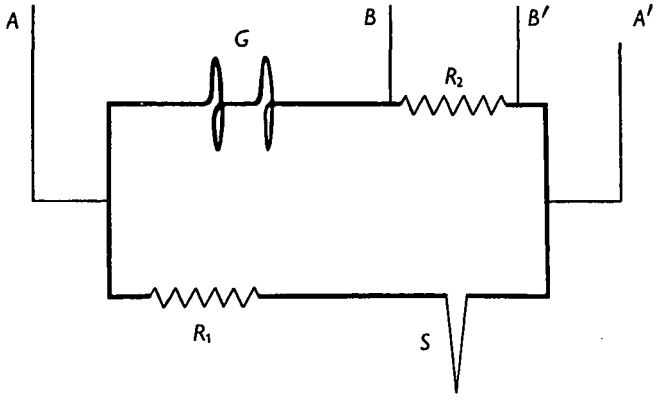


Fig. 5. Circuit for the measurement of resistances of the order of 10^{-7} ohm.

The work was carried out during the tenure by one of us (G.T.P.) of a maintenance award from the Department of Scientific and Industrial Research.

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