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CAN THE U.S. MONETARY POLICY FALL (AGAIN) IN AN EXPECTATION TRAP?

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We propose a model to study monetary policy under discretion. We focus on Markov perfect equilibria, ruling out trigger strategies. The model is simple enough that the determinants of monetary policy under discretion are clear. We also find that for all parameterizations with an equilibrium inflation rate around 2%, there is a second equilibrium with an inflation rate just above 10%. Thus the model can simultaneously account for the low- and high-inflation episodes in the U.S. experience.

Keywords: Time inconsistency, Inflation, Expectation traps

1. INTRODUCTION

After more than 20 years of low inflation in the United States, the study of past high-inflation episodes may not seem a pressing issue. We think otherwise. The high inflation of the 1970s was preceded by a decade of low inflation. Can the current record of low inflation also be suddenly reversed? No doubt, monetary policy has gone a long way since the 1970s, yet the institutional framework is largely unchanged. Could the possibility of high-inflation episodes be an inherent feature of discretionary monetary policy?

The 1970s in the United States is the classic case study for high-inflation episodes in developed economies. The hypothesis that oil shocks were responsible for the high and persistent inflation has long been rejected: see De Long (1997) and Bernanke, Gertler, and Watson (1997) and references herein. Among other evidence, it has been argued that the takeoff of inflation preceded the oil price shocks.

The expectation trap hypothesis conjectures that high inflation during the 1970s was the outcome of a shift in private sector beliefs, which were then validated by monetary policy. For example, Clarida, Gali, and Gertler (2000) argue that

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monetary policy in the pre-Volcker period was highly accommodative to expected inflation. The authors show that the estimated policy rule for the pre-Volcker period gives rise to sunspot equilibria, as it "leaves open the possibility of bursts of inflation and output that result from self-fulfilling changes in expectations."[1] Christiano and Gust (2000) also develop a model of "expectation traps," defined as "a situation in which an increase in private agents' expectations of inflation pressures the central bank into increasing actual inflation."[2] Both papers build on the assumption that monetary policy is given by an exogenous policy rule.

This paper argues instead that monetary policy discretion was responsible for the high U.S. inflation in the 1970s. Monetary policy is modeled as the outcome of a benevolent policy maker who is unable to commit. Thus monetary policy is subject to the time inconsistency problem as in Kydland and Prescott (1977) and Barro and Gordon (1983). Without commitment, the optimal monetary policy may be incompatible with rational expectations.

Monetary policy discretion can also induce multiple equilibria. We pursue the idea that expectation traps arise from the lack of commitment rather than from the properties of a specific policy rule. This theoretical possibility was first explored by Lockwood and Philippopoulos (1994) and Albanesi, Chari, and Christiano (2003) in the context of Markov perfect equilibria, that is, without resorting to trigger strategies.[3]

We argue that expectation traps should be considered more than just a theoretical possibility. For this purpose, we introduce a tractable monetary economy for the study of Markov-perfect equilibria under full policy discretion. The simplicity of the model allows us to discuss the monetary policy decision under discretion in detail. Of course, simplicity and tractability come at the price of a relatively stylized model; yet we think that the channels in operation appear sufficiently general to make the results of importance. We find that the economy features multiple discretionary equilibria and provides a surprisingly good quantitative fit of the U.S. inflation experience. We find that for all parameterizations with an equilibrium inflation rate between 2% and 2.5%, there is an additional discretionary equilibrium with inflation just above 10%. Hence, whenever we try to match the average inflation in the United Sates in the period 1983–2004, a high-inflation equilibrium arises that is intriguingly similar to the U.S. inflation in the 1970s. We note that we do not have a separate set of parameters for monetary policy.

Our model is simple enough to provide an insight into the economics of expectation traps. We assume some firms are financially constrained and have to borrow the wage bill in nominal terms. There is also a subset of monopolistic firms that set the nominal price in advance. Monetary policy has a different impact on each type of firm. For financially constrained firms, inflation drives the cost of money up and hinders their productivity. On the other hand, unexpected inflation eases the monopoly distortion by eroding nominal prices.

There is a low-inflation equilibrium in which the monopoly and financial distortions are balanced. The monetary authority has little to gain from further inflation:

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any sticky price firms' output expansion is nearly offset by the output loss in the financially constrained sector. To have a positive inflation rate in equilibrium, the share of sticky price firms must be larger than the share of financially constrained firms. When the private sector expects high-inflation, the situation is vastly different. Since sticky prices are set according to expectations, low actual inflation would imply very high real prices. On the other hand, if high-inflation expectations are validated, then the financially constrained firm's decisions will be severely distorted. The monetary authority chooses to validate the high-inflation expectations when the sticky price firms predominate over financially constrained firms. Hence, any economy with a 2% equilibrium inflation rate has a high-inflation equilibrium as well.

Our model provides a tractable framework to discuss key issues on credibility and monetary policy. For example, Goodfriend (1993) and Goodfriend and King (2005) discuss how, under Paul Volcker's tenure, the Federal Reserve had to respond to "inflation scares," that is, abrupt changes in the long-term inflation expectations. Such a shift in the private sector beliefs is not at odds with rational expectations in our model. Without doubt, the possibility of high and persistent inflation is a first-order concern for policy makers.

There is a growing literature on expectation traps due to policy discretion. We view our model as tractable as any other in the field with the value added of a reasonable quantitative performance. Albanesi et al. (2003), for example, present a model of limited tractability that does not match the actual U.S. inflation figures. The authors provide three examples with low- and high-inflation pairs of 38% and 107%, 10% and 217%, and 8% and 227%. Khan, King, and Wolman (2001) also present an economy with multiple Markov-perfect equilibria. However, this model has to be solved by backward induction, which complicates the analysis. In a more recent paper, King and Wolman (2004) provide a stylized version of Khan et al. (2001) but do not pursue a quantitative evaluation of the model. Siu (2004) also explores multiple policy equilibria in an economy in which firms can insulate themselves against monetary phenomena. Finally, Armenter (Forthcoming) argues that the necessary conditions for the existence of expectation traps are very general.

The remainder of this paper is as follows. In Section 2 we characterize the private sector equilibrium, and in Section 3 we define the concept of discretionary policy equilibrium. The monetary authority decision is carefully studied in Section 4. Finally Section 5 documents our calibration and explores the set of discretionary equilibria. Section 6 concludes.

2. PRIVATE SECTOR EQUILIBRIUM

The infinite-horizon economy is populated by a representative household, a representative final good firm, a continuum of intermediate good firms, and a monetary authority.

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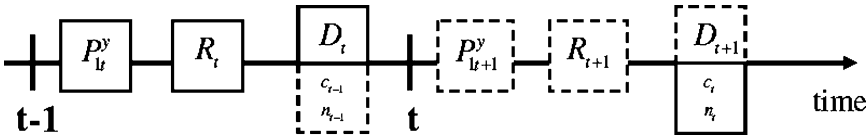


FIGURE 1. Timing of relevant decisions for period t.

Figure 1 illustrates the timing of the model. Several of the variables are set one period in advance. The sequence of decisions within *a period* is also important as it allows us to model the lack of commitment.

First, at the start of period $t-1$ a fraction of the intermediate good firms—the sticky price firms, indexed by $i = 1$—set their nominal price for period t, P^y_{1t}. Second, the monetary authority observes prices P^y_{1t} and chooses the nominal interest rate R_t. Households then decide nominal deposits D_t along with consumption c_{t-1} and labor n_{t-1}. The demand side of the market for nominal deposits is given by a subset of financially constrained firms. As indicated in Figure 1, date t consumption and labor decisions are made at the end of period t.

We assume that the monetary policy instrument is the nominal interest rate R_t that is paid at date t on nominal deposits carried from period $t-1$. The nominal interest rate is implemented via intervention in the market for nominal deposits. As shown below, there is a one-to-one relationship between the nominal interest rate and the inflation rate at date t, $\pi_t \equiv P_t/(P_{t-1})$. Hence, we can think of the inflation rate as the policy instrument.

Before the monetary policy decision, the sticky price firms must form a belief about inflation in period t, denoted π^e_t, in order to set their nominal price P^y_{1t}. Following the literature, we commonly refer to π^e_t as private sector inflation expectations, although "beliefs" would be more accurate.

We show that real prices and allocations in a private sector equilibrium at date t are fully determined by the state of the economy $s_t = (\pi^e_t, \pi_t)$. Neither past nor future policy decisions are relevant, and there is no physical state variable in the economy. By focusing on Markov-perfect equilibria, we can study the monetary authority's decision as a sequence of static problems as in Albanesi et al. (2003). The reader can safely think of our model as a two-period economy in which the monetary authority decision is subject to a subgame perfection requirement.[4]

We do not model money directly. Implicitly, nominal deposits are as good as cash balances. This feature of the model allows us to abstract from money demand considerations and to focus on nominal frictions on the supply side of the economy.[5]

2.1. Households

Household preferences at date t are given by

$$\sum_{j=t}^{\infty} \beta^{j-t} u(c_j, n_j),$$

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with $0 < \beta < 1$. For tractability, we assume quasilinear preferences

$$u(c, n) = c + h(1 - n),$$

where h is a strictly increasing, concave function that satisfies the usual Inada conditions.

The household's problem at date t is

$$\max_{\{c_j, n_j, D_{j+1}\}_{j=t}^{\infty}} \sum_{j=t}^{\infty} \beta^{j-t} u(c_j, n_j), \tag{1}$$

subject to

$$c_j \geq 0,$$
$$0 \leq n_j \leq 1,$$

and

$$P_j c_j + D_{j+1} \leq R_j D_j + W_j n_j + T_j^f \tag{2}$$

for all $j \geq t$, where D_{j+1} are nominal deposits, which pay a nominal interest rate R_j, and T_j^f are profits. Nominal deposits D_j are the unique asset holdings of the household.

The intertemporal Euler equation associated with the household's problem (1) at date t is

$$R_{t+1} = \frac{1}{\beta} \pi_{t+1}.$$

This is the standard Fisher equation. Our timing implies that all uncertainty with respect to the monetary authority's decision at date $t + 1$ has been resolved before the nominal deposit markets clears. Hence, next period's inflation π_{t+1} is known by the time of the household's savings decision.

Labor supply is characterized by the first-order condition

$$h'(1 - n_t) = w_t,$$

where $w_t = W_t / P_t$ is the real wage.

Neither the level of deposits D_t nor the price level P_t appear in the intertemporal Euler equation and the labor supply condition. Therefore we can write both equilibrium conditions in terms of the economy-wide state $s = (\pi^e, \pi)$. First the labor supply condition

$$h'(1 - n(s)) = w(s). \tag{3}$$

As the policy choice for period t is made at period $t - 1$, the relevant pricing equation for the date t private sector equilibrium is given by the household problem at date $t - 1$,

$$R(s) = \frac{1}{\beta} \pi. \tag{4}$$

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We will drop the time subscripts for the remaining of the paper and normalize the last period's aggregate price index to 1 in order to resolve the nominal indeterminacy.

2.2. Firms

There is a representative final good firm that combines a continuum $I = [0, 1]$ of intermediate inputs y_i to produce the final good y according to

$$y(s) = \left[\int y_i(s)^{\eta} di\right]^{\frac{1}{\eta}}, \tag{5}$$

where $\eta < 1$. Its profit-maximization problem is

$$\max_{y,\{y_i\}_I} P(s)y - \int P_i^y(s) y_i di,$$

subject to (5). Hence, the demand for good y_i is given by

$$p_i^y(s) = y(s)^{1-\eta} y_i(s)^{\eta-1}, \tag{6}$$

where $p_i^y(s) = P_i^y/P$.

There is monopolistic competition in the intermediate good sector. Each intermediate good is produced by a single firm i according to a simple linear technology:

$$y_i(s) = n_i(s).$$

All intermediate good firms internalize the demand function for their own good.

There are three types of intermediate good firms. Let μ_i denote the measure of firms of type i. The aggregate measure of intermediate good firms is normalized to one. We assume that the firms' decisions are symmetric within each type.

Firms of type 1—the sticky price firms—set their nominal prices before the monetary authority's policy choice. As a consequence, they set the nominal price $P_1^y(\pi^e)$ according to the private sector inflation expectations π^e. Given our specification for the demand for each good i in (6), profit maximization implies that the nominal price equals a constant markup over the expected marginal cost:

$$P_1^y(\pi^e) = \frac{1}{\eta} w(\pi^e, \pi^e)\pi^e, \tag{7}$$

where $w(\pi^e, \pi^e)\pi^e$ is the nominal wage consistent with the inflation belief π^e.

Firms of type 2 are flexible price setters, that is, they set the nominal price $P_2^y(s)$ after the monetary authority's decision. The price is a function of both π and π^e. Firms of type 2 are financially constrained and must borrow the nominal wage bill Wn one period in advance at the nominal interest rate $R(s)$. Their optimal pricing rule is

$$p_2^y(s) = \frac{1}{\eta} R(s) w\,(s). \tag{8}$$

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The fact that their marginal cost is augmented by $R(s)$ is reflected in the real price.

Finally, firms of type 3 are flexible price setters and financially unconstrained. Therefore we have

$$p_3^y(s) = \frac{1}{\eta} w(s). \tag{9}$$

Note that if the expectation and the actual inflation rate are the same, $\pi^e = \pi$, (7) and (9) imply that prices and output are the same across sticky and non-financially constrained flexible price firms, that is, $p_1^y(\pi, \pi) = p_3^y(\pi, \pi)$ and $y_1(s) = y_3(s)$. Moreover, if $R(\pi, \pi) = 1$, all firms' prices and production are identical. Since the production function for the final good (5) is convex, symmetry across firm types is a necessary condition for production efficiency. In other words, $R(s) > 1$ and $\pi^e \neq \pi$ introduce costly price distortions.

2.3. Market Clearing Conditions and Private Sector Equilibrium Definition

The aggregate resource constraint is

$$c(s) = \left[\int n_i(s)^\eta di \right]^{1/\eta}, \tag{10}$$

where (5) has been combined with each intermediate good production technology. The market clearing condition for the labor market is

$$n(s) = \int n_i(s) di. \tag{11}$$

Equations (3)–(11) are sufficient to solve for all real prices and allocations as functions of $s = (\pi^e, \pi)$. We proceed to define a private sector equilibrium (PSE) given π^e as a collection of allocation and price functions and a sticky nominal price $P_1^y(\pi^e)$.

DEFINITION 1. *Given an inflation rate expectation π^e, a private sector equilibrium (PSE) is a number, $P_1^y(\pi^e)$, and a collection of functions, $\{p_i^y(s), y_i(s), n_i(s)\}_{i \in I}$, $R(s)$, $w(s)$, $n(s)$, $c(s)$, and $y(s)$ over $\pi \geq \beta$, such that*

1. *The household optimality conditions, (3) and (4), are satisfied.*
2. *Firms maximize profits; (7)–(9) are satisfied.*
3. *Markets clear; (6), (10), and (11) hold.*

A PSE outcome in state $s = (\pi^e, \pi)$ is the collection of allocations and prices that occur at a PSE given π^e evaluated at π.

Our definition of the PSE is sufficient to characterize the monetary authority's problem. Note that nominal prices, deposits, and monetary transfers are not included in the PSE. Now we show how to characterize these variables and why they are not relevant for the monetary authority's problem.

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It is straightforward to recover all nominal prices under our normalization $\pi = P(s)$. The nominal deposit market clearing condition is

$$D = W(s) \int_{I_2} n_i(s) di - X(D, s), \tag{12}$$

where $X(D, s)$ are monetary transfers by the monetary authority and I_2 the set of financially constrained firms. Note that for any level of nominal deposits D and state s, there is $X(D, s)$ that clears the nominal deposits market. Hence, for any D and π^e, the monetary authority can implement its policy decision in terms of an inflation rate by setting $X(D, s)$ accordingly. This assumes there is no external constraint in the monetary authority's ability to infuse liquidity in the system other than the zero nominal interest rate bound.

Finally, the household budget constraint (2) gives a law of motion for nominal deposits, $D' = R(s)D$. Since $R(s) \geq 1$, the path for nominal deposits is strictly positive given $D_0 > 0$.

2.4. Solving for the PSE

In our model, the PSE can be solved for analytically. We start by taking P_1^y, a number, as given. Then we solve for the PSE functions that map the actual inflation rate π into allocations and prices. Using these PSE functions, we can characterize the sticky price firms decision as a function of the expected inflation rate, $P_1^y(\pi^e)$.

From the Fisher equation (4), the nominal interest rate and inflation are simply linked by

$$R(s) = \frac{\pi}{\beta}.$$

The relative price of sticky price firms' goods is given by

$$p_1^y(s) = \frac{P_1^y(\pi^e)}{\pi}.$$

Next we solve for relative quantities,

$$\frac{y_i(s)}{y_j(s)} = \left[\frac{p_j^y(s)}{p_i^y(s)}\right]^{\frac{1}{1-\eta}},$$

by combining the demand function (6) for two given goods i and j. Using the pricing formulas (7)–(9)

$$\frac{y_1(s)}{y_3(s)} = \left[\frac{1}{\eta}\frac{w(s)}{p_1^y(s)}\right]^{\frac{1}{1-\eta}},$$

$$\frac{y_2(s)}{y_3(s)} = R(s)^{\frac{1}{\eta-1}}.$$

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These wedges between sectorial production are the result of relative price distortions.

From (5) and the relative quantity relations obtained above, we obtain that

$$y(s) = y_3(s)\left[\mu_3 + \mu_2 R(s)^{\frac{\eta}{\eta-1}} + \mu_1\left[\frac{1}{\eta}\frac{w(s)}{p_1^y(s)}\right]^{\frac{\eta}{1-\eta}}\right]^{\frac{1}{\eta}} \tag{13}$$

and from (11)

$$n(s) = y_3(s)\left\{\mu_3 + \mu_2 R(s)^{\frac{1}{\eta-1}} + \mu_1\left[\frac{1}{\eta}\frac{w(s)}{p_1^y(s)}\right]^{\frac{1}{1-\eta}}\right\}. \tag{14}$$

To characterize the real wage rate, use the previous expression and combine it with the demand function (6) and pricing equation (9) for goods of type 3:

$$\frac{1}{\eta}w(s) = \left\{\mu_3 + \mu_2 R(s)^{\frac{\eta}{\eta-1}} + \mu_1\left[\frac{1}{\eta}\frac{w(s)}{p_1^y(s)}\right]^{\frac{\eta}{1-\eta}}\right\}^{\frac{1-\eta}{\eta}},$$

$$\left(\frac{1}{\eta}w(s)\right)^{\frac{\eta}{1-\eta}} = \mu_3 + \mu_2 R(s)^{\frac{\eta}{\eta-1}} + \mu_1\left[\frac{1}{\eta}\frac{w(s)}{p_1^y(s)}\right]^{\frac{\eta}{1-\eta}},$$

$$\frac{w}{\eta}\left(1 - \frac{\mu_1}{p_1^y(s)^{\frac{\eta}{1-\eta}}}\right)^{\frac{1-\eta}{\eta}} = \left[\mu_3 + \mu_2 R(s)^{\frac{\eta}{\eta-1}}\right]^{\frac{1-\eta}{\eta}}.$$

Hence, the real wage rate can be solved for:

$$w(s) = \eta\left[\frac{\mu_3 + \mu_2 R(s)^{\frac{\eta}{\eta-1}}}{1 - \mu_1 p_1^y(s)^{\frac{\eta}{\eta-1}}}\right]^{\frac{1-\eta}{\eta}}. \tag{15}$$

This expression is the key to solve for the PSE. With knowledge of $w(s)$, the rest of equilibrium allocations and prices follow easily. Labor, $n(s)$, is given by (3). Then, combining (13) with (14),

$$y(s) = \varphi(s)n(s),$$

where

$$\varphi(s) = \frac{\left[\mu_3 + \mu_2 R(s)^{\frac{\eta}{\eta-1}} + \mu_1\left[\frac{1}{\eta}\frac{w(s)}{p_1^y(s)}\right]^{\frac{\eta}{1-\eta}}\right]^{\frac{1}{\eta}}}{\mu_3 + \mu_2 R(s)^{\frac{1}{\eta-1}} + \mu_1\left[\frac{1}{\eta}\frac{w(s)}{p_1^y(s)}\right]^{\frac{1}{1-\eta}}}.$$

The remaining allocation and price functions are straightforward.

To close the PSE, we still need to solve for $P_1^y(\pi^e)$. Given an expectation π^e, (7) implies that $P_1^y(\pi^e)$ will satisfy $p_1^y(s) = p_3^y(s)$. This allows us to write the

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real wage as

$$w(\pi^e, \pi^e) = \eta\left[\mu_1 + \mu_3 + \mu_2 R(\pi^e, \pi^e)^{\frac{\eta}{\eta-1}}\right]^{\frac{1-\eta}{\eta}}$$

and, using (7) again, $P_1^y(\pi^e)$ as

$$P_1^y(\pi^e) = \pi^e\left[\mu_1 + \mu_3 + \mu_2 R(\pi^e, \pi^e)^{\frac{\eta}{\eta-1}}\right]^{\frac{1-\eta}{\eta}}. \quad (16)$$

Note that $P_1^y(\pi^e)$ is increasing in π^e.

3. OPTIMAL AND DISCRETIONARY POLICY

In this section we introduce the policy equilibrium concept. First, we show that the optimal monetary policy with commitment is the Friedman rule. We then state the monetary authority's problem under discretion and the corresponding definition of a discretionary policy equilibrium.

3.1. Friedman Rule

Optimal monetary policy with commitment can be thought as a policy equilibrium under an alternative timing. Let the monetary authority decide once and for all on the inflation rate at the beginning of period $t-1$. Sticky nominal prices and remaining private sector variables are then set with the knowledge of the policy decision. Thus the monetary authority's decision pins down private sector inflation expectations.

Not surprisingly, the optimal monetary policy with commitment turns out to be the Friedman rule. All distortions associated with price dispersion are zeroed by setting the nominal interest rate to zero, $R(s) = 1$. The distortion that arises from monopoly pricing remains. However, there is nothing monetary policy can do to curtail the market power of the intermediate good firms.[6] Hence, labor remains undersupplied.

PROPOSITION 2. *The optimal monetary policy with commitment features* $R(s) = 1$.

Proof. Consider functions $\tilde{\varphi}(\pi) = \varphi(\pi, \pi)$ and $\tilde{w}(\pi) = w(\pi, \pi)$. Simple algebra shows that $\tilde{\varphi}$ and $\tilde{w}$ are decreasing in π, and $\tilde{\varphi}(\pi) \geq \tilde{w}(\pi)$ for all $\pi \geq \beta$. Next we show that the household welfare is increasing in φ and w. Let

$$\tilde{u}(\varphi, w) = \varphi\tilde{n}(w) + h(1 - \tilde{n}(w)),$$

where $\tilde{n}(w)$ is given by (3). $\tilde{u}$ is increasing in $\tilde{\varphi}$. Moreover,

$$\frac{d\tilde{u}}{dw} = (\varphi - w)\frac{d\tilde{n}}{dw},$$

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so given that $\varphi > w$ and the labor supply is upward sloping. Household welfare is also increasing in the wage. Hence any policy choice $\pi > \beta$ is welfare dominated by $\pi = \beta$. ■

3.2. Discretionary Policy Equilibrium

Under discretion, policy is given by the monetary authority's problem given nominal prices $P_1^y(\pi^e)$. While the monetary authority seeks to maximize household welfare, it has no ability to manipulate the private sector inflation expectations.

The choice of the inflation rate is constrained as follows. First, the nominal interest rate is bounded below by one, that is, $R(s) \geq 1$. This bound is implied by the arbitrage condition between nominal bonds and cash balances. The latter are not explicitly modeled here, yet we can use (4) to establish that the lower bound for inflation equals the intertemporal discount rate $\pi \geq \beta$.

Second, the existence of a PSE outcome also imposes an upper bound, $\bar{\pi}(\pi^e)$, on the inflation rate. This upper bound is an increasing function of the private sector inflation expectations. As π approaches the upper bound $\bar{\pi}$, the sticky price firms have unbounded losses.[7]

PROPOSITION 3. *For any $\pi^e \geq \beta$, a PSE outcome exists for all π such that*

$$\pi < \bar{\pi}(\pi^e) = \pi^e \left[\mu_1 + \mu_3 + \mu_2 R(\pi^e, \pi^e)^{\frac{\eta}{\eta-1}}\right]^{\frac{1-\eta}{\eta}} \mu_1^{\frac{\eta-1}{\eta}}.$$

Proof. As long as we have a finite, strictly positive real wage rate, a PSE outcome exists. From (15), $0 < w(s) \leq B$ implies that

$$1 - \mu_1 p_1^y(s)^{\frac{\eta}{\eta-1}} > 0.$$

The above restriction can be rewritten as

$$p_t^y(s) > \mu_1^{\frac{1-\eta}{\eta}},$$

or in terms of π and π^e,

$$\pi < \bar{\pi}(\pi^e) = \frac{P_1^y(\pi^e)}{\mu_1^{\frac{1-\eta}{\eta}}}.$$

■

In the Appendix we show that the policy choice set can be defined without any loss of generality as

$$\beta \leq \pi \leq \bar{\pi}(\pi^e) - \varepsilon$$

for an arbitrarily small $\varepsilon > 0$. First, the upper bound is shown to be never binding. Second, we prove that the policy choice set is never empty as $\bar{\pi}(\pi^e) > \beta$ for all $\pi^e \geq \beta$.

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Because a PSE outcome fully determines the household period welfare, we can state the monetary authority's problem as an intratemporal optimization problem:

$$\max_{\beta \leq \pi < \bar{\pi}(\pi^e)} u\left(c\left(s\right), n(s)\right), \tag{17}$$

where $c(s)$ and $n(s)$ belong to a PSE given π^e. Let $\pi^*(\pi^e)$ be the best policy response function, which solves (17) given any $\pi^e \geq \beta$.[8]

Note that our definition of the monetary authority problem (17) effectively rules out trigger strategies in this setting. Hence our equilibrium will be Markov perfect.

DEFINITION 4. *A discretionary policy equilibrium is a PSE given private sector expectations π^e and an inflation rate π such that the solution to (17) is*

$$\pi^*(\pi^e) = \pi$$

and private sector expectations are rational:

$$\pi^e = \pi.$$

We will say that a policy π is time consistent if there exists a discretionary policy equilibrium with $\pi^e = \pi$. The definition is for an one-period economy. We will spare the reader the corresponding definition for the infinite horizon economy.

4. MONETARY POLICY UNDER DISCRETION

This section provides insight into the monetary policy decision. The economy is simple enough to keep track of the incentives that shape the monetary authority's decision. We discuss how the nature of the monetary authority's problem changes with the level of inflation expectations, providing the basis for expectations traps.

4.1. The Phillips Curve

We start by studying how allocations depend on the policy choice π. In particular, we want to characterize the relationship between inflation and labor demand that forms the basis of the monetary policy transmission mechanism.

If inflation expectations are below a certain level, employment is monotonically increasing with actual inflation and the Phillips curve is upward sloping.

PROPOSITION 5. *For π^e such that*

$$P_1^y(\pi^e) \leq \bar{P}_1^y = \beta\left[\left(1 - \mu_1 + \mu_2\right)\left(\frac{\mu_1}{\mu_2}\right)\right]^{\frac{1-\eta}{\eta}},$$

we have that $dn/d\pi(s) > 0$ for all $\pi \geq \beta$.

Proof. See the Appendix ■

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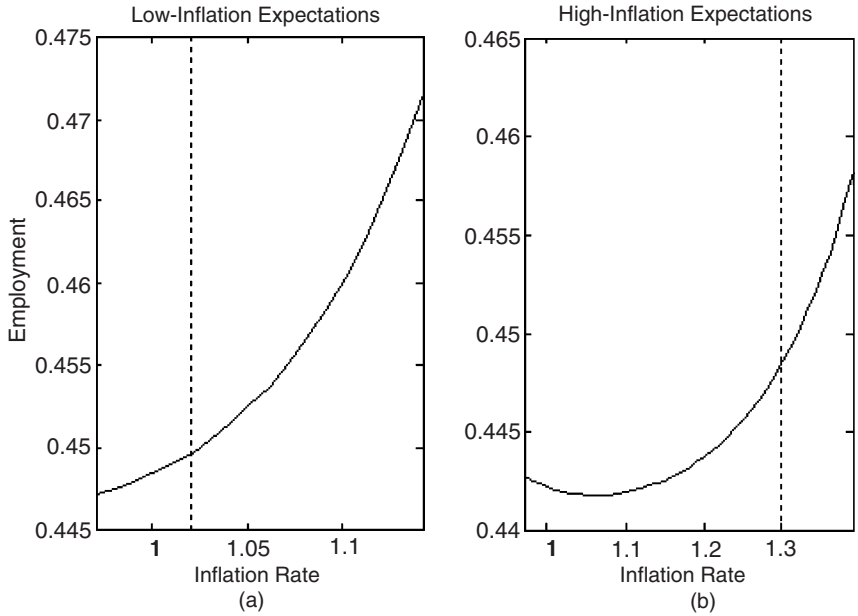


FIGURE 2. Inflation and employment.

Recall that $P_1^y(\pi^e)$ is a strictly increasing function of the private sector inflation expectations π^e. Hence the condition $P_1^y(\pi^e) \leq \bar{P}_1^y$ is equivalent to a cap on inflation expectations.

For high inflation expectations—defined as expectations that exceed the cap—the Phillips curve can be nonmonotone. We illustrate the possibility with a numerical example. Figure 2 displays employment as a function of the inflation rate π for two different inflation expectations π^e.[9] Figure 2a corresponds to an expected inflation of 2%, as indicated by the dashed vertical line. Employment is strictly increasing with inflation.

Figure 2b corresponds to high-inflation expectations, around 30%. The relationship between employment and inflation is not monotone. For any inflation rate below 8%, more inflation actually reduces employment. The Phillips curve has the "wrong" shape only when inflation is clearly below inflation expectations, but it looks standard elsewhere.

Figure 3 shows the output for sticky price firms, y_1, and financially constrained firms, y_2, as a function of the inflation rate for inflation expectations of 2% and 30%. The output of the sticky price firms is strictly increasing in the inflation rate as their relative price decreases with inflation. The output of the financially constrained firms decreases as inflation augments their marginal cost through a higher nominal interest rate. It is easy to see why the welfare benefits and costs of inflation arise from the output response of sticky price and financially constrained firm, respectively.

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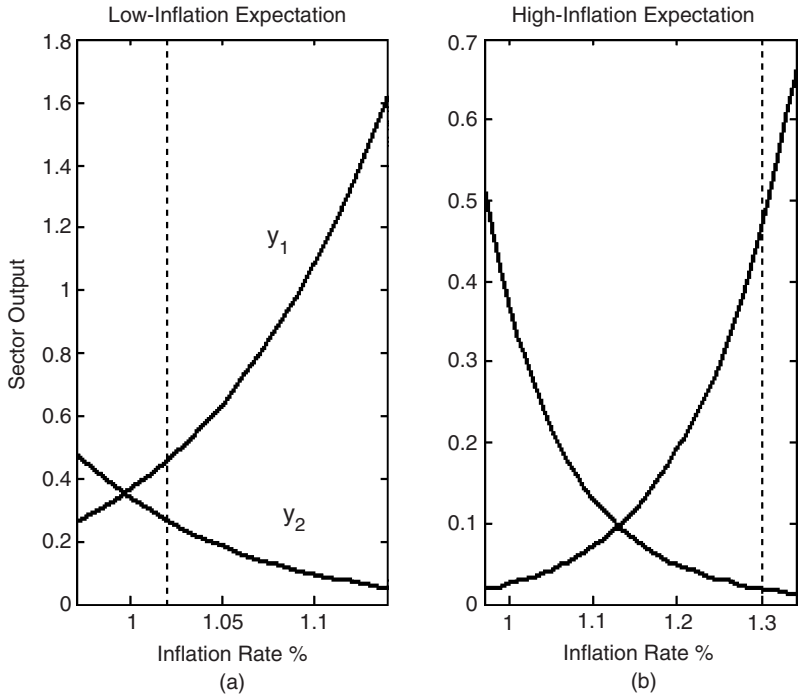


FIGURE 3. Inflation and firm $i = 1, 2$ output.

Figure 3a depicts the case of low-inflation expectations. Around the expected inflation rate, inflation displaces production from financially constrained firms to sticky price firms. As the distortion—and hence employment—shifts from one sector to the other, it is possible that the economy achieves greater efficiency.

Figure 3b makes clear that high-inflation expectations deliver a different scenario. Validating the inflation expectations hurts the financially constrained sector: the output of financially constrained firms y_2 is close to 0 at $\pi = \pi^e$. Low inflation can restore the financially constrained firm sector close to efficient levels, but this comes at the cost of depressing the sticky price firm sector. The nonmontone Phillips curve arises because, in a severely distorted sector, output reacts very little to additional inflation. For example, increasing inflation from 2% to 4% has a large negative impact on the output of financially constrained firms. However, output of sticky price firms barely improves, and there is an aggregate employment loss.

4.2. Inside The Monetary Authority Problem

As the transmission mechanism changes with the level of inflation expectations, so it does the monetary authority problem. When inflation expectations are low, that is, $P_1^y(\pi^e) \leq \bar{P}_1^y$, the monetary policy decision is one of fine tuning. In line with this argument, we need only a mild technical assumption to show that

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the first-order condition of the monetary authority problem (17) is sufficient to characterize the best policy response $\pi^*(\pi^e)$.

PROPOSITION 6. *Consider a PSE with* $P_1^y(\pi^e) \leq \bar{P}_1^y$. *Let* $\theta(\pi^e, n)$ *be the final good production associated with* π *such that* $n(s) = n$. *Then* $\theta(\pi^e, n)$ *is a differentiable function. Moreover, if*

$$\frac{\theta_{nn}(\pi^e, n)}{n\theta_n(\pi^e, n)} \leq -\frac{h''(1-n)}{nh'(1-n)} \tag{18}$$

for all n, then the first-order condition of the monetary authority problem is sufficient to characterize $\pi^*(\pi^e)$. *Moreover,* $\pi^*(\pi^e)$ *is continuous.*

Proof. See the Appendix[10] ■

While the set of first-best allocations is convex given standard assumptions on technology, similar conditions for the convexity of the set of second-best allocations are restrictive and often without of interpretation. The condition (18) is weaker than the convexity of the aggregate resource set spanned by monetary policy. If the aggregate resource set is indeed convex, $\theta_{nn} < 0$, the first-order condition is sufficient because h is a concave function. Quite interestingly, (3) implies that the right-hand side of (18) is the inverse of the Frisch labor supply elasticity. The labor literature has reported very low estimates for the Frisch labor supply elasticity, between 0 and 0.25. Hence (18) does not seem to be a strong restriction. In our preferred calibration, the Frisch labor supply elasticity is one, yet condition (18) is satisfied.

As long as inflation expectations are low, Proposition 6 asserts that it is safe to explore the monetary policy decision on the margin. In the Appendix, we include a discussion of the first-order condition associated with (17). We show that it can be rewritten as

$$\sum_{i=1}^{3} \mu_i n_i \left(p_i^y - w\right) \xi_\pi^i = 0, \tag{19}$$

where

$$\xi_\pi^i = \frac{dn_i}{d\pi} \frac{\pi}{n_i}.$$

This expression is closely related to classic second-best theory. The monetary authority's problem is to solve (19), correctly allocating distortions across sectors. As a result, the policy choice can be expressed in terms of a trade-off between gaps and the employment elasticities with respect to inflation. This is what we understand by "fine-tuning." The sufficiency of the first-order condition (19) also implies that the monetary authority's problem can be studied with a quadratic approximation.

For a low expected inflation of 2%, Figure 4 displays several allocations and welfare in response to changes in the inflation rate. Figure 4a is the Phillips curve. Output also increases with inflation, but labor productivity does not once actual

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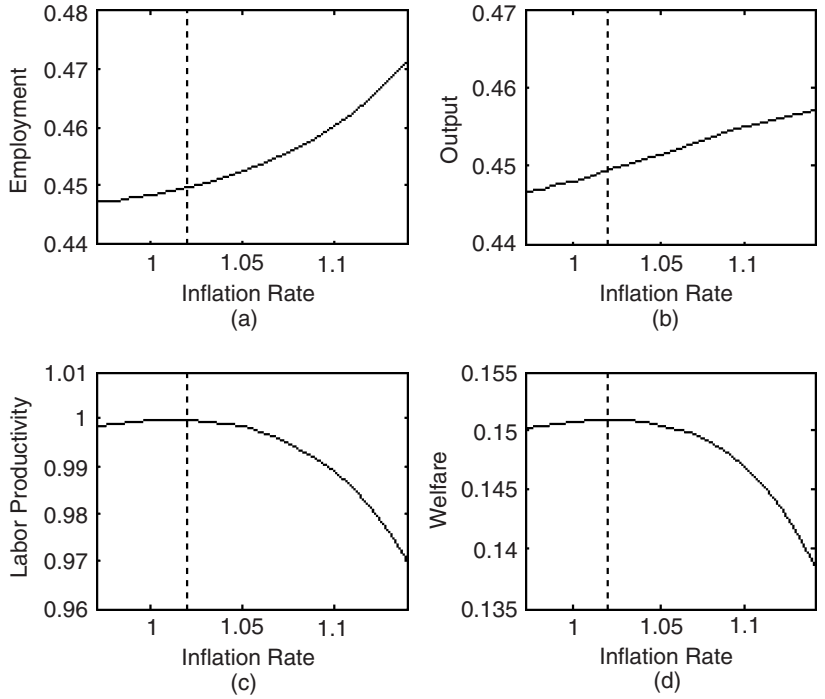


FIGURE 4. Allocations and welfare under inflation expectation $\pi^e = 2\%$.

inflation exceeds expected inflation (marked with a vertical dashed line). Hence while the monetary authority can further increase output, it does so at the cost of reduced production efficiency. Both price distortions and the impact of financial constraints contribute to the latter.

Welfare is displayed in Figure 4d. Welfare peaks somewhere above the inflation expectation. The resulting monetary policy decision reflects the trade-off between stimulating the labor demand and maintaining high labor productivity.

How does the monetary authority problem look when confronted with high-inflation expectations? Fulfilling the high-inflation expectations implies a large wedge between the price and the marginal cost for financially constrained firms. However, low inflation also delivers large distortions—only in this case on the sector of sticky price firms. There is no way out of severe distortions, so the monetary authority only gets to pick its poison.

Figure 5 graphs allocations and welfare as a function of the monetary authority's inflation choice under an expectation of a 30% inflation rate. As discussed earlier, the relationship between employment and inflation is not monotone. Output also displays the same U shape, as shown in Figures 5a and 5b.

Figure 5d depicts welfare. The monetary authority faces a double-peaked indirect utility function. Note that labor productivity also displays two peaks. Low

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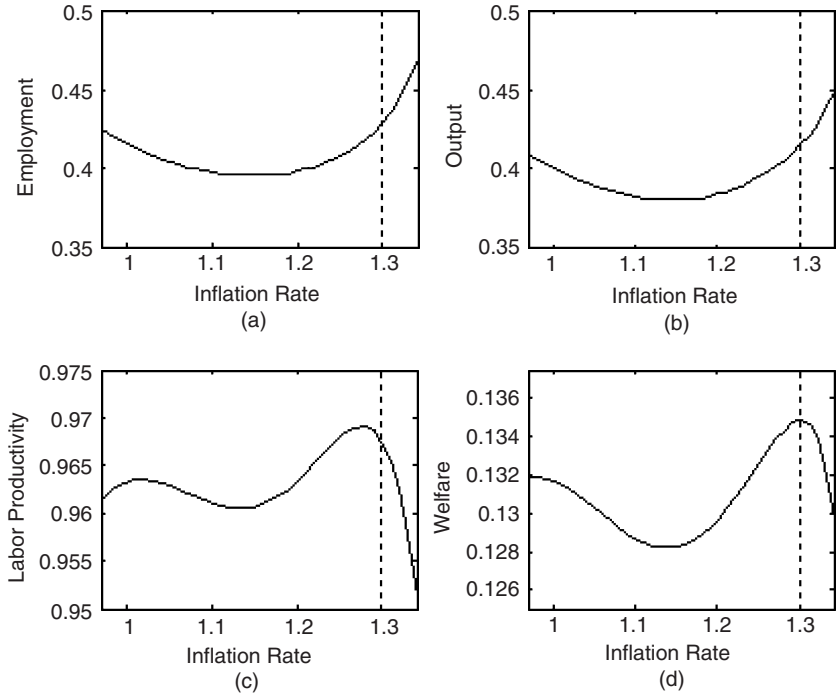


FIGURE 5. Allocations and welfare under inflation expectation $\pi^e = 30\%$.

inflation brings the financially constrained firms close to the optimal production level; high inflation improves production efficiency in the sticky price firms. In each case, the more efficient sector is also larger.

From Figure 5 it is obvious that the monetary authority's problem cannot be characterized by first-order welfare changes. Instead, the monetary authority compares two local maxima: one close to inflation expectations and the other close to the Friedman rule. This is what we understand by pick-your-poison.

4.3. The Best Policy Response

Next we analyze the best policy response $\pi^*(\pi^e)$, that is, the inflation rate that solves the monetary authority's problem given inflation expectation π^e. Nonlinear methods are used: given our discussion above, linear approximation methods are not reliable for high-inflation expectations.

The first relevant question about the best policy response concerns optimal policy: Does the Friedman rule constitute a discretionary equilibrium? Assume that the private sector believes that the Friedman rule will be in place, $\pi^e = \beta$, and sets nominal prices accordingly. Ex-post, the monetary authority considers to set inflation above expectations $\pi > \pi^e$ in order to cut the markup of the

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sticky price firms. Such a move increases price dispersion. The price difference between the sticky and flexible price firms is welcome as it reflects the improved efficiency in the sticky price firms sector. However, financially constrained firms have their marginal cost augmented by $R(s) > 1$. On the margin, the welfare benefits and losses of inflation are weighted by the size of the sticky price firm and financially constrained firm sector, respectively. The next proposition shows that the Friedman rule is not a discretionary equilibrium as long as the sticky price firm sector is strictly larger than the sector of financially constrained firms.

PROPOSITION 7. *If $\mu_1 > \mu_2$, the Friedman rule is time inconsistent.*

Proof. The indirect utility function can be written as

$$u(s) = \varphi(s)n(s) + h\,(1 - n(s)).$$

We will evaluate $du(s)/d\pi$ at $s^* = (\beta, \beta)$. Differentiating,

$$\frac{du(s)}{d\pi} = [\varphi(s) - w\,(s)]\,\frac{dn(s)}{d\pi} + \frac{d\varphi\,(s)}{d\pi}n(s),$$

where we have used (3). The following hold at s^*:

$$\varphi(s^*) = 1,$$
$$w(s^*) = \eta,$$
$$\frac{d\varphi(s^*)}{d\pi} = 0,$$

where the first two equations result from simple algebra. For the last one, note that $\varphi(s) \leq \varphi(s^*) = 1$ on $s \in \Re^2_+$, as $[\int x^\eta di]^{\frac{1}{\eta}} \leq \int x di$. From the fact that φ is differentiable everywhere on $\Re^2_+$, we can conclude that $\varphi(s)$ is concave and it has a maximum at (s^*).

Finally, differentiating (15) and evaluating at s^*,

$$\frac{dw}{d\pi}(s^*) = \frac{w\,(s^*)}{\beta\,(1 - \mu_1)}\,(\mu_1 - \mu_2).$$

Therefore, for $\mu_1 > \mu_2$, $dn(s^*)/d\pi > 0$ as implied by (3). Hence, if $\mu_1 > \mu_2$ the Friedman rule $\hat{R} = R = 1$ is not a discretionary equilibrium, as there is $\pi' > \beta$ such that household welfare is larger, $u(\beta, \pi') > u(s^*)$. Hence, $\pi^*(\beta) \neq \beta$ ■

Figure 6 displays the best policy response for our preferred calibration discussed in Section 5. Private sector inflation expectations, π^e, are displayed on the horizontal axis. The solid line plots the difference between the best policy response and private sector expectations, $\pi^*(\pi^e) - \pi^e$. A discretionary equilibrium is given by $\pi^*(\pi^m) = \pi^m$. The monetary authority seeks to overshoot private sector expectations whenever they are too low (below 2.50%) or too high (past 10%). As discussed earlier, the monetary authority problem under low-inflation

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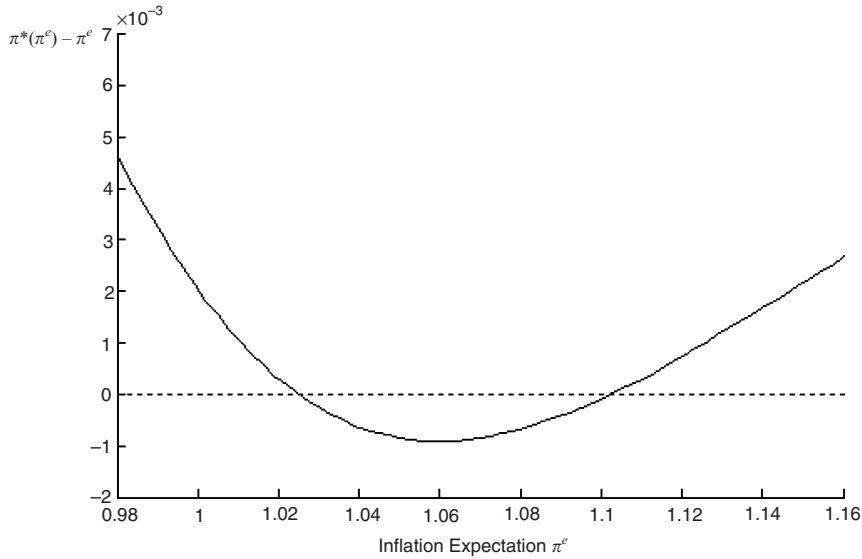


FIGURE 6. Best policy response for preferred calibration.

expectations is a standard fine-tuning problem, and so the best policy response function is downward sloping as in Barro and Gordon (1983). However, under high-inflation expectations, the financially constrained sector is so depressed that there are no longer costs (on the margin) to further inflation. As a result, there is a low-inflation equilibrium at 2.45% and a high-inflation equilibrium of 10.25% .[11]

The best policy response can be potentially complex and, indeed, it is a correspondence for a small set of parameterizations. Figure 7 plots the best policy response function $\pi^*(\pi^e)$ for our preferred calibration and two additional parameterizations. Inflation expectations π^e are on the horizontal axis and the best policy response on the vertical axis. The thick line is the best policy response function. The thin line indicates additional local maxima. A crossing of the best policy response function and the 45-deg (dashed) line indicates a discretionary equilibrium $\pi^*(\pi^m) = \pi^m$.

Figure 7a displays the best policy response function for our preferred calibration displayed in Figure 6. The best policy response tracks inflation expectations closely—indeed, so closely that the reader is referred to Figure 6 to actually spot discretionary equilibria. Note that the best policy response would be well approximated by a linear function, yet a linear approximation would necessarily miss the second equilibrium. The indirect welfare function is double peaked only for very high-inflation expectations, well past high-inflation equilibrium.

Figure 7b corresponds to a parameterization with a very small share of sticky price firms, $\mu_1 = 0.03$. For inflation expectations of 25% and above, there are two local maxima. The local maximum in the low-inflation region always dominates. This rules out the possibility of a high-inflation equilibrium.

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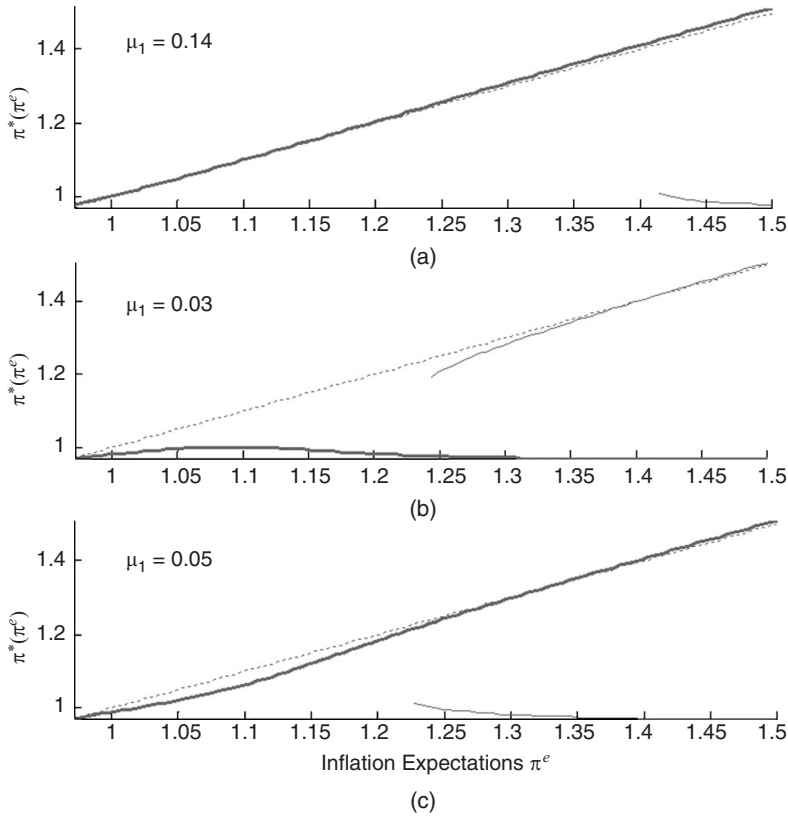


FIGURE 7. Best policy response for preferred calibration.

Figure 7c shows the best policy response function for a very similar parameter choice, $\mu_1 = 0.05$. Again there are two local maxima, but this time high inflation is the best policy response by the monetary authority. Now there are two discretionary equilibria: a low-inflation equilibrium close to the Friedman rule and a high-inflation equilibrium above 30%.

In all three cases the best policy response function is continuous. This does not need to be the case. The best policy response may alternate between local maxima. The discontinuity in the best policy response function must be in the region of high-inflation expectations where the indirect welfare function can be double peaked. By continuity of the indirect welfare function, the monetary authority is actually indifferent between a certain pair of low and high inflation at the discontinuity of the best policy response. This leads us to the possibility that the monetary authority has a mixed strategy in equilibrium, that is, it randomizes between two inflation choices. In the working paper version of the present paper [Armenter and Bodenstein (2005a)], we analyze mixed strategies carefully and show that they arise only in a tiny part of the parameter space.

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5. EXPECTATION TRAPS

The main feature of the economy is the multiplicity of discretionary equilibria. We first briefly discuss the economics of expectation traps. We then document our preferred calibration and then go on to explore other parameterizations. We argue that the model quantitatively matches the U.S. inflation in the past 40 years, including low- and high-inflation episodes.

5.1. The Economics of Expectations Traps

Expectation traps are the rule in our economy—exceptions involve either nonexistence of equilibrium or the Friedman rule being time consistent. The simplicity of the model allows us to discuss the economics behind this property. Expectation traps arise from the heterogeneous impact that inflation expectations have on firms' output levels. The efficiency gains from unexpected inflation stem from the sticky price firms, while the welfare costs of inflation are linked to the financially constrained firms.

Consider first the low-inflation equilibrium. The output of sticky price and financially constrained firms is slightly distorted by monopolistic pricing. The small but positive nominal interest rate further distorts the financially constrained firm's output. The monetary authority's problem is then to fine tune the economy. There are few net welfare gains from shifting the distortion from sticky price to financially constrained firms as both are at a similar distance from efficiency. Hence an equilibrium is achieved at a low-inflation rate at which the cost of price distortion is small.

The situation under high inflation is quite different. Financially constrained firms are severely distorted because of the high cost of money. As a result, financially constrained firms operate at reduced scale. Because in equilibrium sticky price firms anticipate high inflation and set nominal prices accordingly, the distortion on the sticky price firm's output is unchanged.

Hence the sticky price firm's output is larger than that of the financially constrained firms. This leads the monetary authority to validate the high-inflation expectations. Efficiency gains in the sticky price firm sector greatly outweigh the losses in the smaller financially constrained sector. Hence the monetary authority's incentives are also balanced at a high-inflation rate despite the larger costs of price distortion.

5.2. Calibration

To evaluate the model, we first fix all parameters that are not directly related to the monetary policy transmission mechanism. Clarida et al. (2000) and Christiano and Gust (2000) also provide an account of the high inflation in the United States based on policy making. However, they take monetary policy as given by an exogenous Taylor rule. Under our assumption of full discretion, policy decisions are determined by the economy's primitives. We do not have a separate set of parameters to calibrate policy.

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TABLE 1. Preferred calibration

Parameter	Value
β	0.9709
ψ_0	0.5
ψ	1
η	0.91
μ_1	0.14
μ_2	0.04

We start with the utility function, which we assume to be of the form

$$h\,(1-n) = \psi_0 \frac{(1-n)^{1-\psi}}{1-\psi}$$

for $\psi > 0$ and $h(1-n) = \psi_0 \log(1-n)$ for $\psi = 1$. The key parameter is ψ, the inverse of the Frisch labor elasticity, which we set equal to one.[12] Parameter ψ_0 is virtually irrelevant for our results, so we pick it to match the Aristotelian proportion of leisure and work in the first best, $n = \frac{1}{2}$. We define one period to be one year. The inverse of β is the real interest rate in our economy. We take the annual real interest rate to be 3%, implying a value $\beta = 0.9709$. The last of the preset parameters is the constant elasticity of substitution η. We choose to replicate a 10% markup in the price of intermediate goods.

We choose the share of the sticky price and financially constrained firms, μ_1 and μ_2, respectively, such that there is a discretionary equilibrium matching an inflation rate in the range 2%–2.5%. This corresponds to the average U.S. inflation over the period 1983–2004. The growth rate of the gross domestic product (GDP) deflator was 2.45% over 1984–2004. The annual producer price index (PPI) inflation rate is 1.8% for the period, the consumer price index (CPI) around 3%, and the core inflation only a little above 3%.[13]

Our preferred calibration is summarized in Table 1. It features a small amount of nominal frictions. More than 80% of the firms are not subject to any friction, and there are more sticky price firms than financially constrained firms, 14% over 4%. While our economy is too stylized to compare these numbers with empirical evidence on firm-level data, we note that they are not out of line with many of the estimates used in the literature. For example, if we would take the share of sticky price firms as the outcome of a quarterly i.i.d. probability of not changing prices, the median duration would be just above 2 quarters.[14]

5.3. The Set of Discretionary Equilibria

We find that all parameterizations with a discretionary equilibrium in the range of 2%–2.5% inflation also feature a high-inflation equilibrium between 10% and 11%. This is quite surprising since there are many pairs (μ_1, μ_2) that imply a 2%–2.5% discretionary equilibrium.

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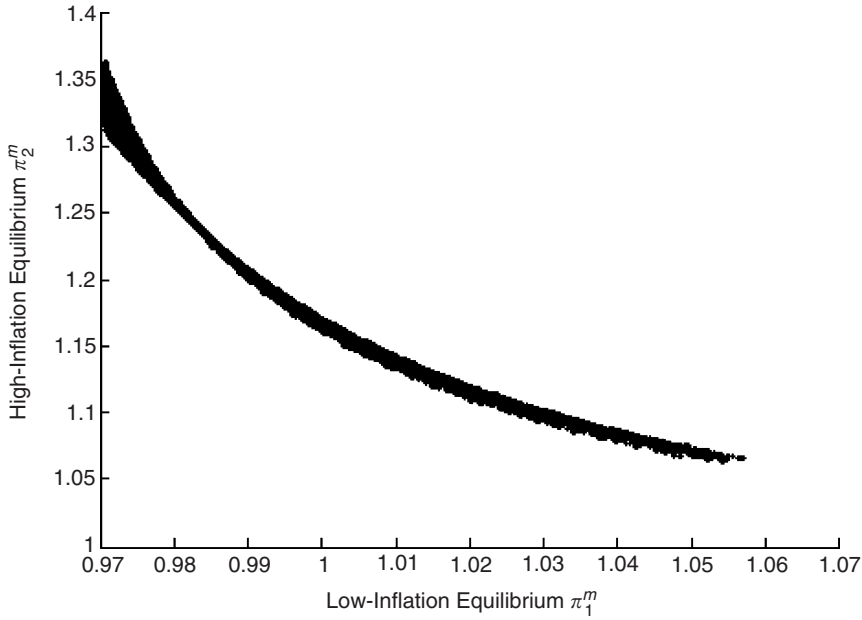


FIGURE 8. Discretionary equilibria: Several economies.

Figure 8 illustrates this noteworthy property of the model. We compute the discretionary equilibria for all feasible pairs (μ_1, μ_2), holding the remaining parameters constant. For the vast majority of parameterizations there are two equilibria. In Figure 8 we plot for each pair (μ_1, μ_2) the set of discretionary equilibria as a point in $\Re^2$. The abscissa is the low-inflation equilibrium, and the ordinate is the high-inflation equilibrium. The only pairs (μ_1, μ_2) not displayed here are those for which there is no equilibrium or the Friedman rule is time consistent. Figure 8 makes clear that the set of discretionary equilibria spanned by all pairs is tightly clustered around a downward-sloping locus. We have two degrees of freedom in μ_1 and μ_2, yet the level of the low-inflation equilibrium pins down the high-inflation equilibrium in a very small interval.

In short, when our stylized model is calibrated to match the average inflation in the United States over the past 20 years, an additional inflation equilibrium arises around a 10.5% inflation rate, a level consistent with the U.S. high-inflation experience in the 1970s. We find that all parameterizations with a low-inflation equilibrium between 2% and 2.5% also have a high-inflation equilibrium in the interval 10%–11%. Hence the model provides an account of U.S. inflation in the 1970s and the 1990s. During the period 1973–1983, inflation rates peaked at 10.5% for the GDP deflator, 13.4% for the CPI, and 10.8% for the personal consumption expenditures (PCE). Averages over the period are all above 7%.

This feature of the model is quite robust. Figure 9 reproduces Figure 8 for different values of parameter η. Values $\eta = 0.89$ to $\eta = 0.93$ cover the most widely

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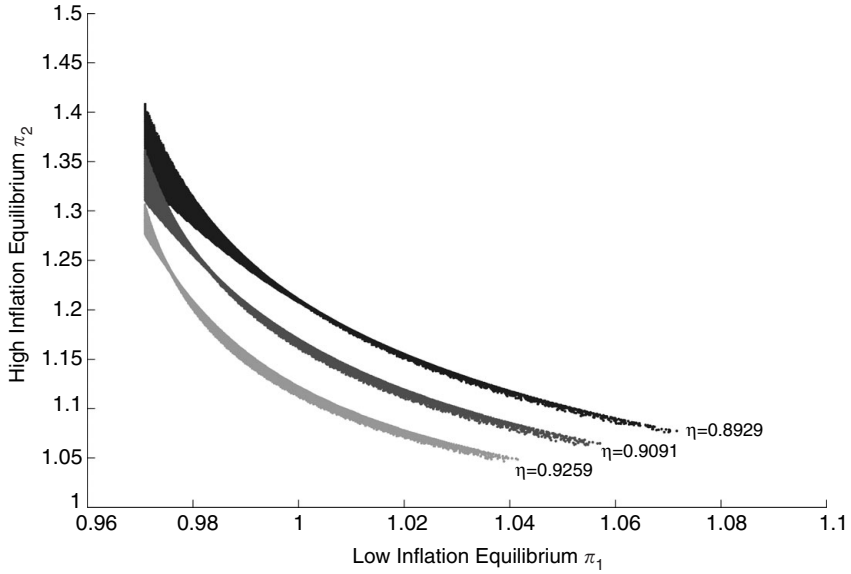


FIGURE 9. Discretionary equilibria: Robustness analysis.

accepted values for the markup, from 8% to 12%. In each case, a calibrated discretionary equilibrium in the range of 2%–2.5% implies an additional equilibrium with higher inflation. The range for the high-inflation equilibrium changes with η, but it stays within reasonable bounds, from 8% to 16%. We have also explored variations in the parameters in the labor supply, and the results remain unchanged. The robustness of our main result does not imply that discretionary equilibria are not sensitive to parameters μ_1 and μ_2.

6. CONCLUSIONS

This paper takes a step forward in exploring the hypothesis that the high inflation experienced by the United States in the 1970s was driven by expectations. We present an economy in which policy discretion gives rise to multiple discretionary equilibria. The model's match of the U.S. inflation experience is noteworthy. For any parameterization featuring an equilibrium inflation rate around 2%, there is an additional high-inflation equilibrium just above 10%. We find this property of the model to be robust.

The model is tractable enough to provide an insight into the economics behind expectation traps. The forces at work are quite general, and the equilibrium multiplicity does not rely on large nominal frictions. These results suggest that expectation traps are more than just a theoretical curiosity.

We acknowledge that the indeterminacy of inflation expectations is unsatisfactory. One possibility is to explore learning in the spirit of Marcet and Sargent (1989) and, more recently, Marcet and Nicolini (2003). Another possibility is

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to relax the common-knowledge assumption. Recent literature has successfully applied global games to several policy problems, as in Morris and Shin (1998), Morris and Shin (2004), Rochet and Vives (2004), and many others.

Expectation traps are specially relevant for applied monetary policy. For example, Armenter and Bodenstein (2005b) discuss exchange rate regimes in the presence of expectation traps: the equilibrium multiplicity makes a stronger case for a fixed exchange rate.

The possibility of a high-inflation equilibrium is a first-order concern for the policy maker. Credibility is not about getting the inflation rate right up to a tenth of a percent. It is about avoiding a 1970s encore.

NOTES

1. Clarida et al. (2000), page 149. See also Judd and Rudebusch (1998).
2. Christiano and Gust (2000), page 22.
3. Chari, Christiano, and Eichenbaum (1998) show that there exist Nash equilibria with trigger strategies that feature expectation traps in an infinite horizon economy. Albeit the term "expectation trap" was introduced in that paper, it is now standard to restrict attention to Markov-perfect equilibria, thereby ruling out trigger strategies.
4. However, the implementation of policy requires that we avoid an end-date condition for nominal deposits.
5. This in the spirit of the cashless economies discussed in Woodford (2003).
6. Dupor (2003) shows that optimal monetary policy with commitment may have a random component that can alleviate the monopoly distortion. This is not the case in this economy.
7. It is possible to allow firms to shut down or reset nominal prices if profits fall below some arbitrary level. A PSE would then exist for all $\pi \geq \beta$. Whether we allow for negative profits does not affect our results.
8. Existence of $\pi^*(\pi^e)$ follows from $u(c, n)$ being bounded above and the closure of the policy choice set previously discussed. However, the solution of (17) can be a correspondence. We will briefly get back to this possibility.
9. All figures in this section are based on our preferred calibration, which is detailed in Section 5.2.
10. We prove the proposition under a weaker requirement, namely that condition (18) needs to be satisfied only in a neighborhood of any critical point associated with (17).
11. The reader may wonder about the stability of the high-inflation equilibrium given that the policy response is upward sloping around it. We note, though, that the high-inflation equilibrium is unstable only to fully coordinated deviations of the agent's beliefs. It is stable, for example, to perturbations in the actions of the agents, like the nominal price set in advance.
12. Most estimates in the labor literature are well below 1; for example, see Altonji (1986). However, recent work has pointed out that estimates may be biased downward. See Domeij and Floden (2004) and references-herein.
13. We used data from the Federal Reserve Bank of St. Louis' FRED database.
14. This number is not far from the median estimates in the literature. For example, Bils and Klenow (2004) report a median duration of just below five months.
15. Again, this set can be closed without loss of generality following the same steps as in Claim 8.

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APPENDIX

A.1. CLOSURE OF THE POLICY CHOICE SET

CLAIM 8. *For any $\pi^e \geq \beta$, the policy choice set for (17) can be bounded from above by $\pi \leq \bar{\pi}(\pi^e) - \varepsilon$ for an arbitrarily small $\varepsilon > 0$ without any loss of generality. Moreover, it is never empty.*

Proof. For any $\pi^e \geq \beta$, consider a monotone sequence $\{\pi_j\}_{j=0}^{\infty}$ such that $\lim_{j\to\infty} \pi_j = \bar{\pi}(\pi^e)$. Let x_j denote the corresponding PSE outcomes associated with π_j given π^e. From (15) it is clear that $\{w_j\}_{j=0}^{\infty}$ is unbounded above, and so $\lim_{j\to\infty} n_j = 1$. Let $u_j = u(c_j, n_j)$. Because of the Inada conditions on $h(1-n)$, $\{u_{j-1} - u_j\}_{j=0}^{\infty}$ is unbounded below while note c is bounded above by 1. Hence, any $\pi > \bar{\pi}(\pi^e) - \varepsilon$ will not be a solution to (17).

To show that the set is not empty, note that

$$\begin{aligned}\bar{\pi}(\pi^e) &= \beta R(\pi^e, \pi^e)\left(\mu_1 + \mu_2 R(\pi^e, \pi^e)^{\frac{\eta}{\eta-1}} + \mu_3\right)^{\frac{1-\eta}{\eta}} \mu_1^{\frac{\eta-1}{\eta}} \\ &= \beta\left(\mu_1 R(\pi^e, \pi^e)^{\frac{\eta}{1-\eta}} + \mu_2 + \mu_3 R(\pi^e, \pi^e)^{\frac{\eta}{1-\eta}}\right)^{\frac{1-\eta}{\eta}} \mu_1^{\frac{\eta-1}{\eta}} > \beta\end{aligned}$$

using $R(\pi^e, \pi^e) \geq 1$, $\mu_1 + \mu_2 + \mu_3 = 1$,

$$\mu_1^{\frac{1-\eta}{\eta}} < 1,$$

which holds true as $\mu_1 < 1$ and $\eta < 1$. ∎

A.2. First-Order Welfare Changes

We drop the dependence on s for convenience. Solving the monetary authority problems in (17) we have the following first-order condition:

$$u^c \frac{dc}{d\pi} + u^n \frac{dn}{d\pi} = 0$$

as long as $\pi > \beta$. The binding case $\pi = \beta$ is relevant only if the Friedman rule is a discretionary equilibrium, and this can be easily ruled out with a parameter choice $\mu_1 > \mu_2$ as asserted by Proposition 7.

With some algebra, the previous expression can be rewritten in terms of wedges and elasticities. Using (10) and (11), we have that

$$\begin{aligned}\frac{dc}{d\pi} &= \sum_{i=1}^{3} \mu_i y^{1-\eta} y_i^{\eta-1} \frac{dy_i}{d\pi} = \sum_{i=1}^{3} \mu_i p_i^y \frac{dn_i}{d\pi}, \\ \frac{dn}{d\pi} &= \sum_{i=1}^{3} \mu_i \frac{dn_i}{d\pi},\end{aligned}$$

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so we can rewrite the necessary first-order condition, using $w = -u^n/u^c$, as

$$\sum_{i=1}^{3} \mu_i n_i \left(p_i^y - w\right) \xi_\pi^i = 0,$$

where

$$\xi_R^i = \frac{dn_i}{d\pi} \frac{\pi}{n_i}.$$

Expression (19) relates to the classic second-best theory. Because of monopolistic competition, there is a wedge between the price and marginal cost in every sector. Each wedge is weighted by the "size" of the sector, that is, $\mu_i n_i$, and then the production elasticity with respect to inflation. Each sector has a different elasticity. We show that for $P_1^y(\hat{\pi}) \leq \bar{P}_1^y$, $\varepsilon_\pi^2 < \varepsilon_\pi^3 < \varepsilon_\pi^1$.

We solve first for $dn_i/d\pi$. Using (9) and (6),

$$n_3 = y \left[\frac{1}{\eta} w\right]^{\frac{1}{\eta-1}};$$

therefore

$$\frac{dn_3}{d\pi} = \frac{dy}{d\pi} \left[\frac{1}{\eta} w\right]^{\frac{1}{\eta-1}} + \frac{1}{\eta - 1} y \left[\frac{1}{\eta} w\right]^{\frac{1}{\eta-1}} \frac{1}{w} \frac{dw}{d\pi}$$

or

$$\xi_\pi^3 = \xi_\pi^y - \frac{1}{1-\eta} \xi_\pi^w,$$

where

$$\xi_\pi^y = \frac{dy}{d\pi} \frac{\pi}{y},$$

$$\xi_\pi^w = \frac{dw}{d\pi} \frac{\pi}{w}.$$

Similarly for $i = 2$, we have

$$n_2 = y \left[\frac{1}{\eta} Rw\right]^{\frac{1}{\eta-1}}$$

and

$$\frac{dn_2}{d\pi} = \frac{dy}{d\pi} \left[\frac{1}{\eta} \pi w\right]^{\frac{1}{\eta-1}} + \frac{1}{\eta - 1} y \left[\frac{1}{\eta} \pi w\right]^{\frac{1}{\eta-1}} \frac{w + \pi \frac{dw}{d\pi}}{Rw}$$

or

$$\xi_\pi^2 = \xi_\pi^y - \frac{1}{1-\eta} \xi_\pi^w - \frac{1}{1-\eta}.$$

For industry $i = 1$,

$$n_1 = y \left[\frac{P^y}{\pi}\right]^{\frac{1}{\eta-1}};$$

hence

$$\frac{dn_1}{d\pi} = \frac{dy}{d\pi} \left[\frac{P^y}{\pi}\right]^{\frac{1}{\eta-1}} + \frac{1}{1-\eta} y \left[\frac{P^y}{\pi}\right]^{\frac{1}{\eta-1}} \frac{1}{\pi}$$

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or

$$\xi_{\pi}^{1} = \xi_{\pi}^{y} + \frac{1}{1-\eta}.$$

A.3. Monetary Authority Problem

A PSE outcome solves a system of nonlinear equations

$$G_{\pi^e}(y, w, n, \pi) = 0,$$

where $G_{\pi^e} : \Re^4 \to \Re^3$ for $\pi \in [\beta, \bar{\pi}(\pi^e))$. We have shown in Claim 8 that interval $[\beta, \bar{\pi}(\pi^e))$ can be closed without any loss of generality.

First we prove Proposition 5.

Proof. Note that n is an increasing function of w as given by (3). We proceed then to show that $dw(s)/d\pi > 0$. We can rewrite (15) as follows:

$$w = \eta \left[\frac{\mu_2 x^{-1} + \mu_3}{1 - \tilde{\mu}_1 x} \right]^{\frac{1-\eta}{\eta}},$$

where $x = R^{\frac{\eta}{1-\eta}}$ and $\tilde{\mu}_1 = \mu_1 (\frac{P_1^y}{\beta})^{\frac{\eta}{\eta-1}}$. Note that it is sufficient to establish that $(\mu_2 x^{-1} + \mu_3)/1 - \tilde{\mu}_1 x$ is increasing in x:

$$\frac{d}{dx} \left[\frac{\mu_2 + \mu_3 x}{x - \tilde{\mu}_1 x^2} \right] \geq 0,$$

$$\frac{\mu_3(x - \tilde{\mu}_1 x^2) - (\mu_2 + \mu_3 x)(1 - 2\tilde{\mu}_1 x)}{(x - \tilde{\mu}_1 x^2)^2} \geq 0,$$

$$\frac{\tilde{\mu}_1 \mu_3 x^2 + 2\tilde{\mu}_1 \mu_2 x - \mu_2}{(x - \tilde{\mu}_1 x^2)^2} \geq 0,$$

$$\tilde{\mu}_1 \mu_3 x^2 + 2\tilde{\mu}_1 \mu_2 x - \mu_2 \geq 0.$$

Note that $x - \tilde{\mu}_1 x^2 > 0$ is guaranteed because the policy choice set bounds. We are interested in a condition such that for all $x \geq 1$, labor is a monotone function of π. Since the left-hand side is increasing in x, this condition is

$$\tilde{\mu}_1 \mu_3 + 2\tilde{\mu}_1 \mu_2 - \mu_2 \geq 0,$$

$$\left(\frac{P^y}{\beta} \right)^{\frac{\eta}{\eta-1}} (1 - \mu_1 + \mu_2) \geq \frac{\mu_2}{\mu_1},$$

or

$$P^y \leq \beta \left[\frac{\mu_1}{\mu_2} (1 - \mu_1 + \mu_2) \right]^{\frac{1-\eta}{\eta}}.$$

■

Proposition 5 has an important implication. Because labor is a monotone function of π, we can think of labor being the policy instrument. This is possible only under the restriction $P_1^y(\pi^e) \leq \bar{P}_1^y$.

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Let

$$\upsilon(\pi^e, n) = \theta(\pi^e, n) + h(1-n),$$

where $\theta(\pi^e, n) = \{y = y(s) : \forall \pi | n(s) = n\}$. Under the condition $P^y(\pi^e) \leq \beta[\mu_1/\mu_2(1-\mu_1+\mu_2)]^{\frac{1-\eta}{\eta}}$, $\theta(\pi^e, n)$ is a differentiable function of n for $n \in [n(\pi^e, 1), 1)$ as implied by Proposition 5.[15] Hence the monetary authority's problem becomes

$$\max_{n \in [n(\pi^e,1),1)} \upsilon(\pi^e, n), \tag{A.1}$$

and the optimal policy decision $\pi^*(\pi^e)$ can be obtained using (3), (4), and (15).

All is set for the proof of Proposition 6.

Proof. We have already argued that θ is a differentiable function: it follows from Proposition 5. The necessary first-order condition associated with (A.1) is

$$\theta_n(\pi^e, n) - h'(1-n) = 0.$$

Let $\{n_i^*\}$ be the set of $n \in (n(\pi^e, 1), 1)$ that solve the first-order condition. If for all n^*,

$$\theta_{nn}(\pi^e, n^*) \leq -h''(1-n^*),$$

then υ has an interior local maximum at each critical point $\{n_i^*\}$. It follows that there is a unique critical point and the first-order condition is sufficient. In order to have (18), the first-order condition implies $\theta_n(\pi^e, n^*) = h'(1-n^*)$ and $n^* > 0$ because of the Inada conditions on h. ■

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