References

1. M. de Villiers, Feedback: More on hexagons with opposite sides parallel, *Math. Gaz*. **90** (November 2006) pp. 517-518.

   (An interactive, dynamic sketch illustrating the concurrency result mentioned at the start can be accessed online at:

   http://dynamicmathematicslearning.com/parahex.html)
2. H. Schumann. *Elementary geometry in a virtual area of action* (A book about teaching and learning with *Cabri* 3D on CD). (Schulgeometrie im virtuellen Handlungsraum. Ein Lehr-und Lernbuch der interaktiven Raumgeometrie mit Cabri 3D auf CD.) 2007, Hildesheim: Franzbecker.
3. M. de Villiers, The role and function of proof in mathematics. *Pythagoras* **24** (November 1990), pp. 17 -24.

10.1017/mag.2018.72

MICHAEL DE VILLIERS

Mathematics Education, University of Stellenbosch, South Africa

e-mail: *profmd@mweb.co.za*

HEINZ SCHUMANN

Mathematics Education, University of Education Weingarten, Germany

e-mail: *schumann@ph-weingarten.de*

102.31 Cyclic gridpoint polygons

Consider a standard circle, centre the origin, with equation $x^2 + y^2 = r^2$, where $x, y, r \in \mathbb{R}$ and $r \geqslant 0$. As r increases from 0, the circle passes through 'gridpoints', where x and y are both integers for specific values of r. Hereafter we shall confine our attention to these circles, which are given by the equation $x^2 + y^2 = k^2$, where x, y, k are integers and k is necessarily the largest number (and the length of the hypotenuse) in a primitive Pythagorean triple. For each value if $r = k$, we call the polygon formed by joining the gridpoints on the circle a 'cyclic gridpoint polygon' (CGP).

Dissect a CGP into strips parallel to one of the axes (say the y-axis). These are either triangles or trapezia. Because the vertices are gridpoints, the dimensions needed to calculate the area of each strip are integers. Furthermore, by symmetry about the x-axis, the lengths of their bases and tops (with reference to the formulae for the area of a triangle and a trapezium) are even. The area of each strip, therefore, and hence the area of the whole CGP, is integral. Let this be A_k for the case $r = k$.

By making r the product of an arbitrarily large number of primitive Pythagorean hypotenuses, $c_1, c_2, c_3, \ldots$ we can construct a CGP with as many sides as we wish. This is by virtue of the fact that the ratio $a : b$ of every primitive Pythagorean triple (a, b, k) is distinct and primitive Pythagorean triples are infinite in number. Figure 1 shows what happens within a sector defined by the gridpoints (4, 3) and (4, −3) as we increase the number of Pythagorean hypotenuses. In fact the set of CGP vertices on the outer circle is the union of all the sets of CGP vertices on the inner

https://doi.org/10.1017/mag.2018.73 Published online by Cambridge University Press

circles, projected onto it. As the polygon grows, new gridpoints (shown circled in Figure 1), and the sides joining them, appear within existing segments and hence increase the fraction of the circle which the polygon occupies.

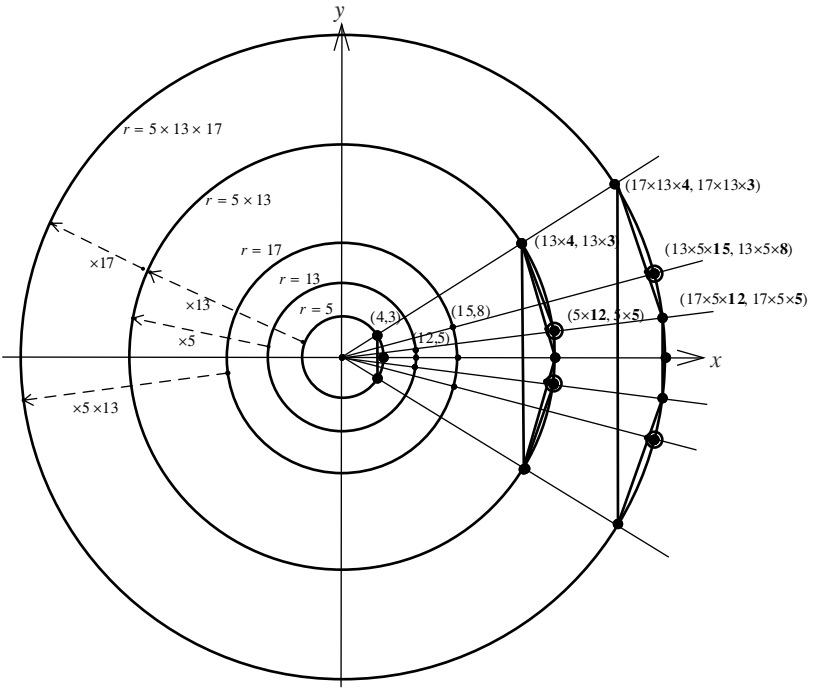


FIGURE 1

As $r = c_1 \times c_2 \times c_3 \times \ldots \to \infty$, $\dfrac{A_k}{\pi r^2} \to 1$, or $\dfrac{A_k}{r^2} \to \pi$. This is a geometric demonstration of the fact that we can find a rational number which is arbitrarily close to π.

10.1017/mag.2018.73

PAUL STEPHENSON
Böhmerstraße 66, 45144 Essen, Germany
e-mail: *pstephenson1@me.com*

102.32 Panning for gold in the streams of a regular pentagon

The golden ratio has fascinated mathematicians for centuries and makes a surprising appearance in number theory and geometry. The proportion is used in architecture, photography, artistic compositions and music, and can be seen throughout nature. In fact, the golden ratio fingerprint is observed with such regularity that many consider it part of the grand mosaic of the universe. The Egyptians applied the golden ratio when designing the

https://doi.org/10.1017/mag.2018.73 Published online by Cambridge University Press