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Waves in a 1D electrorheological dusty plasma lattice

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The behavior of waves in a one-dimensional (1D) dusty plasma lattice where the dust interacts via Yukawa and electric dipole interactions is discussed theoretically. This study is motivated by recent reports on electrorheological dusty plasmas (e.g. Ivlev et al. 2008 *Phys. Rev. Lett.* **100**, 095 003) where the dipole interaction arises due to an external uniaxial AC electric field that distorts the Debye sphere surrounding each grain. Application to possible dusty plasma experimental parameters is discussed.

1. Introduction

Dusty plasmas are plasmas containing small charged solid particulates, or dust grains. The interaction between negatively charged dust grains in low temperature dusty plasmas is generally represented by a screened Coulomb (Yukawa) interaction, with the screening provided by the plasma ions and electrons (Shukla and Mamun 2002; Fortov et al. 2005a). However, since the grains typically reside in plasma sheaths in laboratory experiments where there is an electric field, the grains are subject to other forces. While the electrostatic force due to the sheath electric can levitate the grains, the ions flow along this electric field and can focus downstream of the grains generating wake fields that lead to attractive forces between grains (see e.g. Shukla and Eliasson 2009). Recently, Ivlev et al. (2008) (see also Kompaneets et al. 2009; Ivlev et al. 2010) have proposed using an external AC electric field to generate symmetric wakes around grains. The idea is to use an AC electric field with frequency larger than the dust plasma frequency, but smaller than the ion plasma frequency, so only the ions react to the field (the electrons being a homogeneous neutralizing background). This leads to a distortion of the Debye spheres resulting in an attractive dipole interaction at ranges on the order of Debye radius, which adds to the Yukawa interaction. Such plasmas are referred to as electrorheological complex (dusty) plasmas (Ivlev et al. 2008, 2010). When the ion flow speed is smaller than the ion thermal speed, the attractive dipole interaction arises from the interaction between the charge of one grain and the quadrupole part of the ion-flow induced wake of the other grain, and can be described as an effective dipole–dipole interaction (Ivlev et al. 2008). The first observation of an electrorheological dusty plasma under the action of uniaxial polarized AC electric fields in microgravity experiments was reported by Ivlev et al. (2008), where the dust structure transitioned from an isotropic to an anisotropic (string) structure as the magnitude of the external electric field increased.

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Motivated by the possibility of strongly coupled electrorheological dusty plasmas (see Ivlev et al. 2010), we theoretically consider the dispersion relation of waves in a simple model comprising a 1D dusty plasma lattice where the charged grains interact via a Yukawa interaction and a dipole interaction due to the action of an AC uniaxial electric field. Prior studies on the effects of dipole interactions on dust lattice waves have considered the effects of each grain's electric dipole moment (e.g. Murillo and Rosenberg 1998; Wang and Wang 2006), or magnetic dipole moment (e.g. Yaroshenko et al. 2004; Feldmann et al. 2006). In addition, the effects of asymmetric grain-wake interactions on dust lattice waves have been considered (e.g. Vladimirov et al. 1998; Ivlev and Morfill 2000; Rocker et al. 2014). Also, recently the effect of various types of attractive shadow potential interactions on dust lattice waves have been discussed with application primarily to terrestrial experiments (Verma et al. 2013).

This note is organized as follows. Section 2 derives the the dispersion relation for the waves, and Sec. 3 discusses possible dusty plasma parameters with application to microgravity conditions. Section 4 gives a brief summary.

2. Wave dispersion relations

The model weakly ionized plasma contains electrons, singly charged ions, and negatively charged dust grains with uniform radius R, mass $m_{\rm d}$ and charge $Q = -Z_{\rm d}e$ where e is the elementary charge. There is an external uniaxial AC electric field imposed on the plasma that induces ion flows that lead to symmetric wakes on opposite sides of each grain. The dust grains in our model dusty plasma are assumed to interact via a Yukawa interaction,

$$\phi_Y = \frac{Q^2}{r}\exp\left(-\frac{r}{\lambda}\right), \tag{1}$$

and an effective dipole interaction given by Ivlev et al. (2008)

$$\phi_D = -0.43\frac{Q^2}{r}\,\frac{M_{\rm th}^2\lambda^2}{r^2}(3\cos^2\theta - 1). \tag{2}$$

Here, λ is the plasma screening length, $M_{\rm th}$ is the thermal Mach number of the ions (i.e. the ratio of the ion flow speed to the ion thermal speed) which is assumed to be small, and θ is the angle between the external electric field $\mathbf{E}$ and the vector $\mathbf{r}$ connecting the two grains. We are interested in longitudinal waves propagating along the dipole axis in the 1D system where $\theta = 0^\circ$.

We consider a 1D lattice in the x-direction composed of dust grains with uniform spacing a which we assume to be fixed. (The actual value of a may vary with the external electric field $\mathbf{E}$ which is along the x-direction, but this equilibrium analysis is left for further work). We consider a system with screening parameter $\kappa = a/\lambda \geqslant 1$ so that taking into account only nearest-neighbor interactions for the Yukawa interaction may be a reasonable approximation (see Melandsoe 1996). In this case, the dispersion relation for waves in a 1D lattice can be given by (Kittel 1971; McKelvey 1993)

$$\omega = 2\left(\frac{1}{m_{\rm d}}\frac{d^2\phi_{\rm Tot}(a)}{dx^2}\right)^{1/2}\left|\sin\frac{ka}{2}\right|. \tag{3}$$

Here $\phi_{\rm Tot} = \phi_Y + \phi_D$. The relation (3) can be rewritten as

$$\omega = 2\left(\frac{2Q^2}{m_{\rm d}a^3}\right)^{1/2} S^{1/2}\left|\sin\frac{ka}{2}\right|. \tag{4a}$$

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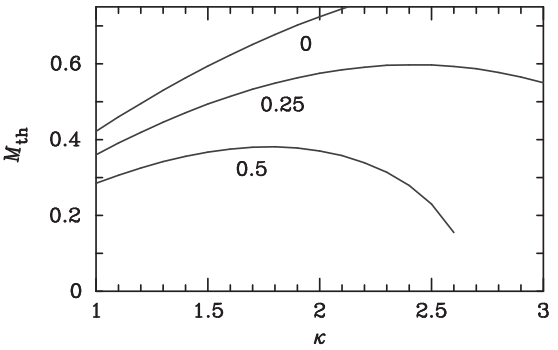


FIGURE 1. Contour plot of the quantity S in (4) as a function of κ and M_{th}.The value of S labels the curves.

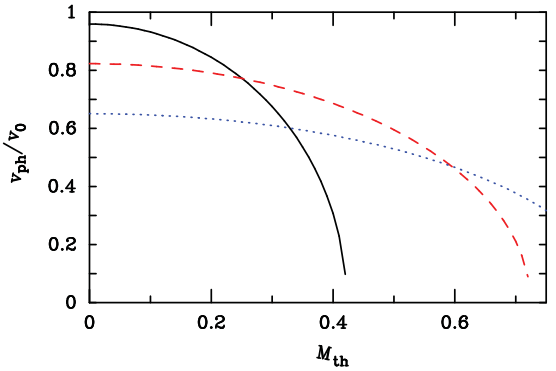


FIGURE 2. Phase speed of the lattice wave (normalized to v_0) as a function of M_{th} for $\kappa = 1$ (black, solid curve), $\kappa = 2$ (red, dashed curve) and $\kappa = 3$ (blue, dotted curve).

with

$$S = e^{-\kappa}\left(1 + \kappa + \frac{\kappa^2}{2}\right) - \frac{258}{50}\frac{M_{th}^2}{\kappa^2}. \tag{4b}$$

Thus, for fixed a, the effect of the induced dipole interaction is to reduce the wave frequency and sound speed. We note that (4) has a similar form to the dispersion relation (25) obtained by Verma et al. (2013) for an intergrain potential comprised of Yukawa and dipole–dipole terms.

From (4b), note that there is a constraint on the allowed values of κ and M_{th} in order that $S > 0$. The latter condition corresponds to a situation where the interaction between two grains is repulsive which allows for a stable equilibrium. Figure 1 shows a contour plot of S as a function of κ and M_{th}. For smaller $\kappa \sim 1$ the region where S is positive corresponds to $M_{th} < 0.4$, while for larger κ this region extends to larger M_{th}. Note, however, that the expression (2) for the dipole–dipole interaction was given under the condition of small M_{th} (Ivlev et al. 2008, 2010).

In the long wavelength limit where $ka \ll 1$, the phase speed of the wave, $v_{ph} = \omega/k$, can be obtained from (4) as

$$\frac{v_{ph}}{v_0} \approx S^{1/2}, \tag{5}$$

where $v_0 = (2Q^2/m_d a)^{1/2}$. This is plotted in Fig. 2 as a function of M_{th} for various values of κ (for parameter regions below the $S = 0$ curve in Fig. 1).

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3. Possible dusty plasma parameters

As can be seen from Fig. 2, the phase speed of the 1D dust lattice wave in this model decreases as a function of $M_{\rm th}$, with a substantial decrease for the smaller $\kappa \sim 1$ case. If it were possible to impose a uniaxial AC electric field on a dusty plasma in the crystalline phase, and excite dust lattice waves along the field (perhaps using laser excitation (Homann et al. 1998)), it might be possible to determine $M_{\rm th}$ by measuring the phase speed of the dust lattice wave, assuming that κ could be estimated (e.g. from imaging the dust). In addition, if it were possible to measure the maximum frequency of the wave (where $ka = \pi$, from (4)), that frequency would also decrease with $M_{\rm th}$ in the same way as the sound speed. We note that measurements of the dispersion of laser-excited dust lattice waves have been used to infer the plasma screening length (Homann et al. 1998), so this is a variation on that theme.

Electrorheological dusty plasma experiments have been reported under microgravity conditions (Ivlev et al. 2008) and have been studied in numerical simulations (e.g. Ivlev et al. 2010; Totsuji et al. 2014). Under microgravity conditions, the electric fields in the plasma can have smaller magnitude than the electric fields needed to levitate grains in terrestrial experiments, so ion flow speeds could be subthermal. We consider a set of dusty plasma parameters that may fall in the range of representative PK-4 microgravity experimental parameters (see Fortov et al. 2005b; Khrapak et al. 2005). These are: neon gas of pressure $\sim$5 Pa, neon ion density $n_{\rm i} \sim 1 \times 10^8$ cm^{-3}, ion temperature $\sim$0.03 eV, electron temperature $T_e \sim$ 5–7 eV, dust (melamine-formaldehyde) of radius $R \sim 0.65$ μm, and dust density $n_{\rm d} \sim 5 \times 10^4$ cm^{-3}. With these parameters, we estimate the negative dust charge to be about $Q \sim -2000\ e$, using orbit-motion-limited along with electron depletion estimates (see Shukla and Mamun 2002; Khrapak et al. 2005). The dust mass $m_{\rm d}$ is estimated to be $\sim 10^{12}$ times the proton mass, using a mass density of $\sim$1.5 g cm^{-3}. Thus the dust plasma frequency, $\omega_{\rm pd} = (4\pi n_{\rm d} Q^2/m_{\rm d})^{1/2}$ is $\sim$590 rad s^{-1}. Also using these parameters, the plasma screening length is approximately the ion Debye length, $\lambda \sim 0.013$ cm, while the average distance between grains given by the Wigner–Seitz radius $a = (3/4\pi n_{\rm d})^{1/3}$ is $a \sim 0.017$ cm, implying that $\kappa \sim 1.3$. If we assume the dust (kinetic) temperature $T_{\rm d} \sim T_{\rm i}$, the Coulomb coupling parameter between neighboring grains, $\Gamma = Q^2/aT_{\rm d} \sim 1100$, so that the Yukawa coupling parameter would be roughly $\Gamma e^{-\kappa} \sim 300$ which would correspond to a crystalline phase for a Yukawa system (see e.g. Hamaguchi et al. 1997).

The phase speed of a 1D dust lattice wave would be given by expression (5), where, using the above parameters, $v_0 \sim 8.1$ cm s^{-1}. The phase speed would decrease from about $\sim$0.93 v_0 when $M_{\rm th} = 0$ to about $\sim$0.6 v_0 as $M_{\rm th}$ increases to 0.4. The long wavelength limit $ka/2 \ll 1$ corresponds to wavelengths $\gg$0.5 mm for the above parameters. However, note that there is an upper bound on the wavelength range of the dispersion curve that might be useful for estimating $M_{\rm th}$. This is due to damping of the wave by dust-neutral collisions. The dust-neutral collision rate is roughly $\nu_{\rm dn} \sim (8\sqrt{2\pi}/3) R^2 n_{\rm n} v_{\rm n} m_{\rm n}/m_{\rm d}$ (Khrapak et al. 2004), where $n_{\rm n}$, $v_{\rm n}$ and $m_{\rm n}$ are the number density, thermal speed, and mass, respectively, of the neutrals. To avoid strong collisional damping of the waves, the condition $\omega > \nu_{\rm dn}$ should hold. Because ω decreases as $M_{\rm th}$ increases, the wavelength region where $\omega > \nu_{\rm dn}$ shrinks as $M_{\rm th}$ increases. As an example, for the parameters considered above, we estimate $\nu_{\rm dn} \sim 25$ s^{-1}. When $M_{\rm th} \sim 0.4$, waves with wavenumber $ka > 0.2$ (wavelength $<$5 mm), would have frequency $>$60 rad s^{-1} which is larger than $\nu_{\rm dn}$.

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4. Summary

The behavior of dust lattice waves in a 1D dusty plasma lattice where the dust interacts via a Yukawa and an electric dipole interaction was discussed theoretically. This was motivated by recent studies of electrorheological dusty plasmas (Ivlev et al. 2008; Kompaneets et al. 2009; Ivlev et al. 2010), where an effective dipole interaction can arise due to an external uniaxial AC electric field that induces ion-flow induced wakes that distort the Debye sphere surrounding each grain. We showed how the frequency and sound speed of the dust lattice wave decreases as the ion flow speed increases. It was proposed that if the dusty plasma screening parameter can be estimated (e.g. from imaging the dust), measurement of the wave dispersion might help determine the ion flow speed. Application to possible dusty plasma experimental parameters under microgravity conditions were discussed.

Future work should include studies of how the lattice spacing may vary as the magnitude of the external electric field varies, and a 3D treatment of the system including analysis of wave propagation oblique to the external electric field direction. In addition, the analysis should be extended to the strongly coupled liquid phase, which may be applicable to microgravity experimental conditions (see Khrapak et al. 2005; Fortov et al. 2005b; Ivlev et al. 2008).

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