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Nonlinear dust–ion acoustic waves in a dusty plasma with non-extensive electrons and ions

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Abstract. Using the reductive perturbation approach, dust–ion acoustic solitons and double layers (DLs) have been studied in a dust–electron–positron–ion (d-e-p-i) plasma composed of q-distributed electrons and positrons, warm fluid ions, and a fraction of immobile dust grains. Existence domains of either solitary waves or DLs are presented and their parametric dependence determined. It is found that particle non-extensivity, dust concentration and positron concentration may drastically affect these existence domains and may play a key role in defining the polarity of these localized structures. Our results should assist in interpreting the nonlinear structures that may occur in astrophysical environments.

1. Introduction

Plasmas are known to support a wide variety of low- and high-frequency wave modes (Chen 1974). One of the most well studied of such modes is the so-called ion–acoustic wave (IAW) which arises due to the restoring force provided by the electron thermal pressure while the inertia is due to the ion mass. Since the influential and pioneering work of Sagdeev (1966), a vast body of literature has sprung up and the nature of the nonlinear IAWs has been studied both theoretically and experimentally (Ikezi 1973; Tran 1979). Theoretical analyses have been derived on the basis of the Sagdeev quasi-potentials or the reductive perturbation theory, leading to dynamical systems being described by the Korteweg–de Vries (K-dV) or derivative nonlinear Schrödinger type equations (Washimi and Taniuti 1966; Su and Gardner 1969). During the last two decades, collective waves in dusty plasmas have attracted a great deal of attention (Verheest 2000; Shukla and Mamun 2002). A dusty plasma is a normal electron–ion plasma with an additional highly charged component of small micron- or submicron-sized extremely massive charged particulates (dust grains). Dust and plasma coexist in a wide variety of cosmic and laboratory environments. Such mixtures of electrons, ions and charged grains distinguish themselves from ordinary plasmas in various ways. Because of the high charge and large mass of the grain particulates compared to those of ions, new time and space scales come into play, giving rise to either new or modified waves and instabilities in the dusty plasmas (Rao et al. 1990; Shukla and Silin 1992). Among the host of modified dusty modes discussed in the literature, the dust–ion acoustic wave (DIAW) (Shukla and Silin 1992) has received wide attention as well as experimental confirmation (Merlino et al. 1998).

It is well known that the presence of grain particulates may give rise to either new or modified waves in the dusty plasmas. It has been shown that the dust dynamics introduces new eigenmodes, such as dust acoustic (DA) mode (Rao et al. 1990) (weak coupling regime), dust lattice (DL) mode (Melandso 1996) (strong coupling regime) and dust Bernstein–Greene–Kruskal modes (Tribeche et al. 2000). On the contrary, it has been shown that the presence of static charged dust grains modifies the existing plasma wave spectra. Shukla and Silin (1992) first theoretically showed that due to the conservation of equilibrium charge density and the strong electron depletion, a dusty plasma with negatively charged static dust grains supports low-frequency DIAWs. The latter is the usual ion-acoustic mode modified by the presence of dust particles. In contrast to the DA waves, the frequency of the DIAWs is much larger (smaller) than the dust (ion) plasma frequency, whereas their phase velocity is much larger (smaller) than the ion and dust (the electron) thermal velocity. That is why the dust grains are taken immobile when dealing with DIA oscillations.

With more empirical data becoming available, it is now realized that the Maxwellian is not a realistic distribution under all circumstances (Christon et al. 1988; Maksimovic et al. 1997; Leubner 2004). Various observations of fast ions and electrons in space environments clearly indicate that such particles have non-Maxwellian velocity distributions exhibiting non-thermal high-energy tails or flat tops with pronounced shoulders (Asbridge et al. 1968; Divine and Garret 1983; Krimigis et al. 1983; Lundlin et al. 1989; Futaana et al. 2003). In fact, a plasma may be subject to a whole variety of different effects, such as spatial variation of physical quantities (e.g. density and temperature),

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background turbulence and external force fields, which can contribute to the appearance of non-thermal distribution functions (Treumann 1999). When a non-Maxwellian distribution function develops into a plasma, the IA mode is modified, as there is a change in the restoring force and/or inertia that controls the oscillation underpinning the wave.

During the past two decades, it has been proved that systems which present long-range interactions, long-time memory, fractality of the corresponding space–time/phase space or intrinsic inhomogeneity are untractable within the conventional Boltzmann–Gibbs (BG) statistics (for detailed reviews see Tsallis 1994, 1995). The main reason for this failure is that BG statistics is an additive or extensive formalism. In dealing with the statistical properties of systems with long-range correlations, Tsallis (1988) consistently extended BG thermodynamics by generalizing the concept of entropy to non-extensive regimes. Non-extensivity means that the entropy of the composition $(A + B)$ of two independent systems A and B is equal to $S_q^{(A+B)} = S_q^{(A)} + S_q^{(B)} + (1-q)\,S_q^{(A)}S_q^{(B)}$, where the parameter q that underpins the generalized entropy of Tsallis is linked to the underlying dynamics of the system and provides a measure of the degree of its correlation. Among the various physical systems where connections with the Tsallis entropy have been found are plasma physics (Lima et al. 2000; Muñoz 2006), long-range Hamiltonian systems (Latora et al. 2001), gravitational systems (Taruya and Sakagami 2003), and many other applications (Abe and Okamoto 2001). As noted by Lima et al. (2000), the q-non-extensive formalism may be very important for systems endowed with long-range interactions such as astrophysics and plasma physics. In this connection, Liu et al. (1994) and Lima et al. (2000) showed that plasma experimental results point to the non-Maxwellian velocity distribution. Since then, the Tsallis q-entropy and the ensuing generalized statistics have been employed with some success in plasma physics (Du 2004; Liu and Goree 2008; Liyan and Du 2008; Liu and Du 2009; Liu et al. 2009; Amour and Tribeche 2010; Tribeche and Djebarni 2010; Tribeche et al. 2010; Ait Gougam and Tribeche 2011a, b; Amour and Tribeche 2011; Bains et al. 2011a, b; Tribeche and Merriche 2011).

The aim of this paper is to study the propagation of nonlinear DIAWs in a plasma consisting of warm fluid ions, non-extensive electrons and positrons, and a fraction of immobile dust grains. The electron–positron (e-p) plasma is believed to exist in a pulsar magnetosphere (Ginzburg 1971; Sturrock 1971; Ruderman and Sutherland 1975; Michel 1982; Michel 1991), bipolar outflows, active galactic nuclei (Begelman et al. 1984; Miller and Witta 1987) and the early universe (Gibbons et al. 1983). Though the dominant constituents of these astrophysical plasma are electrons and positrons, a small fraction of ions and dust grains may also be present, forming therefore a pair–ion–dust (e-p-i-d) plasma system.

2. Theoretical model

Let us consider a collisionless, unmagnetized plasma having warm positive fluid ions, non-extensive electrons and positrons, and a fraction of immobile dust grains of densities n_i, n_e, n_p and n_d, respectively. The nonlinear dynamics of the DIAWs, whose phase speed is much smaller (larger) than the electron and positron (ion) thermal velocities, is governed by the following normalized equations:

$$\frac{\partial N_i}{\partial T} + \frac{\partial}{\partial X}(N_i V_i) = 0, \tag{1}$$

$$\frac{\partial V_i}{\partial t} + V_i\frac{\partial V_i}{\partial X} + \frac{\partial \Psi}{\partial X} + 3\sigma_i N_i\frac{\partial N_i}{\partial X} = 0, \tag{2}$$

$$\frac{\partial^2 \Psi}{\partial X^2} = N_e - N_p - N_i + \alpha, \tag{3}$$

with (Ait Gougam and Tribeche 2011a)

$$N_e = \mu\left[1 + (q_e - 1)\Psi\right]^{\frac{1}{q_e-1}+\frac{1}{2}}, \tag{4}$$

$$N_p = \nu\left[1 - (q_p - 1)\sigma_p\Psi\right]^{\frac{1}{q_p-1}+\frac{1}{2}}. \tag{5}$$

The subscript $j = i, e, p, d$ denotes ions, electrons, positrons and dust grains, respectively. The parameters q_e and q_p denote the strength of electron and positron non-extensivity. Here and in the following, m_j are the masses, T_j the temperatures, $\sigma_i = T_i/T_e$, $\sigma_p = T_e/T_p$, $\mu = n_e^{(0)}/n_i^{(0)}$, $\nu = n_p^{(0)}/n_i^{(0)}$, and $\alpha = Z_d n_d^{(0)}/n_i^{(0)}$, where Z_d is the dust number charge and $n_j^{(0)}$ is the perturbed density of particles of species j. In (1)–(5), the densities N_j, the ion fluid velocity V_i and the electrostatic potential Ψ are normalized, respectively, by the unperturbed ion density $n_i^{(0)}$, the ion-acoustic speed $C_{si} = (k_B T_e/m_i)^{1/2}$ and the thermal potential $k_B T_e/e$. The time T and the space X variables are normalized by the ion Debye length $\lambda_{Di} = (k_B T_e/4\pi e^2 n_i^{(0)})^{1/2}$ and the ion plasma period $\omega_{pi}^{-1} = (4\pi e^2 n_i^{(0)}/m_i)^{-1/2}$, respectively. k_B is the Boltzmann constant and e is the magnitude of the electron charge. The charge neutrality condition at equilibrium is $\mu + \alpha = 1 + \nu$.

To study small but finite amplitude DIAWs, we employ the reductive perturbation method (Washimi and Taniuti 1966). According to the latter, we introduce the stretching space–time coordinates

$$\xi = \varepsilon(X - \lambda T) \quad \text{and} \quad \tau = \varepsilon^3 T, \tag{6}$$

where the small parameter ε is to be regarded as a measure of the strength of the nonlinearity and λ is the linear wave phase velocity normalized by C_{si}. Moreover, all physical quantities appearing in (1)–(5) are expanded as power series in ε around the equilibrium state as follows:

$$F = F^{(0)} + \sum_{m=1}^{\infty}\varepsilon^m F^{(m)}, \tag{7}$$

where $F = (N_i, V_i, \Psi)$ and $F^{(0)} = (1, 0, 0)$. Introducing the variables (6) and the expansion (7) into (1)–(5), the

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lowest order in ε gives

$$N_i^{(1)} = \frac{1}{(\lambda^2 - 3\sigma_i)}\Psi^{(1)} \quad \text{and} \quad V_i^{(1)} = \frac{\lambda}{(\lambda^2 - 3\sigma_i)}\Psi^{(1)} \tag{8}$$

while the Poisson equation provides the compatibility condition

$$\mu\frac{(q_e+1)}{2} + \nu\frac{\sigma_p(q_p+1)}{2} - \frac{1}{(\lambda^2 - 3\sigma_i)} = 0. \tag{9}$$

To the next order in ε, we get

$$V_i^{(2)} = \frac{\lambda(\lambda^2 + 9\sigma_i)}{2(\lambda^2 - 3\sigma_i)^3}[\Psi^{(1)}]^2 + \frac{\lambda}{(\lambda^2 - 3\sigma_i)}\Psi^{(2)}, \tag{10}$$

$$N_i^{(2)} = \frac{3(\lambda^2 + \sigma_i)}{2(\lambda^2 - 3\sigma_i)}[\Psi^{(1)}]^2 + \frac{1}{(\lambda^2 - 3\sigma_i)}\Psi^{(2)}, \tag{11}$$

and the Poisson equation gives

$$\left[\mu\frac{(q_e+1)}{2} + \nu\frac{\sigma_p(q_p+1)}{2} - \frac{1}{(\lambda^2 - 3\sigma_i)}\right]\Psi^{(2)} = \left[-\frac{\mu}{4}(q_e+1)(3-q_e) + \frac{\nu}{4}\sigma_p^2(q_p+1)(3-q_p) + \frac{3(\lambda^2+\sigma_i)}{2(\lambda^2-3\sigma_i)^3}\right][\Psi^{(1)}]^2. \tag{12}$$

In (12), the coefficient of $\Psi^{(2)}$ is identically zero, due to the compatibility condition (9). Since $\Psi^{(1)} \neq 0$, the coefficient of $[\Psi^{(1)}]^2$ could be at least of the order of ε and, therefore, (12) becomes of the order of ε^3; so it should be included in the next order of the Poisson equation. The next order in ε gives a system of equations and making use of the first-order and second-order solutions along with (12), we finally obtain the following modified K-dV (mKdV) equation:

$$\frac{\partial\Phi}{\partial\tau} + (AB\Phi + AC\Phi^2)\frac{\partial\Phi}{\partial\xi} + \frac{1}{2}A\frac{\partial^3\Phi}{\partial\xi^3} = 0, \tag{13}$$

where

$$A = \left[\frac{\lambda}{(\lambda^2 - 3\sigma_i)^2}\right]^{-1}, \tag{14}$$

$$B = -\frac{\mu}{2}(q_e+1)(3-q_e) + \frac{\nu\sigma_p^2}{2}(q_p+1)(3-q_p) + \frac{3(\lambda^2+\sigma_i)}{(\lambda^2-3\sigma_i)^3}, \tag{15}$$

$$C = \frac{3}{4}\left[\frac{5\lambda^4 + 30\lambda^2\sigma_i + 9\sigma_i^2}{(\lambda^2-3\sigma_i)^5} - \frac{\mu}{24}(q_e+1)(3-q_e)(5-3q_e) + \frac{\nu\sigma_p^3}{24}(q_p+1)(3-q_p)(5-3q_p)\right], \tag{16}$$

where we introduce $\Psi^{(1)} = \Phi$ for the sake of simplicity. To obtain the stationary solution of (13), we introduce the transformation $\eta = \xi - U\tau$, where U is a constant velocity normalized by C_{si}. Making use of the following appropriate boundary conditions, viz. $\Phi \to 0$ and $d\Phi/d\eta \to 0$ at $|\eta| \to \infty$, (13) can be integrated to yield the energy integral

$$\frac{1}{2}\left(\frac{d\Phi}{d\eta}\right)^2 + V(\Phi) = 0, \tag{17}$$

where the pseudo-potential

$$V(\Phi) = -\frac{U}{A}\Phi^2 + \frac{1}{3}B\Phi^3 + \frac{1}{6}C\Phi^4 \tag{18}$$

is known as the Sagdeev potential (Sagdeev 1966).

2.1. Solitary wave solution

Looking first at the solitary wave solution, we integrate (17) and use the appropriate boundary conditions for localized perturbations ($\Phi \to 0$ and $d\Phi/d\eta \to 0$ at $|\eta| \to \infty$) to obtain

$$\Phi = \frac{6U}{AB}\left[1 + \sqrt{\Delta}\cosh\left(\sqrt{\frac{2U}{A}}\eta\right)\right]^{-1}, \tag{19}$$

where

$$\Delta = \left(1 + \frac{6UC}{AB^2}\right). \tag{20}$$

The maximum amplitude $\Phi_{\max}$ of the solitary pulse is given by

$$\Phi_{\max} = \frac{6U}{AB}\left(1 + \sqrt{\Delta}\right)^{-1}. \tag{21}$$

2.2. Double-layers solution

Double layers (DLs) solutions of (17) exist if (i) $V(0) = V(\Phi_m) = 0$, (ii) $\left(dV/d\Phi\right)_{\Phi=0} = \left(dV/d\Phi\right)_{\Phi=\Phi_m} = 0$ and $\left(d^2V/d\Phi^2\right)_{\Phi=0,\Phi_m} < 0$, where 0 and Φ_m are two extreme points of the Sagdeev potential $V(\Phi)$. Applying these conditions, we obtain

$$U = -\frac{AC}{6}\Phi_m^2 \quad \text{and} \quad \Phi_m = -\frac{B}{C}. \tag{22}$$

Substituting (22) into (18), we get

$$V(\Phi) = \frac{C}{6}(\Phi_m - \Phi)^2\Phi^2. \tag{23}$$

The DLs solution of (17) is therefore given by

$$\Phi = \frac{1}{2}\Phi_m\left[1 \pm \tanh\left(\frac{\eta}{W_{DL}}\right)\right], \tag{24}$$

where

$$W_{DL} = \frac{\sqrt{-12/C}}{|\Phi_m|} \tag{25}$$

denotes the DLs width.

3. Numerical results

Before proceeding further, note that the propagation of solitary waves or DLs is subject to certain conditions that are imposed, respectively, by (20) and (25). More precisely, the positivity of Δ ensures that the solitary

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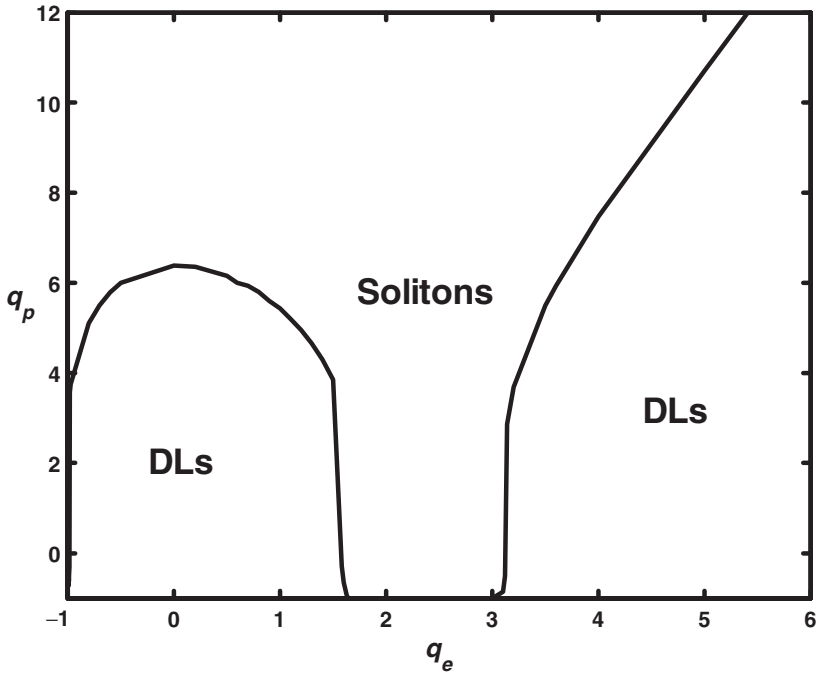


Figure 1. Existence domains of the solitary waves and double layers corresponding to the conditions $C > 0$ and $C < 0$ [represented by (16)] in the q_e–q_p plane, with $\sigma_i = 0.8$, $\sigma_p = 0.5$, $v = 0.01$ and $\alpha = 0.98$.

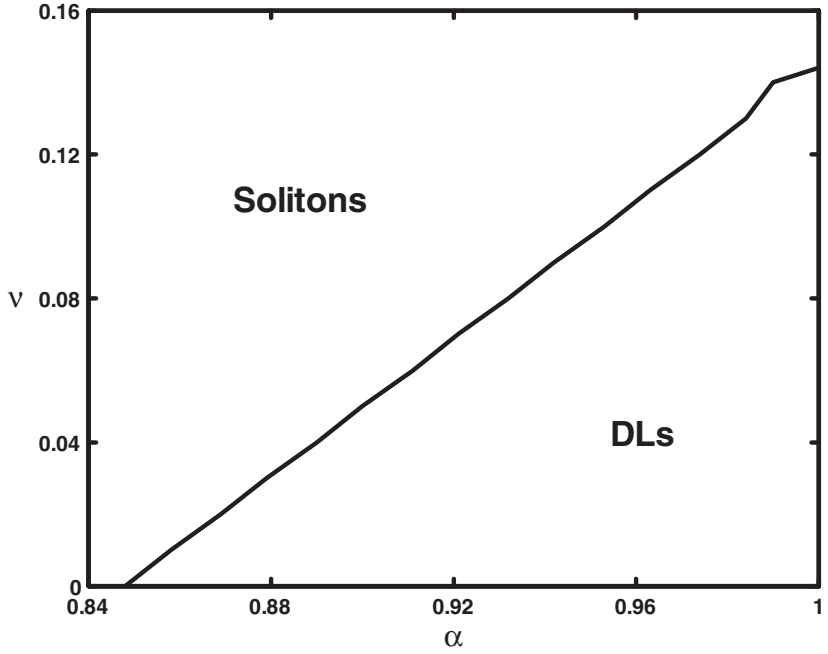


Figure 2. Existence domains of the solitary waves and double layers corresponding to the conditions $C > 0$ and $C < 0$ [represented by (16)] in the α–v plane, with $q_e = 5$, $q_p = 1.1$, $\sigma_i = 0.1$ and $\sigma_p = 0.1$.

wave will propagate, whereas the nonlinear coefficient C should be negative for the existence of propagating DLs. Next, relation (20) is investigated numerically to delimit the regions of existence of solitary waves and/or DLs and determine their parametric dependence. The regions of either solitary waves or DLs are depicted in Figs. 1 and 2. It is found that the particle (electrons and positrons) non-extensivity (Fig. 1), the dust concentration α and the positron concentration v (Fig. 2) may drastically affect the existence conditions of either solitons or DLs.

We have carried out a numerical investigation over a wide range of plasma parameters and found that the coefficient B (15) can be either positive (compressive

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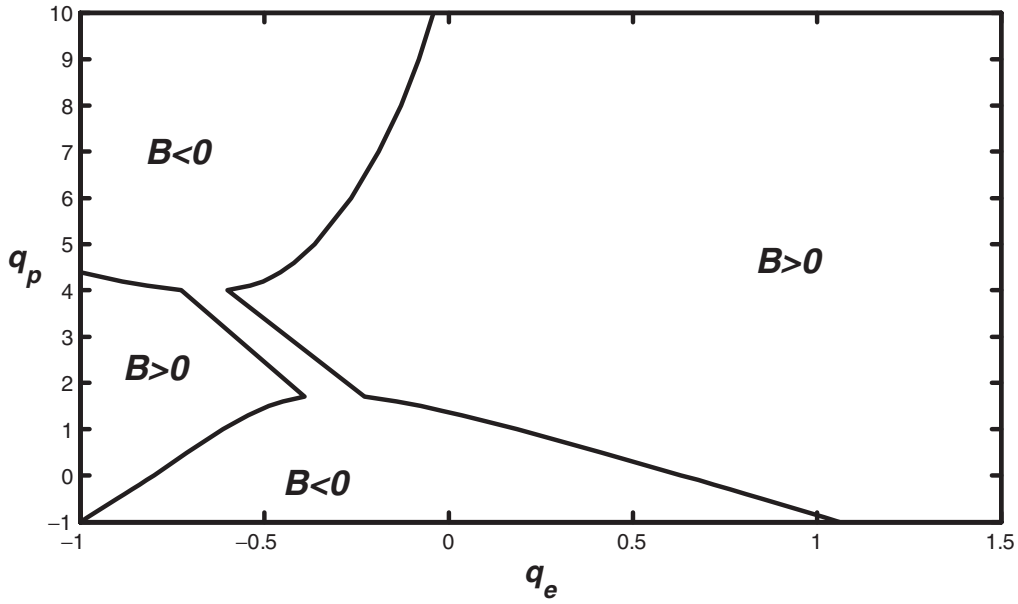


Figure 3. Polarity of the nonlinear pulses corresponding to the sign of B [represented by (15)] ($B > 0 \rightarrow$ positive pulses and $B < 0 \rightarrow$ negative pulses) in the q_e–q_p plane, with $\sigma_i = 0.8$, $\sigma_p = 0.5$, $\nu = 0.01$ and $\alpha = 0.98$.

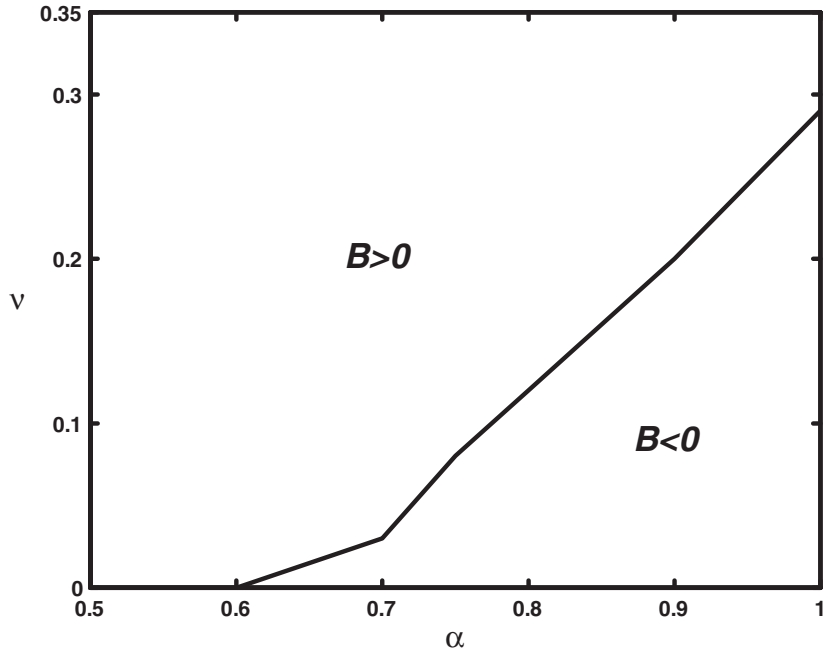


Figure 4. Polarity of the nonlinear pulses corresponding to the sign of B [represented by (15)] ($B > 0 \rightarrow$ positive pulses and $B < 0 \rightarrow$ negative pulses) in the α–ν plane, with $q_e = 1.5$, $q_p = 1.1$, $\sigma_i = 0.1$ and $\sigma_p = 0.1$.

solitary pulses) or negative (rarefactive solitary pulses). Although the dominant propagating pulses are positive in nature, it is clearly seen that the particle (electrons and positrons) non-extensivity (Fig. 3), the dust concentration α and the positron concentration ν (Fig. 4) may play a key role in defining the polarity of the localized solitary pulses.

In agreement with our former results and making use of (19) and (24), it can be seen that owing to particle non-extensivity, our plasma model may admit rarefactive as well as compressive solitons (Fig. 5) and DLs (Fig. 6). As the electron non-extensive parameter q_e increases, the rarefactive DL strength increases, leading (beyond a certain critical value of q_e) to the development of compressive DLs. The latter (in contrast to the rarefactive DLs) shrink as q_e increases, i.e. as the electrons evolve far away from their Maxwellian equilibrium. Physically, increasing the DL strength means that the potential

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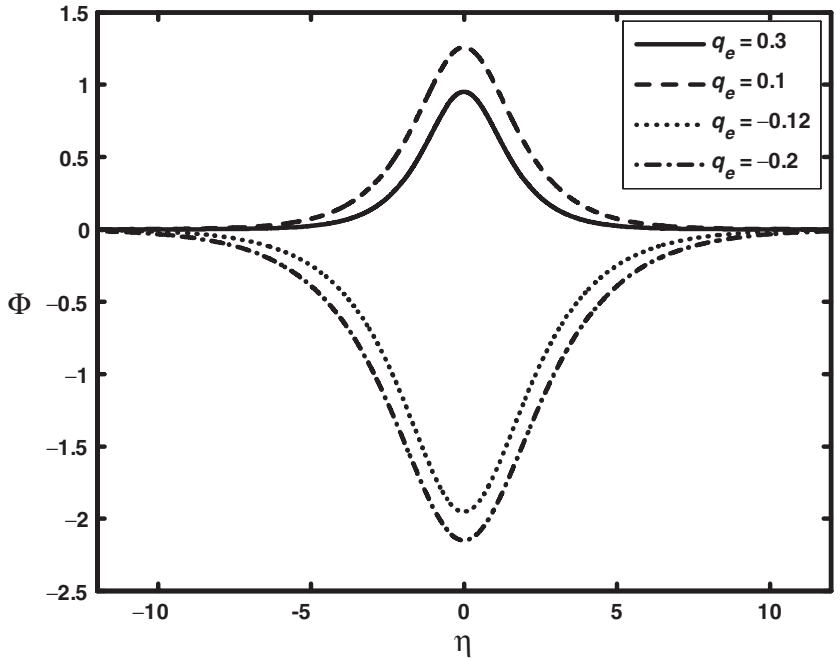


Figure 5. Plot of solitary potential profiles Φ (19) against η for different values of the electron non-extensive parameter q_e, with $q_p = 1.1$, $\sigma_i = 0.6$, $\sigma_p = 0.8$, $\nu = 0.11$, $\alpha = 0.55$ and $U = 1.1$.

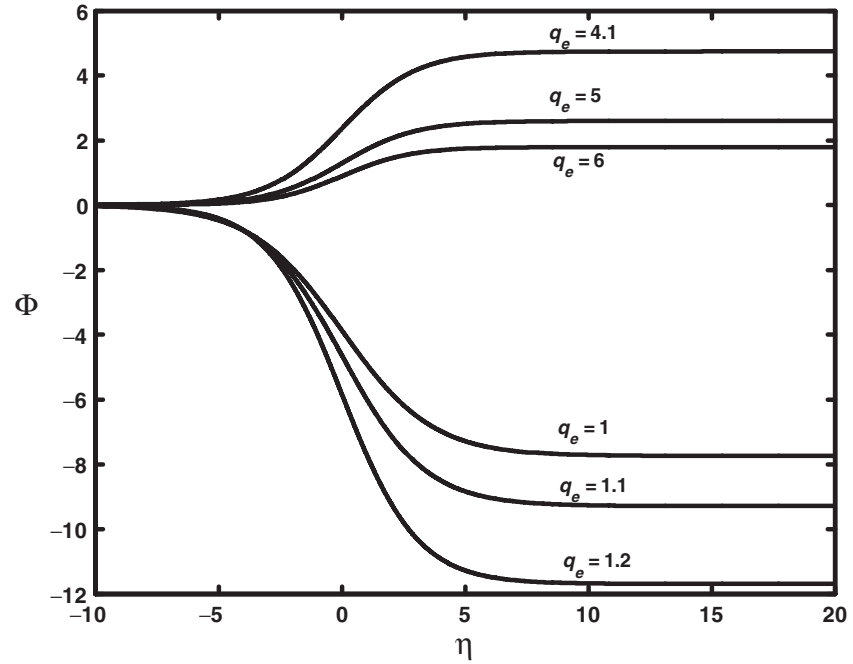


Figure 6. Plot of the DL profiles Φ (24) against η for different values of the electron non-extensive parameter q_e, with $q_p = 1.1$, $\sigma_i = 0.3$, $\sigma_p = 0.03$, $\nu = 0.015$ and $\alpha = 0.95$.

drop across the double layer increases and, therefore, more particles should be accelerated.

4. Conclusion

To conclude, dust–ion acoustic solitons and double layers have been studied in a dust–electron–positron–ion plasma composed of q-distributed electrons and positrons, warm fluid ions and a fraction of immobile dust grains. Employing the reductive perturbation approach, a modified K-deV equation has been derived. Existence domains of either solitary waves or double layers have been presented and their parametric dependence determined. We have found that particle non-extensivity, dust concentration and positron concentration may drastically affect the existence domains and may play a key role in defining the polarity of these localized structures. In particular, it may be noted

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that owing to particle non-extensivity, our plasma model may admit rarefactive as well as compressive nonlinear localized structures. It may be useful to note that if one assumes mobile dust grains, one can get (in the small amplitude regime) two phase velocities, namely the phase velocity of the faster DIAW λ_+ and the phase velocity of the slower DA wave λ_- (Ghosh et al. 2000, 2002; Younsi and Tribeche 2010). In the limit $m_d \to \infty$ (i.e. the dust dynamics is ruled out), we obtain a vanishingly small $\lambda_- \sim 0$. In the nonlinear regime, the dust mobility does not affect qualitatively our results, but leads to a shift of the critical as well as the existing domains. Moreover, the presence of positrons in the dusty plasma is a rather exotic situation from the point of view of the traditional gas discharge dusty plasma. The impact of positrons on the dust charging may be of great interest. Taking into account positron fluxes may cause a faster decrease of the net negative charge residing on the dust grain surface, leading to either a neutralized dust grain (Tribeche and Shukla 2012) or (eventually) a positively charged dust grain at a floating potential. In this case, the anomalous charge variation-induced nonlinear wave damping should be taken into account. On the contrary, an interesting feature of dusty plasmas is that because of the large charges on the individual dust particles, the dust component of the plasma can easily be in the strongly coupled regime in which the electrostatic energy of dust particle interactions greatly exceeds the dust kinetic energy. This strong correlation among the dust particles leads to physical effects such as the formation of ordered dust crystalline patterns. Moreover, the strong dust–dust interactions induce a dissipation that may prevent solitary wave formation giving rise to monotonic shock-like waves (dissipation dominant case). Our results should assist in interpreting the nonlinear structures that may occur in astrophysical environments.

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