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Nonlinear Vertical Vibration of Tension Leg Platforms with Homotopy Analysis Method

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Abstract. One of the useful methods for offshore oil exploration in the deep regions is the use of tension leg platforms (TLP). The effective mass fluctuating of the structure which due to its vibration can be noted as one of the important issues about these platforms. With this description, dynamic analysis of these structures will play a significant role in their design. Differential equations of motion of such systems are nonlinear and providing a useful method for its analysis is very important. Also, the amount of added mass coefficient has a direct effect on the level of nonlinearity of partial differential equation of these systems. In this study, Homotopy analysis method has been used for closed form solution of the governing differential equation. Linear springs have been used for modeling the stiffness of this system and the effects of torsion, bending and damping of water have been ignored. In the study of obtained results, the effect of added mass coefficient has been investigated. The results show that increasing of this coefficient decreases the bottom amplitude of fluctuations and the system frequency. The obtained results from this method are in good agreement with the published results on the valid articles.

AMS subject classifications: 70G99, 70K25, 82C80

Key words: TLP, Homotopy analysis method, nonlinear vibrational equation, added mass, vertical vibration.

1 Introduction

Tension leg platform (TLP) is the newest types of restrained comparative structures. The platform is permanently moored by means of tethers or tendons grouped at each of the structure's corners. A group of tethers is called a tension leg. A schematic diagram of a

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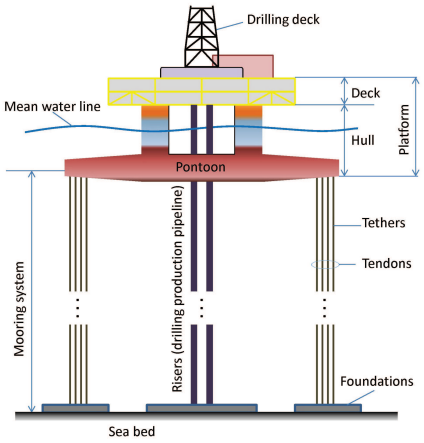


Figure 1: Schematic diagram of tension leg platform (TLP).

typical TLP is shown in Fig. 1. These platforms are connected by the stretched chains or cables to the sea floor that creates more than 15 to 25 percent of buoyancy force. Being stretched the platform, will reduced the vertical and rolling movement very much. These platforms are designed to be able to show great stiffness against the vertical loads. These types of platforms are used usually in very deep seas (approximately 300m to 1500m). As the benefits of tension leg platform can be pointed to its easy transfer during the changing of the platform location due to the completion of oilfield operations, as well as fewer installation costs.

In the last three decades, numerous attempts have been done to provide a better model in terms of costs and useful life of the platform. These platforms must be designed based on a dynamic analysis. Many studies [1–3] have been performed to understand the structural behavior of TLP and determine the effect of several parameters on dynamic response. Tabeshpour et al. [4] examined the effects of added mass by using perturbation method. In their study, many curves are presented to show free vibration response of the system that in these curves, linear solution of system is compared with the first and second order of perturbation method. They concluded that by using the first order perturbation method, the system frequency is equal to its natural frequency. Also, a conceptual discussion on the results of nonlinear dynamic analysis of TLP under arbitrary initial values had been presented by Tabeshpour [5]. Adrezin et al. [6, 7] developed a set of non-linear equations of tension leg platform for a single-tendon undergoing planar motion. The transverse and longitudinal responses of the tendon are included in the equations of motion and the Extended Hamilton's Principle is applied and the Lagrangian is fully developed. Chandrasekaran and Jain [8] studied dynamic analysis of a triangular model TLP to regular waves and considered various nonlinearities produced due to change in the tether tension and nonlinear hydrodynamic drag force. The nonlinear equation of motion is solved in the time domain using Newmark's β integration scheme and

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the effects of different parameters that influence the response investigated. Senjanovic et al. [9] studied global horizontal low frequency motions with large amplitudes as a result of dominant second order wave excitation and small stiffness, and vertical local motions of higher frequency and small amplitudes excited by the first order wave forces.

The effective mass fluctuating of structure on its vertical vibrations can be considered as one of the most important issue in the TLP at the sea. This vertical displacement has a destructive role in the platform and has a great influence on the determining of its fatigue life. In the analysis of these platforms, partial differential equations of motion are nonlinear and having a useful technique for the analysis is very important. The variable coefficient of added mass is entered in the nonlinear equation of vertical vibrations of tension leg platforms and should be analyzed for several values of this coefficient. Due to the complexity of displacement of these platforms, always a simplification is carried out. In fact, because the structure is consists of a deck on the water, pontoon in the water and a set of restrained cable chain, a part of the analysis should be always considered. In this paper, the effect of variable added mass which caused from the buoyancy force is investigated on the dynamic response of the platform. In this model, the platform is considered as a rigid body and the tethers are modeled by linear elastic springs. Tendons have often been modeled as massless springs, which neglect contribution to the response. The effects of radiation damping, torsion and bending are also ignored. By taking these assumptions, the equations of motion and frequency of the system in the case of free nonlinear vibrations is determined. Always, using different methods for solving the nonlinear equations of offshore platforms was the one of discussed issues by the researchers. In contrast to other conventional methods, Homotopy analysis method can solve strongly nonlinear partial differential equations, analytically. Therefore, Homotopy Analysis Method (HAM) has been used to solving the nonlinear equations. Finally, the effect of added mass has been studied on the oscillatory behavior of the system.

2 Dynamical modeling

Structural modeling of a tension leg platform as a moored structure is shown in Fig. 2. It is assumed that the platform structure is a rigid body. Mooring members and tethers are modeled by linear elastic springs, and the effects of damping, torsion and bending are ignored. With these assumptions, free vibration equation of the system is as follows:

$$M\ddot{y}(t)+Ky(t)=0. \tag{2.1}$$

Where t denotes the time and the dot denotes derivative with respect to t. Also, K is the stiffness factor of the entire system that based on the Eq. (2.2), is equal to the total of mooring and buoyancy stiffness

$$K=k_r+k_b. \tag{2.2}$$

Where k_r and k_b are mooring and buoyancy stiffness, respectively. If the structure is not being vertically moored, k_r is equal to zero.

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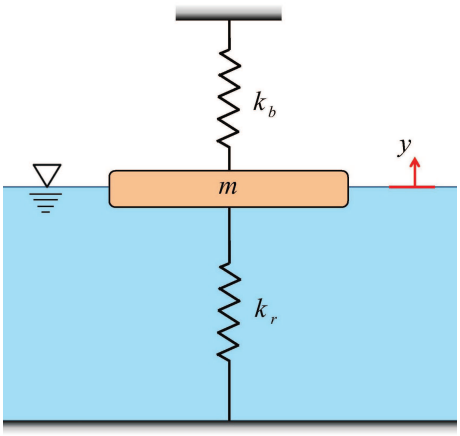


Figure 2: Structural model of TLP.

When the platform is in its balance state, the buoyancy force is equal to the weight of structure. If the platform is drawn to the below of its balance buoyancy depth and be kept by the tether, the force balance equation will be change. This change is due to a tensile force of the tethers as well as fluctuation in the effective mass of platform. The effective mass of platform is sum of its mass and a variable added mass that its value changed with the platform depth. The effective mass of platform may be expressed by

$$M = m_s + m_a(t). \tag{2.3}$$

Where, m_s is the structural mass and $m_a(t)$ is the time varying added mass that will change according to Eq. (2.4).

$$m_a(t) = m_a^o + \varepsilon m_a^o y(t). \tag{2.4}$$

If the coefficients of μ and the natural frequency ω_n are considered as follows;

$$\mu = \frac{m_a^o}{m_s + m_a^o}, \quad \omega_n = \sqrt{\frac{k}{m_s + m_a^o}}. \tag{2.5}$$

Free vibration differential equation of system can be rewritten as follows:

$$(1+\alpha y)\ddot{y} + \omega_n y = 0. \tag{2.6}$$

Where, $\alpha = \varepsilon\mu$. Physically, the frequency ω can be regarded as a time scale. For a linear system, the frequency is independent of the amplitude. However, for a nonlinear system, it is important to know the relationship between the frequency and amplitude [10].

3 Analysis of governing nonlinear equation with HAM

According to Eq. (2.6), can be seen that α is the coefficient of nonlinear term in the differential equation of motion. In this study, Homotopy analytical method has been used to

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solve the nonlinear differential equation of system. Unlike perturbation techniques, it is unnecessary to assume the existence of any small/large parameters in the nonlinear differential equation [10]. It is clear from the physics of problem that the system fluctuates under a periodic motion. Also, for the free vibration of the system, the following initial conditions have been considered:

$$\dot{y}(0)=0, \quad y(0)=A. \tag{3.1}$$

3.1 Zero-order deformation equation

Free oscillations of a nonlinear conservative system can be expressed by the base functions of $\{\cos(m\omega t)|m=1,2,3,\cdots\}$. By taking the transformation $\tau=\omega t$ and $y(t)=u(\tau)$, Eq. (2.6) may be written as:

$$[1+\alpha u(\tau)]\omega^2 u''(\tau)+\omega_n^2 u(\tau)=0. \tag{3.2}$$

Initial conditions can be considered as $u(\tau)=A$ and $u'(\tau)=0$, where the prime denotes derivative with respect to τ. Thus, $u(\tau)$ can be expressed by the base functions as $\{\cos(m\tau)|m=1,2,3,\cdots\}$ in the form of Eq. (3.3)

$$u(\tau)=\sum_{k=1}^{+\infty} c_k\cos(k\tau). \tag{3.3}$$

Where, c_k is a coefficient. This provides us with the so-called rule of solution expression [10]. Initial guess $u_0(\tau)$, the auxiliary linear operator L, the auxiliary parameters $\hbar$ and auxiliary functions $H(\tau)$ can be determined by means of this expression. Obviously, under the rule of solution expression denoted by Eq. (3.3) and using the initial condition, it is easy to choose $u_0(\tau)=A\cos(\tau)$ as the initial guess of $u(\tau)$, where A is the amplitude of oscillation. Under the rule of solution expression denoted by Eq. (3.3), we choose the auxiliary linear operator as follows:

$$L[\Phi(\tau;q)]=\omega_0^2\Big[\frac{\partial^2\Phi(\tau;q)}{\partial\tau^2}+\Phi(\tau;q)\Big]. \tag{3.4}$$

With the property:

$$L[C_1\sin\tau+C_2\cos\tau]=0. \tag{3.5}$$

From Eq. (3.2), the nonlinear operator can be defined as follows:

$$N[\Phi(\tau;q),\Omega(q)]=[1+\alpha\Phi(\tau;q)]\Omega^2(q)\frac{\partial^2\Phi(\tau;q)}{\partial\tau^2}+\omega_n^2\Phi(\tau;q). \tag{3.6}$$

Where, $\Phi(\tau;q)$ is a function of τ and q, and $\Omega(q)$ is a function of q. Let $\hbar$ denote a nonzero auxiliary parameter and $H(\tau)$ denote a nonzero auxiliary function. The zero-order deformation equation can be constructed as Eq. (3.7):

$$(1-q)L[\Phi(\tau;)-u_0(\tau)]=q\hbar H(\tau)N[\Phi(\tau;q),\Omega(q)] \tag{3.7}$$

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subject to the initial conditions that expressed in Eq. (3.8)

$$\Phi(0;q)=A,\quad \frac{\partial\Phi(\tau;q)}{\partial\tau}\Big|_{\tau=0}=0. \tag{3.8}$$

When q=0, it is clear that Eq. (3.7) and (3.8) have the solution of $\Phi(\tau;0)=u_0(\tau)$ and $\Omega(0)=\omega_0$. Also, when $q=1$, it can be provided that $\Phi(\tau;1)=u(\tau)$ and $\Omega(1)=\omega$. Therefore, as q increases from 0 to 1, $\Phi(\tau;q)$ deforms from the initial guess $u_0(\tau)=A\cos(\tau)$ to the exact solution $u(\tau)$, and $\Omega(q)$ varies from the initial guess ω_0 to the exact frequency ω, respectively. Using the initial conditions and Taylor's Theorem, $\Phi(\tau;q)$ and $\Omega(q)$ can be expanded in the power series of q as follows:

$$\Phi(\tau;q)=u_0(\tau)+\sum_{m=1}^{+\infty}u_m(\tau)q^m,\quad \Omega(q)=\omega_0+\sum_{m=1}^{+\infty}\omega_m q^m. \tag{3.9}$$

Where

$$u_m(\tau)=\frac{1}{m!}\frac{\partial^m\Phi(\tau;q)}{\partial q^m}\Big|_{q=0},\quad \omega_m=\frac{1}{m!}\frac{\partial^m\Omega(q)}{\partial q^m}\Big|_{q=0}. \tag{3.10}$$

Note that the zero-order deformation (Eq. (3.7)) contains the auxiliary parameter $\hbar$ and the auxiliary function $H(\tau)$. Also, $\Phi(\tau;q)$ and $\Omega(q)$ are dependent upon them. Assuming that $\hbar$ and $H(\tau)$ are properly chosen so that the above series converge at $q=1$ [11], the accurate approximations of solution can be expressed as Eq. (3.11):

$$u(\tau)=u_0(\tau)+\sum_{m=1}^{+\infty}u_m(\tau),\quad \Omega(q)=\omega_0+\sum_{m=1}^{+\infty}\omega_m q^m. \tag{3.11}$$

3.2 High-order deformation equation

For brevity, the below vectors have been defined:

$$\vec{u}_n=\{u_0(\tau),u_1(\tau),\cdots,u_n(\tau)\},\quad \vec{\omega}_n=\{\omega_0,\omega_1,\cdots,\omega_n\}. \tag{3.12}$$

Differentiating the zero-order deformation equation, m times with respect to q, then setting $q=0$, and finally dividing it by $m!$, the so-called high-order deformation equation may be obtained as Eq. (3.13), subject to the initial conditions $u_m(0)=u_m'(0)=0$

$$L[u_m(\tau)-\chi_m u_{m-1}(\tau)]=\hbar H(\tau)R_m(\vec{u}_{m-1},\vec{\omega}_{m-1}). \tag{3.13}$$

Where, χ_m is defined by

$$\chi_m=\begin{cases}0, & m\le 1,\\ 1, & \text{otherwise},\end{cases} \tag{3.14}$$

and

$$R_m(\vec{u}_{m-1},\vec{\omega}_{m-1})=\frac{1}{(m-1)!}\frac{\partial^{m-1}N[\Phi(\tau;q),\Omega(q)]}{\partial q^{m-1}}\Big|_{q=0}. \tag{3.15}$$

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There are two set of unknowns: $u_m(\tau)$ and ω_{m-1}. However, only Eq. (3.13) exists for $u_m(\tau)$. Thus, an additional algebraic equation is required to determine ω_{m-1}. Under the rule of solution expression denoted by Eq. (3.7), $R_m(\vec{u}_{m-1},\vec{\omega}_{m-1})$ can be expressed by:

$$R_m(\vec{u}_{m-1},\vec{\omega}_{m-1})=\sum_{n=1}^{f(m)} b_{m,n}(\vec{\omega}_{m-1})\cos(n\tau). \tag{3.16}$$

Where, $b_{m,n}(\vec{\omega}_{m-1})$ is a coefficient that depends upon $\vec{\omega}_{m-1}$, and the integer $f(m)$ depends upon m. In order to obey the rule of solution expression $H(\tau)$ should be in the form of

$$H(\tau)=\cos(2\kappa\tau),\quad \kappa=0,1,2,3,\cdots. \tag{3.17}$$

The solution of Eq. (3.16) should not contain the so-called secular term $\cos(\tau)$. So, the right-hand side of this equation should not contain the term $\cos(\tau)$, i.e., the coefficients of $\cos(\tau)$ must be zero [12]. This is extra relationship for solving the higher-order deformation equation.

$$b_{m,1}(\vec{\omega}_{m-1})=0. \tag{3.18}$$

This problem has been solved by means of symbolic computation software such as Maple. Auxiliary functions of $H(\tau)$ for free vibration of the system are assumed to be equal 1. By solving higher order deformation equation for m-th order, any relations have been obtained in terms of $\vec{u}_{m-1}$ and $\vec{\omega}_{m-1}$. Therefore, in the first order we have obtained these relations:

$$u_1(\tau)=\frac{1}{3}\cos(\tau)\alpha A^2\hbar+\frac{1}{6}\alpha A^2\hbar(-3+\cos(2\tau)), \tag{3.19a}$$

$$\hbar(-\omega_0^2A+\omega_n^2A)=0\Rightarrow\omega_0=\omega_n. \tag{3.19b}$$

By continuing trend of solving to the higher order, expansion of relation (3.11) can be continued to reach the proper convergence. For example, the following expressions for u_2 and ω_1 are as follows:

$$\begin{aligned}u_2(\tau)=&\left(\frac{1}{3}\alpha\hbar A^2+\frac{1}{3}\alpha\hbar^2A^2+\frac{1}{36}\alpha^3\hbar^2A^4+\frac{29}{288}\alpha^2\hbar^2A^3\right)\cos(\tau)\\&+\left(\frac{1}{6}\alpha\hbar^2A^2+\frac{1}{9}\alpha^2\hbar^2A^3+\frac{1}{72}\alpha^3\hbar^2A^4+\frac{1}{6}\alpha\hbar A^2\right)\cos(2\tau)\\&+\frac{5}{96}\alpha^2\hbar^2A^3\cos(3\tau),\end{aligned} \tag{3.20a}$$

$$\frac{1}{12}\alpha^2\hbar A^3\omega_n^2-2\omega_nA\omega_1=0\Rightarrow\omega_1=\frac{1}{24}\alpha^2\hbar A^2\omega_n. \tag{3.20b}$$

As can be seen, the coefficients of above expression are dependent upon the auxiliary parameter $\hbar$, which there are great freedom to choose. The validity of Homotopy analysis approach is based on such an assumption that the series in Eq. (3.11) converges at $q=1$ [13–16]. Generally, for any an analytic solution given by the Homotopy analysis

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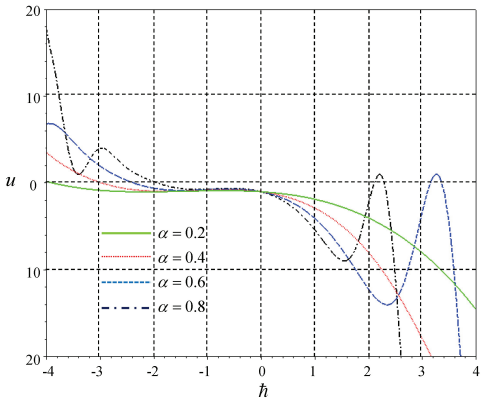


Figure 3: Convergence region for choosing $\hbar$ (for different value of α).

method, one should provide the corresponding region of $\hbar$, in which the given analytic solution is valid [17]. According to Fig. 3, it is found that the Eq. (3.11) is convergent when $-1 \leq \hbar \leq 0$ for any values of α.

4 Results and discussion

For numerical study, the results have been extracted by considering $A = 1$ and $\omega_n = 2\pi$. In Homotopy technique, note that the series of displacement and frequency contains the auxiliary parameter $\hbar$. As shown in Fig. 3, when $-1 \leq \hbar \leq 0$, the series of displacement and frequency converge in the whole region for α. Therefore, $\hbar$ has been assumed to be equal to -1.

In Fig. 4, a comparison with the second-order perturbation method which has been presented in [4] has been made for validation of Homotopy analysis method. According to Fig. 4, the 10th-order approximation given by the proposed approach agrees with the second-order perturbation method. This comparison is valid only for small values of α, because the perturbation method is not able to solve strongly nonlinear problems. Unlike the perturbation method, strongly nonlinear equations can be solved analytically by Homotopy analysis method. This is one of the advantages of Homotopy analysis method.

Time history of response for $\alpha = 0.2, 0.4, 0.6$ and 0.8 is shown in Fig. 5. The Homotopy solution shows that considering nonlinear equation of vertical vibration of TLP leads to change in the bottom of amplitude of platform fluctuations and the frequency of vibration, especially for large value of α.

According to Fig. 5, it is seen that there is no shift in period of vibration in the case of small value of α in spite of a slowly varying shift in the period of vibration in the case of its large value. In addition, large values of α caused a reduction in the bottom of amplitude of fluctuations. In Table 1, it has been observed that the values of α and ω

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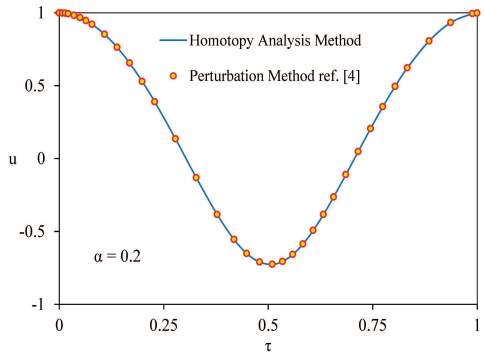


Figure 4: Agreement between Homotopy and perturbation method (see [4]) for $\alpha = 0.2$.

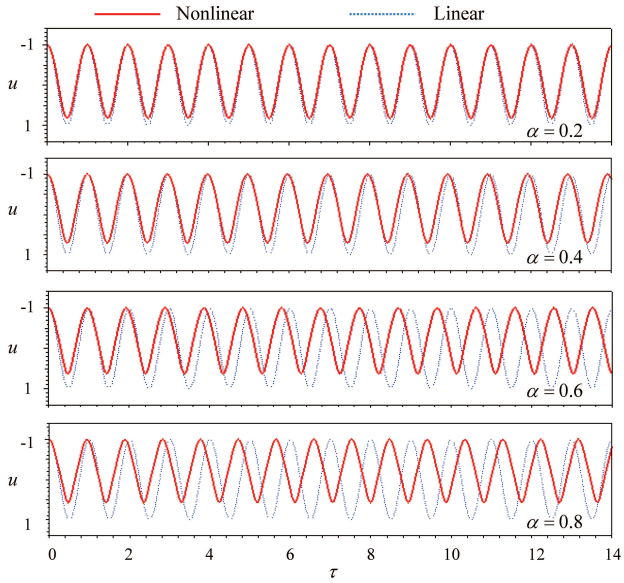


Figure 5: Linear and nonlinear answer of vertical displacement for various α.

have a reverse relation by each other. This means that increasing in α, can be reduced the frequency.

In Fig. 6, phase portrait for different values of α were shown. By comparing the answers, distinguish that difference between linear and nonlinear responses for small value of α is low, but by increasing α, these differences will be grown. Note that these differences consist of difference in the bottom of amplitude and difference in the frequency.

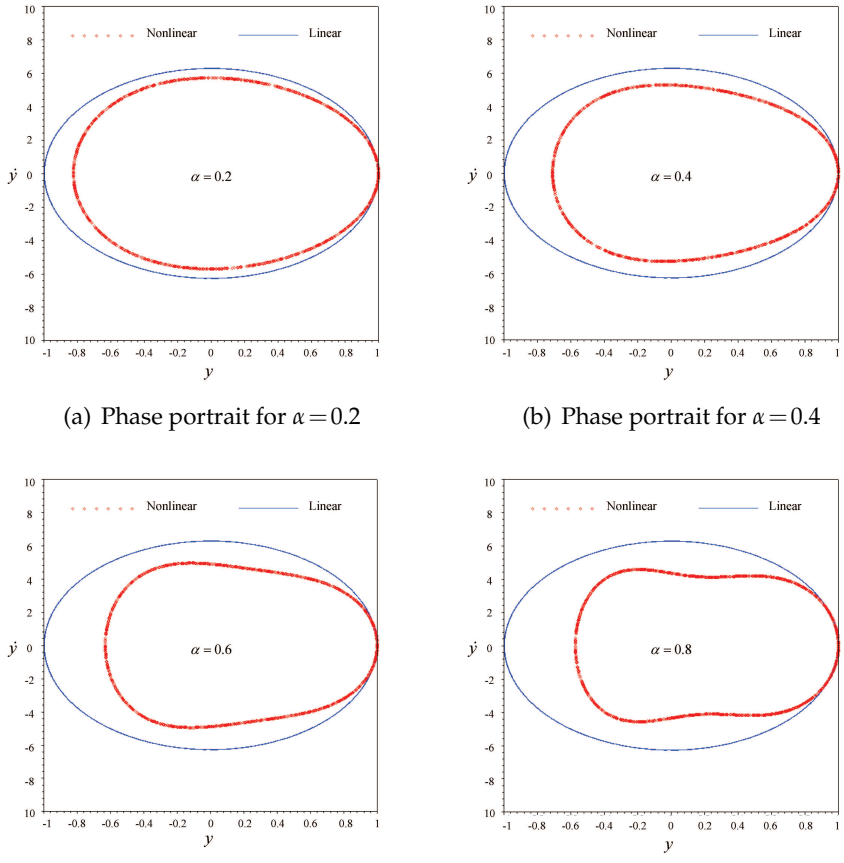
Phase planes which are illustrated in Fig. 6, show the stability and periodicity of the solutions. The differences between the amplitudes related to various values of α are shown in another way. It is seen that for small values of α, the difference between the nonlinear and linear vibrational equations is small and it increases with increasing α. Also increase in parameter α that cause great nonlinearity in vibrational equation, leads

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Table 1: The effect of α on value of frequency.

α	Frequency ω
0.2	6.273575541
0.3	6.246821202
0.4	6.167972035
0.5	6.000170627
0.6	5.743531397
0.7	5.504862914
0.8	5.409052922

to increase in difference between linear response and the Homotopy method solutions. Sensible difference of nonlinear answer get by Homotopy method and linear answer are shown in Fig. 6.



(a) Phase portrait for $\alpha = 0.2$

(b) Phase portrait for $\alpha = 0.4$

(c) Phase portrait for $\alpha = 0.6$

(d) Phase portrait for $\alpha = 0.8$

Figure 6: Phase portrait for different value of α.

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5 Conclusions

The problem of changes in added mass which made by vertical motion of comparative structure in water is analyzed with the Homotopy analysis method and results are defined for free vibration. In the free vibration state, 10th-order analytic approximation of Homotopy analysis method is used. By comparing the answers, perceive that difference between linear and nonlinear responses is low for small value of α, although by increasing in α, these differences will be grown. Also, the value of α and ω have a reverse relation by each other.

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