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Dynamics and model-based control for mobile modular manipulators

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SUMMARY
The dynamic modeling and trajectory following issues are addressed for mobile modular manipulators. Simulations are performed to validate the proposed algorithms.

KEYWORDS: Mobile manipulators; Dynamics; Model-based control.

I. INTRODUCTION

Usually, modular manipulators are mounted on a fixed base whose mobility is constrained. We attach a mobile platform to the bottom of the manipulator to increase workspace. In related research work, dynamic characteristics between platform and the manipulator were studied.[1] Dynamics and control were analyzed for mobile modular manipulators.[2]

II. DYNAMIC MODELING

The end-effector positions (p_x, p_y, p_z) and Z-Y-Z Euler angles (ϕ, θ, ψ) can be obtained as follows:[2]

The nonholonomic constraints for a 3-wheeled mobile platform are

$$\begin{bmatrix} c_m & s_m & -d_m/2 & -r_f & 0 \\ c_m & s_m & d_m/2 & 0 & -r_f \\ s_m & -c_m & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{r_f c_m}{2} & \frac{r_f s_m}{2} & -\frac{r_f}{d_m} & 1 & 0 \\ \frac{r_f c_m}{2} & \frac{r_f s_m}{2} & \frac{r_f}{d_m} & 0 & 1 \end{bmatrix}^T = 0 \tag{1}$$

In short, $A(\xi) \cdot S(\xi) = 0$; $c_m = \cos\phi_m$, $s_m = \sin\phi_m$; x_m, y_m, ϕ_L, ϕ_R, ϕ_m, d_m, r_f are shown in Figure 1.

Define $\xi = [x_m\ y_m\ \phi_m\ \phi_L\ \phi_R]^T$, $\varsigma = [\xi^T\ q_l\ \cdots\ q_n]^T$, $q = [\phi_L\ \phi_R\ q_l\ \cdots\ q_n]^T$ and $x = [p_x\ p_y\ p_z\ \phi\ \theta\ \psi]^T$; the Jacobian can be derived by

$$J = \frac{\partial x}{\partial \varsigma} \cdot \bar{S}(\xi) = \frac{\partial x}{\partial \varsigma} \cdot \begin{bmatrix} S(\xi) & 0_{5\times n} \\ 0_{n\times 2} & I_{n\times n} \end{bmatrix} \tag{2}$$

where $0_{5\times n}, 0_{2\times n}$ are zero-matrices; $I_{n\times n}$ is an identity-matrix.

The dynamics can be determined by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varsigma}}\right)^T - \left(\frac{\partial L}{\partial \varsigma}\right)^T = B(\varsigma) \cdot \tau + C(\varsigma) \cdot \lambda \tag{3}$$

where L is Lagrange function, $\lambda = [\lambda_1\ \lambda_2\ \lambda_3]^T$ are Lagrange multipliers; $\tau = [\tau_L\ \tau_R\ \tau_1\ \cdots\ \tau_n]^T$ are driving torques. $B = [0_{(n+2)\times 3}\ I_{(n+2)\times(n+2)}]^T \in \Re^{(n+5)\times(n+2)}$, $C = [A(\xi)\ 0_{3\times n}]^T \in \Re^{(n+5)\times 3}$.

Equation 3 can be rewritten as

$$M(\varsigma) \cdot \ddot{\varsigma} + V(\varsigma, \dot{\varsigma}) \cdot \dot{\varsigma} + G(\varsigma) = B(\varsigma) \cdot \tau + C(\varsigma) \cdot \lambda \tag{4}$$

where $M(\varsigma)$, $V(\varsigma, \dot{\varsigma})$, $G(\varsigma)$ are inertial matrix, centripetal and coriolis matrix and gravitational force vector.

In task-space,

$$\bar{M}(\varsigma) \cdot \ddot{x} + \bar{V}(\varsigma, \dot{\varsigma}) \cdot \dot{x} + \bar{G}(\varsigma) = \bar{\tau} \tag{5}$$

where $\bar{M} = j^{-T} \cdot \bar{S}^T \cdot M \cdot \bar{S} \cdot J^{-1}$, $\bar{G} = J^{-T} \cdot \bar{S}^T \cdot G$, $\bar{\tau} = J^{-T} \cdot \bar{S}^T \cdot B \cdot \tau$ and $\bar{V} = J^{-T} \cdot \bar{S}^T \cdot (M \cdot \dot{\bar{S}} \cdot J^{-1} + M \cdot \bar{S} \cdot J^{-1} + V \cdot \bar{S} \cdot J^{-1})$; $J^{-T} \cdot \bar{S}^T \cdot C = 0$ is eliminated.

III. MODEL-BASED CONTROLLER DESIGN

Assume $x_d, \dot{x}_d, \ddot{x}_d$ be desired trajectories, velocities and accelerations, the model-based controller is given by Equation 6, and control system block is shown in Figure 2.

$$\tau = (\bar{S}^T B)^{-1} J^T \{\bar{M}[\ddot{x}_d + \dot{K}_D(\dot{x}_d - \dot{x}) + K_P(x_d - x)] + \bar{V} \cdot \dot{x} + \bar{G}\} \tag{6}$$

where $K_D^T = K_D > 0$, $K_p > 0$ are gain matrices.

Substituting Equation 6 into 5, yields

$$\ddot{e} + K_D \cdot \dot{e} + K_p \cdot e = 0 \tag{7}$$

where $e = x - x_d$ is an error vector.

Theorem: The system described by Equation 7 is asymptotically stable, i.e. $e, \dot{e} \to 0$ as $t \to +\infty$.

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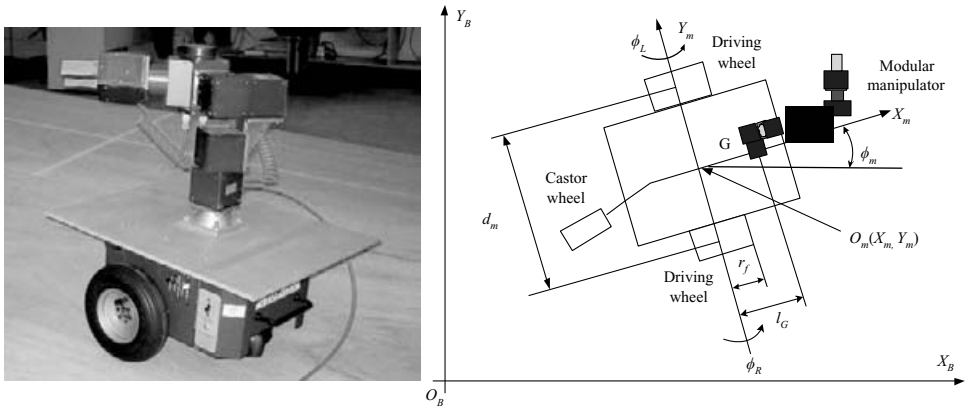


Fig. 1. A mobile modular manipulator.

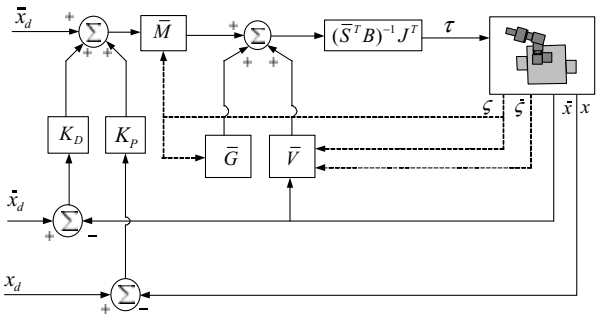


Fig. 2. A model-based controller.

Proof: Considering the nonnegative Lyapunov candidate

$$V_S = \frac{1}{2} \cdot \dot{e}^T \cdot e + \frac{1}{2} \cdot e^T \cdot K_P \cdot e \geq 0 \tag{8}$$

$$\dot{V}_S = \dot{e}^T \cdot \ddot{e} + \dot{e}^T \cdot K_P \cdot e = -\dot{e}^T \cdot K_D \cdot \dot{e} \leq 0 \tag{9}$$

Iff $\dot{e} = 0, \dot{V}_s = 0$, so $\dot{e} \to 0$ as $t \to +\infty$. Substituting $\dot{e} = \ddot{e} = 0$ into Equation 7, yields $e = 0$, i.e., the steady-state-error is zero. So, $e \to 0$ as $t \to +\infty$.

Remark. Assume $\hat{M}, \hat{V}$ and $\hat{G}$ be nominal values of M, V, G, then the steady-state error is

$$e_{sse} = K_P^{-1} \cdot \hat{M}^{-1} \cdot \left[\left(\hat{M} - M \right) \cdot \ddot{x} + \left(\hat{V} - V \right) \cdot \dot{x} + \left(\hat{G} - G \right) \right] \tag{10}$$

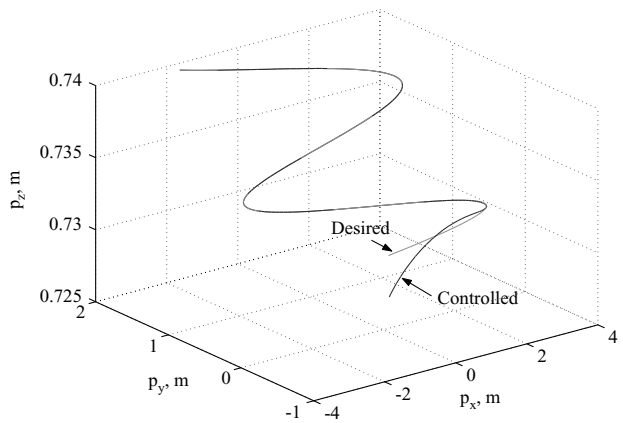


Fig. 3. Desired and controlled locus without uncertainties.

IV. SIMULATION RESULTS

A real robot is shown in Figure 1. The desired trajectory is given by Figure 3. Simulation time is 10-seconds. The gain matrices are: $K_D = 5.0$, $K_p = 20.0$. All the joint variables are initialized to be zeros except $q_2 = \pi/3$.

The desired and control locus are shown in Figure 3. Figure 4 gives tracking errors. If values of dynamic parameters with 10% deviations are adopted, steady-state-errors exist, see Figure 5. It shows that the model-based controller

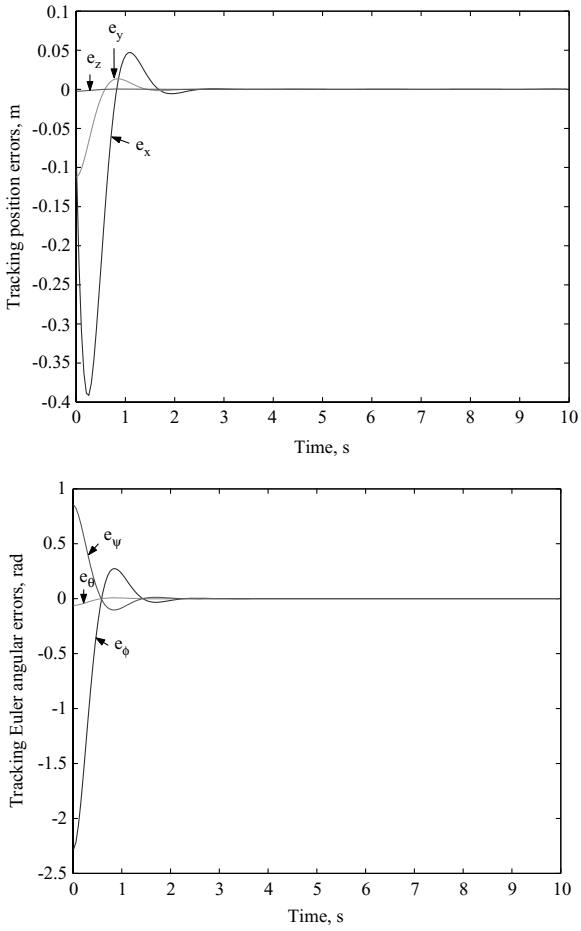


Fig. 4. Tracking errers without uncertainties.

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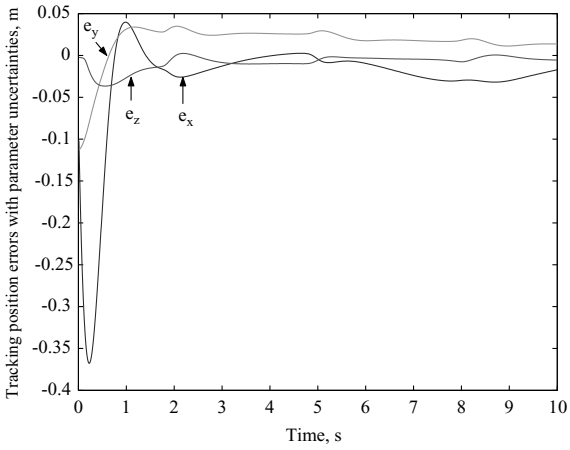


Fig. 5. Tracking position errors with 10% deviations.

is effective if precise dynamic parameters are known in advance.

V. CONCLUSION

In this paper, an integrated dynamic modeling method and a model-based controller are presented. Simulation results for a real robot demonstrate the effectiveness of the proposed algorithms.

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