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Negative energy waves

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Abstract. A brief history of the concept of negative energy waves in fluid and plasma media is given. The theory of microwave tubes is highlighted, where the small signal power theorem was first derived, which led to the interpretation of these devices in terms of the interaction of positive and negative energy waves. Three examples are described to illustrate the use of negative waves in stability problems. The first is concerned with the Kelvin–Helmholtz instability in neutral fluids. The second comes from a kinetic theory of instability in a collisionless shock wave in a magnetized plasma, and the third is concerned with the effect of a resistive wall and plasma flow on the magnetohydrodynamic stability of a magnetically confined plasma. The paper ends with some general conclusions.

1. Introduction

von Laue [1] was the first person to derive the correct expression for the energy density $\mathscr{E}$ of an electromagnetic wave in a dielectric medium,

$$\mathscr{E} = \frac{1}{4}\frac{\partial}{\partial\omega}[\omega\varepsilon(\omega)]|\mathbf{E}|^2 + \frac{1}{4}\mu_0|\mathbf{H}|^2$$

where $\varepsilon(\omega)$ is the dielectric constant of the medium. In thermodynamic equilibrium, $\mathscr{E} > 0$. This follows from the Kramers–Krönig relations as emphasized by Kadomtsev *et al.* [2]. However, the Kramers–Krönig relations do not generally hold in media not in thermodynamic equilibrium. In such media, free energy can be present and it is possible for $\partial\varepsilon/\partial\omega < 0$. In these circumstances, negative energy waves can exist and may be expected to be associated with instability.

The existence of negative energy perturbations is already implied in early work in fluid mechanics [3]. However, the field which could lay claim to the invention of the concept of negative energy waves and which has made the most explicit use of them is in the theory of microwave tubes. This deals with the interaction of an electron beam with a slow electromagnetic wave supported by a circuit. The idea is due to Chu, who in 1951 formulated his small signal kinetic power theorem [4]. A clear discussion of Chu's theorem is given in [5]. Chu found that by ascribing a kinetic power flow to the beam electrons and adding this to the Poynting flux of the associated electromagnetic field, the total power was conserved. There were two surprising features of Chu's analysis. The first was that the conservation theorem involves only small signal (linearized quantities) and the second was that certain beam modes could carry negative power. In particular, it was shown that an arbitrary disturbance on an electron beam could be described in terms of a fast wave and a slow wave where the phase velocities are, respectively, greater or smaller than

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the beam velocity. The kinetic power of the fast wave is positive and that of the slow wave is negative. Since the group velocities have the direction of the beam it follows that the energy densities of the fast and slow waves are positive and negative. The idea of a negative energy wave had an early application through the development of the resistive wall amplifier [6].

The subject of negative energy was further developed by the work of Sturrock [7, 8] who emphasized the significance of the conservation of small signal energy and by means of a Lagrangian analysis showed that all slow waves, in the sense just described, carry negative energy. In order to indicate that the small signal energy may not necessarily correspond to the physical energy, Sturrock introduced the term pseudo energy [7]. However, from the point of view of a linearized stability analysis, the distinction is unimportant. As stressed in [7, 8], the important point is the existence of a small signal energy conservation relation. For a conservative system, obeying such a relation, the existence of growing waves can only occur due to the simultaneous excitation of positive and negative energy waves. Energy flows from the negative energy wave to the positive energy wave, allowing both modes to grow and the total energy to remain constant. Clearly, a negative energy wave is a potential source of instability and for this reason it has been referred to as an active wave. It was also realized that if a negative energy wave is subject to external dissipation, the subsequent removal of energy from the wave will cause it to grow, a point utilized in the resistive wall amplifier [6].

The above information has been utilized by Hasegawa [9] in a classification of longitudinal plasma instabilities. These are divided into two types, reactive and dissipative. The first refers to conservative systems where instability results from the coalescence of a positive energy mode and a negative energy mode. The second describes instabilities which result from Landau damping of a negative energy wave or inverse Landau damping of a positive energy wave, emphasizing the fact that the wave energy is frame dependant.

An early demonstration of instability resulting from dissipation was given by Lamb [3]. He considered the oscillations of a particle inside a spherical bowl of radius a, rotating with constant angular frequency ω about the vertical diameter. For a smooth bowl, the equilibrium of the particle in the lowest position is stable and the rotation of the bowl is irrelevant. However, when the existence of friction between the particle and the bowl is taken into account, a perturbation of the particle from its lowest position is shown to be unstable, provided the rotation frequency of the bowl exceeds the natural frequency of oscillation of the particle in the bowl, i.e. $\omega > (g/a)^{1/2}$. The particle works its way outward into an ever-widening spiral until it reaches the only stable position, given by $\cos\theta = g/\omega^2 a$ where θ is the angular distance from the lowest point.

The subject of the stability of ideal magnetohydrodynamics (MHD) has been greatly facilitated by the energy principle [10]. This principle also results from a conservation theorem based only on the linearized perturbations. A necessary condition for instability is that the potential energy of a linearized perturbation δW should be negative, $\delta W < 0$. The stability boundary is crossed when two ideal modes coalesce for zero frequency. This is clearly different from the negative energy with which we are concerned which involves the oscillatory energy associated with the wave. For ideal MHD, the perturbed motion changes from oscillatory to purely growing (non-oscillatory) as the stability boundary is crossed. However, the ideal MHD energy principle is only valid for a stationary plasma. For a situation in

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which plasma flow is present, ideal MHD instability occurs at a finite frequency and the concept of wave energy again becomes relevant. The concept of negative energy waves has been used to analyse certain MHD stability problems involving flows in solar physics [11].

In the remainder of this brief paper, the use of the concept of negative energy waves for both solving problems and gaining insight into them is illustrated with the aid of three examples. The first example is taken from the work of Cairns [12], who showed how the stability of fluid flows could be analysed by introducing the idea of negative energy waves. The second example is taken from plasma physics [13] and illustrates the two forms of instability, reactive and dissipative, to which a negative energy wave can give rise. The third example is concerned with the resistive wall mode [14], which is a MHD instability of a magnetically confined plasma. It is not the same as the resistive wall amplifier, but in the presence of plasma flow there is a subtle link between them [15].

2. Inviscid parallel fluid flows

The first example is taken from a paper written by Cairns in 1979 [12] and provides a good illustration of some of the many talents he possesses as a theoretical physicist. These are clarity, simplicity, directness and mathematical elegance. It also shows an alertness to the possibility of applying relevant ideas from one field to another.

In [12] Cairns begins by noting that the dispersion relation for an electrostatic wave in a uniform plasma is

$$\varepsilon(\omega, k) = 0,$$

where $\varepsilon(\omega, k)$ is the longitudinal dielectric constant of the plasma. As already mentioned, the wave energy is given by

$$\omega \frac{\partial \varepsilon}{\partial \omega} |E|^2$$

and $|E|^2$ is the intensity of the wave electric field. The wave energy can be obtained from the linear analysis and is used to predict the stability properties of the system.

The aim of the work is to apply this approach to inviscid parallel flows with step function velocity and density profiles. Such flows can give rise to the Kelvin–Helmholtz instability which is due to a non-uniform flow. An incompressible, inviscid fluid is considered whose velocity is assumed to be in the x-direction. The fluid can be bounded, semi-infinite or infinite in the z-direction. The important point is that the density and velocity are assumed to be step functions of z. By considering a surface $z = z_0$ which is then perturbed to

$$z = z_0 + \eta(x, t),$$

where $\eta = A \exp(ikx - i\omega t)$ a dispersion relation for the perturbations to the fluid is obtained. This is accomplished by assuming that the perturbation is the same on both sides of the surface and that the pressure is continuous. The resulting dispersion relation is written

$$D(\omega, k) = D_1(\omega, k) - D_2(\omega, k) = 0. \tag{2.1}$$

The wave energy is obtained by calculating the work done on the fluid by a suitable external force on the surface $z = z_0 + \eta$ where the surface perturbation is again

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taken to be

$$\eta(x,t) = A(t)\exp(ik_0x - i\omega_0 t).$$

The amplitude of the perturbation, now a function of time, satisfies the conditions $A \to 0$ as $t \to -\infty$ and $A \to A_0$ as $t \to \infty$. The time variation of A is assumed to be slow compared with ω_0, and the wave energy is shown to be given by

$$W = \frac{1}{4}\omega_0 \frac{\partial D}{\partial \omega_0}|A_0|^2. \tag{2.2}$$

The Kelvin–Helmholtz instability is analysed by considering a fluid (incompressible and inviscid) of density ϱ_1 in the region $z > 0$, moving with velocity U in the x-direction. In the region $z < 0$, the fluid has a density $\rho_2 > \rho_1$ and is at rest, whilst gravity g acts in the negative z-direction. Including the effect of surface tension γ, the dispersion relation is

$$D(\omega,k) = (\rho_1 - \rho_2)g - k^2\gamma + (\omega - kU)^2(\rho_1/k) + \rho_2(\omega^2/k) = 0 \tag{2.3}$$

where the energy per unit area in a stable wave is

$$W = \frac{1}{4}\omega \frac{\partial D}{\partial \omega}|A|^2 \tag{2.4}$$

and the surface displacement is assumed to be

$$\eta(x,t) = A\cos(kx - \omega t). \tag{2.5}$$

Making use of (2.3) and (2.4), the wave energy is

$$W = \omega\left[\frac{\rho_2\omega}{2k} + \frac{\rho_1}{2k}(\omega - kU)\right]A^2. \tag{2.6}$$

Clearly, the second term will be negative when $\omega < kU$. The solution of the dispersion relation, (2.3) gives

$$\omega = \frac{\rho_1 kU}{\rho_1 + \rho_2} \pm \left\{\frac{C_0^2}{U^2} - \frac{\rho_1\rho_2}{(\rho_1 + \rho_2)}\right\}^{1/2} kU \tag{2.7}$$

where

$$C_0^2 = \frac{(\rho_2 - \rho_1)g}{(\rho_2 + \rho_1)k} + \frac{k\gamma}{\rho_2 + \rho_1}. \tag{2.8}$$

The energy, given by (2.6), is negative when the frequency corresponding to the lower sign becomes positive. Furthermore, the Kelvin–Helmholtz instability occurs when U exceeds a critical velocity given by

$$U > \frac{C_0(\rho_1 + \rho_2)}{(\rho_1\rho_2)^{1/2}}.$$

At the critical velocity

$$U_0 = \frac{C_0(\rho_1 + \rho_2)}{(\rho_1\rho_2)^{1/2}} \tag{2.9}$$

and the frequency is given by

$$\omega = \frac{\rho_1 kU}{\rho_1 + \rho_2} \tag{2.10}$$

for which it is easily seen that the wave energy, $W = 0$, corresponding to the coalescence of a positive energy root with a negative energy root, characteristic of the marginal condition.

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Such systems, with layers of adjacent fluid, are also used to demonstrate the growth of a negative energy wave due to viscosity and the explosive growth due to the nonlinear interaction of three waves, one of which carries negative energy.

3. Instability in a perpendicular collisionless shock

As a second example we describe an instability which might occur at the front of a collisionless shock wave propagating perpendicular to an equilibrium magnetic field [13]. The shock wave is assumed to propagate in the x direction and the z-axis is taken as the direction of the magnetic field. There is then a zero-order electric field in the x direction. The width of the shock front is such that it passes a given point in a time much shorter than the period of an ion-cyclotron oscillation but much longer than the corresponding electron period. As a result, only the electrons undergo the $\mathbf{E}_0 \times \mathbf{B}_0$ drift, which produces a zero-order current in the y direction.

It can be shown that the $\mathbf{E}_0 \times \mathbf{B}_0$ drift is the dominant one. Hence, the gradients in n_0 and $\mathbf{B}_0$ are neglected and electrostatic waves propagating at right angles to both $\mathbf{B}_0$ and the shock front are considered, i.e. perturbations varying as $\exp i(ky - \omega t)$.

Concentrating on frequencies such that $\omega \gg \omega_{ci}$, where ω_{ci} is the ion cyclotron frequency, the ions are treated as being unmagnetized. Hence, the dispersion relation is

$$1 + \frac{\omega_{pi}^2}{k^2 v_{Ti}^2}\{1 + \xi_i Z(\xi_i)\} - \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left\{-1 + e^{-\lambda} I_0(\lambda) + 2(\omega - kv_0)^2 \sum_{n=1}^{\infty} \frac{e^{-\lambda} I_n(\lambda)}{(\omega - kv_0)^2 - n^2\omega_{ce}^2}\right\} = 0, \quad (3.1)$$

where ω_{pi} is the ion plasma frequency, v_{Ti} the ion thermal speed $(\kappa T_i/m_i)^{1/2}$, $\xi_i = \omega/\sqrt{2}kv_{Ti}$, Z the Fried–Conte plasma dispersion function, ω_{pe} the electron plasma frequency, ω_{ce} the modulus of the electron cyclotron frequency, v_{Te} the electron thermal speed $(\kappa T_e/m_e)^{1/2}$, κ is Boltzmann's constant and v_0 the electron drift velocity $-E_0/B_0$. The quantity λ is given by $k^2 v_{Te}^2/\omega_{ce}^2$ and I_n is the nth-order modified Bessel function of the first kind. A feature of the plasma in a collisionless shock wave [16] is that $\omega_{pe} \gg \omega_{ce}$.

The term in the second set of brackets in (3.1) describes the electrostatic Bernstein modes in the presence of the $-E_0/B_0$ drift. The small signal energy density of an electrostatic wave is given by

$$\mathscr{E} = \frac{1}{4}\varepsilon_0 |E|^2 \frac{\partial}{\partial \omega}\{\omega \varepsilon_l(\omega, k)\}_{\varepsilon_l = 0}, \quad (3.2)$$

where $\varepsilon_l(\omega, k)$ is the longitudinal dielectric response function. Utilizing the second of the two brackets in (3.1), which is the electron response, it is easily verified that the condition for the drifted electron Bernstein wave to carry a negative (small signal) energy $\mathscr{E} < 0$ is

$$0 < \omega < kv_0. \quad (3.3)$$

Returning to the general dispersion relation we consider two cases. In the first, let us assume that $\omega \gg kv_{Ti}$, so that the plasma dispersion function for the ions can be expanded asymptotically. If we also assume that the Doppler shifted frequency is close to one harmonic, i.e.

$$\omega - kv_0 \approx \pm n\omega_{ce} \quad (3.4)$$

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the dispersion relation can be reduced to

$$(\omega^2 - k^2 c_s^2)\{(\omega - kv_0)^2 - n^2\omega_{ce}^2\} \simeq 2\omega^2(\omega - kv_0)^2 e^{-\lambda} I_n(\lambda) \tag{3.5}$$

where c_s is the ion sound speed $(\kappa T_e/m_i)^{1/2}$ and we have also assumed short wavelengths such that $kv_{Te}/\omega_{ce} \gg 1$. This dispersion relation describes the coupling between ion sound waves and electron Bernstein waves. Due to the short wavelength assumption the term on the right-hand side of (3.4) is small and may therefore be treated perturbatively. We are interested in the coupling of a positive energy wave with a negative energy wave which can be expected to be unstable. The drifted electron Bernstein waves are described by

$$\omega \simeq kv_0 \pm n\omega_{ce}. \tag{3.6}$$

Clearly, the solution with the negative sign is the wave which carries negative energy since this is the only one which can satisfy (3.3). Instability will result when this wave becomes degenerate with the positive energy sound wave. The condition for this to occur is the resonance condition

$$kc_s = kv_0 - n\omega_{ce}, \tag{3.7}$$

which determines the unstable wave number k. The dispersion relation (3.5) can now be solved perturbatively by assuming

$$\omega = kv_0 - n\omega_{ce} + \delta\omega. \tag{3.8}$$

Substituting (3.8) into (3.5),

$$(\delta\omega)^2 \simeq -\frac{1}{2(2\pi\lambda)^{1/2}} n\omega_{ce}^2 \left(\frac{m_e}{m_i}\right)^{1/2} \tag{3.9}$$

where use has been made of the result

$$I_n(\lambda) \approx e^{\lambda}/(2\pi\lambda)^{1/2} \quad \text{for } \lambda \gg 1.$$

Hence, the growth rate of the resulting instability is

$$\frac{\gamma}{\omega_{ce}} = \frac{n^{1/2}}{(8\pi)^{1/4}} \left(\frac{m_e}{m_i}\right)^{1/4}. \tag{3.10}$$

This instability is of the hydrodynamic (reactive) type since the ion wave is supported by the entire population of the ions in addition to the electrons and the Bernstein wave is due to the collective response of (all) the electrons.

Returning to (3.1) we now consider the second case. The previous case $\omega \gg kv_{Ti}$ requires $T_e \gg T_i$. When $T_i \gtrsim T_e$ the ion sound wave is strongly damped and a reactive instability of the type just described could not occur. However, the electron Bernstein wave still carries negative energy and is therefore a potential source of instability. In fact, ions having velocities comparable to the phase velocity of the slow (negative) energy electron Bernstein wave will extract energy from it and cause it to grow by analogy with the resistive wall amplifier [6]. In other words, the role of the ions in this case is to provide a lossy medium to which the negative energy electron Bernstein wave gives up energy.

In order to solve (3.1) for this case, assume $\omega = \sqrt{2}kv_{Ti}$ and that this frequency is again close to a particular harmonic. The dispersion relation can therefore be

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written in the form

$$\left\{1+\frac{\omega_{\rm pi}^2}{k^2v_{\rm Ti}^2}\{1+Z(1)\}+\frac{\omega_{\rm pe}^2}{k^2v_{\rm Te}^2}\right\}\{(\omega-kv_0)^2-n^2\omega_{\rm c}^2e\}=2(\omega-kv_0)^2\frac{\omega_{\rm pe}^2}{k^2v_{\rm Te}^2}e^{-\lambda}I_{\rm n}(\lambda). \tag{3.11}$$

Substituting (3.8) into (3.11) and approximating $Z(1)\simeq i\pi^{1/2}e^{-1}-1.08$ the growth rate for this case is

$$\frac{\gamma}{\omega_{\rm ce}}=\frac{ne^1(T_{\rm i}/T_{\rm e})}{\sqrt{2}\lambda^{1/2}[(T_{\rm i}/T_{\rm e}-0.08)^2e^2+\pi]} \tag{3.12}$$

where $k^2v_{\rm Te}^2/\omega_{\rm pe}^2$ has been neglected compared with 1.

This second case is an example of a dissipative instability. In contrast to the first case, the negative energy Bernstein wave is driven unstable by a subset of the resonant ions.

4. The resistive wall instability with plasma flow

The final example is taken from ideal MHD. A magnetically confined plasma is subject to the external kink instability [17]. This is a free surface mode and is most unstable in the absence of a wall. The mode can be stabilized by a perfectly conducting wall close to the plasma–vacuum boundary. However, if the perfectly conducting wall is replaced by one with finite resistivity the plasma is again unstable. This is the resistive wall mode [14].

Since tokamaks are often rotating, both toroidally and poloidally, it is interesting to consider the effect of an equilibrium flow of the plasma on the kink and resistive wall modes. A tractable model for this purpose is the sharp boundary, cylindrical system of radius $r=a$ with an equilibrium surface current [17]. The magnetic field is uniform in the plasma and the surface current is parallel to the axis [17]. This cylindrical model has been used to analyse the properties of the resistive wall mode in the presence of a uniform plasma flow [15].

The dispersion relation can be written in the form

$$\omega(\bar{\omega}^2-\omega_0^2)=-i\Gamma(\bar{\omega}^2-\omega_1^2) \tag{4.1}$$

where $\bar{\omega}=\omega-kv_0$ with k the axial wave number, v_0 the axial flow speed and perturbations are assumed to vary as $\exp i(kz+m\theta-\omega t)$. Γ is proportional to the resistivity of the wall and ω_0 and ω_1 have the following significance: $\bar{\omega}^2=\omega_0^2$ is the dispersion relation for the kink modes with a perfectly conducting wall and $\bar{\omega}^2=\omega_1^2$ is the corresponding dispersion relation in the absence of a wall. If the perfectly conducting wall is allowed to recede to infinity then $\omega_0\rightarrow\omega_1$. The classic case for the resistive wall mode is when $\omega_0^2>0$ and $\omega_1^2<0$, i.e. $\omega_1^2=-\gamma_1^2$. In the absence of flow the kink mode can be unstable with growth rate γ, without a wall and for a perfectly conducting wall there will be a position where $\omega_0^2=0$ and for the wall still closer to the plasma $\omega_0^2>0$. This behaviour has interesting consequences in the presence of plasma flow. Since the kink modes for the wall stabilized case are given by

$$\omega=kv_0\pm\omega_0 \tag{4.2}$$

we know from Sturrock's general result that the slow kink mode $\omega=kv_0-\omega_0$ will carry negative energy. Furthermore, the velocity required, $v_0>\omega_0/k$, can be very low as the marginal condition is approached.

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Now consider the solution of (4.1). The resistive wall mode can be shown to be a purely growing mode, even in the presence of flow [15]. Hence, (4.1) can be solved perturbatively since $\Gamma \ll 1$. Perturbing about $\omega = 0$, i.e. $\omega = \delta\omega$, (4.1) becomes

$$\delta\omega(k^2v_0^2 - \omega_0^2) = -i\Gamma(k^2v_0^2 + \gamma_1^2) \tag{4.3}$$

where we have substituted $\omega_1^2 = -\gamma_1^2$ into (4.1). Hence,

$$\delta\omega \simeq i\Gamma\frac{(k^2v_0^2 + \gamma_1^2)}{(\omega_0^2 - k^2v_0^2)}. \tag{4.4}$$

This is the growth rate of the resistive wall mode in the presence of flow. Note that for $v_0 = 0$ the resistive wall instability for a stationary plasma is obtained but the growth rate becomes infinite when $\omega_0 = kv_0$. Clearly, the theory breaks down before this point is reached. However, this situation can be dealt with if we remember that $kv_0 = \omega_0$ is the condition for the slow kink mode to change the sign of its energy. Hence, in our perturbation about zero frequency we must allow for the zero frequency kink mode. The perturbation solution of (4.1) now gives

$$\delta\omega(-2\omega_0)\delta\omega \simeq -i\Gamma(k^2v_0^2 + \gamma_1^2). \tag{4.5}$$

This is a quadratic in $\delta\omega$ whose solution is

$$\delta\omega \simeq \pm\left[\frac{\Gamma(k^2v_0^2 + \gamma_1^2)}{4\omega_0}\right]^{1/2}(1 + i). \tag{4.6}$$

The growth rate is now finite, but much enhanced since it is proportional to $\Gamma^{1/2}$ rather than Γ where $\Gamma \ll 1$. In this case, the resistive wall instability behaves more like an ideal mode since it results from the coalescence of two modes.

As the flow increases beyond this point, the instability changes character again and it is the negative energy, slow kink mode which is driven unstable rather than the wall mode.

Thus, for $kv_0 > \omega_0$, we assume

$$\omega = kv_0 - \omega_0 + \delta\omega \tag{4.7}$$

and substituting in (4.1), we obtain

$$\delta\omega \simeq i\Gamma\frac{(\omega_0^2 + \gamma_1^2)}{2\omega_0(kv_0 - \omega_0)}. \tag{4.8}$$

This case is the analogue of the resistive wall amplifier and the unstable mode has a finite frequency.

5. Conclusion

The concept of a negative energy wave provides an explanation of why a system can become unstable when dissipation is included. An even more cogent argument for the use of negative energy waves in stability problems is the following. For any conservative system possessing a small signal energy invariant, instability can only occur due to the coupling of positive and negative energy waves. In this case, both waves can grow whilst the small signal energy remains constant.

Since negative energy waves can only exist in systems not in thermal equilibrium, they are of particular relevance to stability problems. The identification of a

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negative energy wave provides insight into a stability problem and can be a significant aid in its solution.

Various examples from fluid mechanics and plasma physics have been described to illustrate the use and relevance of negative energy waves. The classic case of the Kelvin–Helmholtz instability relating to non-uniform flow in neutral fluids provided the first example. The second example was taken from the kinetic theory of waves in a hot, magnetized plasma, and the third example refers to the important subject of MHD which is the principle tool for stability analysis of solar and laboratory plasmas. The use of the concept of negative energy waves may be of particular relevance to the study of MHD stability of plasmas in the presence of equilibrium flows.

It has also been noted that the idea of negative energy waves is the basis of a scheme for classifying electrostatic instabilities into reactive and dissipative [9]. This proved helpful in considering the use of feedback stabilization to control confined plasma [18]. Finally a recognition of the energy flows between positive and negative energy waves in hydrodynamic or reactive instabilities could provide insight into their nonlinear evolution.

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I would like to dedicate this paper to Professor R. A. Cairns on the occasion of his 60th birthday. I have had the privilege and pleasure to collaborate with Alan for 25 years, an experience which has enriched and enhanced my work.

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