STUDY OF HAZARD RATE FUNCTIONS ON THE CUMULATIVE DAMAGE PHENOMENON

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ABSTRACT

In this paper different failure mechanisms which yield cumulative damage are investigated through two types of hazard rate functions. They have been studied during the past two decades. Type A was developed early by assuming the hazard rate as a function of reliability. There are two kinds of trend, one follows the negative logistic decay model, the other the negative Gompertz decay. Some modifications are suggested according to the failure tendency and convenience of fittings. Type B is developed recently by assuming the hazard rate as a function of the expected operation time, *T*, which is defined as the integration of reliability over the time, normalized by the mean-time-between-failure. In both types the proposed hazard rates grow with the time monotonically. Typical examples are taken to examine these models, meanwhile the comparisons with the Weibull-typed distribution are also made. The results show that the most of proposed relations are successful in the expression of cumulative damage phenomenon, especially the Type B is a better choice even compared with the Weibull-typed description in some respects. The advantages of the models are discussed based on the physical meanings involved in the parameters.

Keywords : Hazard rate function, Cumulative damage.

1. INTRODUCTION

One of the most obvious failures results in the cumulative damage phenomenon which is occurred in many physical situations, for instance fatigue, wear, corrosion *etc*. Such phenomenon is not easy detected as the system in using, and shows serious consequence suddenly. Two different ways of presentation about the cumulation are observed, see Fig. 1. It depends on the existence of inflection point or not. The difference conducts the manner of awareness about the uncertainty in operating a system. Typical curve with inflection point (Type-A profile) is something like the so-called logistic growth (or Gompertz growth), *i.e.* cumulative failure probability, $P(S \geq S_{st})$, where S and S_{st} denote the applied stress and the material strength, respectively, and $P(\ \ )$ represents the probability of the argument. Type-B profile of cumulation (without inflection point), *e.g.* cumulative damage probability $P(1 \geq D)$ where D is the damage level of a system, is favor in the planning of maintenance consideration. Curve with inflection point, for the case as the derivative of slope changed from positive to negative, takes much longer time to show a remarkable response, or say the threshold which is considered as a warning. On the contrary, for the curve without inflection point, it may start at very beginning with a substantial response, and thus the system is operated with much attention at every moment even as the system in its new situation. For certain failure mechanisms, the cumulative phenomenon can be depicted either by Type A or Type B curve. As a matter of fact, this depends on the quantity chosen in the expression as shown in Fig. 1 which figures the same

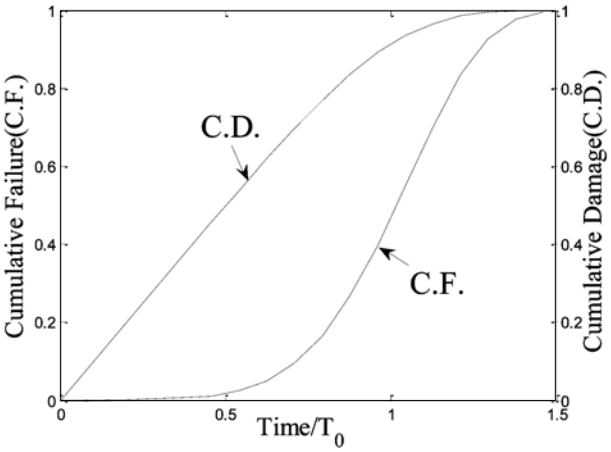


Fig. 1 Different descriptions of cumulative damage. Type A: Cumulative Failure (C.F.); Type B: Cumulative Damage (C.D.)

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failure event. Before doing this, the characteristics of failure probability density (fpd) should be studied first by collected data. By definition fpd is the time rate of reliability, R, thus the understanding of hazard rate, h, becomes an important subject. Usually h can be expressed in terms of a power relation with the time, this yields the Weibull-typed distribution [1].

In the past two decades a sort of relations for h *vs.* R were studied intensively [2-12]. This is based on the consideration that there exists a one-to-one correspondence between R and the time, t, meanwhile the reliability always monotonically decays with t. After a brief review more details of the relation, h *vs.* R, is discussed in Sec. 2.

Another proposed model of h *vs.* T in replacing of h over R is given in Sec. 3, where T is defined as the expected value of operation time, *i.e.* $T = \int_0^t R dt$. T is also shown to be a one-to-one correspondence with the time, and it starts monotonically from zero to a finite value, the so-called the mean-time-between-failure (T_0). This yields a benefit in carrying out the relation $R(D)$, where D is the ratio T/T_0, or $D(R)$, as given in Fig. 1. In a usual sense, D can be considered as the level of the damage of the operating system. Typical examples are taken in the comparisons of these models and the corresponding Weibull- typed distribution.

2. HAZARD RATE AS A FUNCTION OF RELIABILITY

There were two types of h *vs.* R considered in the past. One was named the differential equation model [2] in the form

$$\frac{dh}{dt} = f(R) . \tag{1}$$

Combine with the dynamic reliability equation as

$$\frac{\frac{dh}{dt}}{\frac{dR}{dt}} = \frac{f(R)}{-hR}$$

after integration it leads to

$$\frac{1}{2}(h^2 - h^2(0)) = F(R) , \tag{2}$$

where

$$F(R) = -\int_1^R \frac{f(R')}{R'} dR' .$$

Taking $f(R) = a_0 + a_1(1 - R)$ was studied in the reliability of crack growth problem [2] and the evaluation of a mechanical system reliability in its developing stage [3]. In those cases the hazard rate reduces to

$$\frac{1}{2}\left[h^2 - h^2(0)\right] = -(a_0 + a_1)\ln(R) + a_1(R-1) . \tag{3}$$

This form has its drawback due to its complicated fittings as compared to the other forms discussed later. However this equation shows an algebraic relation with a term proportional to ln(R) which emphasizes the failure tendency as the reliability reduces to zero.

For simplicity the second type of the hazard rate as a power of failure probability [4-6], called (e_0, c_n, n) model, is considered as

$$h = e_0 + c_n(1-R)^n , \tag{4}$$

where e_0 denotes the intrinsic weakness due to some defects during the manufacturing of a system. c_n indicates the process-dependent decay factor which relates to the applied load as well as the material used in the system for a specific power of $(1 - R)$, n which characterizes the memory of damage, *i.e.* for the specific failure mechanism. Usually e_0 is considered to be a very small positive real number as comparing with c_n [4,5]. As n = 1, Eq. (4) was also used to study the crack growth [2], wear failures [6,7]. It was also taken to investigate at the metal fatigue problem based on S-N curve [8]. No matter of c_1 or c_n, they had been correlated to a power function the applied load for metal-fatigue [9], or composite-fatigue failure [10], respectively. The companion of Eq. (4) was also extended to

$$h = e_0 + c_1(1-R) + c_2(1-R)^2 , \tag{5}$$

where the third term in the right-hand-side plays the role as n in Eq. (4) to allow more effectiveness in the adjusting of unsymmetrical distribution character of the failure probability density [11]. Figure 2 depicts the comparisons among Eqs. (4), (5) and Weibull-typed distribution for the reliability decay concerning with the failures of a system about public transportation bus system [13].

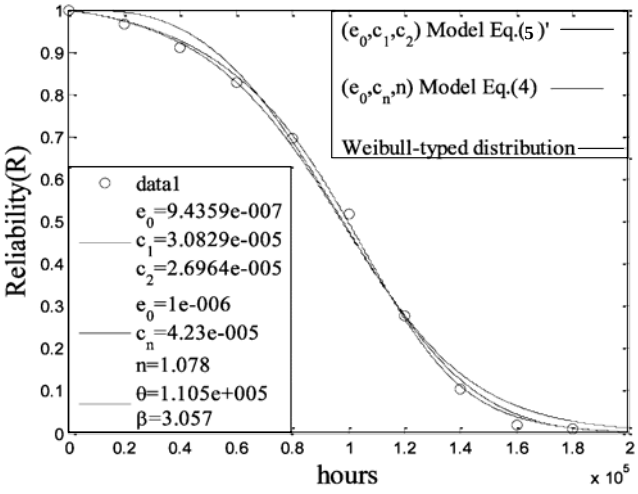


Fig. 2 Reliability decay for a public transportation system, data from Ref. [12]

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The comparison between Eq. (4) with $h = c't^{\beta}$, which leads to a Weibull-typed distribution, was shown in [12]. Little difference was observed besides small R, say $R < 0.1$. This is because that h approaches to a constant, see Eq. (4), but infinite in the Weibull-typed distribution. The behavior of hazard rates as the time approaches to infinite (or the reliability becomes zero) is very difficult to be identified due to less and also much more fluctuated failure data. Thus the hazard rate might be finite instead of infinite. However to account the tendency of h for large time as that shown in the Weibull-typed distribution, another type of hazard rate modified from Eq. (3) was proposed [11], called (e_0, b) model, as

$$h(R) = e_0 - b \cdot \ln R \ , \tag{6}$$

the reliability can be integrated as

$$R(t) = \exp\left[\frac{e_0}{b}(1 - e^{bt})\right] . \tag{7}$$

This relation is the negative growth-rate (or decay rate) Gompertz curve [14]. The comparisons of model (e_0, c_n, n) and (e_0, b) with Weibull-typed distribution are shown in Fig. 3 for the fatigue failure of optical glass fibers NBS under strain 0.525% [15]. For further development of this model, Eq. (5) was suggested as

$$h(R) = e_0 + c_1(1 - R) - b \ln R \ , \tag{8}$$

called (e_0, c_1, b) model, and

$$h(R) = e_0 + c_1(1 - R)(1 - b^* \ln R) \ , \tag{9}$$

called (e_0, c_1, b^*) model. The comparisons of Eqs. (4), (8), (9) with the Weibull-typed distribution for the fatigue failure of optical glass fibers NABS under strain 2.08% are given in Fig. 4. In both figures the fitted results agree with the trend of data. The modification due to the term related to ln(R) shows that the hazard rate increases with time as the occasion in the Weibull-typed distribution. More examinations of Eqs. (4), (5), (6), (8) and (9) can be found in Ref. [16].

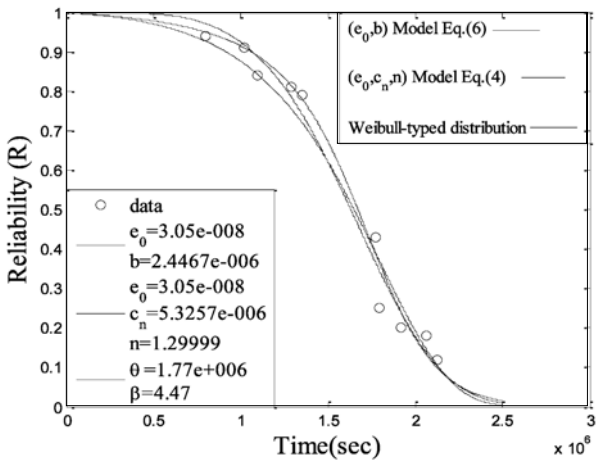


Fig. 3 Reliability decay for the fatigue of optical glass fibers NBS under strain 0.525%, data from Ref. [15]

3. HAZARD RATE AS A FUNCTION OF EXPECTED OPERATED OPERATING TIME

Now consider a mechanical system which transfers the energy from input to output as executing the function of a system. To be a common, the output should be less than the input due to the irreversible process, most because of friction or thermal metallurgy effect. The difference accumulates inside the system which then will become weak more and more, for example, it might induce a propagating crack. This causes failure of the system finally. Thus the expected operating time T can be considered as an index to measure the increment of weakness (or say the damage). In this paper T/T_0 is used to indicate the level of damage of an operating system, where T is defined

$$T = \int_0^t R \, dt \ ,$$

as t goes to infinite, T approaches to the mean-time-between-failure (MTBF) of the system. Denote MTBF as T_0. The benefit of using T instead of R in the expression of hazard rate is due to that this substitution makes the model avoid the necessity of consideration about the parameter playing the role of e_0 introduced in the previous section. At first, assume the hazard rate in the form

$$h = a \, D^m, \ m > 0, \ 0 \le D \le 1$$

Form the definition of operating time, *i.e.*

$$\frac{dT}{dt} = (R) \ , \quad \text{or} \quad \frac{dD}{dt} = \frac{R}{T_0} \ . \tag{10}$$

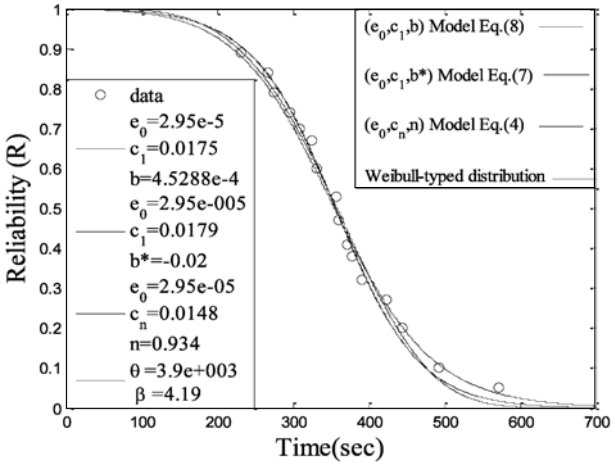


Fig. 4 Reliability decay for the fatigue of optical glass fibers NABS under strain 2.08%, data from Ref. [15]

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The time derivative of $h = aD^m$ leads to

$$\frac{dh}{dt} = a\ m\ D^{m-1}\frac{dD}{dt}\ .$$

Combining the above equation with Eq. (10) and the proposed hazard rate relation it shows

$$\frac{dh}{dt} = \frac{mT_0}{D}hR = -\frac{mT_0}{D}\frac{dR}{dt} = mT_0\left(\frac{a}{h}\right)^{-1/m}\frac{dR}{dt}\ .$$

Thus, we have

$$\frac{dh}{dR} = -mT_0\left(\frac{a}{h}\right)^{-1/m}\ .$$

After integration with the initial condition at $R = 1$, $h = h_0$, the hazard rate becomes

$$h^{m+1/m} - h_0^{\,m+1/m} = (m+1)\ T_0\ a^{-1/m}(1-R)\ . \qquad (11)$$

This relation can be reduced to a one of the type (e_0, c_n, n) model, if $h_0 = e_0 = 0$, and $n < 1$.

To emphasize the cumulative character, a logarithm measure of D would be an another choice [16], thus take the hazard rate in the form

$$h(D) = -\alpha \ln(1-D^{\eta})\ , \qquad (12)$$

it grows from zero to infinite gradually as the time goes on. Consider the system operated from new situation and suffered a sequence of damage as D_0, D_1, D_2, $\cdots$, D_N. The hazard rate can be written as a summation in the form

$$h_N(D_N) = \frac{-f(\eta)}{T_0}\sum_{i=0}^{N-1}\ln\left[\frac{1-D_{i+1}^{\eta}}{1-D_i^{\eta}}\right]\ ,$$
$$= \frac{-f(\eta)}{T_0}\sum_{i=0}^{N-1}h_i\ , \qquad (13)$$

where D_0 can be assumed either zero, if the system is manufactured perfectly, *i.e.* without any defects, or a very small positive real number, *i.e.* $h_0 = e_0$ as defined in Eq. (4). In other words the new system begins to service with little damage which embedded during its manufacturing period. From mathematics point of view the reliability decays naturally from its unstable equilibrium point $R = 1$ for (e_0, c_n, n) model discussed in Sec. 2 or Eq. (12) without the consideration of the existence of h_0 or not. Since the reliability follows the relation

$$dR = -h(D)\ R\ dt = -h(D)\ T_0\ dD\ ,$$

after integrating by parts it shows

$$R(D) = 1-\alpha T_0\ D\left\{\eta\left[1-\sum_{k=0}^{\infty}\frac{D^{k\eta}}{1+k\eta}\right]-\ln(1-D^{\eta})\right\}\ , \qquad (14)$$

where the summation terms are obtained under the constraint $0 \le D < 1$. One of the benefits shows in Eq. (14) is that the damage level can be directly estimated as the failure information, *i.e.* R, of a large amount of mechanical units in service is available, and thus the maintenance can be programmed in time. It exists a relation among α, η and T_0 with the definition of MTBF. Taking an iterative procedure in the convergence of T_0, it is found [18]

$$T_0\ \alpha = 0.619\ \eta + 0.385 = f(\eta) \qquad (15)$$

for the range $0.1 < \eta < 10$ with correlation coefficient 0.99. This model, Eq. (12) with Eq. (15) is called (T_0, η) model. Which indicates that the reliability-decay *vs.* the normalized time (t / T_0) follows the same curve for a given η, see Fig. 5. In other words η carries the same information as the shape factor, β, in the Weibull-typed distribution does. But (T_0, η) model scales the time with the mean-time-between-failure, instead of the scale factor, θ, as defined in the Weibull-typed distribution. Thus the fitting of (T_0, η) model leads directly to show the more meaningful information T_0 insteady of θ in the Weibull-typed distribution. Figures 6(a) and 6(b) present the comparisons of cumulative damage *vs.* time or reliability, respectively, among the four models including Eq. (14), Eq. (A2) (shown in Appendix A), Weibull-typed distribution and the constant failure rate, under the same T_0. Besides the constant failure rate case, little difference is observed for the rest models considered in the plots. The time variation of reliability of (T_0, η) model is obtained by Eq. (10) with Eq. (14). Two sets of failure data [1,19] are used in the fittings of Figs. 7 and 8 among the proposed models, including Eq. (4), (6), (12) and the Weibull-typed distribution. For the above cases the cumulative damage *vs.* time and reliability are presented in Figs. 9(a) and 9(b), respectively. In the plots η can be defined as the damage cumulation index, the larger η is the more serious situation of cumulative phenomenon should be. From physical point of view the wear of automobile brake shows much more cumulative character as compared with the vehicle corrosion failure. Besides the fitted T_0, which can also be correlated to the applied load *e.g.* from N-S curve for fatigue [9], η corresponds to a certain specific failure mechanism. This is shown in Fig. 10, which are plotted for four cases of different fatigue loads [10] under the same value 3.35 of η. As the time is normalized with T_0 the reliability decays follow the same curve. Except the property of superposition of hazard rate shown in Eq. (13), perhaps, this is the most obvious merit of (T_0, η) model this is also shown by considering (e_0, c_n, n) model. To the author's knowledge the shape factor of Weibull-typed distribution can be connected to the specific failure mechanism for a given material, the scale factor related to the applied load. But such representation has not been discussed yet. Detail comparisons of the characteristics of the proposed models are made in Tables 1 and 2. The corresponding formula of cumulative damage *vs.* reliability for the models proposed in Sec. 2 is summarized in the Appendix A.

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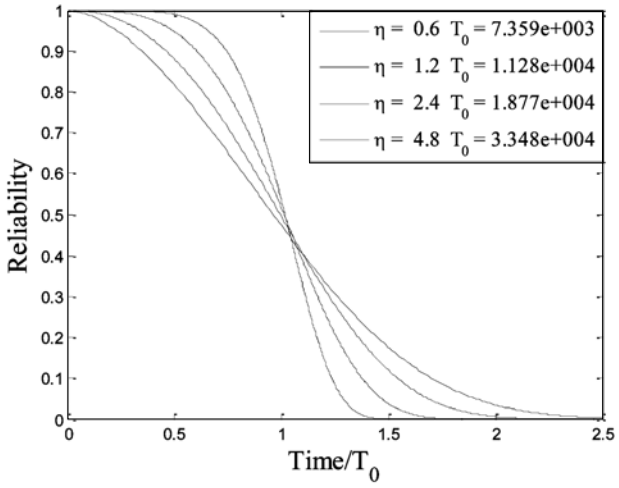
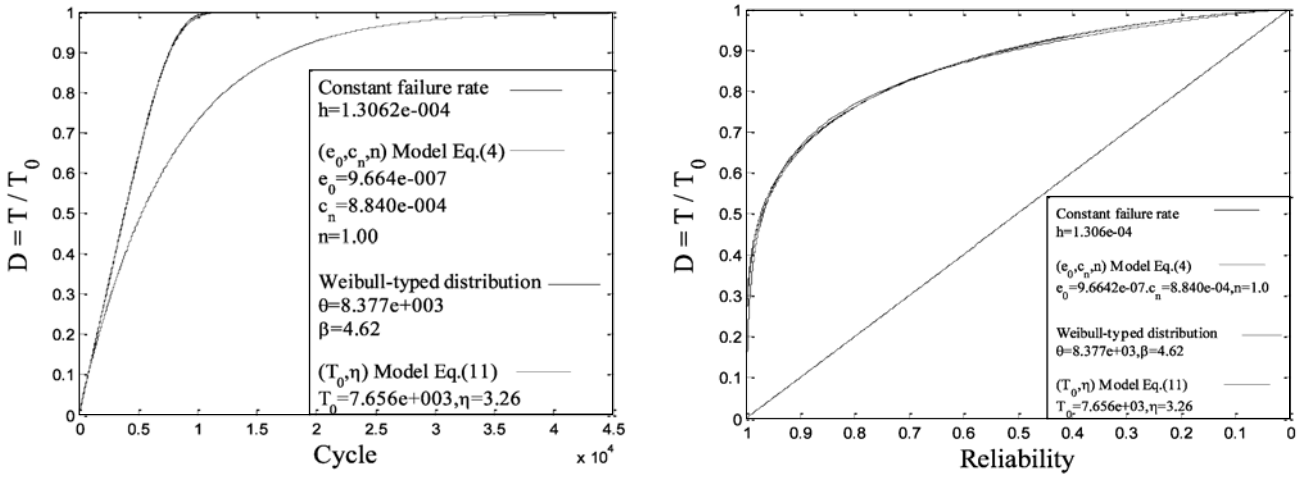


Fig. 5 Reliability decay of (T_0, η) model for different values of T_0 and η



(A) DAMAGE *VS.* CYCLES (B) DAMAGE *VS.* RELIABILITY

Fig. 6 Damage growth for different models under the same T_0

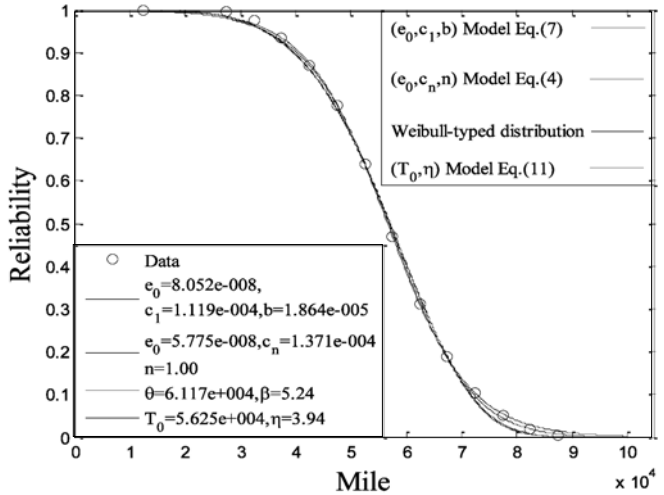


Fig. 7 Reliability decay for automobile brakes failure, data from Ref. [1]

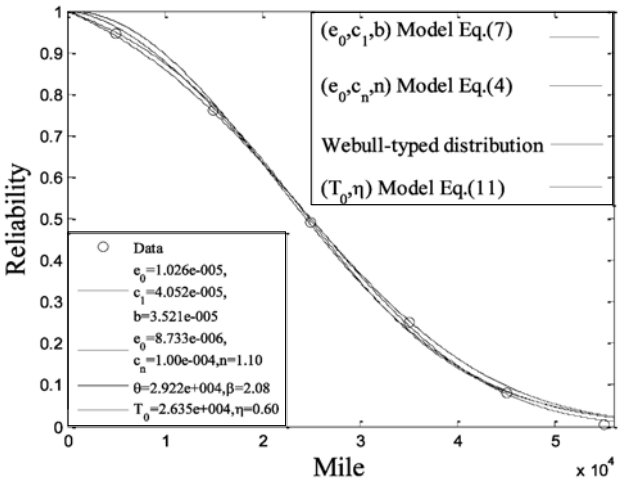
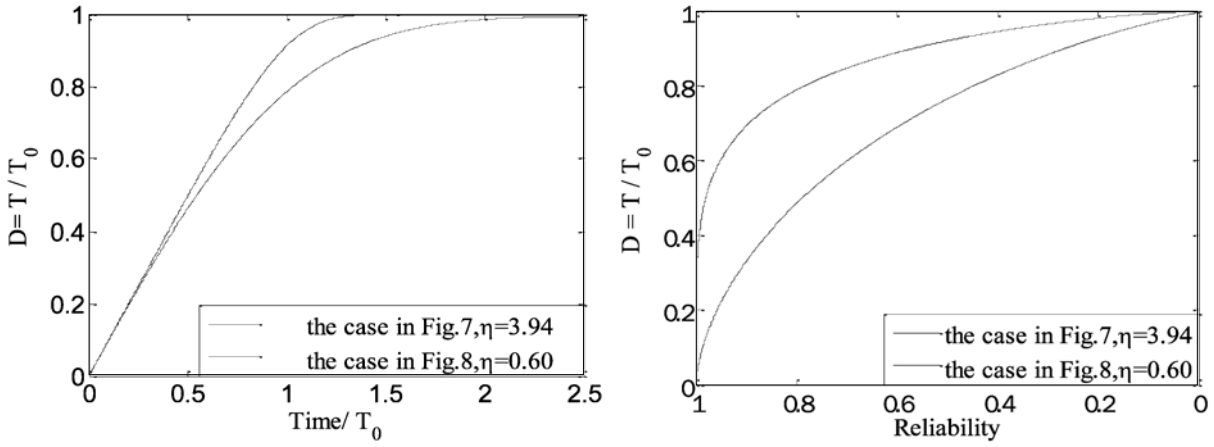


Fig. 8 Reliability decay for the corrosion failure of vehicles, data from Ref. [19]

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(a) Damage *vs.* Normalize time (b) Damage *vs.* Reliability

Fig. 9 The comparisons of damage for the case shown in Figs. 7 and 8

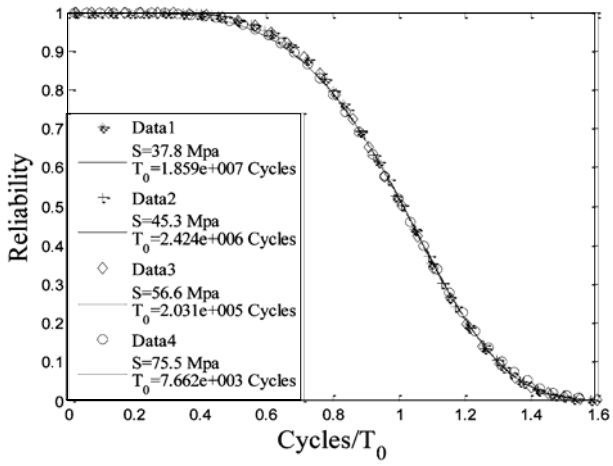


Fig. 10 Reliability decay for the fatigue of composite materials under different loading conditions, data from Ref. [10], η = 3.35

Table 1 The physical meanings of parameters in the proposed models for cumulative damage failure

Model / Parameter involved in the	Eq. (4) (e_0, c_n, n)	Eq. (5) (e_0, c_1, c_2)	Eq. (6) (e_0, b)	Eq. (12) (T_0, η)	Weibull-typed Distribution
Cumulative Damage Character (failure mechanism)	n	c_1, c_2	b	η	β
Initiation of Damage	e_0	e_0	e_0	D_0	–
Applied Load Relation	c_n	c_1	b	T_0	θ
With MTBF (T_0)	c_n, n	c_1, c_2	e_0, b	T_0	β, θ

Table 2 The comparisons of the characterics among proposed models and Weibull-typed distribution

Model / Parameter Involved in the	Eq. (4) (e_0, c_n, n)	Eq. (5) (e_0, c_1, c_2)	Eq. (6) (e_0, b)	Eq. (12) (T_0, η)	Weibull-typed Distribution
Fittings Convenience	Some Experience	Little Experience	Little Experience	Some Experience	Easy
Flexibility for Complicated Loading Situation	Good	Very Good	Moderate	Very Good	Very Good

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Consider a more realistic situation, for example, the applied fatigue load changed stage by stage, the hazard rate in the j-th stage, h_j, is further extended as

$$h_j = \frac{f(\eta)}{T_{j,0}} \ln(1-D_j^{\eta}) , \tag{16}$$

where

$$D_j = \sum_{i=1}^{j-1} \frac{T_i^*}{T_{i,0}} + \frac{T_j}{T_{j,0}} = D_j^* + \tilde{D}_j , \tag{17}$$

and

$$T_i^* = \int_{t_i}^{t_{i+1}} R_{i-1}\, dt ; \quad T_j = \int_{t_j}^{t} R_i\, dt .$$

where t_i and t_{i+1} denote the time of beginning and end of the j-th stage, respectively. η is a certain positive real number according to the specific failure mechanism, *i.e.* fatigue failure in this case. $T_{i,0}$ is the mean-time-between-failure for the i-th fatigue load. R_i is the reliability of the system operated in the i-th period. During the transition, *i.e.* the load changed from high- to low-level or low- to high-level, the reliability remains horizontally with Δn cycles for the former case or drops suddenly ΔR for the latter [20]. Otherwise the damage can be evaluated piecewisely by integrating the reliability over each stage with the corresponding hazard rate. Beyond the transition the cumulative damage *vs.* reliability, *i.e.* Eq. (14) becomes

$$\begin{aligned} R = {} & R_i(t_{i-1}) \\ & - f(\eta)(D_i^* + \tilde{D}_i)\left\{\eta\left[1-\sum_{k=0}^{\infty}\frac{1}{k\eta+1}(D_i^* + \tilde{D}_i)^{\eta}\right]\right. \\ & \left. -\ln\left[1-(D_i^* + \tilde{D}_i)^{\eta}\right]\right\} \\ & + f(\eta)D_i^*\left\{\eta\left[1-\sum_{k=0}^{\infty}\frac{1}{k\eta+1}D_i^{*\eta}\right]\right. \\ & \left. -\ln\ (1-D_i^{*\eta})\right\} \end{aligned} \tag{18}$$

Another relation can be considered as

$$h = \tilde{a}\, L^{\tilde{m}} ,$$

where L is defined to be the expected down time of an operating system as

$$L = \int_0^t (1-R)\, dt ,$$

However it leads to

$$h = e_0(\tilde{a},\ \tilde{m}) + c\ (\tilde{a},\ \tilde{m})\ (R-1-\ln R)^{\tilde{m}/1-\tilde{m}} .$$

This relation has not much flexibility in fittings as Eq. (8).

4. CONCLUSIONS

In this paper two types of hazard rate are proposed to deal with the cumulative damage phenomenon. According to the fitted results most of the models are satisfied as compared with the Weibull-typed distribution. Several remarks of these models are addressed as follows.

1. Reliability-dependent hazard rate function is developed initially from either logistic decay or Gompertz decay. Some modifications are considered to allow the fitting range more flexible and meaningful.
2. Operating time-dependent hazard rate function is developed under the assumption which concerns with the operating time as a manner to measure the cumulative damage of an operating system. In this model, T_0 is related to the applied load and η shows the corresponding cumulative character for specific failure mechanism.
3. From data fitting point of view the proposed relation $h = h(R)$ is more convenient than $h(D)$, but the latter has more potential to deal with the coupling effect of accident failure under normal failure situation or to the case as the applied load changed stage by stage, see Eq. (18). This needs further investigations.

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APPENDIX A
CUMULATIVE DAMAGE *VS.* RELIABILITY FOR MODELS IN SEC. 2

Since the reliability follows

$$\frac{dR}{dt} = -h(R)\ R ,$$

thus, we have

$$R\ dt = dT = -\frac{dR}{h(R)} ,$$

i.e.

$$D = \frac{-1}{T_0}\int_1^R \frac{dR}{h(R)} \tag{A1}$$

For the case, $h(R) = e_0 + c(1 - R)$, Eq. (A1) becomes

$$D = \frac{\ln\left[1+\frac{c}{e_0}(1-R)\right]}{\ln\left(1+\frac{c}{e_0}\right)} . \tag{A2}$$

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For the case, $h(R) = e_0 + c_1(1 - R) + c_2(1 - R)^2$, the cumulative damage becomes

$$D = \frac{\ln\left[\frac{a_1 + 2a_2R - \sqrt{\Delta}}{a_1 + 2a_2R + \sqrt{\Delta}}\right] - \ln\left[\frac{a_1 + 2a_2 - \sqrt{\Delta}}{a_1 + 2a_2 + \sqrt{\Delta}}\right]}{\ln\left[\frac{a_1 - \sqrt{\Delta}}{a_1 + \sqrt{\Delta}}\right] - \ln\left[\frac{a_1 + 2a_2 - \sqrt{\Delta}}{a_1 + 2a_2 + \sqrt{\Delta}}\right]} \quad \text{(A3)}$$

where $a_0 = e_0 + c_1 + c_2$, $a_1 = -c_1 - 2c_2$, $a_2 = c_2$ and the necessary condition

$$\Delta = a_1^2 - 4a_0\, a_1 > 0 \quad i.e.\ c_1^2 - 4e_0\, c_2 > 0$$

For the case,

$$h(R) = e_0 + c_n(1 - R)^n \ ,$$

the cumulative damage becomes

$$D = \frac{x^p \sum_{j=1}^{\infty} \frac{(j-1)!\ (c_n\, q)^{j-1} x^{(j-1)q}}{\prod_{l=1}^{j-1}(p + lq) y^j}}{\sum_{j=1}^{\infty} \frac{(j-1)!\ (c_n\, q)^{j-1}}{\prod_{l=1}^{j-1}(p + lq) y_0^j}} \ , \quad \text{(A4)}$$

where n is a positive real number represented as $n = q/p$, $x = (1 - R)^{1/p}$, $y = e_0 + c_n$. The product notation $\prod_{l=1}^{j-1}(p + lq)$ is defined as

$\prod_{l=1}^{m}(p + lq) = (p + q)(p + 2q)\cdots(p + mq)$, where $m \geq 1$,
and
$\prod_{l=1}^{m}(p + lq) = 1$, where $m < 1$.

For n as a positive integer, then Eq. (4) becomes

$$D = \frac{x \sum_{j=1}^{\infty} \frac{(j-1)!\ (c_n\, n)^{j-1} x^{(j-1)q}}{\prod_{l=1}^{j-1}(l_n + 1)(e_0 + c_n\, x^n)^j}}{\sum_{j=1}^{\infty} \frac{(j-1)!\ (c_n\, n)^{j-1}}{\prod_{l=1}^{j-1}(l_n + 1)(e_0 + c_n)^j}} \quad \text{(A5)}$$

where $x = 1 - R$.

For the case,

$$h(R) = e_0 - b \ln R \ ,$$

the cumulative damage becomes

$$D = \frac{\sum_{j=1}^{\infty}(j-1)!\ (-b)^{j-1}\left\{\frac{1}{e_0^j} - \frac{R}{(e_0 - b\ln R)^j}\right\}}{\sum_{j=1}^{\infty}(j-1)!\ (-b)^{j-1}\frac{1}{e_0^j}} \quad \text{(A6)}$$

For Weibull-typed-distribution the cumulative damage can be calculated as

$$D = \frac{\int_0^t \exp\left(\frac{t}{\theta}\right)^{\beta} dt}{\theta\, \Gamma\left(1 + \frac{1}{\beta}\right)} \ , \quad \text{(A7)}$$

where Γ is Gamma function as

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt \ .$$

In this situation D can not be represented by a function of reliability directly.

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