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Nonlinear effects in Alfvén resonance

I. S. DMITRIENKO
Institute of Solar–Terrestrial Physics (ISTP), Irkutsk 33, PO Box 4026, 664033, Russia

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A study is made of the influence of nonlinearity upon the structure of a steady-state disturbance brought about by low-frequency ($\omega \ll \omega_i$) monochromatic magnetosound propagating in a transversally inhomogeneous low-pressure plasma ($\beta \ll m_e/m_i$), in the vicinity of the Alfvén resonance point for the case when the principal linear mechanism for forming the transverse structure is provided by magnetic field diffusion due to small plasma resistivity. It is shown that the nonlinear properties of the fundamental harmonic of a disturbance are determined by its self-action through a plasma density variation that it produces. The disturbance structure in the region of the Alfvén resonance is described.

1. Introduction

The phenomenon of Alfvén resonance occurs in a plasma that is inhomogeneous across the magnetic field on the penetration of fast magnetosound from a region where conditions are present for its propagation across the magnetic field, far beyond the turning point deep into the opacity region, near the surface $x = x_0$ where the dispersion relation is satisfied for Alfvén oscillations $\omega^2 = k_z^2 A^2$ (where ω and k_z are the frequency and longitudinal wavenumber of an incident FMS wave, and A is the Alfvén velocity, which is a function of the coordinate x) (Chen and Hasegawa 1974; Southwood 1974).

An analytical description of the structure of such a disturbance as a function of the coordinate along the inhomogeneity, in terms of ideal magnetohydrodynamics, leads to a solution with a singularity that manifests itself most dramatically in the y component of the field and the velocity (the outer field is along z): $B_y \propto v_y \propto (x - x_0)^{-1}$ (Southwood 1974). Singularity of a resonant disturbance is known to be a consequence of an abstraction from the real properties of the medium and disturbance and is eliminated by taking into account such effects as the energy dissipation due to the presence of resistance (Southwood 1974) and viscosity (Davila 1987), the linear transformation to kinetic Alfvén waves (Chen and Hasegawa 1974), and the appearance near the resonance surface of oscillations caused by finiteness of the electron inertia (Swanson 1975; Mazur *et al.* 1979). The above-mentioned effects are characterized by small parameters: μ (resistivity), ν_α (ion viscosity coefficients), ρ_i (ion Larmor radius) and s_e (electron skin length); therefore a resonant disturbance acquires a thickness, which is finite but small. The transverse structure is a result of a competition of the above small effects. Hence, on the one hand, it carries information about parameters of the medium; on the other, it is of immediate interest in the context of the research on resonant energy absorption and the interpretation of phenomena associated with particle

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precipitation accompanying geomagnetic pulsations. However, in terms of linear theory, the development of which has lead to the above scheme for formation of the transverse structure of Alfvén resonance, the question as to whether nonlinear effects can come into play as well as linear effects, and what this will bring about, remains open. The goal of this paper is to attempt an answer to this question in the case when plasma and disturbance parameters ensure predominance, among linear structure-forming factors, of the magnetic field diffusion due to the finite plasma resistivity ($\mu \neq 0$). To accommodate nonlinearity correctly, together with the dominant linear effect and the inhomogeneity of the medium, a sequential procedure is adopted on the basis of different-scale methods.

2. Input equations

The input equations that we use are the magnetohydrodynamic equations in the form

$$\left.\begin{aligned} \partial_t \mathbf{B} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \mu \nabla^2 \mathbf{B}, \\ d_t \mathbf{v} &= -\frac{T \nabla n}{m_i n} + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi m_i n} + \frac{\mathbf{F}_v}{m_i n}, \\ \partial_t n &+ \nabla \cdot n\mathbf{v} = 0. \end{aligned}\right\} \tag{1}$$

Here $T = T_e + T_i$ ($T_e \lesssim T_i$), μ is the magnetic resistivity, and $\mathbf{F}_v$ is the force due to the presence of ion viscosity. According to Braginsky (1965), $\mathbf{F}_v = \partial_{x_\beta} \pi_{\alpha\beta}$, where the tensor $\pi_{\alpha\beta}$ has a very complicated form, but for the disturbance under consideration the expression for $\mathbf{F}_v$ allows a substantial simplification. This, as well as the inclusion of the ion frictional force, will be discussed in what follows.

The linearized system (1), for disturbances $\Psi_i'(\mathbf{x}, t)$ of the quantities Ψ_i ($i = 1, ..., 7$) appearing in it, in an inhomogeneous (in the coordinate x) medium has solutions of the form

$$\Psi_i'(\mathbf{x}, t) = \Psi_{iL}'(x, \theta) = \Psi_{iL}'(x) e^{i\theta} + \text{c.c.}, \qquad \theta = -\omega t + k_z z + k_y y.$$

Seeking to determine the influence of nonlinearity upon the dependence of Ψ_{iL} on x in the vicinity of $x = x_0$, we shall assume the Ψ_i' that satisfy the unlinearized equations (1) to be functions of x and θ:

$$\Psi_i = \Psi_{i0}(x) + \bar{\Psi}_i'(x) + \tilde{\Psi}_i'(x, \theta), \qquad \tilde{\Psi}_i'(x, \theta) = \sum_{n=1}^{\infty} \Psi_{in}'(x) e^{in\theta} + \text{c.c.}$$

Henceforth, overbars ($\bar{}$) and tildes ($\tilde{}$) indicate the aperiodic and periodic parts of the quantities on which they appear, and $\Psi_{i0}(x)$ is the (singled out for convenience) contribution of an unperturbed state: $\mathbf{B}_0 = (0, 0, B_0)$, $\mathbf{v}_0 = 0$ and $n_0 = n_0(x)$. From the relationship $n_0(x) + B_0^2 m_i / 8\pi T = \text{const}$, B_0 has a dependence on x that is weak because $\beta = B_0^2 m_i / 8\pi T n_0$ is assumed to be small. Further, Ψ_i' will be considered to mean the dimensionless quantities $(\mathbf{b}, \mathbf{u}, r)$. $\mathbf{B} = \mathbf{B}_0 + B_0(x_0)\mathbf{b}$, $\mathbf{v} = \omega k^{-1} \mathbf{u}$ and $n = n_0(x) + n_0(x_0) r = n_0(x_0)[r_0(x) + r]$. We shall also put $\mathbf{f} = k_z \omega^{-2} \mathbf{F}_v / m_i n_0$.

The nonlinear terms N_i in (1) have the component form

$$N_i = \begin{cases} \omega B_0 n_i & (i = 1, 2, 3), \\ \dfrac{\omega^2}{k} n_i & (i = 4, 5, 6), \\ \omega n_7 & (i = 7), \end{cases} \tag{2}$$

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where

$$n_1 = \partial_\theta \left[\frac{k_y}{k_z}(u_x b_y - u_y b_x) - u_z b_x + u_x b_z \right],$$

$$n_2 = \partial_\theta (u_y b_z - u_z b_y) - \frac{1}{k_z}\partial_x (u_x b_y - u_y b_x),$$

$$n_3 = \frac{1}{k_z}\partial_x (u_z b_x - u_x b_z) - \frac{k_y}{k_z}\partial_\theta (u_y b_z - u_z b_y),$$

$$n_4 = \frac{r}{r_0}\left[\partial_\theta u_x - \left(\mathbf{u} \cdot \frac{\nabla}{k_z} \right) u_x \right] - \frac{1}{2k_z}\partial_x u_x^2 - \frac{1}{2}\partial_\theta u_z^2 - \frac{k_y}{k_z} u_y \partial_\theta u_x$$
$$+ \frac{k_z^2}{q^2}\left[b_z \partial_\theta b_x + \frac{k_y}{k_z} b_y \partial_\theta b_x - \frac{1}{2k_z}\partial_x (b_y^2 + b_z^2) \right],$$

$$n_5 = \frac{r}{r_0}\left[\partial_\theta u_y - \left(\mathbf{u} \cdot \frac{\nabla}{k_z} \right) u_y \right] - \frac{1}{k_z} u_x \partial_x u_y - \frac{k_y}{2k_z}\partial_\theta u_y^2 - u_z \partial_\theta u_y$$
$$+ \frac{k_z^2}{q^2}\left[\frac{1}{k_z} b_x \partial_x b_y - \frac{k_y}{2k_z}\partial_\theta (b_x^2 + b_z^2) + b_z \partial_\theta b_y \right],$$

$$n_6 = \frac{r}{r_0}\left[\partial_\theta u_z - \left(\mathbf{u} \cdot \frac{\nabla}{k_z} \right) u_z \right] - \frac{1}{k_z} u_x \partial_x u_z - \frac{k_y}{k_z} u_y \partial_\theta u_z - \frac{1}{2}\partial_\theta u_z^2$$
$$+ \frac{k_z^2}{q^2}\left[\frac{1}{k_z} b_x \partial_x b_z + \frac{k_y}{k_z} b_y \partial_\theta b_z - \frac{1}{2}\partial_\theta (b_x^2 + b_y^2) \right],$$

$$n_7 = -\frac{1}{k_z}\partial_x (r u_x) - \partial_\theta \left(\frac{k_y}{k_z} r u_y + r u_z \right),$$

and $q^2(x) = 4\pi m_i n_0(x)\omega^2 / B_0^2$. They can be represented for each x as Fourier series expansions:

$$N_i = \bar{N}_i(x) + \sum_{n=1}^{\infty} N_{in}(x) e^{in\theta} + \text{c.c.} = \bar{N}_i(x) + \tilde{N}_i(x, \theta).$$

The relation between disturbances of the transverse components of the magnetic field and the velocity has the form

$$\partial_\theta \tilde{u}_{x,y} = -\partial_\theta \tilde{b}_{x,y} - \frac{\mu}{\omega}\nabla^2 \tilde{b}_{x,y} - \tilde{n}_{1,2}. \tag{3}$$

For a disturbance of the transverse components of the magnetic field, one can obtain the equations

$$\partial_{xx}^2 \tilde{b}_x - \frac{k_y^2 (\partial_x q^2)\partial_x \tilde{b}_x}{(q^2 - k_z^2)(q^2 - k_z^2 - k_y^2)} - (q^2 - k_z^2 - k_y^2)\partial_{\theta\theta}^2 b_x$$
$$= -\frac{q^2 - k_z^2 - k_y^2}{q^2 - k_z^2}\left[k_y \partial_x \left(\frac{q^2 \tilde{P}_2}{q^2 - k_z^2 - k_y^2} \right) + q^2 \partial_\theta \tilde{P}_1 \right], \tag{4}$$

$$\partial_\theta \left\{ \partial_x \left(\frac{\partial_x [(q^2 - k_z^2)\tilde{b}_y]}{q^2 - k_z^2} \right) - (q^2 - k_z^2 - k_y^2)\partial_{\theta\theta}^2 \tilde{b}_y \right\}$$
$$= \partial_x \left\{ \frac{1}{q^2 - k_z^2}[\partial_x (q^2 \tilde{P}_2) - k_y q^2 \partial_\theta \tilde{P}_1] \right\} - q^2 \partial_{\theta\theta}^2 \tilde{P}_2, \tag{5}$$

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where

$$P_{1,2} = -n_{1,2} + n_{4,5} - \frac{\mu}{\omega}\nabla^2 b_{x,y}.$$

We shall be considering the case when finite resistivity ($\mu \neq 0$) is the main linear effect; in what follows, we shall therefore neglect the influence upon $\tilde{\Psi}_i'(x,\theta)$ of linear effects associated with the inclusion of T and $\mathbf{F}_v$. As far as the approximation $T = 0$ in (4) and (5) is concerned, this is known to signify the neglect of drift effects and the relation of a resonant disturbance to slow magnetosound; the structure-forming role played in resonance by thermal effects (a linear transformation to Alfvén waves propagating perpendicularly to the magnetic field) is negligibly small when $\beta \ll m_e/m_i$, and this will be assumed in what follows. Our consideration of this case at frequency $\omega \ll \omega_i$ has dictated our choice of the equation for $\mathbf{B}$ in the form of the first equation of (1) rather than the more exact equation containing the term $\nabla \times \mathrm{d}\mathbf{v}_i/\mathrm{d}t$ on its right-hand side. The discarded linear effects of ion viscosity are taken into account in (4) and (5) by the linear additions of P_{1v} and P_{2v} to P_1 and P_2. Near $x = x_0$, we have

$$\partial_x \tilde{P}_{2v} - \partial_y \tilde{P}_{1v} \sim \partial_x \tilde{f}_y - \partial_y \tilde{f}_x.$$

Neglecting the longitudinal velocity and assuming $\partial_x \tilde{u}_x + \partial_y \tilde{u}_y = 0$ (this condition at the main order of a resonant disturbance will subsequently be shown to be satisfied), we obtain under the condition $\omega_i \tau_i > 1$, according to Braginsky (1965),

$$\tilde{f}_x = \left[\frac{\nu_1}{\omega}(\nabla^2 \tilde{u}_x + 3\partial^2_{zz}\tilde{u}_x) + \frac{\nu_3}{\omega}(\nabla^2 \tilde{u}_y + \partial^2_{zz}\tilde{u}_y)\right],$$

$$\tilde{f}_y = \left[\frac{\nu_1}{\omega}(\nabla^2 \tilde{u}_y + 3\partial^2_{zz}\tilde{u}_y) - \frac{\nu_3}{\omega}(\nabla^2 \tilde{u}_x + \partial^2_{zz}\tilde{u}_x)\right];$$

hence the largest viscosity coefficient, ν_0, and the next largest, ν_3, are not involved in (4) and (5) in the linear case (that ν_0 does not influence the regularization of resonance was pointed out by Hollweg and Yang 1988). Thus, when $\nu_1 \ll \mu$, ion viscosity can be neglected compared with magnetic viscosity: because $\nu_1 \sim T_i\tau_i/m_i(\omega_i\tau_i)^2$ and $\mu \sim c^2 m_e/4\pi n e^2 \tau_e$ (Braginsky 1965), $\nu_1 \sim \mu\beta_i(m_i/m_e)^{1/2}$; and, when $\beta \ll m_e/m_i$, the condition $\nu_1 \ll \mu$ is satisfied. Hence, neglecting the influence of $T \neq 0$ and $\mathbf{F}_v$ upon small (in Alfvén resonances) disturbances $\tilde{b}_z$, $\tilde{u}_z$ and $\tilde{r}$, we obtain for $\tilde{b}_z$ the equation

$$\partial^2_{xx}\tilde{b}_z - (\partial_x \tilde{b}_z)\partial_x[\ln(q^2 - k_z^2)] + (q^2 - k_z^2 - k_y^2)\partial^2_{\theta\theta}\tilde{b}_z$$
$$= \frac{1}{k_z}\{\partial_x(q^2\tilde{P}_1) - q^2\tilde{P}_1\partial_x[\ln(q^2 - k_z^2]\} + \frac{k_y}{k_z}\partial_\theta(q^2\tilde{P}_2). \tag{6}$$

and for $\tilde{u}_z$ and $\tilde{r}$

$$-\partial_\theta \tilde{u}_z = \tilde{n}_6, \tag{7}$$

$$\partial_\theta\left[-\tilde{r} + r_0\left(\frac{k_y}{k_z}\tilde{u}_y + \tilde{u}_z\right)\right] + \frac{1}{k_z}\partial_x(r_0\tilde{u}_x) = \tilde{n}_7. \tag{8}$$

For the aperiodic part of the disturbance $\bar{\Psi}_i'(x)$, from (1), taking account of $\mathbf{F}_v$, for $\mathbf{u} = \mathbf{u}(x)$ (Braginsky 1965) and $\nabla \cdot \mathbf{b}$ we get the equations

$$\partial_x \bar{b}_x = 0, \tag{9}$$

$$-\frac{\mu}{\omega}\partial^2_{xx}\bar{b}_y = \bar{n}_2, \tag{10}$$

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$$-\frac{\mu}{\omega}\partial_{xx}^2\bar{b}_z - \frac{1}{k_z}(\partial_x \ln r_0)\,\bar{u}_x = \bar{n}_3 - \frac{\bar{n}_7}{r_0}, \tag{11}$$

$$\frac{k_z}{q^2}\partial_x(\bar{b}_z + \beta^*\bar{r}) - \frac{1}{\omega}\left(\frac{\nu_0}{3} + \nu_1 + \frac{\nu_3^2}{\nu_1}\right)\partial_{xx}^2\bar{u}_x - \frac{\nu_3}{\omega}\partial_{xx}^2\bar{u}_y = \bar{n}_4 - \frac{\nu_3}{\nu_1}\bar{n}_5, \tag{12}$$

$$-\frac{\nu_1}{\omega}\partial_{xx}^2\bar{u}_y + \frac{\nu_3}{\omega}\partial_{xx}^2\bar{u}_x = \bar{n}_5, \tag{13}$$

$$-4\frac{\nu_1}{\omega}\partial_{xx}^2\bar{u}_z = \bar{n}_6, \tag{14}$$

$$\frac{1}{k_z}\partial_x(r_0\bar{u}_x) = \bar{n}_7, \tag{15}$$

where $\beta^* = T/m_i A^{-2} = \frac{1}{2}\beta$.

Assuming that the magnitude of the disturbance is characterized by a small parameter ϵ (the particular meaning of ϵ will be ascertained later), we represent all disturbed quantities as series expansions in ϵ:

$$\Psi_i' = \epsilon\Psi_i'^{(1)}(x,\theta) + \epsilon^2\Psi_i'^{(2)}(x,\theta) + \epsilon^3\Psi_i'^{(3)}(x,\theta) + \ldots, \tag{16}$$

in which Ψ_{i1}' is the solution of a linear problem $\Psi_{i1}'^{(1)}(x,\theta) = \Psi_{iL}'(x,\theta)$. A necessary condition for $\Psi_{iL}'(x,\theta)$ to be a correct approximation to the solution of a nonlinear problem is the smallness of nonlinear corrections. This condition is most readily violated when Ψ_{iL}' is large, that is, near the resonance surface $x = x_0$; far from the surface $x = x_0$, nonlinear corrections are small (provided, of course, that ϵ is not too large). We shall deal with precisely such a case and consider primarily a nonlinear disturbance in the outer region where an expansion of the form (16) is applicable.

3. Outer region

3.1. Fundamental harmonic

Equations (3)–(6) and (7) and (8) with zero right-hand sides constitute a linear approximation of the problem under consideration, without taking account of plasma resistivity, and their solutions near the resonance surface are well known (Southwood 1974). Assume near $x = x_0$ that $r_0(x) = 1 + \xi$, $\xi = (x - x_0)/l$, and $l^{-1} = (dr_0/dx)|_{x=x_0}$; then $q^2 - k_z^2 = k_z^2\xi$. When $\xi \to 0$, the solution of the differential equation for $b_{zL0}(x)$ (namely (6) with zero right-hand side; $\partial_\theta = i$) is representable as

$$b_{zL0}(x) = A_\pm(\xi'^2 + \ldots) + B_\pm(1 + \xi'^2\ln|\xi'| + \ldots) + \ldots, \qquad \xi' = 2^{-1/2}k_y l\xi, \tag{17}$$

where $A_\pm$ and $B_\pm$ are constants, the + and − signs refer to $x > x_0$ and $x < x_0$ respectively and the subscript 0 on b_{zL0} denotes the approximation $\mu = 0$. The other disturbance components are expressed in terms of $b_{zL0}(x)$ by the relationships

$$\left.\begin{aligned} b_{xL0} &= \frac{i}{k_z l}\frac{\partial_\xi b_{zL0}}{\xi}, \qquad b_{yL0} = -\frac{k_y}{k_z}\frac{b_{zL0}}{\xi}, \\ \mathbf{u}_{\perp L0} &= -\mathbf{b}_{\perp L0}, \qquad u_{zL0} = 0, \qquad r_{L0} = (1+\xi)b_{zL0} - \frac{1}{(k_z l)^2}\frac{\partial_\xi b_{zL0}}{\xi}, \end{aligned}\right\} \tag{18}$$

and as $\xi \to 0$, $b_{xL0} \sim u_{xL0} \sim r_{L0} \sim \ln\xi$ and $b_{yL0} \sim u_{yL0} \sim \xi^{-1}$. On taking resistivity into account, singularities in the solution are eliminated. In the region

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$|\xi| \gg \Delta_\mu$, where Δ_μ is a typical scale of the linear solution regularized in view of $\mu \neq 0$, the 'resistive' solutions themselves are exponentially small, and corrections to the solutions (17) and (18) are derivable from perturbation theory. Representing $\mathbf{\Psi}_{iL}(x)$ as $\mathbf{\Psi}_{iL} = \mathbf{\Psi}_{iL0} + \mathbf{\Psi}_{iL1} + \ldots$, we get

$$\left.\begin{aligned} b_{yL1} &= \frac{i\mu}{\omega l^2}\frac{\partial^2_{\xi\xi} b_{yL0}}{\xi}, \\ \partial_\xi b_{xL1} &= -ik_y l b_{yL1}, \\ b_{zL1} &\sim \frac{\mu}{\omega l^2}\frac{1}{\xi^2}, \\ \mathbf{u}_{\perp L1} &= -\mathbf{b}_{\perp L1} + \frac{i\mu}{\omega l^2}\partial^2_{\xi\xi}\mathbf{b}_{\perp L0}, \\ r_{L1} &= \frac{i}{kl} b_{xL1} + O\left(\frac{\mu}{\omega l^2}\frac{1}{\xi^2}\right). \end{aligned}\right\} \tag{19}$$

Comparison of the corrections with the zeroth-order solution (or the 'resistive' ($\sim \mu$) and 'inhomogeneous' ($\sim \xi$) terms in (4) or (5)) gives (as was to be expected bearing the results of Davila (1987) in mind) $\Delta_\mu = (\mu/\omega l^2)^{1/3}$.

3.2. Second harmonic

At second order in ϵ, the interaction of two fundamental harmonics leads to generation of the second harmonic, $\mathbf{\Psi}'^{(2)}_{i2}(x,\theta) = \mathbf{\Psi}^{(2)}_{i2}(x)e^{2i\theta}$ + c.c.; zeroth order in μ will suffice for us. For $b^{(2)}_{y2}(x)$, (5) gives

$$\partial_\xi\left[\frac{\partial_\xi(\xi b^{(2)}_{y2})}{\xi}\right] + 4(k_z l)^2\left(\xi - \frac{k_y^2}{k_z^2}\right) b^{(2)}_{yz} = -\frac{i}{2}\partial_\xi\left\{\frac{1}{k_z^2\xi}[\partial_\xi(q^2 P^{(2)}_{22}) - 2ik_y l P^{(2)}_{12}]\right\} - 2iq^2 l^2 P^{(2)}_{22} \tag{20}$$

where

$$\partial_\xi(q^2 P^{(2)}_{22}) - 2ik_y l P^{(2)}_{12} = -i\partial_\xi\left[k_z k_y \xi (b_{yL0})^2 - ik_z \xi b_{xL0}\partial_x b_{yL0} + \frac{ik_z}{l} b_{xL0} b_{yL0} - 2k_z^2 b_{zL0} b_{yL0} + o\left(\frac{1}{\xi}\right)\right].$$

Note that the 'largest' ($\sim \xi^{-3}$) terms, involved in $\partial_\xi P^{(2)}_{22}$ from n_5 (the term $\propto k_y \partial_\theta u_y^2$) and in $P^{(2)}_{12}$ from n_4 (the term $\propto \partial_x b_y^2$), compensate each other, and do not produce any driving force at the second harmonic.

The main degree in $b^{(2)}_{y2}$ (which grows fastest than others as $\xi \to 0$) is determined from the relationship

$$\partial_x(\xi b^{(2)}_{y2}) = -\frac{1}{2}\partial_x\left[\frac{k_y}{k_z}\xi b^2_{yL0} - \frac{i}{k_z}\xi b_{xL0}\partial_x b_{yL0} - 2b_{zL0} b_{yL0} + \frac{i}{k_z l} b_{xL0} b_{yL0} + o\left(\frac{1}{\xi}\right)\right].$$

Hence $b^{(2)}_{y2}$ has the form

$$b^{(2)}_{y2} = \frac{\alpha_{1\pm}\ln\xi}{\xi^2} + \frac{\alpha_{2\pm}}{\xi^2} + o\left(\frac{1}{\xi^2}\right), \tag{21}$$

where $o(1/\xi^2)$ contains the solution of the homogeneous equation (20) with two arbitrary constants that make it possible to satisfy the boundary conditions for

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the second-harmonic radiation from the resonance region (or, more precisely, its absence from the incident wave). The particular form of the coefficient multiplying $(\ln \xi)/\xi^2$ and ξ^{-2}, which involves parameters characterizing the medium and the perturbation, is unimportant. Note that the generation of the second harmonic $b_{y2}^{(2)}$ when $\xi \gg \Delta_\mu$ is a consequence of the inhomogeneity of the medium. If $b_{z2}^{(2)}$ is sought in the form

$$b_{z2}^{(2)}(x) = -\tfrac{1}{2}[b_{yL}(x)]^2 + \Delta b_z(x),$$

then for $\Delta b_z^{(2)}$ we obtain from (6) the equation

$$\partial_\xi\left(\frac{\partial_\xi \Delta b_z}{\xi}\right) - 4(k_y l)^2\frac{\Delta b_z}{\xi} + 4(k_z l)^2\,\Delta b_z = -2(k_z l)^2 {b_{yL}}^2 - k_z l\partial_\xi\left(\frac{q^2\Delta P_1}{k_z^2\xi}\right) - 2i\frac{k_y}{k_z}\frac{q^2 l^2\,\Delta P_2}{\xi},$$

where

$$q^2\,\Delta P_1 = q^2 P_1 + \frac{k_z}{2l}\partial_\xi (b_{yL})^2,$$

$$q^2\,\Delta P_2 = q^2 P_2 + k_z k_y (b_{yL})^2.$$

It is easy to see that $\Delta b_z^{(2)} \sim \xi^{-1}$; hence

$$b_{z2}^{(2)} = -\tfrac{1}{2}(b_{yL0})^2 + O\left(\frac{1}{\xi}\right), \tag{22}$$

and from $\nabla \cdot \mathbf{b}_2^{(2)} = 0$ we get

$$b_{x2}^{(2)} \sim \frac{\ln \xi}{\xi}. \tag{23}$$

It follows from the relationships (3) and (2) that the velocity components $\mathbf{u}_{\perp 2}^{(2)}$ do not contain degrees higher than the corresponding components $\mathbf{b}_{\perp 2}^{(2)}$:

$$u_{x,y2}^{(2)} = b_{x,y2}^{(2)} + o(b_{x,y2}^{(2)}). \tag{24}$$

For $u_{z2}^{(2)}$, it is evident from (7) and (2) that

$$u_{z2}^{(2)} = \tfrac{1}{2}(b_{yL})^2 + o\left(\frac{1}{\xi^2}\right). \tag{25}$$

For the second harmonic of the density perturbation, we have the expression

$$r_2^{(2)} = (1+\xi)(b_{z2}^{(2)} + u_{z2}^{(2)}) - \frac{i}{2k_z l}u_{x2}^{(2)} + (1+\xi)\left(\frac{1}{4k_z}\partial_x n_1 + i\frac{k_z}{2k_y}n_2\right) + \frac{i}{2}n_7,$$

and, in view of (22), (24) and (2)

$$r_2^{(2)} \sim \frac{\ln \xi}{\xi}. \tag{26}$$

3.3. Zeroth harmonic

As a result of the interaction of the first two harmonics of the disturbance, a zeroth harmonic also occurs at second order in ϵ. Using the relationships (17)–(19) and

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(2), we obtain for n_{i0}^2 the expressions

$$\bar{n}_1 = 0,$$

$$\bar{n}_2 = -i\frac{\Delta_\mu}{k_z l}\partial_\xi(b_{yL0}^*\partial_{\xi\xi}^2 b_{xL0} - b_{xL0}^*\partial_{\xi\xi}^2 b_{yL0}) + \text{c.c.},$$

$$\bar{n}_3 - \bar{n}_7 \sim \frac{\Delta_\mu}{\xi^3},$$

$$\bar{n}_4 = -\frac{1}{k_z l}\partial_\xi |b_{yL0}|^2 + o\left(\frac{1}{\xi^2}\right) + o\left(\frac{\Delta_\mu}{\xi^6}\right),$$

$$\bar{n}_5 = -i\frac{\Delta_\mu}{k_z l}\partial_{\xi\xi}^2 b_{xL0}^*\partial_\xi b_{yL0} + \text{c.c.} + o\left(\frac{\Delta_\mu \ln \xi}{\xi^3}\right),$$

$$\bar{n}_6 = i\Delta_\mu \frac{k_y}{k_z} b_{zL0}\frac{\partial_{\xi\xi}^2 b_{yL0}^*}{\xi} + \text{c.c.} + o\left(\frac{\Delta_\mu \ln \xi}{\xi^2}\right),$$

$$\bar{n}_7 \sim \frac{\Delta_\mu}{\xi^4}.$$

It follows immediately from these that

$$b_{x0}^{(2)} = \text{const.} \tag{27}$$

As is evident, small ($\propto \mu$) ponderomotive forces arise owing to diffusion of the fundamental harmonic of the magnetic field disturbance. To obtain a closed system of equations for calculating the zeroth harmonic, it is necessary to retain in (12)–(14) $T\nabla n$ and $\mathbf{F}_v$ as forces that are implicated, along with ponderomotive forces, in establishing an equilibrium (at second order in ϵ).

From (10), we obtain

$$b_{y0}^{(2)} \sim \frac{\ln \xi}{\xi^2}. \tag{28}$$

It is clear from (15) that

$$u_{x0}^{(2)} \sim \frac{\Delta_\mu}{\xi^3}, \tag{29}$$

so that, with the required accuracy, $u_{x0}^{(2)} = 0$, and the contribution of the ion frictional force due to the plasma motion along x can be neglected ((12) and (13)). From (13), we get

$$u_{y0}^{(2)} \sim \frac{\mu}{\nu_1}\frac{1}{\xi^2}, \tag{30}$$

and (14) yields

$$u_{z0}^{(2)} \sim \frac{\mu}{\nu_1}\frac{1}{\xi^2}. \tag{31}$$

Assuming that the disturbance scale

$$\Delta > \xi_0 = \frac{k_y}{k_z}\frac{\omega}{\omega_i}\frac{1}{k_z l}\left(\frac{m_e}{m_i}\right)^{1/2},$$

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we can neglect the force $\nu_3\partial^2_{xx}u^{(2)}_{yo}$ in (12). Since, according to (11),

$$b^{(2)}_{z0} = O\left(\frac{1}{\xi}\right), \tag{32}$$

the magnetic pressure gradient produced by the fundamental harmonic can be equilibrated in (12) only by the plasma pressure gradient due to the density gradient $r^{(2)}_0$:

$$r^{(2)}_0 = -\frac{1}{\beta^*}|b_{yL0}|^2 + O\left(\frac{1}{\xi}\right) \tag{33}$$

3.4. Nonlinear correction to the fundamental harmonic

At third order in ϵ, a correction to the first harmonic $\Psi^{(3)}_{i1}(x,\theta)$ occurs as a result of the interaction of the first harmonic with the zeroth and second harmonics, as well as between the first three. The contributions of the different components of the second harmonic on the right-hand side of (5) at third order in ϵ compensate each other; when $T_e > T_i \gg (m_e/m_i)^{1/2}T_e$, we have $\mu/\nu_1 \ll 1/\beta^*$, and can neglect $b^{(2)}_{y0}$, $u^{(2)}_{y0}$ and $u^{(2)}_{z0}$ is comparison with $r^{(2)}_0$. We obtain

$$-\xi b^{(3)}_{y1} \approx r^{(2)}_0 b^{(1)}_{y1},$$

whence

$$b^{(3)}_{y1} = \frac{1}{\beta^*}\left[-\left(\frac{k_y}{k_z}B_\pm\right)^3\frac{1}{\xi^4} + O\left(\frac{1}{\xi^3}\right)\right] + O\left(\frac{\mu}{\nu_1}\frac{1}{\xi^2}\right), \tag{34}$$

and, when $\xi \sim (\epsilon^2/\beta^*)^{1/3}$, the nonlinear correction $b^{(3)}_{y1}(x)$ to the fundamental harmonic $b_{yL}(x)$ is comparalle to the fundamental itself.

4. Inner region

We now introduce the variable $\zeta = \epsilon^{-2/3}\xi$; for the time being, the parameters μ/ν_1, $1/\beta^*$ and $\epsilon^{-2/3}\Delta_\mu$ are taken to be arbitrary. Taking into account the relationships (17)–(19), (21)–(26) and (27)–(33), $\Psi'_i(x,\theta)$ in the inner region is representable as an expansion in terms of $\epsilon^{1/3}$:

$$b_x = \epsilon b^{(1)}_x(\zeta,\theta) + \epsilon^{4/3}b^{(4/3)}_x(\zeta,\theta) + \dots,$$

$$b_y = \epsilon^{1/3}b^{(1/3)}_y(\zeta,\theta) + \epsilon^{2/3}b^{(2/3)}_y(\zeta,\theta) + \epsilon b^{(1)}_y(\zeta,\theta) + \dots,$$

$$b_z = \epsilon^{2/3}b^{(2/3)}_z(\zeta,\theta) + \epsilon b^{(1)}_z(\zeta,\theta) + \dots,$$

$$u_x = \epsilon u^{(1)}_x(\zeta,\theta) + \epsilon^{4/3}u^{(4/3)}_x(\zeta,\theta) + \dots,$$

$$u_y = \epsilon^{1/3}u^{(1/3)}_y(\zeta,\theta) + \epsilon^{2/3}u^{(2/3)}_y(\zeta,\theta) + \epsilon u^{(1)}_y(\zeta,\theta) + \dots,$$

$$u_z = \epsilon^{2/3}u^{(2/3)}_z(\zeta,\theta) + \epsilon u^{(1)}_z(\zeta,\theta) + \dots,$$

$$r = \epsilon^{2/3}r^{(2/3)}(\zeta,\theta) + \epsilon r^{(1)}(\zeta,\theta) + \dots \quad .$$

In this case $r_0(x) = 1+\epsilon^{2/3}\zeta$ and $\omega^2/A^2 - k_z^2 = \epsilon^{2/3}k_z^2\zeta$. An asymptotic representation of the inner solution as $|\zeta| \to \infty$ is given by the solution in the outer region.

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Equations (4), (5) and (3) together with $\nabla \cdot \mathbf{B} = 0$ give

$$\omega^2 = k_z^2 A_0^2, \qquad u_x^{(1)} = -b_x^{(1)}, \qquad u_y^{(1/3)} = -b_y^{(1/3)}, \qquad \partial_\zeta b_x^{(1)} + k_y l \partial_\theta b_y^{(1/3)} = 0.$$

The relationships (3) also yield

$$\tilde{u}_x^{(4/3)} = -\tilde{b}_x^{(4/3)}, \qquad \tilde{u}_y^{(2/3)} = -\tilde{b}_y^{(2/3)}.$$

For periodic parts of the main orders in b_z, u_z and r, we obtain from (6)–(8)

$$\tilde{b}_z^{(2/3)} = -\tfrac{1}{2}(b_y^{(1/3)})^2, \qquad \tilde{u}_z^{(2/3)} = \tfrac{1}{2}(b_y^{(1/3)})^2, \qquad \tilde{r}^{(2/3)} = 0.$$

Since the asymptotic representation of the variables $b_x^{(4/3)}$, $u_x^{(4/3)}$ and $b_z^{(2/3)}$ as $|\zeta| \to \infty$ involves no aperiodic parts, they must also be absent from the inner solution. By this is meant that for the equation

$$\partial_\zeta \bar{b}_x^{(4/3)} = 0$$

the solution $\bar{b}_x^{(4/3)} = 0$ should be chosen. Similarly, it follows from (15) that

$$\partial_\zeta \bar{u}_x^{(4/3)} = \bar{n}_3^{(2/3)} = 0,$$

which allows the solution $\bar{u}_x^{(4/3)} = 0$. And from (11) we obtain the equation

$$-\frac{\mu}{\omega l^2 \epsilon^2} \partial_{\zeta\zeta}^2 \bar{b}_z^{(2/3)} = \bar{n}_3^{(4/3)} - \bar{n}_7^{(4/3)}.$$

It follows from (2) that

$$\bar{n}_3^{(4/3)} - \bar{n}_7^{(4/3)} = \frac{\omega}{k_z} \partial_\zeta \overline{[(u_z^{(1)} + b_z^{(1)} - r^{(1)}) b_x^{(1)}]},$$

and it follows from (6)–(8) (or from appropriate input equations) that

$$-\omega \partial_\theta (\tilde{u}_z^{(1)} + \tilde{b}_z^{(1)} - \tilde{r}^{(1)}) = \frac{\omega}{k_z} b_x^{(1)} \partial_\zeta (-\zeta - \bar{r}^{(2/3)} + \bar{u}_z^{(2/3)} + \bar{b}_z^{(2/3)});$$

consequently, the variable $u_z^{(1)} + b_z^{(1)} - r^{(1)}$ contains only the first harmonic (although $u_z^{(1)}$ and $b_z^{(1)}$ involve, generally speaking, also the third harmonic – this is easy to verify by calculating the third harmonic of these variables in the outer region), and it is shifted in θ by $\frac{1}{2}\pi$ with respect to $b_x^{(1)}$, so that $\bar{n}_3^{(4/3)} - \bar{n}_7^{(4/3)} = 0$. To ensure a matching with the solution in the outer region, we pick $\bar{b}_z^{(2/3)} = 0$ as the solution of the equation $\partial_{\zeta\zeta}^2 \bar{b}_z^{(2/3)} = 0$. In view of the relationships already obtained, we obtain from equation (5) the following equation for $b_y^{(1/3)}$:

$$\frac{\Delta_\mu^3}{\epsilon^2} \partial_{\zeta\zeta}^2 b_y^{(1/3)} + \left\{ \zeta + \bar{r}^{(2/3)} - 2 \left[\bar{u}_z^{(2/3)} + \frac{k_y}{k_z} (\bar{b}_y^{(2/3)} + \bar{u}_y^{(2/3)}) \right] \right\} \partial_\theta b_y^{(1/3)} = F(\theta),$$

F being an arbitrary function of θ. It follows from the solution in the outer region that $b_y^{(1/3)}$ is representable as

$$b_y^{(1/3)} = H_y(\zeta) e^{i\theta} + \text{c.c.},$$

and as $\zeta \to \pm\infty$, according to (18),(19) and (34),

$$\begin{aligned} H_y &\sim -\frac{k_y}{k_z} B_\pm \frac{1}{\zeta} - 2i \frac{\Delta_\mu^3}{\epsilon^2} \frac{k_y}{k_z} B_\pm \frac{1}{\zeta^4} - \frac{1}{\beta^*} \left(\frac{k_y}{k_z} B_\pm \right)^3 \frac{1}{\zeta^4} \\ &= -\frac{k_y}{k_z} B_\pm \frac{1}{\zeta} + O\left(\frac{1}{\zeta^4} \right). \end{aligned}$$

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One can obtain from (10), (13) and (14) (in view of the equation for $b_y^{(1/3)}$) the following equations:

$$\partial^2_{\zeta\zeta}\bar{b}_y^{(2/3)} = \frac{i}{k_z l}\partial_\zeta(H_y^*\partial^2_{\zeta\zeta}H_x - H_x^*\partial^2_{\zeta\zeta}H_y) + \text{c.c.},$$

$$\partial^2_{\zeta\zeta}\bar{u}_y^{(2/3)} = -\frac{\mu}{\nu_1}\frac{k_y}{k_z}\partial_\zeta H_y^*\partial_\zeta H_y + \text{c.c.},$$

$$\partial^2_{\zeta\zeta}\bar{u}_z^{(2/3)} = -\frac{\mu}{4\nu_1}H_y^*\partial^2_{\zeta\zeta}H_y + \text{c.c.}.$$

(we put $b_x^{(1)} = H_x(\zeta)e^{i\theta} + \text{c.c.}$). Thus, as in the outer region, $b_y^{(2/3)}$, $u_y^{(2/3)}$ and $u_z^{(2/3)}$ are characterized by parameters small in comparison with $1/\beta^*$. For $\bar{r}^{(2/3)}$ we obtain from (12)

$$\bar{r}^{(2/3)} = -\frac{1}{2\beta^*}|H_y|^2 + \text{const.}$$

In order for $\bar{r}^{(2/3)}$ to be matched as $\zeta \to \pm\infty$ with the corresponding aperiodic part of the density disturbance ($r_0^{(2)}$), we must put const = 0. Now putting $F(\theta) = Ce^{i\theta} + \text{c.c.}$, where C is a constant, we have for $H_y(\zeta)$ the equation

$$\frac{\Delta_\mu^3}{\epsilon^2}\partial^2_{\zeta\zeta}H_y + i\left(\zeta - \frac{1}{\beta^*}|H_y|^2\right)H_y = C.$$

It is clear that $B_+ = B_- = B$ and $C = -ik_yB/k_z$; the constant B can be taken to be real. The resulting equation describing the transverse structure can be written as

$$i\lambda\partial^2_{\zeta'\zeta'}G - RG = 1. \tag{35}$$

Here

$$R = \zeta' - |G|^2$$

represents a transverse plasma density profile in the resonance region:

$$n = n_0(x_0)\left\{1 + \frac{(\epsilon k_yB/k_z)^{2/3}}{\beta^{*1/3}}R\right\} = n_0(x_0)\left(1 + \xi - \frac{\epsilon^{2/3}}{\beta^*}|H_y|^2\right),$$

$$\zeta' = \beta^{*1/3}\left(\frac{k_z}{k_yB}\right)^{2/3}\zeta,$$

$$G = \beta^{*-1/3}\left(\frac{k_z}{k_yB}\right)^{1/3}H_y,$$

$$\lambda = \frac{\Delta_\mu^3}{\Delta_N^3},$$

$$\Delta_N = \beta^{*-1/3}\left(\frac{k_y}{k_z}\epsilon B\right)^{2/3},$$

Δ_N being a nonlinear scale.

Because it has been assumed above that the turning point lies far from the region of our concern, i.e. $|\xi| \ll k_y^2/k_z^2$, the following constraint on the value of β^* must hold:

$$\beta^* \gg \left(\frac{k_z}{k_y}\right)^8(\epsilon B)^2.$$

The quantity ϵB (B can be taken to be 1) characterizes the magnitude of the

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incident FMS wave amplitude on the resonance surface:

$$b_z \approx \epsilon b_z^{(1)} = \epsilon \left(B + |H_y|^2 H_y\right) = \epsilon \left(B + \beta^* \frac{k_y B}{k_z} |G|^2 G\right);$$

the penetration is largest when $k_y/k_z \sim 1$ and decreases with increasing k_y, so $b_z \approx \epsilon B$, hence the right-hand side of (35) owes its existence to the magnetosonic wave field.

Equation (35) contains only one parameter characterizing the relative roles of nonlinearity and resistivity in the formation of the transverse structure of a disturbance. When $\lambda \gg 1$, the resistivity effect is dominant; when $\lambda \ll 1$, nonlinearity has a substantial influence upon the disturbance structure. The method of establishment was used to obtain a numerical solution of (35), i.e., instead of (35), we solved the equation

$$\partial_\tau G = \lambda \partial^2_{\zeta'\zeta'} G + iRG + ia(\tau) \tag{36}$$

with boundary conditions

$$G(\zeta')_{|\zeta'|\to\infty} = -\frac{a(\tau)}{\zeta'} + \frac{2i\lambda a(\tau)}{\zeta'^4} - \frac{\partial_\tau a(\tau)}{\zeta'^2} + \frac{a(\tau)^3}{\zeta'^4}$$

and the function $a(\tau)$ such that $a(\tau) = 1$ when $\tau \geqslant \tau_0$. Obviously, the solutions of (36) that are independent of τ as $\tau \to \infty$, are also solutions of (35).

When $\tau = 0$, $a(0)$ was chosen such that (36), after renormalization, corresponded to (35) with $\lambda \gg 1$; therefore, when $\tau = 0$, $G(\zeta')$ was specified as the solution of a linear equation

$$i\lambda \partial^2_{\zeta'\zeta'} G - \zeta' G = a(0)$$

(Abramowitz and Stegun 1964) Figures 1–3 show the solutions obtained for $\lambda = 1$, $\lambda = \frac{1}{27}$ and $\lambda = \frac{1}{125}$. The dotted lines in Figs 2 and 3 represent the solution of an algebraic equation resulting from (35) when $\lambda = 0$. Numerical investigations showed that as $\lambda \to 0$, the region of transition from one branch of the algebraic solution to another becomes narrow and tends to $\zeta'_0 = 3/2^{1/3}$, corresponding to a minimum distance between the branches, and at the same time to the condition

$$\partial_{\zeta'} G(\zeta'_0 - 0) = \partial_{\zeta'} G(\zeta'_0 + 0).$$

Since direct numerical solution of (35) is impossible because of instability of its integration using standard algorithms, to determine whether the existence of other solutions is possible for a given λ, a numerical investigation was carried out using the method of establishment. To accomplish this, solutions were obtained for different λ not only by approaching the required amplitude from below (as described above) but also by approaching it from above. In either case (varying the functional specification of $a(\tau)$, the rate of approaching a given amplitude, and the starting amplitude), one solution was obtained for each value of λ.

We now discuss qualitatively the characteristic properties of the manifestation of the nonlinearity in Alfvén resonance that follow both directly from (35) and from the preceding inference. First, it should be noted that, although the resonance condition $\omega^2 = k_z^2 A^2$ is satisfied not only for the fundamental harmonic but also for higher harmonics (just as for resonance phenomena like wave–particle interaction and shear flow–where the resonance condition is defined by the relationship $\omega = kv$, v being the particle and flow velocities in the former and latter cases, respectively), the generation of higher harmonics in a resonance disturbance is reduced

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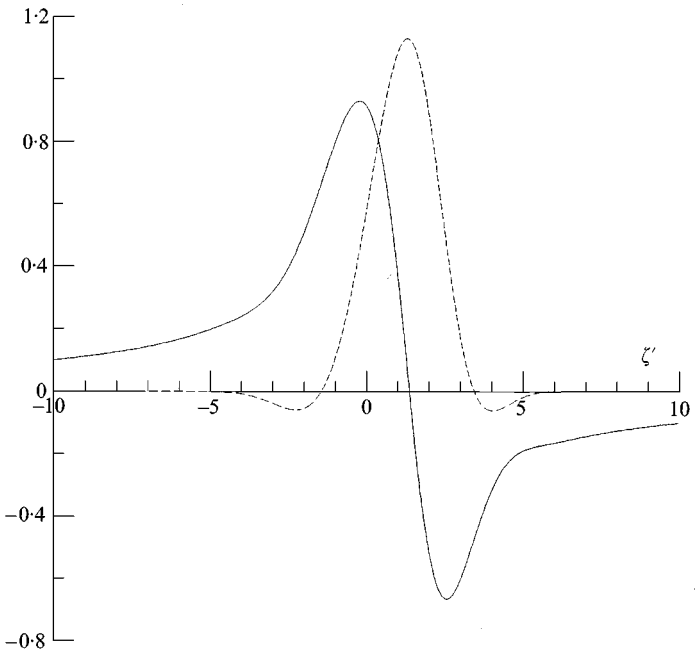


Figure 1. $G(\zeta')$ for $\lambda = 1$: ——, $\mathrm{Re}\, G$; – – –, $\mathrm{Im}\, G$.

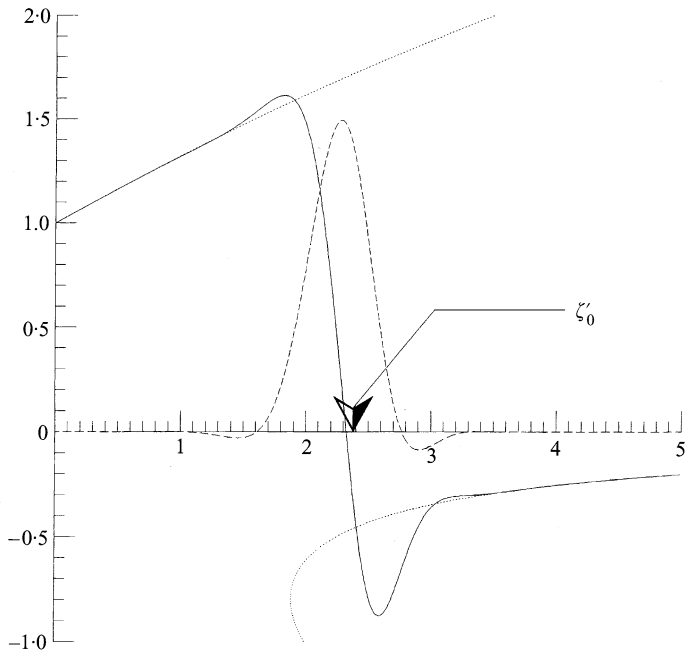


Figure 2. $G(\zeta')$ for $\lambda = \frac{1}{27}$: ——, $\mathrm{Re}\, G$; – – –, $\mathrm{Im}\, G$; · · · · · ·, algebraic solution.

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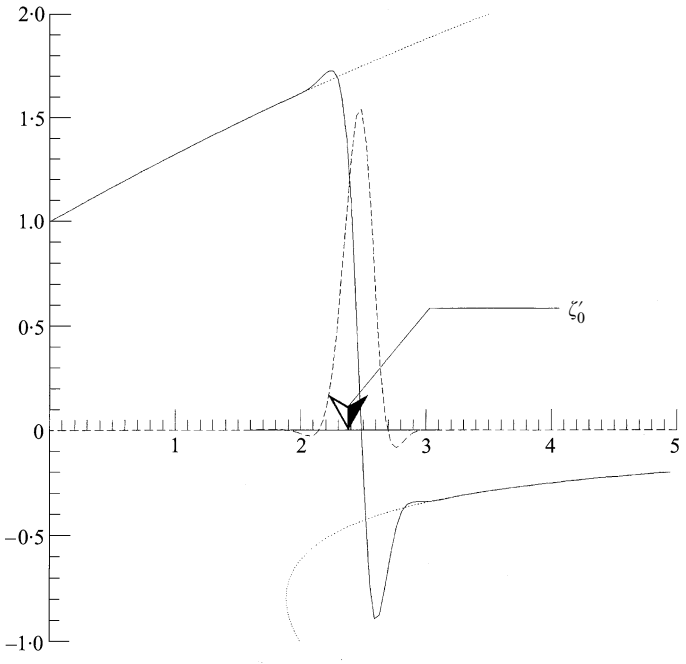


Figure 3. $G(\zeta')$ for $\lambda = \frac{1}{125}$: ——, $\mathrm{Re}\, G$; – – –, $\mathrm{Im}\, G$; · · · · · ·, algebraic solution.

because driving forces, associated with the interaction of transverse components of both the magnetic field and the velocity, compensate each other and manifest themselves only as quantities proportional to small parameters that differentiate the plasma from a homogeneous medium described by cold ideal magnetohydrodynamics. This is a consequence of the relationship $\mathbf{v}_\perp = -\omega k^{-1}\mathbf{b}_\perp$ that holds for Alfvén waves in ideal MHD, and manifests itself, in particular, in the smallness of the nonlinear interaction of Alfvén waves. The growths of the second and zeroth harmonics on approaching the resonance surface are characterized by the same degree ($\propto (x - x_0)^{-2}$). The main nonlinear effect is the self-action of the fundamental harmonic through second and zeroth harmonics. Note that a nonlinearity of the same type appears in the nonlinear resonant absorption of surface hydromagnetic waves described in terms of incompressible MHD equations in the absence of magnetosound (Lundberg and Brodin 1995). In the case considered here the second and zeroth harmonics are dominated by the plasma density disturbance at the zeroth harmonic, which is indeed the main nonlinear factor determining the disturbance structure at the fundamental harmonic. This picture is related to that of nonlinear effects in plasma resonance (Gildenburg and Fraiman 1975) arising when a high-frequency electromagnetic wave penetrates an inhomogeneous plasma. Both in the case of Alfvén resonance and in the case of plasma resonance, the nonlinearity leads to a modification of the linear equation by adding a nonlinear term taking account of plasma density variation under the action of the disturbance. To the linear terms on the left-hand side of (35) there corresponds a dispersion equation

$$1 - \frac{\omega^2}{k^2 A^2(x)} - i\Delta_\mu^3 l^2 k_x^2 = 0.$$

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Its form is determined by the fact that the main linear regulating factor was supposed to be the resistivity. If instead of the resistivity the effect of transformation to kinetic Alfvén waves is included then the dispersion equation becomes

$$1 - \frac{\omega^2}{k^2 A^2(x)} + \rho^2 k_x^2 = 0,$$

where

$$\rho^2 = \left(\frac{3}{4} + \frac{T_e}{T_i}\right) \rho_i^2.$$

On substituting into (35) the term corresponding to the inclusion of resistivity for the term corresponding to the inclusion of $\rho \neq 0$, we obtain the equation

$$-\lambda_* \partial^2_{\zeta'\zeta'} G - RG = 1,$$

where $\lambda_* = \Delta_\rho^3/\Delta_N^3$ and $\Delta_\rho = (\rho/l)^{\frac{2}{3}}$. This equation is similar to that describing the disturbance that forms when a high-frequency electromagnetic wave penetrates an inhomogeneous plasma, with due regard for the nonlinearity and transformation to plasma waves. The problem of nonlinear plasma resonance is also solved by taking into account the non-stationarity of the disturbance (Gourdin and Adam 1984). A non-stationary equation for Alfvén resonance can be obtained following the same principles as for (36). Although the analogy between the two problems seems to be reasonably obvious (for the linear case, it has long been known: (Chen and Hasegawa 1974; Southwood 1974)), we shall refrain from extending the results obtained for plasma resonance to the case of Alfvén resonance, bearing in mind that the nonlinear term in (35) is determined (albeit indirectly) by the interplay of small regularizing factors and, generally speaking, can be different with different regularization mechanisms. Therefore the ultimate answer to the question of the structure of a resonant disturbance excited by magnetosound can be found only through equations obtained consistently from the starting nonlinearized equations, taking into account both of the above-mentioned linear effects.

5. Conclusions

We have studied the influence of nonlinearity upon the structure of a resonant disturbance (Alfvén resonance) driven in an inhomogeneous plasma by fast magnetosound. We have obtained a nonlinear equation describing the transverse structure of such a disturbance, by taking into account one of the possible linear regularizing factors, namely the resistivity ($\mu \neq 0$). It has been shown that the relative roles of resistivity and nonlinearity are characterized by the parameter $\lambda = \Delta_\mu^3/\Delta_N^3$, where Δ_μ and Δ_N are the linear and nonlinear scales: $\Delta_\mu \propto \mu^{1/3}$ and $\Delta_N \propto \epsilon^{2/3}$. The disturbance structure obtained allowing for nonlinearity differs substantially from the linear one. The disturbance maximum B_y is shifted, as compared with the linear case, by an amount of order Δ_N, and the scale of variation of B_y in the resonance region (Δ) depends on the amplitude of driven magnetosound: $\Delta \sim \lambda^{1/2}$. Higher harmonics are present in the disturbance; the second harmonic of the y and z components of the magnetic field disturbance are close in magnitude: $B_{z2} \sim B_{y2}$, with the second harmonic B_{z2} exceeding the fundamental harmonic B_z in magnitude.

The results obtained indicate an analogy between the manifestation of nonlinearity in the problem considered here and that in the problem of penetration

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of a high-frequency electromagnetic wave into a plasma where plasma resonance occurs. It is an extension of the well-known analogy between the two linear problems. In both cases there occurs a self-action of the fundamental harmonic of the disturbance through a variation (which it produces) of mean plasma parameters (strictional nonlinearity). In the case of Alfvén resonance examined here there is a predominant nonlinear effect of plasma ejection out of the resonance region under the action of magnetic pressure produced by the wave, such that the plasma is expelled from the plasma resonance region under the action of the Miller force. Although this paper has taken into account only one of the linear regularizing factors, namely the resistivity, the independence on μ of the disturbance at the second harmonic suggests that the nonlinearity must also have a strictional character when other linear regularizing factors are included.

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