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# CONSEQUENCES OF MODELING HABIT PERSISTENCE

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In this paper, we study the stationary and non-stationary equilibria of a deterministic, pure exchange, two-period overlapping generations model with habit persistence. We show that preferences with multiplicative habits can lead to quite different equilibrium outcomes compared to subtractive ones. The two most commonly adopted habit specifications can differ in terms of homotheticity, gross substitutability, and uniqueness of equilibria. We illustrate these differences in terms of steady-state equilibria, as well as local dynamics.

**Keywords:** Multiplicative and Subtractive Habit Persistence, Multiple Equilibria, Equilibrium Dynamics

## 1. INTRODUCTION

A recent strand of the economic literature has analyzed theoretical and empirical implications of endogenous preferences, those that depend on time, personal experience, or social conditions. Among the different forms of endogenous preferences the ones displaying habits have received particular attention. The literature on habit persistence specifies preferences either using subtractive (SH) or multiplicative habits (MH), according to the terminology introduced by Carroll (2000).[1] Although habits have helped shed some light on several economic phenomena, not much attention has been paid to the economic consequences of these two alternative formulations. For example, Abel (1990) and Constantinides (1990) reach similar conclusions using the two alternative formulations in the context of risk premium. This may seem to suggest, *prima facie*, that the qualitative properties of models with SH and MH are pretty similar. However, Wendner (2003), shows that the two most commonly used habit specifications may easily lead to opposite implications regarding household savings behavior. He shows that in response to

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an increase in the strength of habits, young households may increase savings in the case of SH, whereas they may lower savings in the case of MH. Furthermore, Carroll (2000) points out that under plausible parameter values, SH may give rise to a not well-defined utility in stochastic environments, whereas this is not the case using MH.

Recently, Chen and Ludvigson (2006) point out the lack of theoretical studies on the functional form of habit persistence. Using semi-parametric and structural econometric approaches, they "reverse engineer" the habit specification that best matches the cross-sections of asset returns on a relatively large portfolio. Their empirical findings suggest that the habit function should be nonlinear in current and past consumption rather than linear. Hence, they seem to suggest that a MH specification is empirically preferred to a SH one. A theoretical investigation of the underlying properties of the two habit formulations is thus needed.

The objective of this paper is to study the two alternative formulations adopted in the literature and to examine their differences in terms of multiplicity of steady states, dynamic indeterminacy, and local dynamics within an overlapping generation framework. Understanding how optimal savings decisions change according to different preference specifications is crucial because intertemporal consumption decisions are at the core of macroeconomic analysis. We explore the consequences of specifying different forms of habit persistence in a pure exchange, two-period lived overlapping generations model. Habit persistence in this setting has been previously explored. Lahiri and Puhakka (1998) consider SH in a pure exchange overlapping generations framework and show that increasing the strength of habits raises desired savings and might lead to endogenous cycles. In a similar framework, Bunzel (2006) argues that both SH and MH yield dynamic behaviors that are qualitatively very similar. In this study, we show that this finding does not always hold.

The main contribution of this paper is to show that modeling preferences with MH or SH yields theoretical predictions that are not necessarily equivalent. We find that modeling habit persistence with MH may not always yield concave or homothetic preferences over consumption. This modeling choice can give rise to multiple monetary steady states, and hence stationary sunspots equilibria may arise. In addition, the stability properties of an economy with MH change drastically as we vary the strength of habits. Adopting the hyperbolic absolute risk aversion (HARA) class of preferences, we find instances in which there is a unique steady state under both specifications. However, we also find cases in which there are multiple steady states under MH, whereas there is a unique steady state under SH. Thus, the resulting qualitative properties of the economy under SH and that under MH can be quite different. If the choice of how to model savings behavior in the presence of habits is not innocuous, the predictions and policy prescriptions based on the two types of habit persistence could be quite different as well.

In the next section, we introduce the general model with MH and SH and study the corresponding concavity and homotheticity properties of these preferences.

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Then, we analyze the steady state equilibria and their associated dynamics. In Section 3, we provide a general result on the difference in the dynamics under the Classical case. In Section 4, we present numerical examples that suggest, once again, nonnegligible discrepancies across MH and SH in terms of local stability properties of the steady states. The last section summarizes our main findings and concludes. The proofs can be found in the appendix.

## 2. THE MODEL

This paper builds on Gale (1973) and Lahiri and Puhakka (1998) by considering a pure exchange overlapping generations model. Complex dynamics and endogenous cycles have been shown to emerge in overlapping generation models with production or a variety of different assumptions concerning market imperfections.[2] Here we abstract from technology to reinforce the point we are trying to make: with habits, differences in dynamics and in steady-states properties need not to be driven by the production side of the economy but, rather, by the modeling choice of preferences alone. Economic activity takes place over infinite discrete time. Each generation lives for two periods and has perfect foresight. Agents are endowed with $w_1$ units of the single good when young, and with $w_2$ units when old.

Utility is derived from consumption in both periods. However, because of the presence of habit formation, utility of a given level of consumption when old depends on consumption when young.[3] Formally:

$$V(c_1, c_2) = u(c_1) + \beta u(c_1, c_2; \gamma), \tag{1}$$

where $\gamma \in (0, 1)$ denotes the strength of habits in the instantaneous utility function and $\beta$ is the discount factor. We assume that the function $u(.)$ is well behaved, that is, strictly increasing, strictly concave, homogeneous, and twice continuously differentiable. If one considers SH, then preferences are typically given by $u(c_1, c_2; \gamma) = v(c_2 - \gamma c_1)$ as in Lahiri and Puhakka (1998).[4] If, instead, the instantaneous utility is specified with MH, we have $u(c_1, c_2; \gamma) = v(c_2/c_1^{\gamma})$. Regardless of the specification considered, the importance of past consumption in determining the utility derived by the "effective" consumption is increasing with $\gamma$.[5] Each agent maximizes utility subject to budget constraints; $c_1 = w_1 - s_t$ when young, and $c_2 = w_2 + R_t s_t$ when old. $s_t$ and $R_t$ denote savings when young and the gross nominal interest rate on savings at time $t$, respectively. As pointed out by Lahiri and Puhakka (1998), we need to impose conditions on the parameters to ensure that the indifference curves are downward sloping. These conditions for the subtractive and multiplicative case are, respectively,

$$\frac{u'(c_1)}{v'(c_2 - \gamma c_1)} - \gamma\beta > 0 \tag{2}$$

$$\frac{u'(c_1)}{c_2 c_1^{-(1+\gamma)} v'\left(c_2 c_1^{-\gamma}\right)} - \gamma\beta > 0. \tag{3}$$

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In what follows, we first study the concavity, homothetic properties, and the steady-state equilibria that emerge in each case. We then explore the local dynamic properties of the equilibria under each habit specification.

## 2.1. Preferences

One of the most important differences across alternative habit specifications is in terms of their implications for the underlying preferences. Alonso-Carrera, Caballé, and Raurich (2005) show that when habits are introduced multiplicatively in a capital accumulation model, the consumers' objective function might fail to be concave. In this paper, we find a similar result for pure exchange overlapping generation models as stated in Proposition 1.

LEMMA 1 (Concavity). *Consider a pure exchange two-period lived overlapping generations model. Then we have that*: (i) *under SH persistence*, *the utility function is strictly concave in consumption*, *and* (ii) *under MH persistence*, *concavity is not always ensured.*

Proof. See appendix. ■

Lemma 1 implies that one needs to be cautious when solving for the consumption plan that maximizes consumers' utility under MH. This result, as well as the next one, applies to all sort of models; not only those set up in an overlapping generation framework. The next proposition outlines another crucial implication in terms of the savings function resulting from the consumer's problem. The adoption of MH or SH can greatly affect the savings behavior, even if other fundamentals remain the same.

PROPOSITION 1 (Homotheticity). *If the instantaneous utility function* $u(.)$ *is homogeneous of degree* $n$, *then the SH specification yields homothetic preferences whereas the MH specification does not.*

Proof. See appendix. ■

The result on homotheticity of preferences is important for overlapping generations economies. Homotheticity of the utility function implies existence of a nontrivial steady state in this model of pure exchange, as it will be pointed out later in the proof of Proposition 2.[6] Furthermore, it is well known that, in overlapping generations models, the saving function is linear in income/endowments if the lifetime utility function is homothetic [de la Croix and Michel (2002)]. Thus, in general, we expect to observe substantial differences in the underlying dynamics between SH and MH specifications. This particular comparison is carried out in Sections 3 and 4.

Nonhomotheticity has also important implications for economies in different frameworks. When preferences are homothetic the indirect utility function can be written as a linear function of wealth. Hence, wealth elasticities of demand are

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constant and the Engel curves are linear. When preferences are nonhomothetic, instead, the Engel curves are typically non linear in wealth. This difference is of crucial importance for researchers interested in public finance and applied microeconomics in general. The behavior of the Engel curves is critical in understanding the impact of tax reforms. For example, Bossi, Gomis-Porqueras, and Kelly (2007) are the first to characterize the conditions under which taxation of addictive goods might differ from taxes on leisure and other consumption goods in a dynamic Ramsey setting with habit persistence. Banks, Blundell, and Lewbel (1997) also point out that by failing to model the Engel curve correctly, one can badly misspecify the distribution of welfare losses at the empirical level. Thus, the modeling choice for habits is not harmless.

## 2.2. Steady State Equilibria

In this section, we show that if one adopts SH, then there exists a unique monetary steady state while under MH multiple monetary equilibria cannot be ruled out. The optimal savings function for a young agent in this model with MH is defined as follows:

$$s^* = \arg\max u(w_1 - s) + \beta v\left(\frac{Rs + w_2}{(w_1 - s)^\gamma}\right). \tag{4}$$

Thus, the equation that defines the optimal savings can be written as

$$F(s) = -u'(w_1 - s) + \beta v'\left(\frac{Rs + w_2}{(w_1 - s)^\gamma}\right)\left[\frac{R(w_1 - s) + \gamma(Rs + w_2)}{(w_1 - s)^{\gamma+1}}\right] = 0. \tag{5}$$

The implicit function for optimal savings under SH, by contrast, is given by the following equation:

$$G(s) = -u'(w_1 - s) + (R + \gamma)\beta v'((R + \gamma)s + w_2 - \gamma w_1) = 0. \tag{6}$$

To close the model and to analyze the steady states, we introduce an outside asset into the economy. Following Lahiri and Puhakka (1998), we assume that there is a government that borrows from and lends to the public. This approach is clearly equivalent to injecting valueless fiat money into the economy. Government liabilities in period $t$ are denoted by $b_t$, and the real deficit by $d_t$. The government's budget constraint for period $t$ is then given by

$$d_t + R_{t-1}b_{t-1} = b_t. \tag{7}$$

Asset market equilibrium requires that $b_t = s_t$ for all $t$. Setting the government's deficit to zero, the law of motion for this economy is defined as follows:

$$s_{t+1} = s_t R_t, \tag{8}$$

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which also represents the offer curve for this economy. If $b_t > 0$, then the economy is in the Samuelson case (i.e., agents have positive savings) according to the terminology first coined by Gale (1973). Instead, if $b_t < 0$, then the economy is in the Classical case (i.e., agents are borrowing).

Using a general instantaneous utility function does not always yield an explicit expression for optimal savings nor for the gross interest elasticity of savings. Thus, it is not possible to explicitly analyze equation (8). However, we can characterize a crucial difference between SH and MH in a general setting where no explicit functional form assumptions regarding the utility are made. The following proposition states that the standard results on uniqueness of steady states do not necessarily apply to the case with MH. This result will then have crucial implications for the local dynamics of the economy.

PROPOSITION 2 (Uniqueness). *Restrict the parameter space such that the consumer's problem is concave. Under SH, there always exists a unique monetary steady state. Under MH, on the other hand, multiplicity of monetary steady states cannot be ruled out.*

Proof. See appendix. ■

The economic intuition behind this result is related to the possible lack of gross substitutability under the MH specification. It is well known that when consumption in the first and the second periods are weak gross substitutes, then the steady state is unique.[7] This condition can be verified by studying the sign of the intertemporal elasticity of substitution (IES). Under the subtractive specification, IES is always positive, whereas under the multiplicative case it can be negative. Formally, the IES between consumption when young and old can be written as

$$\Psi(c_1, c_2) = \frac{\frac{1}{c_1 \frac{\partial V}{\partial c_1}} + \frac{1}{c_2 \frac{\partial V}{\partial c_2}}}{-\frac{\frac{\partial^2 V}{(\partial c_1)^2}}{\left(\frac{\partial V}{\partial c_1}\right)^2} + 2\frac{\frac{\partial^2 V}{\partial c_1 \partial c_2}}{\frac{\partial V}{\partial c_1}\frac{\partial V}{\partial c_2}} - \frac{\frac{\partial^2 V}{(\partial c_2)^2}}{\left(\frac{\partial V}{\partial c_2}\right)^2}}. \tag{9}$$

The crucial term in $\Psi$ is $\frac{\partial^2 V}{\partial c_1 \partial c_2}$; if this is negative then gross substitutability could break down. One can verify that under SH

$$\frac{\partial^2 V}{\partial c_1 \partial c_2} = -\gamma\beta v''(c_2 - \gamma c_1) > 0.$$

When we consider MH, instead, we obtain the following:

$$\frac{\partial^2 V}{\partial c_1 \partial c_2} = -\gamma\beta c_1^{-\gamma-1}\left[v'\left(c_2 c_1^{-\gamma}\right) + c_2 c_1^{-\gamma} v''\left(c_2 c_1^{-\gamma}\right)\right].$$

The sign of this expression depends on the underlying parameter values of the model.

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Under intertemporally nonseparable preferences, the elasticity of substitution between current and future consumption is time variant. In particular, different functional forms for the period utility imply different degrees of IES. A subtractive specification implies that consumption in the first and the second periods are always gross substitutes, whereas this is not necessarily true when a multiplicative formulation is adopted. Because of the lack of gross substitutability, it is not possible to rule out the existence of multiple monetary steady states under MH. In the next section, we derive a general result on the dynamics in the Classical case. After that, we present numerical examples that back up our theoretical findings ans that highlight some of the differences between MH and SH modeling choices.

## 3. DYNAMICS

One clear message so far is that we should expect differences in the local dynamics between the two specifications because of homotheticity or the lack thereof. Before we proceed any further, let us establish a general result for the Classical case, which follows directly from a theorem by Gale (1973). The following lemma, which is taken from Gale (1973), together with Proposition 1 suggest that alternative habit formulations can yield drastically different results in terms of dynamic indeterminacy in the Classical case.

LEMMA 2 (Gale, 1973). *In the Classical case*, *if the utility function is either* (i) *separable*, *or* (ii) *homothetic then there exist a unique path approaching the steady state given an initial condition.*

Proof. See the proof of Theorem 5 by Gale (1973, p. 25). ■

Weak gross substitutability implies that, if it exists, there is at most one equilibrium price sequence which converges to the steady state. Lemma 2 and the following proposition suggest that this is not necessarily the case for MH.

PROPOSITION 3. *In the Classical case*, *SH lead to determinate dynamic equilibrium whereas MH may display dynamic indeterminacy.*

Proof. Recall that both habit specifications imply nonseparable preferences. In addition, Proposition 1 proved that preferences are nonhomothetic under MH while they are homothetic under SH. These facts together with Lemma 2 complete the proof. ■

In general, the local dynamic properties are determined by the slope of the offer curve given by

$$\frac{ds_{t+1}}{ds_t} = R_t\left(1+\frac{1}{\epsilon}\right) \tag{10}$$

where $\epsilon$ is the gross interest rate elasticity of savings. Because we can not obtain a closed form solution for the offer curve under MH nor under SH, it is not possible to study its behavior analytically. For this reason, in the next section we provide

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numerical examples that illustrate how the dynamics can differ. In particular, we show that multiple monetary steady states are possible under MH.

## 4. NUMERICAL EXAMPLES

In previous sections, we have highlighted the possibility of crucial differences in terms of number of monetary steady states, local dynamics, and volatility between SH and MH specifications. These differences are indeed important since they have drastic consequences for the underlying observables of the economy. In this section, we provide numerical examples that illustrate these differences both qualitatively and quantitatively. For our numerical examples, let us consider the class of preferences with hyperbolic absolute risk aversion (HARA). This is a general class that nests, as special cases, the family of utility functions with a constant coefficient of relative risk aversion (CRRA), the one with a constant absolute risk aversion (CARA), and the quadratic utility. The HARA family is commonly used in the finance literature, which deals with asset pricing and savings' behavior.[8] Carroll and Kimball (1996) show how CARA and CRRA specifications imply a savings behavior that is qualitatively very different from the one corresponding to a HARA formulation. In particular, they show that under HARA preferences and uncertainty, the consumption function is concave, whereas it is linear when CARA and CRRA specifications are adopted.[9] Therefore, the underlying choice of preferences is crucial.

The HARA utility function takes the following form:

$$u(c) = \frac{1}{1-\sigma}\left[\sigma\left(A + \frac{\alpha c}{\sigma}\right)^{1-\sigma}\right], \tag{11}$$

where $A$ and $\sigma$ are real numbers, and $\alpha > 0$. Incorporating MH in the HARA family yields the following utility function for our example:

$$V(c_1, c_2) = \frac{1}{1-\sigma}\left[\sigma\left(A + \frac{\alpha c_1}{\sigma}\right)^{1-\sigma}\right] + \beta\frac{1}{1-\sigma}\left[\sigma\left(A + \frac{\alpha}{\sigma}\frac{c_2}{c_1^{\gamma}}\right)^{1-\sigma}\right]. \tag{12}$$

Typically, to characterize the dynamic properties of this economy, we need to determine the slope of the offer curve at each of the steady states. The marginal rate of substitution at the initial endowment point should be studied as well. However, the HARA utility function may not always be defined at the origin. Formally, the utility function is defined if $A + \frac{\alpha c}{\sigma} \geq 0$. By substituting the budget constraint for each period into the respective flow utilities we obtain an upper and a lower bound on savings:

$$A + \frac{\alpha(w_1 - s_t)}{\sigma} \geq 0, \tag{B1}$$

and

$$A + \frac{\alpha}{\sigma}\frac{(w_2 + R_t s_t)}{(w_1 - s_t)^{\gamma}} \geq 0. \tag{B2}$$

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**TABLE 1.** Benchmark example

| | SH | MH |
|---|---|---|
| $s^{s,s,\cdot}$ | −0.078 | −0.747 |
| $c_1^{s,s,\cdot}$ | 1.0778 | 1.747 |
| $c_2^{s,s}$ | 1.024 | 0.356 |
| $\epsilon$ | −7.296 | −1.347 |
| $(1+\frac{1}{\epsilon})$ | 0.863 | 0.258 |
| PM | −0.271 | −0.069 |
| detH | 0.072 | 0.134 |
| MRSC | 0.55 | 0.817 |
| $\Psi(c_1^{s,s,\cdot}, c_2^{s,s})$ | 1 | 1 |

Therefore, we need to make sure that these additional restrictions are met when studying the examples.

In the benchmark case, we provide a simple economy that is in line with some empirical regularities. Throughout our numerical analysis, we set $\beta = 0.55$, which corresponds to an annual discount rate of 0.97 when the generation considered lasts 20 years. We also use data from Gourinchas and Parker (2002) for the life-cycle profiles of income. The endowment parameters of our economy are set to mimic their income data. On the basis of their data, we computed the ratio of average income between the ages of 45 and 65 to the average income between the ages of 26 and 44. This procedure yields 1.102 as the ratio of average income when old to young. We assign the following parameter values to the utility function: $A = -1$, $\alpha = 1$, and $\sigma = 0.226$. Because there is a lack of evidence in terms of the empirically plausible range of parameter values measuring the strength of habits, we decided to adopt the following calibration strategy for our benchmark example. We force the IES to be one, a value confirmed by most recent studies,[10] across specifications and we retrieve the resulting $\gamma$. Given our previous results, in particular the one concerning homotheticity, it is not surprising to find that the strength of habits that matches an IES of 1 is different across specifications. The resulting calibrated values are $\gamma = 0.45$ for the SH and $\gamma = 0.415$ for the MH specification, respectively. In Table 1, we report our benchmark example.

The tables throughout our paper provide information regarding steady state allocations for savings ($s^{s,s,\cdot}$), consumption when young ($c_1^{s,s,\cdot}$) and when old ($c_2^{s,s}$), the gross interest rate elasticity of savings ($\epsilon$), and the slope of the offer curve evaluated at the steady state $(1+\frac{1}{\epsilon})$. The tables also report the principal minor (PM) and the determinant of the associated Hessian at steady state (detH) as well. We do so in light of Lemma 1 that points out the potential lack of concavity under the MH case. Finally, we report the conditions established in equations (2) and (3); that is, we have downward sloping indifference curves and meaningful economic equilibria. These conditions are satisfied whenever MRSC is positive. In addition, we provide the steady state intertemporal elasticity of substitution for each example ($\Psi(c_1^{s,s,\cdot}, c_2^{s,s})$).[11]

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Clearly, both economies are in the Classical case where steady states savings are negative. In the benchmark case, the local dynamics of the monetary steady states are similar: they are both stable since $0 < (1 + \frac{1}{\epsilon}) < 1$. However, we find that by varying slightly the intertemporal elasticity of substitution, $\Psi(c_1^{s.s.}, c_2^{s.s})$, the dynamic properties of the steady state can greatly differ as illustrated by the following examples. As we decrease the intertemporal elasticity of substitution, the number of steady-state monetary equilibria and the corresponding local dynamics vary depending on the specific habit formulation adopted.

**Example 1.** *Holding all other parameters constant, if we set the IES* $= 0.929$ *for both specifications we find that the implied strength of habits are* $\gamma = 0.8297$ *for SH and* $\gamma = 0.45825$ *for MH. Furthermore, for these parameter values, we find two monetary steady states under MH and a unique monetary steady state under SH. The local dynamics associated with these two different sets of steady states are quite different.*

It is interesting to note that, as we decrease the intertemporal elasticity of substitution by only 7.1% relative to our benchmark example, two steady states with opposite stability properties emerge under MH, whereas the steady state for SH remains unique but becomes unstable. Moreover, optimal savings across the two specifications are starkly different. Under SH agents are saving, whereas under MH agents are borrowing in the first monetary steady state and saving in the second one. Thus, the implications for the resulting consumption throughout the life cycle depend crucially on the underlying specification of habits.

An important difference between the two specifications, shown in this paper, is the possibility of multiple monetary steady states. Whenever multiple equilibria exist, sunspot equilibria can be constructed as a lottery between two deterministic steady states as suggested by Cass and Shell (1983). Sunspot economies deliver more volatility, thus we would expect to observe smoother consumption paths under SH than under MH. Furthermore, because multiple steady states exist under MH, bifurcation phenomena are possible as we vary the strength of habits.

The implications for the evolution of the economy in this case are different, SH predict monotone divergence from the steady state given that the eigenvalue $[(1 + \frac{1}{\epsilon})]$ is outside the unit circle. Similarly, under MH Steady State II is unstable with diverging monotone dynamics as well. Steady state I, by contrast, exhibits monotone convergence to the steady state since it has the eigenvalue inside the unit circle. We can conclude then that the underlying dynamic properties crucially depend on the modeling choice of habits.

Another interesting observation is that MH exhibits a Classical steady state and a Samuelsonian steady state, suggesting that there are discontinuities in the offer curve probably due to the aforementioned lack of concavity. Depending on their initial conditions, agents may end up perpetually borrowing (Steady State I) or continuously changing their saving patterns (Steady State II). The transitional dynamics also predicts that as long as the discontinuity

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**TABLE 2.** Multiplicity of equilibria under MH but not under SH

| | SH | MH | |
|---|---|---|---|
| | Steady State | Steady State I | Steady State II |
| $s^{s.s.}$ | 0.26 | −0.744 | 0.771 |
| $c_1^{s.s.}$ | 0.738 | 1.744 | 0.229 |
| $c_2^{s.s}$ | 1.36 | 0.358 | 1.873 |
| $\epsilon$ | 1.348 | −1.330 | 0.007 |
| $(1+\frac{1}{\epsilon})$ | 1.742 | 0.248 | 142.677 |
| PM | −0.502 | −0.068 | −221.299 |
| detH | 0.072 | 0.135 | 15.753 |
| MRSC | 0.547 | 0.796 | 0.337 |
| $\Psi(c_1^{s.s.}, c_2^{s.s})$ | 0.929 | 0.929 | 0.012 |

in the offer curve is small enough and the appropriate initial conditions are in place, it is also possible that agents' savings jump from being positive to being negative.

As we can see from Table 2, multiple steady states are possible when MH is considered. This situation is nongeneric because the only way to have multiple monetary steady states is for consumers to be indifferent between two (or more) levels of savings when $R = 1$ holds. In other words, we make sure that the maxima we obtain from the consumers' optimization problem yield exactly the same utility level. Recall from Proposition 2 that multiple maxima never occur under SH. If the choice of how to model savings behavior in the presence of habits is not innocuous, as we have shown, the predictions and policy prescriptions based on the two types of habit persistence are going to be quite different.

### 4.1. Endowment Profiles or Outer Utility?

In a recent paper, Bunzel (2006) argues that, regardless of whether one adopts SH or MH, the dynamical properties of the pure exchange economy are very similar. She imposes an outer utility function that is CRRA and for tractability she considers only the case in which the second period endowments are zero. Here we show that Bunzel's irrelevance results are driven by the specific choice of the utility function and not by the particular endowment profile considered. If one adopts the HARA utility function, then the MH specification can indeed affect the qualitative properties of the dynamical system even when there is no second period endowment. The intuition behind our result can be seen by computing the coefficient of relative risk aversion (RRA) with respect to the second period effective consumption, $\hat{c}_2$, under the two alternative habit specifications using HARA and CRRA:

$$\text{RRA}_{\text{SH}}^{\text{HARA}} = \frac{\alpha\sigma(c_2 - \gamma c_1)}{M\sigma + \alpha(c_2 - \gamma c_1)}$$

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$$\mathrm{RRA}_{\mathrm{MH}}^{\mathrm{HARA}} = \frac{\alpha\sigma\left(c_2/c_1^{\gamma}\right)}{M\sigma + \alpha\left(c_2/c_1^{\gamma}\right)}$$

$$\mathrm{RRA}_{\mathrm{SH}}^{\mathrm{CRRA}} = \mathrm{RRA}_{\mathrm{MH}}^{\mathrm{CRRA}} = \sigma.$$

Thus, in this framework, when one adopts CRRA utility function the implicit assumption is that relative risk-aversion with respect to the effective consumption is constant and independent of wealth. On the contrary, with an HARA specification the relative risk-aversion with respect to second period effective consumption is time varying and contingent on the initial endowments. These facts clearly impact saving's behavior across specifications. Assuming an outer utility function of the CRRA type imposes an extra restriction, which is questionable and should at least be tested empirically in the context of overlapping generations models with habit persistence.

**Example 2.** *Keeping all the other utility parameters as in example 1, but setting the endowments to be* $w_1 = 3$, *and* $w_2 = 0$, *and calibrating the strength of habits such as that the IES is 1 in steady state we find two monetary steady states under MH and a unique monetary steady state under SH. In particular,* $\gamma = 0.293$ *for the SH specification and* $\gamma = 0.989$ *for the MH specification. Local dynamics associated with these two sets of steady states are quite different.*

As we can see from Table 3, optimal savings across the two specifications are positive. Both under SH and MH agents are saving, which is not too surprising given the specific endowment profile proposed by Bunzel (2006). As far as the local dynamics of the economy are concerned, all steady states are unstable under both habit specifications. The evolution of savings around the steady state under SH displays diverging monotonic dynamics. This is also the case for the Steady State II under MH but savings around Steady State I display nondamped oscillations, instead.

**TABLE 3.** Multiplicity of equilibria under MH but not under SH

| | SH | MH | |
|---|---|---|---|
| | Steady State | Steady State I | Steady State II |
| $s^{s.s.}$ | 2.10 | 0.377 | 2.761 |
| $c_1^{s.s.}$ | 0.90 | 2.623 | 0.239 |
| $c_2^{s.s}$ | 2.10 | 0.377 | 2.761 |
| $\epsilon$ | 0.015 | −0.090 | 0.020 |
| $(1+\frac{1}{\epsilon})$ | 66.5 | −10.044 | 50.64 |
| PM | −0.353 | −0.046 | −27.096 |
| detH | 0.024 | 0.076 | 0.858 |
| MRSC | 0.545 | 1.341 | 0.411 |
| $\Psi(c_1^{s.s.}, c_2^{s.s})$ | 1 | 1 | 0.075 |

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The point that we wanted to convey with these numerical examples is to illustrate how multiplicity of equilibria, savings patterns, and the stability properties of an economy can differ, depending on whether the utility specification is SH or MH. In other words, the numerical examples above help us substantiate and illustrate better the theoretical findings of the previous sections.

## 5. CONCLUSION

The objective of this paper is to examine the differences between two alternative formulations used in the habit persistence literature. In particular, we explore the consequences of adopting multiplicative versus subtractive habits in terms of multiplicity of steady states, dynamic indeterminacy, and local dynamics. We find that in a pure exchange two-period lived overlapping generations model, adopting MH as opposed to SH can result in very different outcomes. First, we show that in general under MH, the optimization problem may not be concave and that preferences are not homothetic. Second, we find that multiple monetary steady states cannot be ruled out under MH. Third, we provide numerical examples that illustrate how the local dynamic properties of the economy crucially depend on the type of habits considered as well as their strength.

Finally, it is worth to point out that we did not fully explore the differences in the global dynamics of the two alternative specifications. Because discontinuities in the offer curve are possible under MH because of the nonconvexity of preferences, dynamic equilibrium may not exist for some initial conditions. However, the offer curve under SH is continuous. Our educated guess, therefore, is that SH and MH could yield very different time series patterns. This suggests that the results on the irrelevance of modeling habits by Bunzel (2006) may not hold under a more general specification of preferences even at the global level. We leave this for future research.

Our findings indicate that there are important differences on how savings behave across the two most commonly adopted habit specifications. The choice of how to model savings behavior in the presence of habits is not as innocuous as it might seem. Hence, the predictions and policy prescriptions based on the two types of habit persistence modeling choices could be quite different.

### *NOTES*

1. Some examples of MH are Abel (1990) and Carroll, Overland, and Weil (2000) and, for SH, Campbell and Cochrane (1999) and Constantinides (1990).

2. See Boldrin and Woodford (1990) for a survey.

3. Specifically, both the absolute level of consumption in the second period and the change in consumption between the two periods are important. The higher the consumption when young, the more consumption in the following period is required to derive a given level of utility.

4. In this case, one needs to impose also $\gamma < c_1/c_2$.

5. By second period "effective" consumption, we mean $\hat{c}_2 = c_2 - \gamma c_1$ in the SH case, and $\hat{c}_2 = c_2/c_1^{\gamma}$ in the MH case.

6. See Konishi and Perera-Tallo (1997).

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7. The first to study the gross substitutability condition in a two-period overlapping generation model is Grandmont (1985).

8. See Ingersoll (1987) for a survey of this literature.

9. Carroll (2000, p. 68) points out other reasons to avoid CRRA in habit formation models. He states that: "CRRA utility in combination with the subtractive formulation . . . has several theoretical problems, the gravest of which is that for microeconomically plausible parameterizations of consumption variation the equation [of consumption] accumulation can easily lead to a zero or negative argument to the [utility] function . . . generating infinite negative utility."

10. See Guvenen (2006) for more details.

11. Note that the qualitative properties of the offer curve and that of the steady states can be replicated under many other parameterizations.

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# APPENDIX

## PROOF OF LEMMA 1

**Proof**. (i) The principal minor of the Hessian associated with the SH specification is given by $u''(c_1) + \gamma^2\beta v''(c_2 - \gamma c_1)$. Note that this expression is always negative. The determinant is given by $\beta u''(c_1)v''(c_2 - \gamma c_1)$, which is always positive. Therefore, the Hessian is negative definite and concavity is ensured.
(ii) Repeating the process for the case of MH, we find that the first principal minor is negative if and only if

$$u''(c_1) + \gamma\beta(\gamma+1)\frac{c_2}{c_1^{\gamma+2}}v'\left(\frac{c_2}{c_1^{\gamma}}\right) + \gamma^2\beta\frac{c_2^2}{c_1^{2(\gamma+1)}}v''\left(\frac{c_2}{c_1^{\gamma}}\right) < 0,$$

and the determinant is positive if and only if

$$v''\left(\frac{c_2}{c_1^{\gamma}}\right)u''(c_1) + \gamma(1-\gamma)\beta\frac{c_2}{c_1^{\gamma+2}}v'\left(\frac{c_2}{c_1^{\gamma}}\right)v''\left(\frac{c_2}{c_1^{\gamma}}\right) > \gamma^2\beta\frac{1}{c_1^2}\left[v'\left(\frac{c_2}{c_1^{\gamma}}\right)\right]^2.$$

Therefore, the Hessian is negative semidefinite only under certain parameter restrictions. To check that these conditions could be violated in some cases, we need to provide at least one explicit example where the objective is not concave. Consider the HARA instantaneous utility together with MH:

$$V(c_1, c_2) = \frac{1}{1-\sigma}\left[\sigma\left(A + \frac{\alpha c_1}{\sigma}\right)^{1-\sigma}\right] + \beta\frac{1}{1-\sigma}\left[\sigma\left(A + \frac{\alpha}{\sigma}\frac{c_2}{c_1^{\gamma}}\right)^{1-\sigma}\right].$$

Let $w_1 = 3$, $w_2 = 1$, $\gamma = 0.75$, $\sigma = 0.39$, $A = -1$, $\alpha = 0.79$, and $\beta = 0.55$. Then for $s_t \in [0.974, 2.289]$, concavity is violated. ■

## PROOF OF PROPOSITION 1

**Proof**. It it sufficient to show that the utility function in the subtractive case is homogeneous of degree $n$, whereas the one in the multiplicative case is not. Let $\lambda > 0$ be a scalar. If we multiply the first and second period consumption by $\lambda$ in the subtractive case we obtain: $V(\lambda c_1, \lambda c_2) = u(\lambda c_1) + \beta v(\lambda c_2 - \gamma\lambda c_1) = u(\lambda c_1) + \beta v(\lambda(c_2 - \gamma c_1))$. Assuming that the felicity functions $u(.)$ and $v(.)$ are homogeneous of degree $n$, then the overall utility function is clearly homothetic, that is, $\lambda^n u(c_1) + \lambda^n\beta v(c_2 - \gamma c_1) = \lambda^n V(c_1, c_2)$.

For the multiplicative case, instead, we show that the utility function is not homogeneous of degree $n$ regardless of the degree of homogeneity of $u(.)$ and $v(.)$. Specifically,

$$V(\lambda c_1, \lambda c_2) = u(\lambda c_1) + \beta v\left(\frac{\lambda c_2}{(\lambda c_1)^{\gamma}}\right) = u(\lambda c_1) + \beta v\left(\lambda^{(1-\gamma)}\frac{c_2}{{c_1}^{\gamma}}\right).$$

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And thus it follows that if one assumes that the felicity functions $u(.)$ and $v(.)$ are homogeneous of degree $n$:

$$\lambda^n u(c_1) + \lambda^n \beta v\left(\frac{c_2}{c_1^\gamma}\right) \neq \lambda^n V(c_1, c_2).$$

■

## PROOF OF PROPOSITION 2

**Proof**. The implicit function that defines the optimal level of savings, $\hat{s}$, under SH is given by

$$\beta(R+\gamma)v'((R+\gamma)s + w_2 - \gamma w_1) = u'(w_1 - s).$$

Because both the left- and the right-hand side of the first order condition (FOC) are continuous and differentiable, we can study the sign of the derivatives. More precisely, we find the following expressions:

$$\frac{\partial u'(w_1 - s)}{\partial s} = -u''(w_1 - s) > 0$$

$$\frac{\partial \beta(R+\gamma)v'((R+\gamma)s + w_2 - \gamma w_1)}{\partial s} = \beta(R+\gamma)^2 v''((R+\gamma)s + w_2 - \gamma w_1) < 0$$

Note that the second expression (derivative of the left-hand side of the FOC) is strictly decreasing in $s$ and the first expression (derivative of the right-hand side of the FOC) is strictly increasing in $s$. By the intermediate value theorem, there exists a unique level of optimal savings, $\hat{s}$. A nontrivial equilibrium always exist: to prove this, we can advocate Corollary 1 on page 535 of Konishi and Perera-Tallo (1997). They provide separate conditions on preferences and the production function that ensure the existence of a nontrivial steady state. Because in our framework the condition on the production function is trivially satisfied, we just need to worry about the requirement on preferences. This requirement is that the utility function is homothetic; and this is exactly the case for SH as proved here.

However, the same conclusion cannot be drawn under MH because of a lack of homotheticity. Furthermore, the implicit function that defines $s^*$, the optimal level of savings in the MH case, is given by

$$\beta v'\left[\frac{Rs + w_2}{(w_1 - s)^\gamma}\right]\left[\frac{R(w_1 - s) + \gamma(Rs + w_2)}{(w_1 - s)^\gamma}\right] = u'(w_1 - s).$$

Whereas the right-hand side is strictly increasing in $s$, the sign of the slope for the left-hand side depends on the underlying parameters of the economy. After simplifications, we have the following expression:

$$\frac{\partial \beta v'\left[\frac{Rs + w_2}{(w_1 - s)^\gamma}\right]\left[\frac{R(w_1 - s) + \gamma(Rs + w_2)}{(w_1 - s)^\gamma}\right]}{\partial s} = \beta\left[\frac{R(w_1 - s) + (Rs + w_2)\gamma}{(w_1 - s)^{\gamma+1}}\right]$$
$$\times \left\{v''\left[\frac{Rs + w_2}{(w_1 - s)^\gamma}\right]\left[\frac{R(w_1 - s) + \gamma(Rs + w_2)(w_1 - s)^{\gamma-1}}{(w_1 - s)^{2\gamma}}\right]\right.$$
$$\left. + v'\left[\frac{Rs + w_2}{(w_1 - s)^\gamma}\right]\left(\frac{3\gamma + 1}{w_1 - s}\right)\right\}.$$

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Thus, whether the left-hand side is increasing or decreasing in $s$ depends on the following condition:

$$-\frac{v'\left(\frac{Rs+w_2}{(w_1-s)^\gamma}\right)}{v''\left(\frac{Rs+w_2}{(w_1-s)^\gamma}\right)} \gtreqless \frac{R(w_1-s)+\gamma(Rs+w_2)(w_1-s)^{\gamma-1}}{(3\gamma+1)(w_{1-}s)^{2\gamma-1}}.$$

Therefore, we cannot rule out the possibilities for multiple or a continuum of equilibrium values of $s^*$. Note that we cannot rule out the possibility of no equilibria either. ■

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