How Determinism Can Fail in Classical Physics and How Quantum Physics Can (Sometimes) Provide a Cure

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Various fault modes of determinism in classical physics are outlined. It is shown how quantum mechanics can cure some forms of classical indeterminism.

1. Introduction. In the minds of most otherwise well informed philosophers of science, determinism should be thought of as Nature's default mode in classical (= nonquantum) physics, and some sort of irreducibly stochastic behavior, presumably of a quantum origin, has to come into play to displace this default mode. I will argue that this mind-set is mistaken in two ways. First, the fault modes of determinism in classical physics are so numerous and various as to make determinism in this context seem rather surprising. Second, various forms of classical indeterminism can be cured by quantizing the relevant classical system or by applying Hilbert space techniques to the classical problem. Not only is quantum physics more deterministic (in certain respects) than classical physics, but it is also more predictable, owing to the absence of sensitive dependence on initial conditions. I will not take up this last point since determinism must be distinguished from predictability, and it is the former that is my exclusive concern here.

2. Some Fault Modes of Determinism in Classical Physics.[1]

2.1. Failure of Uniqueness for Solutions to the Initial Value Problem for Ordinary Differential Equations. The uniqueness proof for the initial value problem for ordinary differential equations (odes) uses a Lipschitz con-

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1. A more detailed review of the issues discussed in this section can be found in Earman 2007.

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dition. When this condition fails, uniqueness can (but need not) fail. As an example where uniqueness does fail, consider Hamilton's equations

$$m\dot{x}(t) = p(t), \qquad \dot{p}(t) = -\nabla V(x(t)) \tag{1}$$

for a Hamiltonian of the standard form $H = p^2/2m + V(x)$, describing a particle of mass m moving on the real line $-\infty < x < +\infty$ in a potential V. Take the potential $V(x)$ to have the form $-9/2|x|^{4/3}$ near the origin. Assume for simplicity that m is unity, and set the initial conditions to $x(0) = 0 = p(0)$. The expression ∇V does not satisfy the relevant Lipschitz condition, and there are various solutions of (1) corresponding to the given initial conditions. Here are three: $x(t) \equiv 0$, $x(t) = \pm t^3$, and $x(t) = 0$ for $t < k$ and $x(t) = \pm(t - k)^3$ for $t \geq k$, where k is any positive constant. John Norton's (2008) dome example belongs to this class of examples.

Of course, the fortunes of determinism could be restored by declaring that the Lipschitz condition is a constitutive feature of classical physics. Such a declaration strikes me as ill advised. But it is one of a long line of similar moves that shore up determinism by issuing fiats.

2.2. The Breakdown of Solutions. Now restrict attention to cases where the appropriate Lipschitz condition holds and the initial value problem has a unique solution. This does not assure the fortunes of determinism for all future (or past) times, since the standard existence and uniqueness theorems for odes are local in time and the unique solution for the given initial data can break down after a finite time. Two ways in which the breakdown can happen are illustrated by the case of point mass particles obeying Newton's laws of motion and interacting via a $1/r^2$ attractive force. The initial value problem does have locally in time a unique solution, but the solution can break down either because of (i) collision singularities or (ii) noncollision singularities where the particles run off to spatial infinity in a finite time.

The latter phenomenon can be illustrated even more simply by the case of a particle moving on the positive real line $0 < x < +\infty$ in an external potential $V(x)$. Suppose that an appropriate Lipschitz condition is imposed to guarantee a unique solution $(x(t), p(t))$ near $t = 0$ to equation (1) satisfying given initial conditions $x(0) = x_o$, $p(0) = p_o$. It is intuitively obvious—and can be rigorously proven (see Reed and Simon 1975, Proposition 1, 147)—that a temporally local solution extends to a temporally global solution unless the particle reaches the boundary of the configuration space in a finite time. For some initial conditions the particle will reach $x = 0$ in a finite time if and only if (iff) $V(x)$ is bounded above

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near $x = 0$, and for some initial conditions it will run to $x = +\infty$ in a finite time iff it is bounded above for $x \geq 1$ and

$$\int_t^\infty dx\sqrt{K - V(x)} < \infty \quad \text{for some } K > \sup_{x \geq 1} V(x)$$

(Reed and Simon 1975, Theorem X.5).

The existence of escape solutions, where the particles reach the boundary of the configuration space in a finite time, give a violation of determinism toward the past since such a solution and the trivial empty solution, devoid of particles for all time, agree for all times after the escape time but disagree in the past. The time reverse of an escape solution leads to a violation of determinism toward the future.

2.3. Systems with an Infinite Number of Particles. A series of examples due to Pérez Laraudogitia show how determinism can be violated for an infinite system of particles that interact via elastic collisions. Consider a system of point mass particles that are confined to one spatial dimension and are Zeno packed into the unit interval $0 < x \leq 1$: initially P_1 is at $x = 1$, P_2 is at $x = 1/4$, P_3 is at $x = 1/8$, etc. ad infinitum. Stipulate that all of the particles, save for P_1, are initially at rest and that P_1 is moving to the left with a finite speed. Assume that collisions between two particles obey the classical laws of elastic impact and that P_1 undergoes only one collision—that with P_2—while every other P_i undergoes exactly two collisions—first with P_{i-1} and then with P_{i+1}. If all of the P_i have the same mass, then after a finite time an infinite ripple of collisions has gone through the Zeno string, and all of the particles are at rest, producing a violation of determinism toward the past. The time reverse of the process produces a violation of determinism toward the future. Although energy and momentum are conserved in each collision, the initial energy and momentum carried by P_1 has mysteriously dissipated.

One objectionable feature of this example is that the total mass of the system is infinite. Atkinson (2007) and Pérez Laraudogitia (2007) have shown that it is possible to assign masses to the P_i in such a way as to make the total mass finite while still producing a violation of determinism—with some subtle differences. In the Newtonian analysis it can be arranged that all of the particles are at relative rest after the infinite ripple of collisions has passed through the Zeno string. As with the previous example, the initial and final energies are not the same; but with finite total mass, the initial and final momenta must be the same so that the inertial frame in which all of the particles are at rest post collision is not the same as the frame in which $P_2, P_3, \ldots$ are initially at rest. In the relativistic analysis it can also be arranged that eventually all the particles

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are at relative rest. But since the relativistic conservation of energy and momentum stand or fall together, both fall.

Determinism can be restored by ruling out the Pérez Laraudogitia examples with the declaration that conservation of energy and momentum (respectively, energy-momentum) is a constitutive principle of Newtonian physics (respectively, relativistic physics). Such a declaration would also sweep away the escape solutions of the previous section. Again I take such a declaration to be ill advised. At every moment in which the particles of an escape solution exist (= exist at some finite spacetime location), conservation of energy does hold in the sense that the sum of their kinetic energies and potential energies is the same. This can be taken as a constitutive principle—or at least it is a general feature of time independent Hamiltonians. But what the possibility of the determinism killing escape solutions shows is that the stronger sense of conservation of energy—same energy for all times—does not follow from the weaker, "constitutive" sense. A parallel remark applies to the Pérez Laraudogitia examples. Conservation of energy and momentum in pairwise collisions might plausibly be promoted as a constitutive principle of elastic collisions. But what the Atkinson–Pérez Laraudogitia examples show is that this sense of conservation does not entail the strong sense.

2.4. Fields and Fluids in Newtonian Physics. Newton himself professed to be uncomfortable with the notion of action-at-a-distance. Perhaps the discomfort can be assuaged by rewriting Newtonian gravitational theory as a field theory. A mass density ρ generates a gravitational field with potential φ according to the Poisson equation

$$\nabla^2\varphi = \rho. \tag{2}$$

Then a test particle of mass m responds locally to the field by obeying Newton's first law of motion with $\mathbf{F}_{grav} := -m\nabla\varphi$.

A drawback of this rewriting is to introduce a further form of indeterminism. If φ is a solution to Poisson's equation, then so is $\varphi' = \varphi + g(\mathbf{x})f(t)$ where $g(\mathbf{x})$ is a linear function of the spatial variables and $f(t)$ is an arbitrary function of t. If f is chosen so that $f(t) = 0$ for $t \leq 0$ but $f(t) > 0$ for $t > 0$, then after $t = 0$ in the primed solution the test particle will experience an extra gravitational force, proportional to $f(t)$, even though the primed and unprimed solutions agree for all past times. A staunch believer in determinism may react to this phenomenon by denying to the gravitational field the status of an autonomous degree of freedom and backing up this denial by excluding homogeneous solutions to (2).

Determinism-buttressing moves are also needed for other Newtonian equations of motion-governing fields and fluids. For example, uniqueness of solutions to the initial value problem for the classical heat equation

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fails unless boundary conditions at infinity are imposed or it is stipulated that the temperature never turns negative. As another example, the global existence of smooth solutions of the initial value problem for the Navier-Stokes equations is still an open question, the answer to which carries a million dollar reward from the Clay Mathematics Institute. Weak solutions exist for all times but can fail to be unique.

2.5. Fields and Fluids in Special Relativistic Physics. The prospects of determinism for fields/fluids improve dramatically when the setting is changed from Newtonian spacetime to Minkowski spacetime. In the latter setting the appropriate field equations are hyperbolic partial differential equations (pdes), and such equations may have an initial value problem for which a unique solution exists for all times and, moreover, depends continuously on the initial ("well-posedness"). In contrast to the Newtonian case, declarations of constitutive principles, boundary conditions at infinity, or other determinism-saving devices need not be employed; the determinism of the equations of motion speaks for itself—which is what determinism was supposed to do in the first instance.

It should be emphasized, however, that only restricted classes of hyperbolic pdes are known to have well-posed, global initial value problems. Nevertheless, physicists are so convinced of determinism for classical relativistic physics that they sort "fundamental" from "nonfundamental" matter fields according as the field does or does not have a well-posed, global initial value problem. By this criterion the Klein-Gordon field and the source-free Maxwell electromagnetic field count as fundamental while a field of dust matter does not.

2.6. General Relativistic Physics. When they obtain for a field propagating in Minkowski spacetime, the global existence and uniqueness proofs can usually be generalized to curved spacetimes that are *globally hyperbolic*.[2] Global hyperbolicity is equivalent to the existence of a *Cauchy surface*, a spacelike hypersurface that intersects every maximally extended causal curve exactly once.[3] A drastic failure of global hyerbolicity is illustrated by a spacetime that does not admit *global time slices*, that is, spacelike hypersurface without edges (e.g., Gödel spacetime). For a field propagating in such a spacetime the global initial value problem doesn't even make sense. Alternatively, the spacetime may be *stably causal*[4] and

2. A choice has to be made as to how the field couples to the spacetime curvature. Typically "minimal coupling" is chosen.

3. If there is one such surface, then the spacetime can be foliated by such surfaces.

4. See Wald 1994, 198, for a definition of stable causality and a proof that it is equivalent to the existence of a global time function.

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thus admit a *global time function* in the sense of a smooth map $t : M \to \mathbb{R}$ (where M is the spacetime manifold) such that t increases along every future-directed timelike curve; but the level surfaces $t = const.$ may lack the Cauchy property. Two illustrations of this phenomenon are given by anti–de Sitter spacetime and negative mass Schwarzschild spacetime. In the former case causal influences can escape to or appear from spatial infinity without registering on some $t = const.$ slices. In the latter case, maximally extended causal curves fail to register on some $t = const.$ slices because they fall into or emerge from the singularity at the origin $r = 0$. The failure of possible causal processes to leave footprints on a $t = const.$ slice is the intuitive explanation of why initial data for the field on such a slice cannot be expected to determine the field throughout the spacetime.

Making the scene friendly to determinism by ruling out some forms of nonglobal hyperbolicity may have a non-question-begging rationale; for example, the paradoxes of time travel might serve as a motivation for ruling out spacetimes with closed timelike curves (CTCs; but see Earman, Smeenk, and Wüthrich 2008). But global hyperbolicity sits at the top of a hierarchy of causality conditions that are much stronger than requiring no CTCs, and I do not see any non-question-begging a priori justification for climbing to the summit.

From the point of view of the general theory of relativity (GTR) the idea of a matter field propagating on a fixed spacetime background must be regarded as an idealization that is justified only to the extent that the back reaction of the matter field on the spacetime geometry can be neglected. The problem that must eventually be tackled is the co-evolution of the matter field and the spacetime geometry according to the coupled Einstein-matter-field equations. To keep things simple, I will only talk about the opposite extreme to the matter field propagating in a fixed spacetime background, namely, the case of the vacuum Einstein equations which describe the evolution of curved empty spacetime. And because the issues about determinism I want to emphasize here are independent of the issues raised by the infamous "hole argument," I will simply stipulate that the "up to diffeomorphism" proviso on uniqueness of solutions to the Einstein equations does not indicate any breakdown in determinism because the diffeomorphism invariance of the theory is to be interpreted as a gauge symmetry.

In rough form, the existence and uniqueness proof for the vacuum Einstein field equations states that for any appropriate initial data on a three manifold Σ, there exists a unique (up to diffeomorphism) maximal spacetime M, g_{ab} such that Σ can be embedded as a Cauchy surface in such a way that the data induced by g_{ab} on the embedded Σ agree with the given initial data. This is a powerful and impressive result, but it

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doesn't establish global determinism because the unique maximal spacetime for which (the embedded) Σ is a Cauchy surface may fail to be maximal simpliciter. That is, the maximal spacetime for which (the embedded) Σ is a Cauchy surface may be properly extendible as a spacetime satisfying the vacuum Einstein equations. This might happen because of a poor choice of initial value hypersurface. More interestingly, it might happen because a naked curvature singularity develops to the future of Σ or because of the development of some causal pathology, such as CTCs, wrecks global hyperbolicity. Roger Penrose's cosmic censorship conjecture is an attempt to capture the intuition that the development of such pathologies is highly nongeneric among solutions of the vacuum Einstein equations and, more generally, among solutions of the coupled Einstein-matter-field equations, at least for fundamental matter fields (i.e., those that have well-posed global initial value formulations in Minkowski spacetime). Stating and proving precise versions of this conjecture remains one of the most important outstanding problems in classical GTR.

3. Making Life More Deterministic with Quantum Physics.[5]

3.1. Overcoming the Failure of Uniqueness for Solutions to the Initial Value Problem for Ordinary Differential Equations. Quantize the system considered in Section 2.1 by taking the Hilbert space to be the standard $\mathcal{H} = L^2_{\mathbb{C}}(\mathbb{R}, dx)$ and the Hamiltonian operator to be $\hat{H} = -\hbar^2 d^2/dx^2 + V(x)$. If the potential is suitably modified away from the origin (so that it is bounded below or square integrable), it can be shown that not only is $\hat{H}$ symmetric on a suitable dense domain of $\mathcal{H}$ but it is essentially self-adjoint (SA), that is, there is a unique self-adjoint extension, which will also be denoted by $\hat{H}$. The exponentiation of $\hat{H}$ to $U(t) := \exp(-i\hbar\hat{H}t)$ defines a one-parameter family of unitaries, which encapsulates the quantum dynamics (the Schrödinger equation being the infinitesimal form). The time evolute $\psi(t) := U(t)\psi$ of *any* vector $\psi \in \mathcal{H}$ well defined for *all* t. So whatever quantum state ψ you choose to mimic the classical initial conditions $x(0) = 0 = p(0)$, the quantum evolution of ψ is unique even though there are many classical solutions corresponding to the given initial data.

3.2. Overcoming the Breakdown of Solutions. Kato's theorem shows that the Hamiltonian operator for a system of N point mass particles interacting via a $1/r^2$ attractive force is essentially SA on a suitable dense domain of $\mathcal{H} = L^2_{\mathbb{C}}(\mathbb{R}^{3N}, d^{3N}x)$. Since the time evolute $\psi(t) := U(t)\psi$ of any

5. Details of the issues discussed in this section can be found in Earman 2008.

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vector $\psi \in \mathcal{H}$ is well defined for *all* t, the quantum dynamics cures both the collision singularities and the noncollision singularities that produce a breakdown in classical solutions.

Rauch and Reed (1973) give another example of how quantization can overcome the breakdown of classical solutions, and in contrast to the preceding example one can identify the quantum mechanism that is responsible. They construct a potential that is a descending sequence of plateaus smoothly connected by steep cliffs. A classical particle moving in this potential is accelerated off to spatial infinity in a finite time. But the cliffs serve to reflect the quantum wave so that it does not escape to infinity. Again, the technical result is that the Hamiltonian operator is essentially SA.

These examples, together with the one from Section 3.1, indicate that the classical-quantum correspondence is more complex than textbooks would lead one to believe.

3.3. Overcoming the Indeterminism in Systems with an Infinite Number of Particles. In contrast to the two preceding cases, the argument that quantum mechanics cures some forms of classical indeterminism for infinite systems of particles is not based on a neat formal result but rather on heuristic considerations. In the setup described in Section 2.3 the particles $P_2, P_3, \ldots$ are all supposed to be at rest at $t = 0$ and to be Zeno packed into the interval $0 < x \leq 1$. Trying to rig a quantum analogue of this situation would seem to produce a violation of the uncertainty principle $\Delta x \Delta p \geq \hbar$. For a wave packet to adequately represent a classical particle, Δx must be smaller than some prescribed macroscopic standard. And for the Zeno packing to work, $\Delta_n x$ for the wave packet that represents P_n must decrease without bound as $n \to \infty$. On the other hand, $\Delta_n p$ must not increase without bound as $n \to \infty$, for otherwise whatever macroscopic standard one sets for a particle being at rest will be exceeded. The combination of these two requirements is inconsistent with the uncertainty principle.

In other examples the implications of quantum mechanics are far from clear. Consider a countably infinite linear array of unit mass particles with neighboring masses connected by a spring. Newton's Second Law of motion and Hooke's spring law are in effect, giving rise to an infinite system of coupled odes. Norton (1999) demonstrated that this system is indeterministic—corresponding to the equilibrium state (all particles at rest and all the springs unstretched and uncompressed) there are many solutions: one in which the system remains in equilibrium for all time, others in which the system self-excites at varying moments in the future. Quantizing this system is not a mechanical exercise because an inner product has to be chosen for the infinite tensor product space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots$

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($\mathcal{H}_n$ the Hilbert space of particle n) to make it into a Hilbert space $\mathfrak{H}$ for the infinite system, and this can be done in an infinite number of ways. Can it happen that the inner product of $\mathfrak{H}$ can be chosen so that the Hamiltonian operator $\hat{H}$ acting on $\mathfrak{H}$ is essentially self-adjoint (in which case the indeterminism of the classical evolution is cured)? If so, can it also be the case that the quantum system can still exhibit the supertask aspect of the classical system, namely, that initially only a finite number of particles are excited and subsequently an infinite number become excited within a finite time? I conjecture that both questions cannot have positive answers, that is, quantum mechanics cannot restore determinism without damping out the classical excitability of the system.

3.4. Overcoming Failures of Determinism due to Nonglobal Hyperbolicity. So called semiclassical quantum gravity does not have the ambition of quantizing the gravitational-metric field but confines itself to calculating the back reaction effect on the metric of quantum fields propagating in the spacetime. The most spectacular result in this area is Hawking's finding that black holes are not in fact black but radiate with a thermal spectrum. If the Hawking radiation leads to a complete evaporation of the black hole, the upshot, insofar as it can be described by a classical general relativistic spacetime, is a naked singularity.[6] Thus, it would seem that adding quantum considerations to gravitational physics makes the situation for determinism worse rather than better.

However, semiclassical quantum gravity is at best a stepping stone to a full quantum theory of gravity, and one of the pious hopes of research in quantum gravity is that the sought after theory will contain mechanisms that smooth out the spacetime singularities encountered in classical GTR. Some of the proponents of loop quantum gravity (LQG) have claimed that indications of this Holy Grail are indeed found in loop quantum cosmology (LQC), which results from applying LQG to the restricted classical phase space of spatially homogeneous and isotropic situations. In the classical $k = 0$ (flat space) Friedmann-Robertson-Walker (FRW) models, the line element takes the form $ds^2 = a(t)(dx^2 + dy^2 + dz^2) - dt^2$. In the subclass of such models with a big bang the scale factor $a(t) \to 0$ as $t \to 0$. Various curvature scalar "blow up" as the big bang is approached (e.g., the Ricci curvature scalar scales as $1/a$), and there is no physically sensible way to continue the classical evolution through the big bang singularity. By contrast, the LQC treatment seems to cure the big bang singularity both because the spectrum of the inverse scale factor

6. This statement, as well as practically everything about black hole evaporation, is controversial. For an overview of the issues, see Belot, Earman, and Ruetsche 1999.

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operator is bounded from above and because the quantum evolution does continue through the classical singularity (see Bojowald 2001).

However, the issue of curvature singularity resolution in LQG turns out to be much more delicate and complicated than it initially seemed. For one thing, the happy result on the boundedness from above of the inverse scale factor operator depends crucially on the idealization of the absence of inhomogeneous perturbations (see Brunnemann and Thiemann 2006).[7] In any case, even if LQG does provide mechanisms for avoidance of the big bang curvature singularity, this would not necessarily indicate that these mechanisms also serve to avoid the naked spacetime singularities that threaten classical determinism by undermining global hyperbolicity; for the FRW models are globally hyperbolic so that the big bang singularity offers no threat to classical determinism.

Moreover, it might seem that LQG simply isn't up to the challenge of avoiding naked singularities; for it produces a quantum theory of gravity by applying a version of the canonical quantization program to classical GTR, and as such it simply assumes deterministic evolution at the classical level. This first impression may be overly pessimistic. Goswami, Joshi, and Singh (2006) have argued that the quantum effects implied by LQG can produce an avoidance of the naked singularity that develops in the classical general relativistic collapse of a homogeneous and isotropic scalar field. It is thus conceivable that a quantum gravity version of Penrose's cosmic censorship conjecture could hold even if the classical version fails.

It is worth mentioning the less interesting but also less speculative issue of how quantum considerations can be used to cure the indeterminism of fields propagating in a fixed background spacetime that is not globally hyperbolic. Consider, for example, a real-valued scalar field ϕ that propagates according to the Klein-Gordon equation; if the spacetime on which the field propagates is nonglobally hyperbolic, say, because there are timelike singularities, then initial data cannot be expected to determine a unique solution throughout the spacetime. Wald (1980) showed that on a static and stably causal spacetime, the Klein-Gordon field equation can be turned into a Hilbert space equation of the form

$$\frac{d^2\phi}{dt^2} = -\hat{A}\phi, \tag{3}$$

where $\hat{A}$ is a symmetric operator. Horowitz and Marolf (1995) showed that in some nonglobally hyperbolic spacetimes, $\hat{A}$ is essentially self-ad-

7. And for another thing, the scale factor is not a genuine "observable" in LQG. For other difficulties, see Green and Unruh 2004.

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joint and, hence, that in these cases there is a unique prescription for a deterministic dynamics.[8]

4. How Quantum Physics Can Fail to Be Deterministic. The preceding section illustrated respects in which quantum physics *can* be more deterministic than its classical counterpart. Needless to say, this does not mean that quantum physics *is* more deterministic in cases that matter to us. Turning to the other side of the coin, there are respects in which quantum physics *can* be more indeterministic than classical physics. But in parity with the opposite side of the coin, this does not mean that quantum physics *is* more indeterministic in cases that matter to us.

The most common association with the phrase "quantum indeterminism" is collapse of the state vector. Collapse has been bruited as a solution to the measurement problem, and there are various proposals for mechanisms that would affect the collapse by means of a stochastic mechanism that introduces an indeterministic evolution of the quantum state. It is worth noting, however, that such mechanisms play no role in mainline physics research in either quantum field theory or quantum gravity.

Resolving the measurement problem without state vector collapse seems to require a rule for assigning determinate values to quantum observables that breaks open the eigenvalue-eigenvector link. Call the compendium of determinate values the *value state* of the system. Of course, if the value state is uniquely fixed by the quantum state and the quantum state evolves deterministically, then the total state—quantum state plus value state—also evolves deterministically. But in orthodox versions of the family of modal interpretations, while the quantum state determines which observables have determinate values, it does not fix those values, and the evolution of the value state follows one or another of a number of possible stochastic dynamics. In this way the total state evolves indeterministically even though there is no state vector collapse and the quantum state enjoys a deterministic evolution.

Here I want to call attention to a third way in which indeterminism can enter the quantum description; namely, the Hamiltonian operator for the system is not essentially SA. This can happen even if the classical system is fully deterministic in the sense that the initial value problem has a unique global solution. As an example, return to the case of Section 2.2 of a particle moving on the positive real line. The potential can be chosen so that the appropriate Lipschitz condition is satisfied and, thus, there is a unique classical solution locally in time. And the potential can also be chosen so that the classical particle cannot reach the origin or

8. However, there substantive assumptions that underlie this prescription; see Ishibashi and Wald 2003.

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spatial infinity in a finite time and, thus, the temporally local solution extends to a unique global solution. But if $V(x)$ has the form Cx^{-2} with $C < 3/4$, then the Hamiltonian operator is not essentially SA. It is possible to construct cases where all of the self-adjoint extensions of a nonessentially SA $\hat{H}$ give dynamics that agree in the classical limit, in which case the quantum indeterminism would not be detectable.

Cases where the quantum Hamiltonian operator fails to be essentially SA are perhaps not best described as cases of quantum indeterminism but by some other terminology. I am not overly concerned with labeling. But there is the following analogy with the kind of classical indeterminism that occurs when the particles or fields can escape to the boundaries of the configuration space. Classical determinism can be restored by adding boundary conditions at edges of the configuration space. Similarly, the different SA extensions of a nonessentially SA $\hat{H}$ correspond to different conditions at the boundaries of the configuration space. Choosing one of these boundary conditions fixes a particular SA extension and, thus, a dynamics that is globally deterministic.

5. Conclusion. Determinism still manages to spring surprises on us. And apart from the surprise factor, investigating the triumphs and pitfalls of determinism remains one of the most fruitful methods of probing the foundations of physical theories, classical and quantum.

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