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# Electron-acoustic waves in the laboratory: an experiment revisited

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**Abstract.** High-frequency electrostatic waves have been observed in a two-electron-temperature plasma. Both bi-Maxwellian and Maxwellian-waterbag models were found to be inadequate in explaining the observed dispersion and damping rates. However, modelling of the hot electron component with a $\kappa$-distribution function confirms that the experiments represent observation of the electron-acoustic wave in the laboratory.

## Dedication

It is with great pleasure that this paper is dedicated to John Dougherty, in recognition of his contribution to the development of plasma physics through this journal. One of us (M.A.H.) is particularly grateful to John for introducing him to the beauty of the kinetic theory of waves in plasmas.

## 1. Introduction

The kinetic dispersion relation for electrostatic waves in an unmagnetized, homogeneous, collisionless plasma of stationary ions and electrons yields an infinite set of complex solutions. The small group of solutions that exhibit weak damping and thus describe the time-asymptotic behaviour of the plasma are termed the normal modes, while the others are called higher-order modes.

Usually, the latter are unimportant physically because of the high level of damping that they experience. However, conditions may arise in which the damping of the higher-order modes is considerably reduced, making them observable in the laboratory. That could arise from situations such as the presence of a drift or an additional species in the plasma. Fried and Gould (1961), for instance, showed that higher-order ion-acoustic modes became manifest under certain circumstances in a two-ion plasma. The criteria governing the observability of the higher-order modes in a two-ion plasma were subsequently explored in more detail by a number of authors. In particular, Gledhill and Hellberg (1986) developed a numerical technique

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for studying the saddle points of the dispersion function in the complex-frequency plane, which turned out to play a significant role in governing wave behaviour.

Plasmas that are not in thermodynamic equilibrium may exhibit two electron temperatures. Such plasmas are commonly found in space, for example in the undisturbed solar wind, (Feldman et al. 1975), within the Earth's bow shock (Feldman et al. 1982, 1983a) and near interplanetary shocks (Feldman et al. 1983b). Similarly, such plasmas may be found in the laboratory (Armstrong et al. 1979; Chakravarti and Sen Gupta 1977).

It has been shown that in an unmagnetized two-electron-temperature plasma, with $T_h/T_c$ above a critical value, a new normal mode may arise between the electron and ion plasma frequencies. Because of its relatively high frequency and its sound-like dispersion characteristics, it has been called the electron-acoustic wave (Watanabe and Taniuti 1977). Gary and Tokar (1985) carried out a parameter survey and calculated existence domains for the electron-acoustic wave (EAW) in a two-electron-temperature plasma. Using an analogous approach to that of Gledhill and Hellberg (1986), Mace and Hellberg (1990a) investigated in detail the role of the relative electron density ratio in determining the dispersion and damping characteristics of the EAW, and its interplay with the Langmuir wave. They also demonstrated its development from a higher-order mode. This work extended a preliminary study by Dell et al. (1987). Key characteristics of the EAW will be outlined below in Sec. 4.

It should be noted that a similar mode, propagating perpendicularly to the magnetic field, may be found in a hot-ion magnetized plasma involving a single electron species only, in which $T_i \gg T_e$ (Aref'ev 1970; Lashmore-Davies and Martin 1973). However, in this paper, we shall deal only with the EAW in a two-electron-temperature plasma.

The EAW may be driven unstable by an electron or an ion beam, and the resulting forms of electron-acoustic instability have been studied *inter alia* in attempts to explain the electrostatic hiss in the polar cusp of the Earth's magnetosphere (Tokar and Gary 1984), and the broadband electrostatic noise (BEN) in the magnetotail (Schriver and Ashour-Abdalla 1987). Other studies of the EA instability in a magnetic field include those of Gary (1987), Mace and Hellberg (1990b) and Bharuthram (1991). Coupling of the EAW to the electron-cyclotron mode to yield an electron-cyclotron-sound wave has also been invoked to explain BEN and hiss in the cusp/cleft region (Mace and Hellberg 1993).

Despite the fact that the EAW may play an important role in space physics, there has been little experimental work on the mode in the laboratory. The pioneering work of Treguier and Henry (1975), and Nakamura and co-workers (Ikezawa and Nakamura 1981) must be acknowledged even though they do not make explicit use of the terminology 'electron-acoustic waves'. The dearth of studies is no doubt related to the fact that the EAW is in many instances strongly damped, and thus not easily observable. As we shall see below, a further reason may relate to the difficulty in interpreting the data using Maxwellian distributions.

Some years ago, high-frequency electrostatic-wave experiments were carried out in the relatively homogeneous target plasma of a large DP device (Karlstad et al. 1984). Two modes were found, consistent with the observed two-temperature distribution. While one wave could be identified as the electron plasma wave, there were indications that the second mode provided the first laboratory demonstration of the electron-acoustic wave. Unfortunately, however, attempts to fit the kinetic

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dispersion relation to the experimental results were not wholly successful. The authors considered two alternative models for the electron distributions, but neither combination yielded results that matched both the dispersion and the damping characteristics of the observed waves.

We now revisit this experiment, and model the hot electrons by a generalized Lorentzian (or $\kappa$) distribution. This family of distributions, which is power-law-like above the thermal speed, has been found to be useful in modelling space plasmas with a non-Maxwellian tail of excess superthermal particles (Vasyliunas 1968; Leubner 1983; Summers and Thorne 1992). It includes a parameter $\kappa$ that may be varied to obtain a best fit. A modified plasma dispersion function, first introduced for integer $\kappa$ by Summers and Thorne (1991), has been used in a number of wave studies (see e.g. Thorne and Summers 1991; Meng et al. 1992; Xue et al. 1993; Summers et al. 1994).

An important development was the generalization of the plasma dispersion function to arbitrary $\kappa$ by Mace and Hellberg (1995). The generalized plasma dispersion function $Z_\kappa$ has been used to develop a dielectric tensor for electromagnetic waves in a magnetoplasma (Mace 1996a, 1996b), to study a whistler instability in the Earth's foreshock (Mace 1998), for investigations of electrostatic fluctuations (Mace et al. 1998), and, more recently, a parameter study of the electron-acoustic wave in a two-temperature plasma with cool Maxwellian and hot $\kappa$-distribution electrons (Mace et al. 1999).

It should be noted that Mace and Hellberg (1995) showed that the generalized plasma dispersion function $Z_\kappa$ could be represented in closed form by the Gauss hypergeometric function. It is this form that we use below, and it enables us to narrow down the range of values of the power $\kappa$ appropriate for the hot electrons observed in the experiment.

Results confirm that electron-acoustic waves were indeed observed by Karlstad et al. (1984). In particular, it turns out that, whereas the cool component is Maxwellian, the hot electron component in their experiment has a distribution with a non-integer value of $\kappa$ around 3–4, i.e. it is found to have a significant superthermal component.

In Sec. 2, we present a brief outline of the experiment, while in Sec. 3, we discuss the theoretical approach taken to model it. In Sec. 4 we present the results, and summarize our conclusions in Sec. 5.

Preliminary results of this work were presented at the 1998 ICPP and 25th EPS Conference on Controlled Fusion and Plasma Physics (Hellberg et al. 1998).

## 2. The experiment

The experiment was conducted in the Tromsø DP device, which has a stainless-steel chamber 1.2 m long and with a diameter of 0.6 m. No wall magnets were used, and indeed this may be important for the observation of the electron-acoustic mode. It should be noted that the filament currents produced local magnetic fields, but that no perturbation from the Earth's magnetic field or any other extraneous field was found. The pressure was in the range $10^{-2}$–$10^{-1}$ Pa, while the lowest residual pressure was $10^{-7}$ Pa.

Waves were excited with a variety of antennas, including a small shielded grid, a T-bar probe and a simple one-sided circular disc (Langmuir probes). Both lock-in and mixer detection were used. Typical values of the plasma parameters were

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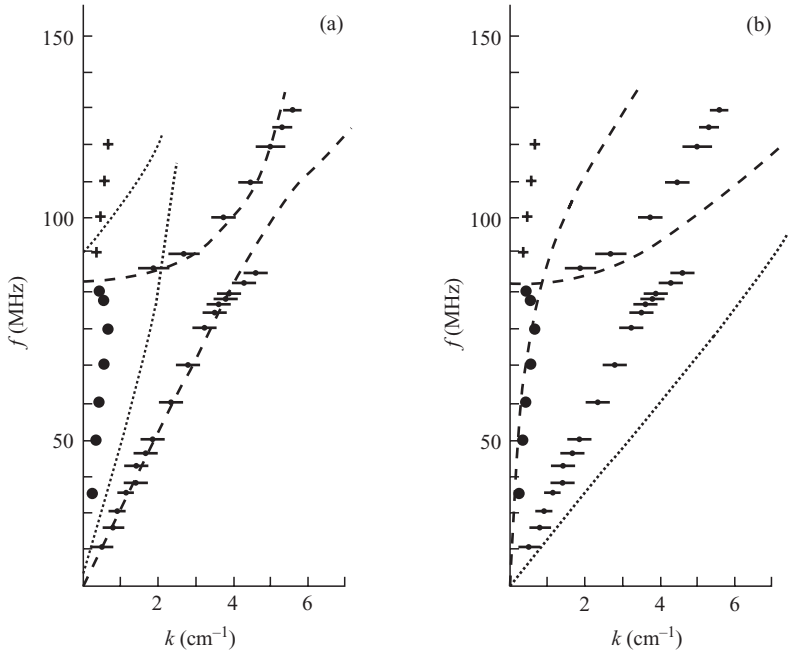



**Figure 1.** Experimental results and calculated dispersion curves for (a) a bi-Maxwellian distribution, and (b) a cool Maxwellian and a hot waterbag component. Adapted from Karlstad et al. (1984); and from Hellberg et al. (1998).

densities in the range $n_e = 5 \times 10^{13}$–$1 \times 10^{15}$ m$^{-3}$ and temperatures in the range $T_c = 0.8$–$3$ eV. There was clear evidence for a second electron component with apparent temperatures of up to about $T_h = 12$ eV. Spatial dispersion and damping were measured using standard interferometric techniques. It was found that the waves were usually rapidly damped.

We illustrate typical results in Fig. 1. The plasma data for this case are $n_c = 6 \times 10^{13}$ m$^{-3}$; $n_h = 2 \times 10^{13}$ m$^{-3}$; $T_c = 0.7$ eV and $T_h = 2.1$ eV.

Measurements of the real wavenumber $k_r$ (shown with error bars) clearly demonstrate the existence of two modes. For the high-frequency wave, the inverse damping length $k_i$ is shown by + signs, and for the lower-frequency mode by filled circles.

The higher-frequency mode has the weak damping and the dispersion characteristics of electron plasma waves, with a plasma frequency $\omega_{pe}/2\pi$ a little above 80 MHz.

The lower-frequency mode is seen to have an acoustic-like dispersion character, with a phase speed for small $k$ of about $1.3 \times 10^6$ m s$^{-1}$ or about $2v_h$, where $v_h = (T_h/m_e)^{1/2}$ is the usual thermal speed of the hot electrons. The damping curve shows strong damping for small values of the wavenumber, but a region of reduced damping in the vicinity of the plasma frequency. While it is damped within a couple of wavelengths for frequencies of up to about 50 MHz and wavenumbers of up to about 2.5 cm$^{-1}$, damping drops to low values ($k_i \approx 0.2$ cm$^{-1}$) at a real wavenumber of about 4 cm$^{-1}$, measured at a frequency (81 MHz) close to, but just below the plasma frequency.

It should be noted that, compared with all other wave experiments conducted in

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the device, both with and without a magnetic field, these experiments were found to be particularly difficult to carry out.

In an attempt to explain the observations, modelling calculations were carried out with (a) a bi-Maxwellian electron distribution and (b) a cool Maxwellian, together with a waterbag distribution for the hot electrons. The theoretical dispersion is represented by dashed curves, and the damping by dotted curves. It is clear from the figure that while the bi-Maxwellian distribution provides good agreement with the dispersion for both modes, it leads to overestimates of the damping (Fig. 1a). On the other hand, the hot waterbag distribution (Fig. 1b) models the damping reasonably, but only provides a poor approximation to the dispersion.

## 3. Theoretical modelling

The generalized Lorentzian or $\kappa$ distribution has the form

$$f(v) = (\pi\kappa\theta^2)^{-3/2}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})}\left(1+\frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)}, \tag{1}$$

where $\Gamma(x)$ is the gamma function, and $\theta$ is a generalized thermal speed, related to the usual thermal speed $v_{\rm th} = (T/m)^{1/2}$ by $\theta = [(2\kappa-3)/\kappa]^{1/2}v_{\rm th}$. It follows from the latter relation that, for physical reasons, one would normally expect to have $\kappa > 1.5$, as otherwise a thermal speed cannot be defined.

As one can see from the form of (1), an important feature of the generalized Lorentzian is that the parameter $\kappa$ provides an additional degree of freedom. The Maxwellian distribution is simply a special case, namely it is represented by the value $\kappa \to \infty$. The distributions tend in general to be quasi-Maxwellian for subthermal particles. Low values of $\kappa$ are associated with excess superthermal particles, as may be found when there is some accelerating mechanism, leading to a non-equilibrium distribution on the time scale of the experiment, with a hard spectrum. Near a Fermi shock, theory would predict that one would observe a value of $\kappa < 2$ (Mace and Hellberg 1995), while (non-integer) values of $\kappa$ in the range 4–6 have been reported from satellite experiments (see e.g. Christon et al. 1988). For values of $\kappa > 10$, the behaviour is essentially Maxwellian.

The generalized plasma dispersion function $Z_\kappa$ is a generalization of the usual $Z$ function, with the normalized $\kappa$ distribution replacing the Maxwellian in the integrand, namely

$$Z_\kappa(\xi) = \frac{1}{\pi^{1/2}\kappa^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})}\int_{-\infty}^{\infty}\frac{ds}{(s-\xi)(1+s^2/\kappa)^{\kappa+1}}, \qquad \mathrm{Im}(\xi) > 0. \tag{2}$$

As shown by Mace and Hellberg (1995) it may be expressed in the form

$$Z_\kappa(\xi) = i\frac{(\kappa+\frac{1}{2})(\kappa-\frac{1}{2})}{\kappa^{3/2}(\kappa+1)}\,{}_2F_1\left(1, 2\kappa+2; \kappa+2; \frac{1}{2}\left(1-\frac{\xi}{i\kappa^{1/2}}\right)\right). \tag{3}$$

Here ${}_2F_1$ is the Gauss hypergeometric function, which is well known both analytically and numerically, and is available, for instance, as a standard function in *Mathematica*, thereby simplifying calculations.

As experimental data indicate that naturally occurring distributions, both in space and in the laboratory, usually do not have integer values of $\kappa$, it is obviously valuable to use $Z_\kappa$, as above.

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To model the experiment, we consider a cold ion background and two electron species with $\kappa$ distributions. On substitution of the distribution functions, the general expression for the dispersion relation,

$$1 + \sum_j \frac{\omega_{pj}^2}{k^2} \int \frac{\mathbf{k}\cdot\nabla f_j^{(0)}}{\omega - \mathbf{k}\cdot\mathbf{v}}\, d^3\mathbf{v} = 0, \tag{4}$$

becomes (Mace and Hellberg 1995)

$$1 + 2\sum_j \frac{\omega_{pj}^2}{k^2\theta_j^2}\left[\frac{2\kappa_j - 1}{2\kappa_j} + \frac{\omega}{k\theta_j} Z_{\kappa_j}\left(\frac{\omega}{k\theta_j}\right)\right] = 0, \tag{5}$$

which can then be written in the form

$$1 - \sum_j \frac{(\kappa_j - 1)^2}{\kappa_j(\kappa_j - \frac{3}{2})} \frac{\omega_{pj}^2}{k^2\theta_j^2} Z'_{\kappa_j - 1}\left(\left(\frac{\kappa_j - 1}{\kappa_j}\right)^{1/2} \frac{\omega}{k\theta_j}\right) = 0. \tag{6}$$

Here $Z'_\kappa(\zeta)$ is the usual derivative of the $Z_\kappa$ function with respect to its own argument. This turns out to be (Mace and Hellberg 1995)

$$Z'_\kappa(\xi) = -2\frac{(\kappa + \frac{1}{2})(\kappa - \frac{1}{2})}{\kappa^2}\left[1 + \frac{\kappa + 1}{\kappa + \frac{1}{2}}\left(\frac{\kappa + 1}{\kappa}\right)^{1/2} \xi\, Z_{\kappa+1}\left(\left(\frac{\kappa + 1}{\kappa}\right)^{1/2}\xi\right)\right],$$

which reduces to the well-known expression $Z'(\xi) = -2[1 + \xi\, Z(\xi)]$ for $\kappa \to \infty$.

## 4. Results

The EAW is very strongly damped in a simple electron–ion plasma, and has been shown to be a Landau higher-order mode of the electron plasma wave. In a plasma with a two-temperature electron distribution and/or drifting electrons or ions, however, it may manifest itself in weakly damped form and become a normal mode (Watanabe and Taniuti 1977; Gary and Tokar 1985; Mace and Hellberg 1990a).

Calculations assuming real $k$ and complex $\omega$ have shown that the EAW is characterized by the following: (i) its dispersion is acoustic-like, with a phase speed given approximately by $v_{ea} = (n_c/n_h)^{1/2} v_h$ for a bi-Maxwellian distribution, i.e. it is of the order of the hot-species thermal speed $v_h$ (Gary and Tokar 1985); (ii) although strongly damped at very low $k$ and high $k$ (i.e. having $|\omega_i/\omega_r| > 1/2\pi$), it may be more weakly damped for intermediate $k$ ($k\lambda_{Dc} \approx 0.2$–$0.5$), the range of weak damping depending on both the hot fraction of the total electron density ($f = n_h/n_e$) and the ratio of hot to cool electron temperatures, $T_h/T_c$ (Mace and Hellberg 1990a; Mace et al. 1999). Further aspects of the EAW in a plasma with a hot $\kappa$-distribution and cool Maxwellian electrons may be found in Mace et al. (1999), who have drawn attention to the fact that damping in such a plasma yields a different observable parameter range from that obtained for a bi-Maxwellian electron distribution.

To compare with the experiment, rather than assume real wavenumber and complex frequency, we now assume real $\omega$ and complex $k$, and solve the dispersion relation (6), using the parameter values of the experiment reported in Fig. 1. In Fig. 2, we present the dispersion and damping curves for different values of $\kappa$ for the hot component, with the cool electrons being effectively chosen as Maxwellian by using $\kappa = 50$. Figure 2(a) shows $\omega$ versus $k_r$, and Fig. 2(b) $\omega$ versus $k_i$. We note that both dispersion and damping curves vary with the value $\kappa_h$ of the hot component,

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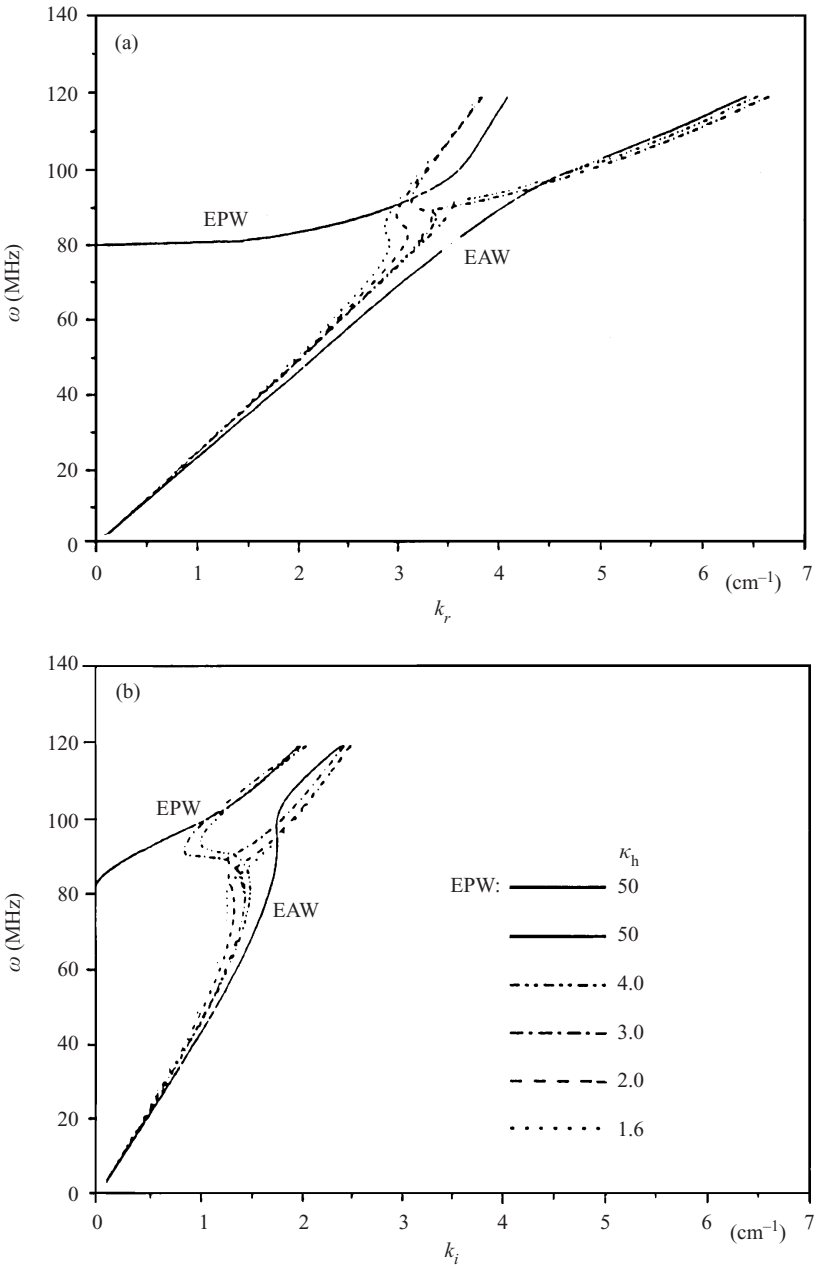



**Figure 2.** Theoretical dispersion curves for varying $\kappa_h$ with $\kappa_c = 50$: (a) plot of frequency $\omega$ versus $k_r$; (b) plot of $\omega$ versus $k_i$. From Hellberg et al. (1998).

as may be expected. In particular, for a bi-Maxwellian ($\kappa_h = \kappa_c = 50$) there are two distinct modes in both curves: the higher-frequency electron plasma wave (EPW) and the lower-frequency EAW. As the superthermal component of the hot species is increased (i.e. $\kappa_h$ is decreased), increased coupling between the two modes is observed, leading to reduced damping in a region near the plasma frequency, unlike what was found for the bi-Maxwellian (Karlstad et al. 1984). Minimum damping is

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observed for a value of $\kappa_h \approx 3$– 4, while for smaller $\kappa$, damping again increases. The minimum damping ($k_i < 0.8$ cm$^{-1}$) is found in a narrow range near the plasma frequency, i.e. at $f \approx 88$ MHz, with the wavenumber at an intermediate value, $k_r \approx 3.2$ cm$^{-1}$. From the acoustic part of the dispersion curve in Fig. 2(a) one may calculate a phase velocity $v_{ea} \approx 1.5 \times 10^6$ m s$^{-1}$ or about $2.5v_h$. Qualitatively, the calculation reproduces the experimental curve, and it even provides reasonable quantitative agreement with the experimental results. It is also qualitatively characteristic of the EAW.

We had noted earlier that in the paper by Karlstad et al. (1984), the bi-Maxwellian model had provided good agreement with the experimental dispersion curve. That may be understood because the value of $\kappa$ does not have a strong influence on the dispersion. Assuming complex $\omega$, Mace et al. (1999) have shown that the dispersion relation for EAW in a plasma with hot $\kappa$-distributed electrons is given approximately by

$$\omega_r^2 = \omega_{pc}^2 \frac{1 + 3k^2\lambda_{Dc}^2}{1 + 1/k^2\lambda_{\kappa h}^2}. \tag{7}$$

This differs from the usual dispersion relation (Gary and Tokar 1985) only in the fact that the usual hot-electron Debye length $\lambda_{Dh}$ in the denominator is replaced here by the $\kappa$-based screening length found independently by Bryant (1996) and Mace et al. (1996, 1998), namely

$$\lambda_{\kappa h} \equiv \lambda_{Dh} \left( \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}} \right)^{1/2}. \tag{8}$$

The same $\kappa$-related factor also appears in the expression for the phase speed, which for small $k$ in such a plasma, is given approximately by

$$\left( \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}} \right)^{1/2} \left( \frac{n_c}{n_h} \right)^{1/2} v_h$$

(Mace et al. 1999). Using the experimental parameter values, this expression yields an approximate phase speed of about $1.5v_h$ for small $k$, the $\kappa$-related factor being $0.83 \approx 1$.

The analytical expression for the temporal damping ($\omega_i/\omega_r$ for complex $\omega$), on the other hand, is a complicated function of $\kappa$, and in addition depends on such factors as $n_h/n_c$ and $T_h/T_c$ in exponential and power terms. As discussed by Mace et al. (1999), analytically its behaviour is not transparent. There is, of course, a further difference, as we are here interested in spatial damping, with real $\omega$ and complex $k$.

We have observed that as $\kappa_h$ is varied, the topology of the curves clearly changes in the region of $\kappa_h \approx 3$–4. This accords with the observation (Gledhill and Hellberg 1986; Dell et al. 1987; Mace and Hellberg 1990a) that a transition from one principal mode to another, yielding a new normal mode, is associated with a saddle point in the imaginary part of the dielectric function, observed in the complex $\omega$ (or, in this case, $k$) plane.

The value of using the dispersion function $Z_\kappa$ in the form (3) given above lies particularly in one's ability to evaluate it for non-integer $\kappa$. Closer study of the form of the curves as $\kappa_h$ was varied in steps of 0.1 showed that the lowest value of $k_i$ for the EAW was found for $\kappa_h = 3.8$, i.e. the value at which the saddle point

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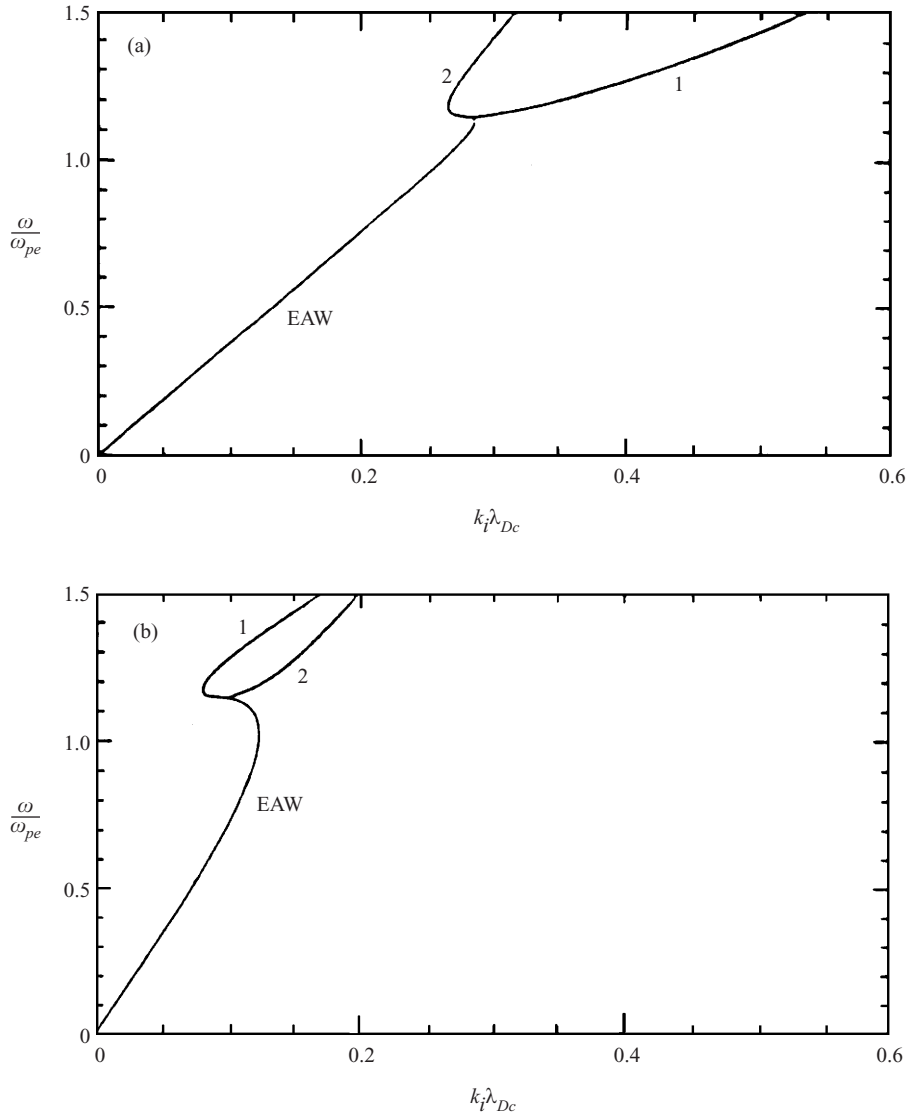



**Figure 3.** Dispersion and damping curves, showing the saddle point in the dispersion function $\epsilon(\omega, k_r, k_i)$ for $\kappa_h = 3.8$, in the normalized space of (a) $\omega$ versus $k_r$ and (b) $\omega$ versus $k_i$. From Hellberg et al. (1998).

occurs. Figure 3 illustrates the saddle point, with dispersion and damping plotted in the normalized $(\omega, k_r)$ and $(\omega, k_i)$ planes. The normalizations are with respect to the total electron plasma frequency and $\lambda_{Dc}$, the Debye length for the cool electrons, respectively.

The possibility that the cool electrons may be non-Maxwellian was explored by varying $\kappa_c$, but it was found that the highest value of $\kappa_c$ used, namely 50, gave the best agreement with the experimental curves. Thus we conclude that the cool component is definitely Maxwellian.

Similar results to the above were found when modelling other experimental runs,

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with different parameter values. In each case, the cool electrons were found to be Maxwellian, while the hot component had a distribution with $\kappa_h \approx 3$–4.

## 5. Conclusions

The electron-acoustic wave is in general strongly damped, but there are conditions under which the damping is weak and it becomes physically observable. It has been invoked in explaining a variety of observations made in space. Despite its importance in that arena and the existence of a theoretical understanding of the wave, its existence as the sub-plasma-frequency electron wave sometimes observed has not hitherto been explicitly explored in the laboratory.

The modelling that we have carried out confirms that electron-acoustic waves were indeed observed in the experiments reported earlier (Karlstad et al. 1984), although in that paper agreement between experiment and theory was not good. We have recognized that the lack of agreement lay in the assumptions regarding the distribution function of the hot electron component in that paper.

The curve-fitting flexibility provided by the parameter $\kappa$ when using the generalized Lorentzian distribution enables one to obtain agreement with both the dispersion and the damping curves measured in the experiments. The fact that we found good agreement between experiment and theory for $\kappa_h = 3$–4 and $\kappa_c \approx 50$ clearly confirms that the hot electrons have a marked superthermal tail, while the cool component is Maxwellian. Thus we propose that modelling of such wave experiments can be used as a diagnostic for the form of the distribution functions.

Finally, the fact that the best agreement is obtained for non-integer $\kappa$ underlines the value of the general plasma dispersion function expressed as a hypergeometric function, as reported by Mace and Hellberg (1995).

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