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Experimental evaluation of a model reference adaptive control for a hydraulic robot: a case study

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SUMMARY

This paper presents the implementation of an explicit model reference adaptive control (MRAC) for position tracking of a dynamically unknown robot. An auto regressive exogenous (ARX) model is chosen to define the plant model and the control input is optimised in a H_2 norm to reduce computational time and to simplify the algorithm. The theory of MRAC falls into a description of the various forms of controllers and parameter estimation techniques, therefore, applications may require very complicated solution methods depending on the selected laws. However, in this study, the proposed MRAC shows that applications may be as easy as classical control methods, such as PID, by guaranteeing the stability and achieving the convergency of the plant parameters. Despite the selected simple control model, simple optimisation method and drawbacks of the robot the experimental results show that MRAC provides an excellent position tracking compared with conventional control (PID). Many experimental implementations have been done on the robot and one of them is included in the paper.

KEYWORDS: Hydraulic robot; MRAC; Position tracking; ARX model.

1. INTRODUCTION

Studies on the model reference adaptive control (MRAC) started in the 1950s in connection with the design of autopilots operating at a wide range of speed and high performance.[1] Although the theory of MRAC improved in developments in the control area, especially those in dynamic programming, stochastic control and parameter estimation, but the stability problem of the system was still needed to be solved in 1970s.[2–5] Developments and progress in computers and electronics that made implementation of the MRAC possible in the late 1980s and therefore, many new techniques and modifications were proposed to overcome the stability problem, such as robust adaptive control, to guarantee that the control input and the system output remain bounded for all time whether the system minimum phase or not.[6–9] In another optimal control study proposed by Hang and Chen, LQG was connected to MRAC designs and the difference between the reference model and the process was minimised in LQG. This strategy was accomplished by using a RLS method for plant identification and calculating the controller parameters accordingly.[10] Some novel studies were proposed by the researchers in the last ten years, such as improvement of the transient performance by modifying the control law, solution of singularity problem for MIMO systems and complete stability analysis using a signal synthesis method.[11–13]

Although MRAC has a rich literature, most of these studies are about design and analysis of different techniques or include some computer simulations.[14–16] However, there are just a few records of real implementation for complex systems.[17–19] This is mainly due to the fact that the MRAC theory is based on assumptions that are generally not met; complex algorithms are used to achieve improvements in theory, but in practice they make the system more sensitive to violations of the assumptions made. For example, it is observed that a smaller number of adjustable parameters is favoured, because convergence is generally slower if more parameters are used. Besides, violation of linearity requirement may prohibit convergence altogether, especially if there are too many number of adjustable parameters.[20]

The purpose of this study is to present an implementation of model reference adaptive control to a real system for position tracking and to give some useful information about practical applications, concluded from the experimental results.

2. EXPERIMENTAL ARRANGEMENT

The hydraulic robot is designed and manufactured in the Mechanical Engineering Department of the University of Gaziantep. The manipulator has three degrees of freedom including two revolute and one prismatic joint. A view of the experimental set-up is shown in Figure 1. The joints of the robot are driven by hydraulic actuators controlled by electro-hydraulic servo valves and these valves are controlled by a microcomputer. The supply of hydraulic pressure to the servo valves is provided by a 22 kW hydraulic power unit capable of supplying up to 95 litres /min. at 0.12 MP.

For the communication between the computer and the manipulator a MacADIOS II interface card is used. The card is a general purpose high speed and high resolution analog-digital converter installed into one of the bus of the computer's main board. The position feedback is provided by three rotary potentiometers.[21]

3. MRAC: SYSTEM DESCRIPTION

The structure of the model reference adaptive control is illustrated in Figure 2. The desired system response to a

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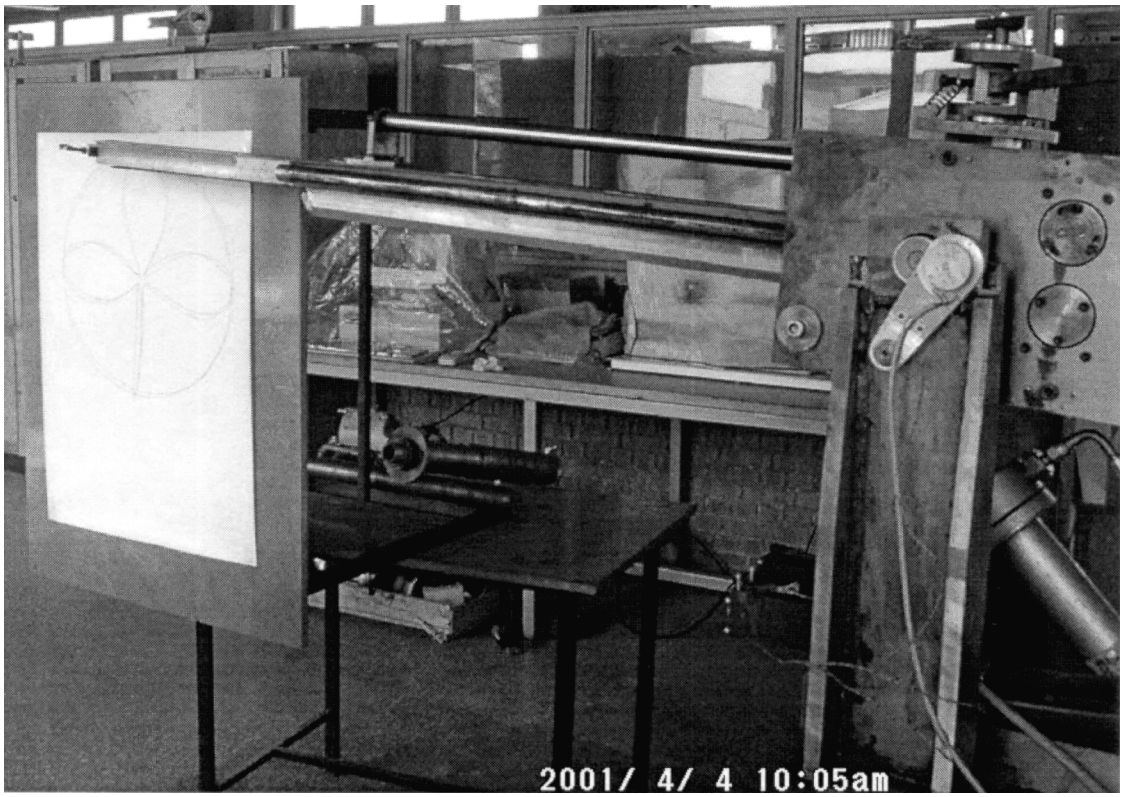


Fig. 1. A view of the experimental set-up.

command signal is specified by means of a parametrically defined reference model. Intuitively, the reference model provides the ideal system response which the adaptation mechanism should seek in adjusting the parameters. The reference model should reflect the performance specifications and should be achievable for the adaptive control system.[22] This model is explicitly implemented in a real system and is subjected to the same desired value signal y_{ref} as is the feedback controller. Its output y_m is the reference signal of the name "*model-reference*".

The actual output of the system is compared with the reference model output and the parameters of the controller

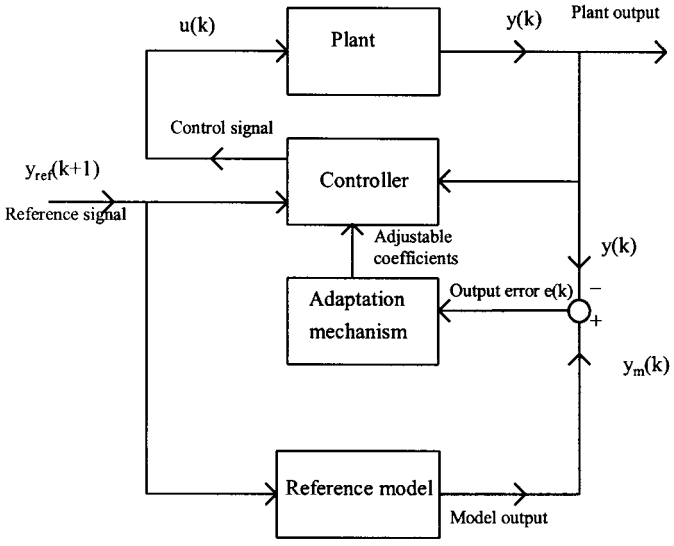


Fig. 2. Structure of the model reference adaptive control.

are modified in such a way that the error between the system and the reference model approaches zero. This is accomplished by the adaptation mechanism which takes the form of an adjustment of controller parameters so as to force the response of the resulting closed-loop control system towards that of desired model output. Therefore the adaptation mechanism must be designed so as to reduce to zero the measured output error and keep the system stable under all operating conditions.

The model reference adaptive control is one of the practical and simple control methods. Because it has only two tuning parameters (G_1 and G_2) and has a straightforward polynomial solution algorithm which includes some polynomial multiplication and a solution of a single diophantine equation.

A single-input/single-output (SISO) linear discrete time-series of an ARX[23,24] model may be used to represent the plant with unknown parameters.

$$A(z^{-1})y(k)=z^{-d}B(z^{-1})u(k) \tag{1}$$

It is assumed that $A(z^{-1})$ and $B(z^{-1})$ have no common unstable hidden modes, where

$$A(z^{-1})=1+a_1z^{-1}+a_2z^{-2}+,\ldots,+a_{na}z^{-na} \tag{1a}$$

$$B(z^{-1})=b_0+b_1z^{-1}+,\ldots,+b_{nb}z^{-nb} \qquad b_0\neq 0 \tag{1b}$$

k is a sampling instant, $u(k)$ is the control input for the plant and $y(k)$ is the plant output. The positive integer d is the time delay between the input and output and z^{-1} is a delay operator. The orders of polynomials $A(z^{-1})$ and $B(z^{-1})$ are represented, respectively, by n_a and n_b. These polynomials are relatively coprime and assumed that d ($d\geq 1$), n_a and n_b are known.

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3.1. Reference model

The input/output relationship between $y_{ref}(k)$ and $y_m(k)$ for a reference model can be written as

$$A_m(z^{-1})y_m(k)=z^{-d}B_m(z^{-1})y_{ref}(k) \tag{2}$$

The reference model is selected as a stable and minimum phase, such that

$$M(z^{-1})=\frac{B_m(z^{-1})}{A_m(z^{-1})} \tag{3}$$

The model output is rewritten as

$$y_m(k)=z^{-d}M(z^{-1})y_{ref}(k) \tag{4}$$

3.2. Controller design

The goal of the controller is to make the output of the system track the desired reference model output and to guarantee the stability of the closed loop system, always regardless of whether or not the plant is unstable or non-minimum phase. The general structure of the controller is given in Figure 3. The input and output relationship for a general controller may be written as

$$R(z^{-1})u(k)=T(z^{-1})y_{ref}(k)-S(z^{-1})y(k) \tag{5}$$

where $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$ are the unknown polynomials of the controller.

The closed-loop system relationship between $y_{ref}(k)$ and $y(k)$ is obtained from Equations (1) and (5).

$$y(k)=z^{-d}B_-(z^{-1})L(z^{-1})y_{ref}(k) \tag{6}$$

In the above equation $L(z^{-1})$ is the part of the transfer function of the system and its open form is given as

$$L(z^{-1})=\frac{B_+(z^{-1})T(z^{-1})}{[R(z^{-1})A(z^{-1})+z^{-d}B(z^{-1})S(z^{-1})]} \tag{7}$$

where the polynomial $B_-(z^{-1})$ represents the unstable part of polynomial $B(z^{-1})$ and $B_+(z^{-1})$ represents the stable part of polynomial $B(z^{-1})$, i.e. $B(z^{-1})=B_+(z^{-1})B_-(z^{-1})$.

The next step is to obtain an optimal controller consisting a stable and minimum phase transfer function to calculate the unknown parameters of the polynomials $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$. The H_2 norm is used for the optimisation of the control input, since other methods with polynomial solutions have more numerical solution steps, more tuning parameters and need at least solutions of two diophantine equations with spectral factorisations such as LQG and H_∞ methods.[25]

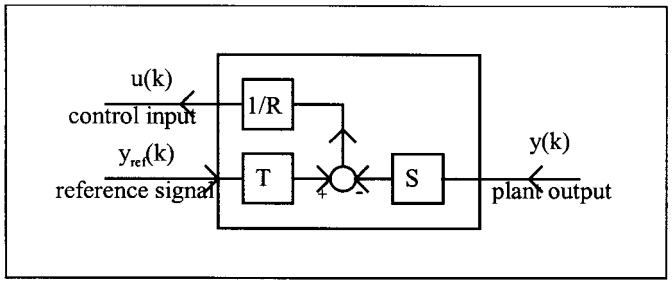


Fig. 3. The inner structure of the controller.

The cost index in the H_2 norm is[10,26]

$$J=\|E(z^{-1})+z^{-d}G(z^{-1})U(z^{-1})\|_2 \tag{8}$$

where $G(z^{-1})$ is a rational weighting function, $E(z^{-1})$ and $U(z^{-1})$ are polynomial representations of error $e(k)$ and control input, $u(k)$, respectively.

In the cost index not only is the output error minimised, the weighted control input is also minimised in order to prevent large ripples in the control input that may damage the actuator in a mechanical system.[27] Also, an unsuitable choice of the weighting function, $G(z^{-1})$ may increase the effect of plant parameter uncertainty.[28]

The term $e(k)$ may be written using Figure 1, Equations (4) and (6) as

$$E(z^{-1})=z^{-d}[M(z^{-1})-B_-(z^{-1})L(z^{-1})]\frac{N(z^{-1})}{D(z^{-1})} \tag{9}$$

The control input $u(k)$ may be written as

$$U(z^{-1})=\frac{A(z^{-1})L(z^{-1})N(z^{-1})}{B_+(z^{-1})D(z^{-1})} \tag{10}$$

where $N(z^{-1}/D(z^{-1})$ is the stable reference model having all zeros on $|z^{-1}|<1$.

Substituting Equations (9) and (10) into cost index function (Equation (8)) gives,

$$J=\left\|z^{-d}\left[M(z^{-1})-B_-(z^{-1})L(z^{-1}\right.\right.$$

$$\left.\left.+\frac{G(z^{-1})A(z^{-1})L(z^{-1})}{B_+(z^{-1})}\right]\frac{N(z^{-1})}{D(z^{-1})}\right\|_2 \tag{11}$$

In order to obtain an optimal solution for the controller $J_{\min}$ must be zero.[10] So, the optimal $L_o(z^{-1})$ is obtained as

$$L_o(z^{-1})=\frac{M(z^{-1})B_+(z^{-1})}{B(z^{-1})-G(z^{-1})A(z^{-1})} \tag{12}$$

The weighting function $G(z^{-1})$ can be defined as a rational function

$$G(z^{-1})=\frac{G_1(z^{-1})}{G_2(z^{-1})} \tag{13}$$

Substituting Equations (3) and (13) into Equation (12), we can obtain $L_o(z^{-1})$ such as

$$L_o(z^{-1})=\frac{B_m(z^{-1})G_2(z^{-1})B_+(z^{-1})}{A_m(z^{-1})[B(z^{-1})G_2(z^{-1})-G_1(z^{-1})A(z^{-1})]} \tag{14}$$

This equation may be simplified as

$$L_o(z^{-1})=\frac{B_m(z^{-1})G_2(z^{-1})B_+(z^{-1})}{A_m(z^{-1})C(z^{-1})} \tag{15}$$

where

$$C(z^{-1})=B(z^{-1})G_2(z^{-1})-G_1(z^{-1})A(z^{-1}) \tag{16}$$

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Since the transfer function of the system is a stable function, $C(z^{-1})$ does not have poles on the unit circle. Note that only $G_1(z^{-1})$ and $G_2(z^{-1})$ have an effect on the poles of the $C(z^{-1})$, therefore, to guarantee the closed loop stability of the system, $G_1(z^{-1})$ and $G_2(z^{-1})$ should be selected that the zeros of the $C(z^{-1})$ are inside the unit circle.

Equation (7) includes unknown controller polynomials $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$; on the other side, Equation (14) is the optimal solution with no any unknown parameters. In order to calculate the optimal values of $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$, the numerators and denominators of both the equations are taken as equal to each other.[29] The resultant equations are

$$R(z^{-1})A(z^{-1})+z^{-d}S(z^{-1})B(z^{-1}) = A_m(z^{-1})[B(z^{-1})G_2(z^{-1})-G_1(z^{-1})A(z^{-1})] \quad (17)$$

$$T(z^{-1})=B_m(z^{-1})G_2(z^{-1}) \quad (18)$$

Equation (17) is a Diophantine equation and can be solved numerically using the algorithm given in references [30, 31] to obtain the values of $R(z^{-1})$ and $S(z^{-1})$. The coefficients of polynomial $T(z^{-1})$ are obtained from Equation (18) by a simple polynomial multiplication.

3.3. Parameter estimation

Parameter estimation is the most important part of the adaptive control algorithm. There are several on-line parameter estimation methods such as recursive least squares, maximum likelihood, stochastic approximation, gradient and prediction error methods.[32] A recursive parameter estimator is used in this study because a covariance resetting approach[23] and random walk operations[32] provide robust and quick convergence. The main idea is to update continuously (recursively) the unknown parameters of the plant model. If the degrees of the polynomials n_a and n_b are known then the unknown plant model Equation (1) is rewritten as

$$y(k)=\phi(k-1)^T.\theta \quad (19)$$

where

$$\phi(k-1)^T=[-y(k-1)-y(k-2)\ldots-y(k-n_a); u(k-d)u(k-d-1)\ldots u(k-d-n_b)] \quad (20)$$

$$\theta^T=[a_1\ a_2\ldots a_{n_a},\ b_0\ b_1\ldots b_{n_b}] \quad (21)$$

where θ^T is a vector consisting of the parameters to be estimated, $\phi(k-1)^T$ is a vector of delayed control inputs and system outputs.

The unknown parameters of model in Equation (19) are estimated recursively on-line using Beirman's U-D factorisation method. The parameter vector is estimated at each sample time and denoted by θ. For further information see references [33–35].

4. IMPLEMENTATION OF MRAC

An independent joint control method is used to drive the hydraulic robot. The following calculations are given for the first axis of the robot as an implementation of MRAC. The implementation for the second axis is the same as with the first axis. The plant model and reference model orders are selected as second order ($n_a=n_b=n_{ma}=n_{mb}=2$). All the off-line studies related with degree of the reference model show that a second order reference model matches all the desired trajectories. Time delay is one ($d=1$). The reference model polynomials are

$$A_m(z^{-1})=1+a_{m1}z^{-1}+a_{m2}z^{-2} \text{ and}$$

$$B_m(z^{-1})=b_{m0}+b_{m1}z^{-1}+b_{m2}z^{-2}$$

where the polynomial coefficients of the reference model a_{m1}, a_{m2}, b_{m0}, b_{m1} and b_{m2} are calculated off-line, as shown below:

$$a_{m1}=0.195,\ a_{m2}=0.005,\ b_{m0}=1.0,\ b_{m1}=0.8 \text{ and } b_{m2}=-0.6$$

The forgetting factor γ is selected as 0.98 for each joint.

The initial diagonal and upper triangle matrix are chosen as

$$D(0)=1000I_m \text{ and } U_p(0)=I_m$$

where m is the number of unknowns in the model equation.

The weighting polynomial is chosen as $G_2(z^{-1})=1$ and $G_1(z^{-1})=0.5$.

The initial values of estimated plant parameters are set to the following:

$$\hat{a}_1(0)=-1,\ \hat{a}_2(0)=0,\ \hat{b}_0(0)=-0.5,\ \hat{b}_1(0)=0.5 \text{ and } \hat{b}_2(0)=0.5$$

The following initial values of input and output values in the data vector are set to zero.

$$y(-1)=y(0)=u(-2)=u(-1)=u(0)=0.$$

Finally, the model parameters of the robot manipulator are estimated on-line by applying the U-D factorisation method using the initial values system input/output and initial values of estimated system parameters.

The polynomial $C(z^{-1})$ is obtained for given $G_1(z^{-1})$ and $G_2(z^{-1})$

$$C(z^{-1})=B(z^{-1})-0.5A(z^{-1})$$

This yields

$$C(z^{-1})=(b_0+b_1z^{-1}+b_2z^{-2})-0.5(1+a_1z^{-1}+a_2z^{-2})$$

Since $C(z^{-1})$ is a second order polynomial, it may be rearranged as

$$C(z^{-1})=c_0+c_1z^{-1}+c_2z^{-2}$$

Hence, the coefficients of the polynomial $C(z^{-1})$ are calculated from the estimated values of polynomials $A(z^{-1})$ and $B(z^{-1})$.

$$c_0=b_0-0.5,\ c_1=b_1-0.5a_1 \text{ and } c_2=b_2-0.5a_2$$

Equations (17) and (18) are rewritten in terms of the estimated polynomials.

$$\hat{R}(z^{-1})\hat{A}(z^{-1})+z^{-1}\hat{S}(z^{-1})\hat{B}(z^{-1})=A_m(z^{-1})\hat{C}(z^{-1})$$

$$T(z^{-1})=B_m(z^{-1})G_2(z^{-1})$$

where

$$\hat{C}(z^{-1})=\hat{c}_0+\hat{c}_1z^{-1}+\hat{c}_2z^{-2}$$

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$$\hat{R}(z^{-1})=\hat{r}_0+\hat{r}_1z^{-1}+\hat{r}_2z^{-2}\ (n_r=n_b)$$

$$\hat{S}(z^{-1})=\hat{s}_0+\hat{s}_1z^{-1}\ (n_s=n_a-1)$$

To obtain $\hat{R}(z^{-1})$ and $\hat{S}(z^{-1})$ values the following equation is used

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \hat{a}_1 & 1 & 0 & \hat{b}_0 & 0 \\ \hat{a}_2 & \hat{a}_1 & 1 & \hat{b}_1 & \hat{b}_0 \\ 0 & \hat{a}_2 & \hat{a}_1 & \hat{b}_2 & \hat{b}_1 \\ 0 & 0 & \hat{a}_2 & 0 & \hat{b}_2 \end{bmatrix} \begin{bmatrix} \hat{r}_0 \\ \hat{r}_1 \\ \hat{r}_2 \\ \hat{s}_0 \\ \hat{s}_1 \end{bmatrix} = \begin{bmatrix} \hat{c}_0 \\ \hat{c}_1+a_{m1}\hat{c}_0 \\ \hat{c}_2+a_{m1}\hat{c}_1+a_{m2}\hat{c}_0 \\ a_{m1}\hat{c}_2+a_{m2}\hat{c}_1 \\ a_{m2}\hat{c}_2 \end{bmatrix}$$

The solution $T(z^{-1})$ is obtained directly as,

$$T(z^{-1})=b_{m0}+b_{m1}z^{-1}+b_{m2}z^{-2} \text{ since } G_2(z^{-1})=1.$$

The optimal control signal is calculated by putting the values estimated parameter of $\hat{R}(z^{-1})$ and $\hat{S}(z^{-1})$ and $T(z^{-1})$ into Equation (5) as

$$u(k)=\frac{1}{\hat{r}_0}[\hat{b}_{m0}y_{ref}(k)+\hat{b}_{m1}y_{ref}(k-1)$$

$$+\hat{b}_{m2}y_{ref}(k-2)-\hat{s}_0y(t)-\hat{s}_1y(k-1)]$$

$$+\frac{1}{\hat{r}_0}[-\hat{r}_1u_i(k-1)-\hat{r}_2u(k-2)]$$

4.1. MRAC algorithm

(i) Enter model polynomials $A_m(z^{-1})$ and $B_m(z^{-1})$

(ii) Define weighting function polynomials, $G_1(z^{-1})$ and $G_2(z^{-1})$

(iii) Measure input $u(k)$ and output $y(k)$

(iv) Estimate plant polynomials $\hat{A}(z^{-1})$ and $\hat{B}(z^{-1})$ using a recursive estimation algorithm

(v) Calculate the strictly Hurwitz polynomial $\hat{C}(z^{-1})$

$$\hat{C}(z^{-1})=\hat{B}(z^{-1})G_2(z^{-1})-\hat{A}(z^{-1})G_1(z^{-1})$$

(vi) Solve the following diophantine equation for the controller polynomials $\hat{R}(z^{-1})$ and $\hat{S}(z^{-1})$

$$\hat{R}(z^{-1})\hat{A}(z^{-1})+z^{-d}\hat{S}(z^{-1})\hat{B}(z^{-1})=A_m(z^{-1})\hat{C}(z^{-1})$$

(vii) Calculate the other controller polynomial $T(z^{-1})$

$$T(z^{-1})=B_m(z^{-1})G_2(z^{-1})$$

(viii) Calculate the new control signal u(k)

$$u(k)=\frac{T(z^{-1})}{\hat{R}(z^{-1})}y_{ref}(k)-\frac{\hat{S}(z^{-1})}{\hat{R}(z^{-1})}y(k)$$

(ix) Go to step (iii).

4.2. Experimental results

Several examples are presented of the hydraulic robot to demonstrate the capability of the MRAC algorithm and one of these examples is given below. All the trajectories used in this study have been designed by the motion design software called "MOTDES".[36] The robot has three axes, but axes 1 and 2 were used for the implementation of example trajectories in the vertical plane (see Figure 1).

The results for revolute and prismatic joints are shown in the first and second column of Figure 4 respectively. The robot is tested for a step input signal change and the rise time is obtained as 1.6 seconds; during this time interval eight measurements are taken to analyse the response and the sample time is selected as *0.2* seconds as given in reference [32]. Figure 4-a and Figure 4-b are obtained by using a conventional control method, where the dashed lines correspond to actual measurement output of each joint variables, while solid lines are the reference model output. It is seen that there is a steady state tracking error between the response and the reference model. An acceptable execution of the job process may not be expected from the system with this amount of error. However, the curves in Figure 4-c and Figure 4-d show the results achieved by using the MRAC method.

Figure 4-e and Figure 4-f show the variation of the model parameters for revolute and prismatic joints, respectively. Naturally, model parameters of a prismatic joint (Figure 4-f) have more fluctuations than that of revolute joint (Figure 4-e) because of load effects and the built up of disturbances of both joints. The controller parameters for each joint are shown in Figure 4-g and Figure 4-h. Figures 5 and 6 show the responses of the end effector of the robot when conventional and MRAC are applied, respectively. A small oscillation is seen at the beginning of the motion when MRAC is applied; however, after a few seconds it settles down and then the robot produces an excellent tracking for the rest of the motion.

5. CONCLUSION

In this paper, the feasibility of a model reference adaptive control for a dynamically unknown hydraulic robot was experimentally explored. The use of an adaptive algorithm was mainly justified by some important simplifications of the control structure; simple model (ARX), simple optimisation (H_2 norm) and simple control (independent joint control) were used to simplify the algorithm and reduce computational time. The ARX model is the simplest model and does not include any noise term in the plant model. Similarly, optimisation of the control input in a H_2 norm requires fewer solution steps, less tuning parameters and fewer diophantine equations compared with a H_∞ norm or linear quadratic Gaussian (LQG) method. On the other hand, the hydraulic robot used in this study is not an industrial type and has many deficiencies such as high friction between the links, highly unbalanced masses, etc. Very good position tracking was obtained in spite of these simplifications. All the experimental studies showed that the input and output were bounded all the time and stability was obtained under any circumstances in the system controlled by MRAC. In conclusion, this study shows that MRAC can be an alternative to the classical control methods and it may be used in practice for controlling linear or non-linear

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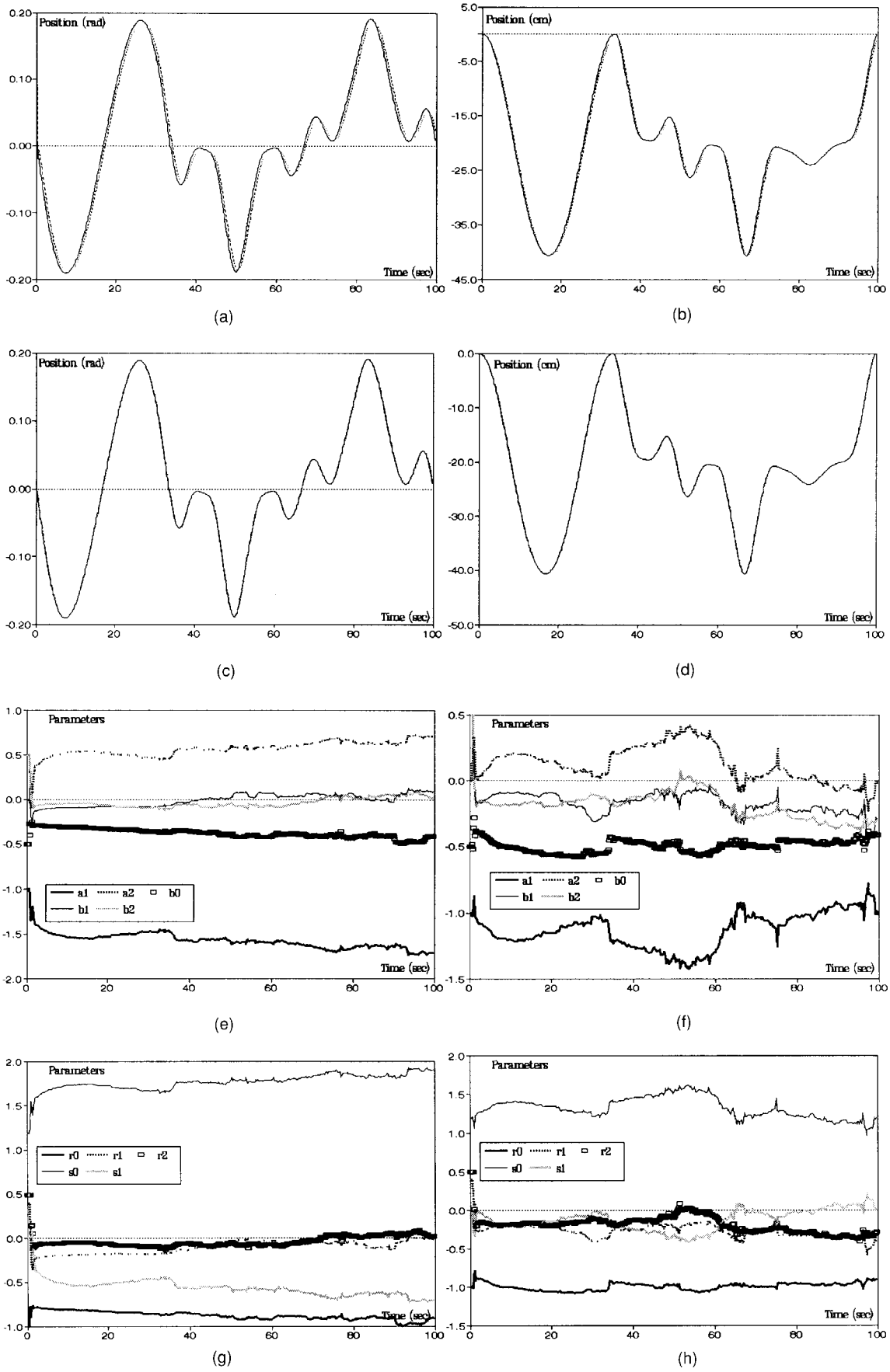


Fig. 4. Variations of model and the controller parameters for axes 1 and 2.
(a) Response of the revolute joint and reference model; (b) Response of the prismatic joint and reference model; (c) Response of the revolute joint and reference model; (d) Response of the prismatic joint and reference model; (e) Changing of plant parameters for a revolute joint; (f) Changing of plant parameters for a prismatic joint; (g) Changing of controller parameters for a revolute joint: (h) Changing of controller parameters for a prismatic joint.

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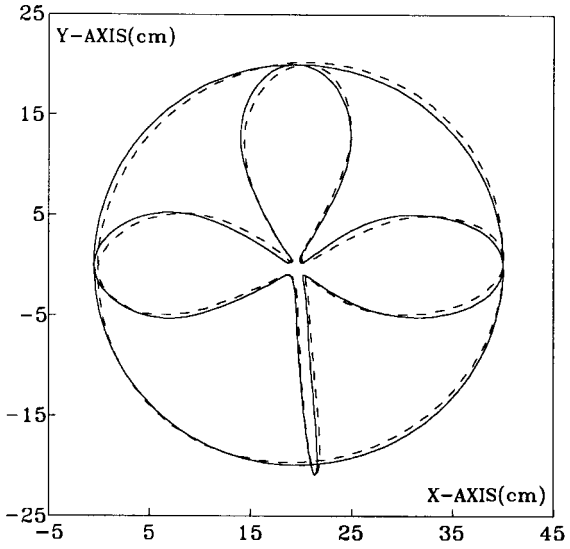


Fig. 5. Response with conventional control.

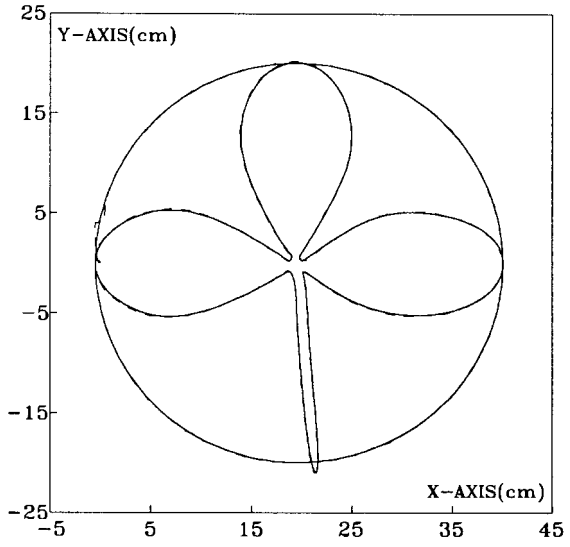


Fig. 6. Response with MRAC.

hydraulic systems which may have completely unknown dynamics. However, it is known that the physical properties of this robot manipulator, such as high friction in the hydraulic pistons, produces a constructive effect on the stability of the system. On the contrary, the same MRAC algorithm has been applied to a servo motor driven system and stability was achieved in limited conditions, which is the subject of another study.

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