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# Asset Variance Risk Premium and Capital Structure

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## Abstract

This article investigates how the asset-return variance risk premium changes leverage. I find that the premium reduces leverage by increasing risk-neutral bankruptcy probability and costs in a model where asset returns have stochastic variance with the risk premium. Empirically, the model calibrations verify a significant reduction in optimal leverage, closer to observed leverage than the model without the premium. In model-free regressions, I document that leverage correlates negatively with the variance premium. The highest negative correlation is among investment-grade firms with low asset beta and historical variance but high variance premiums because their assets have high exposure to the market's variance premium.

## I. Introduction

Observed leverage among firms is lower than theoretically expected; empirical studies report the underleverage puzzle where "firms with low expected distress costs use debt conservatively" (Graham (2000), p.1901). In this observation, historical asset-return variance that measures business uncertainty plays a key role: Underleveraged firms usually have low historical asset variance. This implies low expected distress costs (Zhang, Zhou, and Zhu (2009)). Low distress costs provide an opportunity to have more debt, which these firms seemingly do not utilize. However, these firms' asset variance is time varying (Choi and Richardson (2016)) at its low levels and risky (similar to market variance, e.g., Volatility Index (VIX) index). It is not yet clear how the risky time variations in asset variance that create the variance risk premium (VRP) affect leverage or if the VRP has a role in the underleverage puzzle.

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In response to this question, this article investigates the effect of the asset VRP on leverage by addressing two challenges. First, it is complicated to theoretically consider risky variance in the already-complex capital structure models (Huang and Huang (2012)). Hence, the relationship between the VRP and leverage remains unclear, which leads to the second challenge: It is not obvious empirically which firms' leverage is affected by the asset VRP. This article tackles these challenges and models the VRP in the capital structure, where risk-neutral (RN) asset variance is greater than historical variance. I find that the asset VRP negatively affects leverage, and in the cross section of the firms, the strongest negative relationship is for investment-grade (IG) firms. The VRP reduces the optimal leverage significantly, on top of the variance level itself, because the firms borrow conservatively to hedge against the risk of future increases in variance.

By introducing the missing factor of the VRP, this article helps to ease the empirical underleverage puzzle. Most of the empirical leverage studies consider asset variance without any risk and any VRP.[1] But Du, Elkamhi, and Ericsson (2019) report that the asset VRP has a first-order effect on debt's credit risk premium. The impact is largest among the highly rated firms. If the asset VRP has a critical impact on the credit risk premium, then, analogically, this critical role also implies that the asset VRP affects leverage. To check for this potential connection, I follow the use of the ratings. I find that IG firms are large and profitable and that they have the lowest leverage and historical asset variance, all of which match the descriptions of an underleveraged firm noted by Graham (2000). I confirm that the largest difference between observed and theoretically expected optimal leverage is for IG firms. In the data between 2002 and 2015, for example, an average IG firm has leverage close to 37%, but the model without the VRP predicts[2] optimal leverage close to 53%. However, with the VRP, the model yields lower optimal leverage that is closer to observed leverage. In the case of IG firms, the model implies 38% leverage on average because of their high asset VRP. Hence, the VRP is an important missing factor in the capital structure analysis.

Through further empirical analysis, I verify that the asset VRP reduces leverage with a first-order effect, particularly for IG firms. Model-free regressions with controls for other factors, such as cash holdings and profitability, also produce the two following results: i) The VRP has a stronger negative effect on IG firms than on an average firm, and ii) the VRP proxy has a negative correlation with the target leverage of an average firm, which also has a relatively stronger negative effect than the profitability factor. Because profitability has a first-order effect on leverage (Titman and Wessels (1988)), this result supports the importance of the VRP–leverage relationship.

[1] See also Denis (2012), Welch (2004), and Frank and Goyal (2007) for a broader analysis. Without the VRP, this article's calibrations are limited to historical variance and collapse to earlier models that propose higher optimal leverage for the firms compared with observed leverage.

[2] The main calibration method is similar to those in Jones, Mason, and Rosenfeld (1984) and Huang and Huang (2012). The findings remain robust to calibrations on individual firms in each rating class rather than calibrations to an average firm. Individual firm calibration is similar to that in Elkamhi, Ericsson, and Parsons (2012). Elkamhi et al. (2012) also report that IG firms have the lowest historical asset variance.

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Theoretically, this article introduces the concept of hedging the asset VRP to determine leverage in corporate finance. Stochastic asset variance and the hedging asset VRP have recently been used only in asset pricing to explain some puzzles, such as the credit-premium puzzle (Du et al. (2019)) motivated by the empirical evidence of Huang and Huang (2012). I model asset variance to have the VRP and relax the deterministic asset variance assumption in the earlier leverage studies.[3] This is also inspired by the observations that asset variance is time varying on the basis of equity variance (Choi and Richardson (2016)), and equity variance has the VRP (Barras and Malkhozov (2016)). In the model, I focus only on trade-off theory in the extended settings of Brennan and Schwartz (1978), although the results are complementary to other capital structure theories. Without the VRP, the model nests Leland's (1998) dynamic capital structure, which has optimal default, leverage, and debt rollover, but there are no upward restructuring and transaction costs. The model with the VRP results in tractable formulas for debt, equity, and optimal default policy, thus making the optimal leverage calculations simpler. The model shows that a high VRP increases the RN default probability. A high RN default probability reduces tax benefits and increases debt costs, which lowers optimal leverage.

This article also analyzes why RN variance remains relatively high while historical variance is low for IG firms. The spread between the variances, or the VRP, increases for IG firms because they hold asset portfolios with low idiosyncratic variance and high exposure to market variance.[4] With constant market variance and no VRP, only the level of market variance and fear of direct shocks to the market reduce leverage via asset beta. Chen, Xu, and Yang (2013) and Schwert and Strebulaev (2014) document several effects of systematic variance level on the capital structure. This article complements these studies by proposing a high VRP as a new effect of exposure to market variance on leverage; with the VRP, the shocks to asset variance are additional to the direct shocks to asset return. The idiosyncratic variance of assets does not change asset beta. Rather, idiosyncratic variance cushions the market-variance shocks and VRP to be transferred to the firm. Although a firm may have a low asset beta and total variance, it chooses conservative leverage to hedge exposure to the market VRP because of low idiosyncratic asset variance and a high asset VRP. In the case of IG firms, relatively high RN variance from exposure to the market VRP lowers IG firms' leverage more than the leverage of other firms.

Finally, in a supporting analysis, although IG firms have the lowest asset beta and historical variance, this article verifies that IG firms' asset variance has the largest correlation and exposure to market variance among all the rated firms. Because market variance is priced, high correlation explains these firms' high asset VRP. Robust calibrations with this exposure also support the earlier results. In sum,

[3]See, for example, Correia and Población (2015) and Hackbarth, Hennessy, and Leland (2007), where historical and RN asset variances are identical and constant. See Strebulaev and Whited (2011) and Sundaresan (2013) for reviews.

[4]A possible explanation that requires further research is the case of holding a large and diverse portfolio of the assets. Idiosyncratic variance is also called *diversifiable variance.* Market variance is also called *systematic* or *nondiversifiable variance.*

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the empirical results confirm the following proposition: The asset VRP reduces leverage and, consequently, underleverage in situations where the highest VRP belongs to IG firms.

This article contributes to the earlier empirical work on firm leverage in three ways. First, it shows how ratings highlight firms with more severe underleverage and connect underleverage to the credit-spread puzzle. In the finance literature, there are separate reports of the mismatch in credit premium and leverage versus their theoretical levels without noticing any connection across ratings. For example, a gap in credit spreads is reported by Huang and Huang (2012). The gap in leverage is implicit in capital structure studies without a thorough analysis (e.g., in Elkamhi et al. (2012)). The big picture that this article portrays combines these familiar pieces in the literature with a thorough analysis of leverage. Chen (2010) also reports a connection between the credit spread and leverage puzzles, but only in time series without priced variance risk because variance depends only on macroeconomic conditions. However, this article shows that the puzzles are cross-sectionally connected. Without the VRP, this article finds an almost one-to-one correlation between underleverage and credit-spread puzzles across IG firms. This finding supports the idea that ignoring the VRP creates misspecification that is not limited to only the credit premium; Du et al. (2019), for example, report a high unexplained credit premium without considering the VRP.

Second, this article adds the asset VRP as a factor to determine optimal leverage.[5] The calibrations with the VRP produce results closer to the observed leverage than the calibrations excluding the VRP. The regressions also show that the asset VRP is as important as profitability in determining the target leverage for all firms. Third, I propose that the asset VRP and hedging the exposure to market variance risk complement credit-score targeting, as identified by Kisgen (2006), (2009) in determining the capital structure of the rated firms.

This article also makes three contributions to the literature on structural models and optimal capital structure. It is the first article, to my knowledge, to extend the work on hedging variance risk from asset pricing[6] to capital structure both theoretically and empirically. Some structural models suggest that a dynamic capital structure can explain the leverage choice of firms. However, they have difficulty explaining the observed abnormal low leverage because they are limited to the constant-variance assumption. This article relaxes the constant-variance assumption and finds that the VRP reduces optimal leverage by increasing the RN default probability and hedging variance risk. Second, using a framework similar to that of Tahani (2005), I derive approximate closed-form solutions for asset prices and the default policy with stochastic variance. The tractable formulas are simpler to derive compared with the perturbation method suggested by Fouque, Sircar, and Solna (2006). Hence, this article's method technically makes it easier to calculate optimal leverage because the closed form is used for the optimal default. Third, this article appends model misspecification and a missing factor to the earlier literature

[5]Lists of the other factors are available in many articles (e.g., Öztekin (2015), Fan, Titman, and Twite (2012), Graham and Leary (2011), Frank and Goyal (2009) and Rajan and Zingales (1995)).

[6]See, for example, Campbell, Hilscher, and Szilagyi (2008), Campbell, Giglio, Polk, and Turley (2018), and McQuade (2018).

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concerning the sources of underleverage, such as macro conditions (Korajczyk and Levy (2003)) or errors in tests (Strebulaev (2007)). I confirm that ignoring the VRP in a constant-variance framework implies puzzling higher optimal leverage than observed leverage and, thus, puzzling underleverage.

## II. The Model

Let us consider a firm with an unlevered-asset-return process that follows stochastic variance:

$$(1)\quad \mathbb{P}:\begin{cases}\dfrac{dv}{v}=(\mu-\delta)dt+\sqrt{V}dW_1^p\\ dV=\kappa(\theta-V)dt+\sigma\sqrt{V}dW_2^p\end{cases}\qquad \mathbb{Q}:\begin{cases}\dfrac{dv}{v}=(r-\delta)dt+\sqrt{V}dW_1\\ dV=\lambda(\theta^*-V)dt+\sigma\sqrt{V}dW_2,\end{cases}$$

where $\mathbb{P}$ is a physical measure, $\mathbb{Q}$ is an RN measure, $\mu$ is the return drift, $\delta$ is the payout rate of cash or a leak in the cash flows, $V$ is variance, $\sqrt{V}$ is volatility, $\kappa$ is the mean-reversion speed, $\sigma$ is variance volatility, $\theta$ is the mean variance (long-run or long-term variance), $r$ is the risk-free rate, $\theta^*$ is the RN mean variance, and $\lambda$ is the RN speed of mean reversion. $W_1$ and $W_2$ are independent Brownian motions under $\mathbb{Q}$. $W_1^p$ and $W_2^p$ are independent Brownian motions under $\mathbb{P}$ (see Appendix A for a description of all variables). $W_1^p$ and $W_1$ are direct shocks to asset returns. $W_2^p$ and $W_2$ are shocks to variance.

With stochastic variance, the $\mathbb{Q}$ measure is not unique, and this article assumes that the firm's asset-return process under $\mathbb{Q}$ is a process similar to that noted by Heston (1993), where $\theta^*$ is $\kappa\theta/\lambda$. Cox, Ingersoll, and Ross (1985) present the general equilibrium model behind these processes. The difference between the RN and physical mean-reversion speeds ($\lambda-\kappa$) represents the VRP. If, hypothetically, one could invest on variance as a security, the VRP is intuitively the return premium of this security, the spread between physical and RN returns. The VRP is unit-less and negative ($\lambda<\kappa$), which implies that variance's mean under the pricing measure is higher than the historical mean. Carr and Wu (2009) and Bakshi and Kapadia (2003) report empirical evidence on the negative VRP. The negative VRP holds because, intuitively, risk-averse agents dislike risky variance and consider a higher RN mean for variance when variance is stochastic with priced risk. For example, no VRP sets RN variance equal to historical variance. This article reports the absolute value of the VRP in the calibrations. Similar to what is found by McQuade (2018), Stein and Stein (1991), and Hull and White (1987), variance is not asymmetric for the asset returns, $\mathrm{CORR}_{\mathrm{ASSET}}(dv/v,dV)=0$, which helps to keep the model tractable.[7]

[7]Unlevered asset value, $v$, is the present value of all future asset cash flows (see Appendix A in the Supplementary Material for details). Appendix B in the Supplementary Material describes the underlying economic assumptions in more detail, such as risk premiums ($\mu-r$ and $\lambda-\kappa$), and why $\lambda-\kappa$ represents the VRP. Despite the assumption about correlation, the model replicates asymmetric stock variance for the equity process ($\mathrm{CORR}_{\mathrm{EQUITY}}<0$) as a stylized fact in the markets (see Appendix C in the Supplementary Material).

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## A. VRP and Market-Variance Risk

Let us assume that market variance has priced risk according to the following process:

$$(2)\qquad \begin{cases} \mathbb{P}: & dV_s = \kappa_s(\theta_s - V_s)dt + \sigma_s\sqrt{V_s}dW^p_{\text{SYS}} \quad \theta^*_s = \kappa_s\theta_s/\lambda_s, \\ \mathbb{Q}: & dV_s = \lambda_s(\theta^*_s - V_s)dt + \sigma_s\sqrt{V_s}dW_{\text{SYS}} \quad \text{VRP}_s = \lambda_s - \kappa_s, \end{cases}$$

where $V_s$ is market variance, $\kappa_s$ is the market-variance mean-reversion speed, and $\theta_s$ is mean market variance in physical measure. $\sigma_s$ is the volatility of market variance, $\lambda_s$is the market variance's mean-reversion speed under the pricing measure, $\theta^*_s$ is the mean market variance in the pricing measure, and $\text{VRP}_s$ is the market VRP. $W^p_{\text{SYS}}$ and $W_{\text{SYS}}$, respectively, are variance shocks under physical and RN measures. For the firm, the total variance in equation (1) breaks down into systematic priced variance and idiosyncratic variance:

$$(3)\qquad \mathbb{P}: \begin{cases} \dfrac{dv}{v} = (\mu-\delta)dt + \sqrt{bV_s}dW^p_s + \sqrt{V_i}dW^p_i & \beta = \sqrt{b}, \quad \alpha = 1 - \dfrac{V_I}{V}, \quad \kappa = \alpha\kappa_s, \\ V = V_i + bV_s, \quad dV = bdV_s, & \sigma = \sigma_s\sqrt{\alpha b}, \quad \theta = b\theta_s/\alpha, \end{cases}$$

$$\mathbb{Q}: \begin{cases} \dfrac{dv}{v} = (r-\delta)dt + \sqrt{bV_s}dW_s + \sqrt{V_i}dW_i & \lambda = \alpha\lambda_s, \quad \theta^* = b\theta^*_s/\alpha, \\ V = V_i + bV_s, \quad dV = bdV_s, & \theta^* = \kappa\theta/\lambda, \quad \text{VRP} = \alpha\text{VRP}_s, \end{cases}$$

where $V_i$ is constant idiosyncratic variance, $\beta$ is asset beta, and $b$ is a positive constant. $\alpha$ is the exposure and mean correlation of asset variance with market variance, which matches the proportional systematic variance. $W^p_i$ and $W_i$, respectively, are idiosyncratic shocks under physical and RN measures independent from $W^p_s$ and $W_s$, which are shocks to the market. Although it has no effect on asset beta, which transfers market-return shocks, idiosyncratic variance reduces the impact of market-variance shocks and VRP to be transferred to the firm.[8]

If I shut down the stochastic variance and its priced risk, the formulas collapse into a model similar to that of Strebulaev (2007). When market variance is constant and has no priced risk, only the market-variance level would affect the firms' leverage decisions via total asset variance and beta. But if market variance has priced risk, having proportionally high asset-return exposure to market variance makes a firm more likely to have a high asset VRP: Although market variance may remain low, the market VRP can be high because of the uncertain future of market variance. The VRP channel complements the direct effect of exposure to market-return shocks suggested by the earlier studies through beta. With the asset VRP, the assets are additionally exposed to the variance shocks. Hence, RN asset variance and the asset VRP also depend on exposure to market variance.

There is no qualitative difference between decomposing variance or using only total variance. Therefore, I abstract from the decomposition for brevity in the

[8]Table B.1 in Appendix D of the Supplementary Material reports a numerical example based on empirical values.

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derivations and present only the model and calibrations with total variance using equation (1).

## B. Capital Structure

In this scenario, the firm with unlevered assets is all equity and optimally recapitalizes some equity with debt. For simplicity, the firm faces no information asymmetry and agency costs because the manager has aligned objectives with the shareholders. Unlevered asset value is not affected by financial decisions, and the financial manager cannot control the business risk of the firm, $V$, which is exogenously imposed by the business environment. According to the classical framework of Modigliani and Miller (1958), tax savings from interest payments, default costs, and debt rollover are the only frictions. The only effect of the VRP is on optimal default policy and RN default probability, which affects the value for tax savings and default costs. The levered value of the firm is the unlevered value plus the expected net debt benefits (tax savings of interest less the cost of bankruptcy). The levered firm continuously issues and rolls over debt with infinite maturity. However, average actual debt maturity is $M = 1/m$ because the firm replaces old perpetual debt at a continuous rate $m$ by issuing new perpetual debt with coupon $c$ and face value $p$. The firm effectively has outstanding debt with coupon payments $C$ and face value $P$ at any point in time after issuance, where $p = mP$, $c = mC$. The coupon payments symbolically represent the interest expenses of the firm. Due to rollover, the continuous debt service is the total coupon rate, $C$, plus the net partial principal repayment, $mP$. The debt rollover has no transaction costs but creates extra continuous debt service equal to $mP$.

After recapitalization, shareholders decide when to file for bankruptcy. They pick a boundary, $L$. For the asset value below $L$, they file for bankruptcy because there is no value in servicing debt anymore. Shareholders fix the boundary for a period, $T$. At the beginning of each period, they set a boundary, and they commit to declaring bankruptcy when the unlevered value falls below the boundary. Most of the earlier research assumes a constant boundary due to constant variance. A very large $T$ results in a constant boundary. Changing the boundary at certain points in time creates a degree of freedom over fixing the boundary for an infinite horizon. At default, shareholders recover 0 value by losing the firm to creditors. Creditors liquidate the firm and receive the value of the firm less the bankruptcy costs, $(1-\rho)L$; $\rho$ is the proportional bankruptcy cost (PBC) rate.

When equity holders decide about the boundary, they maximize the equity's value. The shareholders' optimization problem requires an optimal boundary, $L^*$, to satisfy the "smooth-pasting" condition ( $L^*: \partial \mathrm{EQ}/\partial v|_{(v=L^*)} = 0$, where EQ is the equity value function). $L^*$ maximizes equity value, and $\partial \mathrm{EQ}/\partial L^* = 0$. The smooth-pasting condition ensures that the equity derivative remains continuous at $L^*$ and is equal to 0.

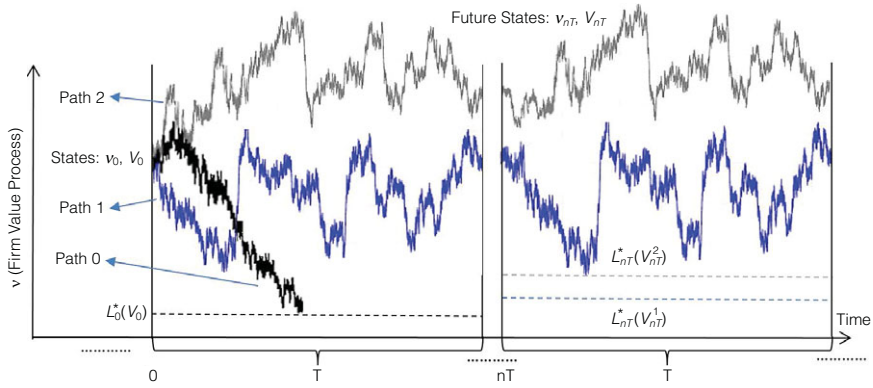
Without the VRP, the model reduces to the dynamic model of Leland (1998) without optimal risk shifting, cash-flow-triggered default, transaction costs, and rebalancing. These assumptions are not in the model because, qualitatively, they do not change the results, but they add extra layers of complexity to the model. Even with the current simplifying assumptions, the model equations are hard to set up and solve because of stochastic variance and require applying approximations to derive the optimal policies.

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FIGURE 1

Setup of the Model Based on State Variables

In Figure 1, all the paths start at the same state with the same default boundary. Path 0 ends in early default. Path 1 changes the boundary to $L_1$. Path 2 changes the boundary to $L_2$. If, by coincidence, $V_{nT} = V_0$ in a path, only then is the optimal boundary for that path $L^*_{nT} = L^*_0$ because variance is the only state variable that determines the optimal boundary.




The assumption about outstanding debt being perpetual allows for time homogeneity and for formulating equity and debt functions from a recursive relation: Given states $v_{nT} \in [L^*(V_{nT}), \infty]$ and $V_{nT} \in (0, \infty]$ at time $nT$, where $n = \{0, 1, 2, \ldots\}$, shareholders choose optimal $L^*_{nT}$ as only a function of the current variance state, $V_{nT}$. Then, at time $kT$, $L^*_{kT}$ is a function of $V_{kT}$, whereas $v$ is only checked for crossing the boundary $L^*_{kT}$. The time homogeneity at decision points means that if, by coincidence, $V_{kT} = V_{nT}$, then optimal policies are the same, $L^*_{kT} = L^*_{nT}$, because variance is the only state variable to determine the optimal boundary. If the firm is not in default, the optimal default boundary is dependent only on variance and is independent of the current value of the firm; shareholders do not change their minds about the boundary if the firm value changes. This result has already been used in previous studies (e.g., Leland (1994), Leland and Toft (1996)), for constant-variance cases.[9] Figure 1 shows the structure of the model.

## C. Securities Valuation

### 1. Debt Valuation

The perpetuity of debt allows writing the formula in recursive form, given time homogeneity. At the beginning of each period, the debt value, $d$, is:

$$(4) \quad d(v_0, V_0) = E^{\mathbb{Q}}\left(\frac{c+mp}{m+r} + (1 - I_{\tau<T})\left[e^{-rT}\left(d(v_T, V_T) - \frac{c+mp}{m+r}\right)\right] + I_{\tau<T}\left[e^{-r\tau}\left((1-\rho)mL - \frac{c+mp}{m+r}\right)\right]\right),$$

[9] Proposition 1 in Appendix E of the Supplementary Material shows that the same result is valid for a more general case.

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where $\tau$ is default time, and $I_{\tau<T}$ is 1 if default happens prior to time $T$. The value of debt is simply the cash flow from coupons and debt rollover until the holder reevaluates it at time $T$ or receives the recovery value at default. The first term is the present value of risk-free perpetuity with debt retirement. The second term is the present value of debt at time $T$ given no default. The final term is the present value of debt in default. The coupon and face value of outstanding debt have a linear relation with the coupon and face value of $d$ ($c = mC, p = mP$), which implies $d = mD$. Using the approach of Romano and Touzi (1997), I define $\zeta$ as $ln(v/L)$ and future variance, $\hat{V}$, as the average variance between time 0 and time $T$. Thus (see Appendix B):

$$(5)\qquad \begin{cases} d\zeta = (r-\delta-\frac{1}{2}\hat{V})dt+\sqrt{\hat{V}}dW \\ \hat{V} = (\int_0^T V_s ds)/T \end{cases}.$$

The transformed state variables facilitate the valuation. However, both new variables are still random. I condition the term inside the expectation operator in equation (4) on future variance. Using this conditioning, the debt formula is (see Appendix C.1 for details):

$$(6)\qquad d = \frac{c+mp}{m+r}+\left((1-\rho)mL-\frac{c+mp}{m+r}\right)E(e^{-H_b\zeta_0}),$$

$$h = \frac{r-\delta-\frac{1}{2}\hat{V}}{\sqrt{\hat{V}}},\quad H_b = \frac{\sqrt{h^2+2(r+m)}+h}{\sqrt{\hat{V}}}.$$

The only random variable is future variance within the expectation operator (the last term in the equation). Based on the solution for the expectation, the debt value is:

$$(7)\quad d = \frac{c+mp}{m+r}+\left((1-\rho)mL-\frac{c+mp}{m+r}\right)\left(1-\frac{1}{2}(A_b\zeta_0-B_b{\zeta_0}^2)\right)e^{-\zeta_0\widehat{H_b}}$$

$$A_b = \widehat{H_b''}\times E[(\hat{V}_0-E[\hat{V}_0])^2],\quad B_b = \widehat{H_b}'^2\times E[(\hat{V}_0-E[\hat{V}_0])^2]$$

$$\widehat{H_b} = H_b|_{\hat{V}_0=E[\hat{V}_0]},\ \widehat{H_b'} = \frac{\partial H_b}{\partial \hat{V}_0}|_{\hat{V}_0=E[\hat{V}_0]},\ \widehat{H_b''} = \frac{\partial^2 H_b}{\partial \hat{V}_0^2}|_{\hat{V}_0=E[\hat{V}_0]}.$$

There is no available closed-form solution for the future variance moments. Appendix D derives the formulas for the moments. Taylor expansion up to the second order approximates the expectation term around the expected variance. Hull and White (1987) and Sabanis (2003) have also used a similar approximation technique.

Table 1 compares the model with some of the studies and highlights model features. For example, Hsu, Saa-Requejo, and Santa-Clara (2010) use a stochastic boundary, but it has just random variations without an underlying factor. This article considers stochastic asset variance to be the underlying cause for the time-varying optimal default boundary. Du et al. (2019) show the effect of the asset VRP on credit risk, and McQuade (2018) analyzes the asset VRP and Fama–French factors. This

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TABLE 1

Model Features Compared with Earlier Structural Models of the Firm

In Table 1, the *X*s indicate the model features. The last row describes the model costs. In the model costs, numerical integration yields accurate results for asset prices but makes finding optimal leverage difficult because of optimization complexity. Fast mean reversion limits the analysis by restricting the range for the variance mean-reversion speed. Zero correlation implies no asset variance asymmetry, although the model still replicates equity variance asymmetry as a stylized fact in the equity markets (see Appendix C in the Supplementary Material).

| Model | Leland (1994) | Leland and Toft (1996) | Hsu et al. (2010) | Du et al. (2019) | McQuade (2018) | This Article |
|---|---|---|---|---|---|---|
| Endogenous leverage | X | X | — | — | — | X |
| Endogenous default | X | X | — | X | X | X |
| Stochastic variance | — | — | — | X | X | X |
| Time-varying boundary | — | — | X | X | X | X |
| Term structure of credit spread | — | X | X | X | X | — |
| Model cost | No stochastic variance | No stochastic variance | No stochastic variance | Numerical integration | Zero correlation and fast mean reversion | Zero correlation |

article extends their work from asset pricing into the capital structure using a novel approximation technique that allows me to find optimal leverage and determine the effect of the asset VRP on leverage.

Compared with the numerical integration of Du et al. (2019), the approximation method provides direct formulas for debt and equity. Without the formulas, optimal leverage calculations require three levels of numerical calculations: Calculating optimal leverage requires the calculation of optimal default, which, itself, depends on the calculation of debt and equity values. Basically, the calculations of debt and equity values add one layer of numerical complexity. However, this article's formulas even allow for derivation of the semiclosed form for optimal default; only optimal leverage requires numerical calculation, which substantially reduces the numerical complexity. The perturbation method based on Fouque et al. (2006) requires a fast mean-reversion speed to deliver an accurate estimation. But the calibrations in this article show that volatility has a very slow mean-reversion speed, especially under the RN measure. This slow mean-reversion speed may reduce the accuracy of the perturbation method. In this article, I extend the technique of Romano and Touzi (1997) to calculate debt and equity values; this method is less sensitive to mean-reversion speed. The formulas perform reasonably and converge.[10]

## 2. Equity and Optimal Default Boundary

Similar steps produce the values for the tax benefits and bankruptcy costs of debt (see Appendix C.2), where TAX is the tax rate:

$$\text{(8)} \qquad \text{TB}(\zeta_0, \hat{V}_0) = \text{TAX} \times \frac{C}{r} - \text{TAX} \times \frac{C}{r} E\left(e^{-H\zeta_0}\right), \quad H = \frac{\sqrt{h^2 + 2r} + h}{\sqrt{\hat{V}}},$$

[10]Appendix F of the Supplementary Material compares the values from the simulation and approximation results to show that i) the formulas converge to the simulation results, and ii) the second order is acceptable.

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$$\text{BC}(\zeta_0, \hat{V}_0) = \rho LE\left(e^{-H\zeta_0}\right). \tag{9}$$

In the contingent-claim framework discussed by Ericsson and Reneby (1998), the expectation term is the value of a claim that pays \$1 at default, or the discounted RN default probability:

$$E\left(e^{-H\zeta_0}\right) \simeq e^{-\zeta_0 \hat{H}}\left[1 - \frac{1}{2}\left(A\zeta_0 - B\zeta_0^2\right)\right], \tag{10}$$

where all the parameter definitions, such as A and B, and derivations are provided in Appendix D). Equity is simply the unlevered value plus the tax benefits minus debt and bankruptcy costs:

$$\text{EQ}(\zeta_0, \hat{V}_0) = v_0 + \text{TB}(\zeta_0, \hat{V}_0) - D(\zeta_0, \hat{V}_0) - \text{BC}(\zeta_0, \hat{V}_0). \tag{11}$$

All the valuations depend on the optimal default level. The optimal boundary is as follows:

$$\frac{\partial \text{EQ}}{\partial v}\Big|_{(v=L^*)} = 0 \Rightarrow L^* = \frac{\frac{C+mP}{r+m}\left(\widehat{H_b} + \frac{1}{2}A_b\right) - \frac{\text{TAX} \times C}{r}\left(\hat{H} + \frac{1}{2}A\right)}{1 + (1-\rho)\left(\widehat{H_b} + \frac{1}{2}A_b\right) + \rho\left(\hat{H} + \frac{1}{2}A\right)}. \tag{12}$$

Here, the boundary adjusts for the asset VRP and nests earlier models. If $m$ is 0 and without the VRP, the optimal boundary is very similar to the formula given by Leland (1994).

## D. Comparative Statics: Optimal Leverage and Default Boundary

The comparative statics show that the asset VRP reduces both the optimal default boundary and leverage. In the statics, the model parameters match the empirical values. Risk-free and asset payout rates are equal, respectively, to 5% and 3% to match the historical rates between 1997 and 2015. Variance volatility is 30% to match the standard deviation of variance in equity data. The historical variance mean-reversion speed, $\kappa$, is set to 4 to make sure that the Feller condition is met in a way similar to that of Du et al. (2019). The average asset variance is 0.04, 20% squared, close to the asset variance in low-risk firms reported by Elkamhi et al. (2012). The managers update their decision for the boundary every year, $T = 1$. The effective tax rate is 25%, and the debt rollover rate is 10%, which are the tax and minimum rollover rates used by Leland (1998). The tax rate is lower than the conventional 35% to include personal taxes. The PBC rate is in the 30%–60% range, with an average of 45%, as calculated by Glover (2016). Earlier studies have used lower costs (e.g., Bris, Welch, and Zhu (2006), Davydenko, Strebulaev, and Zhao (2012)), but Glover (2016) argues that the earlier low estimates have selection bias and that fixing for the bias increases the bankruptcy costs. Elkamhi et al. (2012) also estimate 50% cost, which includes other early distress costs, such as lost customers. For optimal leverage, the decision variable is the amount of debt to borrow, $P$, and

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maximizes the total levered value of the firm, $P: \partial(\mathrm{EQ}+D)/\partial P = 0$. The coupon rate is set to make the face value of debt, $p$, match the market value, $d$. If both face value and coupons are optimized, the managers choose 0 face value to avoid debt rollover, and the answer is degenerated; the model collapses into a static model. Finally, the default boundary is at the optimal level.

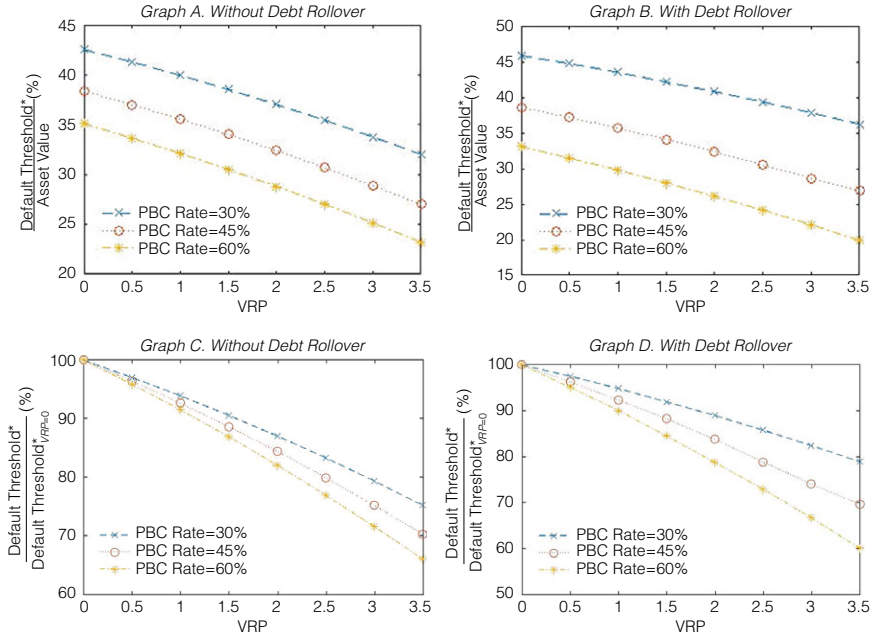
The VRP decreases the optimal default boundary (see Figure 2). Intuitively, with a high VRP, equity holders are more patient in filing for bankruptcy in the hope of getting out of trouble because of the uncertainty in variance. Equity is analogous to a call option on the assets, and a call's value increases with the RN variance of the underlying asset. This relation makes equity holders more likely to wait because high RN variance implies a high value for their call option that benefits only from the upside risk. Thus, shareholders are more patient when the VRP is high, and they lower the optimal boundary.

Another explanation for the negative effect of the VRP on the boundary is through the real-option theory. In the real-option framework of Dixit and Pindyck

FIGURE 2

Reduction in the Optimal Default Boundary Due to the Asset VRP

The $x$-axis shows the variance risk premium (VRP). In the Graphs A and B, the $y$-axis shows the ratio of the optimal default boundary, $L^*$, to the unlevered asset value, $v$. In Graphs C and D, the $y$-axis shows the relative optimal default boundary with and without the VRP. The optimal boundary maximizes equity value. The boundary is the unlevered asset level at which the firm files for bankruptcy (see equation (12)). The firm has optimal leverage. The debt coupon rate is set to make the debt's face value match the market value, $c: d = p$. The VRP is $|\lambda - \kappa|$. The proportional bankruptcy cost (PBC) rate, $p$, is the proportion of assets lost at default. The initial and mean variances are the same, $\theta = V_0$, and set to 4%, 0.2 squared. The initial asset value is \$100 and is scalable. The historical variance mean-reversion speed, $\kappa$, is 4; the risk-free rate, $r$, is 5%; the asset-payout rate, $\delta$, is 3%; and the tax rate is 25%. The debt-rollover rate, $m$, is 10%.




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(2012), filing for bankruptcy is a real option held by shareholders, and they compare it with the costs of serving debt (Geske (1977)). Ceteris paribus, an option experiences appreciation when its underlying assets' VRPs go up. This principle applies to the real option to default: A high VRP increases the value of holding the real option to default, which reduces the optimal default threshold.

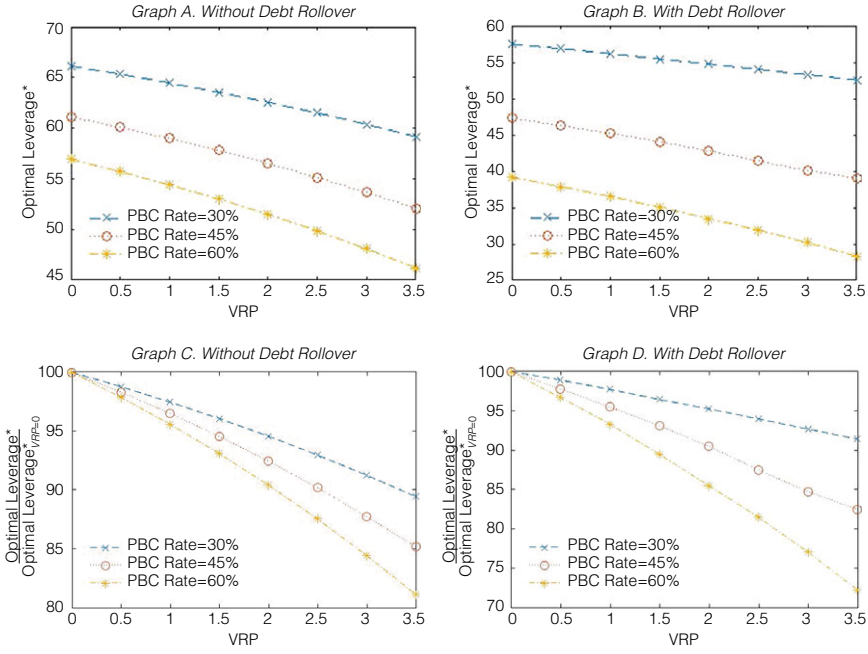
The optimal market leverage ratio decreases with s high asset VRP in Figure 3. A high VRP implies a high RN variance because it represents the relative difference between RN and historical variances. Even if the default boundary is lower for high RN variance, a high RN variance increases the RN default probability overall. Hence, the VRP increases the present value of the bankruptcy costs and lowers the tax benefits, which both reduce optimal market leverage. These results lead to the following hypothesis:

*Hypothesis 1 (H1).* The asset variance risk premium decreases target leverage, ceteris paribus.

FIGURE 3

Reduction in the Optimal Leverage Ratio Due to the Asset VRP

The *x*-axis shows the variance risk premium (VRP). In Graphs A and B, the *y*-axis shows the optimal market leverage, $D/(D+EQ)$. In Graphs C and D, the *y*-axis shows the relative optimal market leverage with and without the VRP. Optimal leverage maximizes the total levered firm value by choosing optimum debt, $P: \partial(EQ+D)/\partial P=0$. The debt coupon rate is set to make the debt's face value match the market value, $c: d=p$. The firms follow the optimal default policy. The VRP is $|\lambda-\kappa|$. The proportional bankruptcy cost (PBC) rate, $\rho$, is the proportion of assets lost at default. The initial and mean variances are the same, $\theta=V_0$, and set to 4%, 0.2 squared. The initial asset value is \$100 and is scalable. The historical variance mean-reversion speed, $\kappa$, is 4; the risk-free rate, $r$, is 5%; the asset-payout rate, $\delta$, is 3%; and the tax rate is 25%. The debt rollover rate, $m$, is 10%.




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# III. Empirical Analysis

## A. Data, Puzzles, and Variance Risk

Model calibrations with data show that the asset VRP decreases optimal leverage to a level closer to actual leverage, especially for IG firms. I also compare the contribution of the model with the VRP versus the model without the VRP in explaining the empirical trends.

The sample includes matched fiscal-year-end financial data of the U.S. firms from Compustat, the ratings from Standard & Poor's (S&P), and equity historical volatilities for 365 days from Optionmetrics. Data are limited between 1997 and 2015 because of the availability range for Optionmetrics data. I drop financial firms (Standard Industrial Classification (SIC) codes 6000–6999), utility firms (SIC codes 4900–4999), nonpublic firms, small firm-years with book asset value (AT) below \$10 million, and subsidiaries (STKO = 1 and 2) because they have different bankruptcy policies (e.g., see Luciano and Nicodano (2014)). Equity market capitalization is shares times share price. Equity volatility is the 365-day standard deviation of equity returns. The linear interpolation fills the equity-volatility time series for missing data dates, where volatility represents variance with the square-root transformation. The time series of the U.S. Treasury notes as the risk-free rate and yield spreads is from the Federal Reserve. The yield data are available from the Bank of America Merrill Lynch (BOA-ML) U.S. Corporate Option-Adjusted Spreads (OAS) for an average firm. In each rating, the spread is between the OAS index of all the bonds and the spot risk-free rate. BOA-ML spreads are available only for the average firms (e.g., see Hong and Sraer (2013), who also use the same data). Therefore, this article calibrates the model to the representative firms in each category instead of each firm-year data point. Later, in a robustness check, I use firm-year data from a different source, and the results do not change. The simple averages of the firm-years' characteristics in each rating represent the firms in the rating. Columns 1–4 of Table 2 report the statistics for all the firm-years in the sample. In the data, an average IG firm's equity is larger, and the financial risks are smaller, with lower equity volatility, yield spread, and financial leverage. The trends in the data across ratings are similar to those in earlier studies (e.g., Huang and Huang (2012)). Further discussion of Table 2 follows.

### 1. Leverage and Credit Premium without the VRP

The calibration results for all the representative firms are also provided in Table 2. The calibration without the VRP in this article is the standard procedure in the literature (Jones et al. (1984), Elkamhi et al. (2012)). Table 3 summarizes the calibration. In brief, the calibrations adjust the asset value and volatility to match the model-implied equity value and volatility with empirical values (2-by-2 calibration: 2 equations, 2 unknowns). The remaining parameters equal their empirical values. Calibration details (e.g., procedure and formulas) are provided in Appendix E. For example, an average BB-rated firm has a \$3,160 million equity market cap and 45% equity volatility. To produce the equity market cap and volatility in the model

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TABLE 2

Sample Statistics and Calibration Results for Representative Firms across the Ratings in the Model without the VRP (1997–2015)

In Table 2, columns 1 and 2 report the sample averages used as target values in calibration for the representative firm in each rating. Volatility is the square root of variance. Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap (equity value) is common shares times stock price. Columns 7 and 8 report the outcomes of 2-by-2 calibration in the model without the variance risk premium (VRP) to the sample averages in columns 1 and 2 (details are in Appendix E). Size, $v_0$, is the unlevered value of the firm's assets. Mean volatility is the square root of the mean asset variance, $\sqrt{\theta}$. Columns 5 and 6 report the yield spread and optimal leverage implied by the calibrated parameters in columns 7 and 8. Columns 5 and 6 are comparable with the empirical leverage and yield spread in columns 3 and 4. Empirical yield spreads are the Bank of America Merrill Lynch (BOA-ML) U.S. Corporate Option-Adjusted Spreads for an average rated firm. Empirical leverage is book liabilities divided by book liabilities plus equity market cap. Table A2 in Appendix A has the details of the variable calculations.

| | | | 1997–2015 Data Statistics | | | | Model Implied | | Calibrated Values in Model Without VRP | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Equity Value | Equity Volatility | Yield Spread | Leverage | Optimal Leverage | Yield Spread | Size | Mean Volatility |
| Rating | No. of Obs. | Statistic | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| AAA | 113 | Mean | 183,896 | 26.5% | 0.8% | 24.8% | 45.0% | 0.1% | 226,943 | 21.3% |
| AA | 369 | Mean | 82,053 | 28.2% | 1.1% | 24.2% | 43.6% | 0.1% | 100,954 | 22.8% |
| A | 1,975 | Mean | 26,614 | 30.9% | 1.4% | 30.2% | 43.3% | 0.3% | 35,142 | 23.1% |
| BBB | 3,681 | Mean | 9,308 | 35.1% | 2.1% | 39.6% | 43.4% | 0.5% | 13,969 | 23.0% |
| Investment-grade (IG) firms | 6,138 | Mean | 22,464 | 33.1% | 1.6% | 35.3% | 43.3% | 0.4% | 31,725 | 23.1% |
| BB | 3,741 | Mean | 3,160 | 45.2% | 3.9% | 46.1% | 40.7% | 1.3% | 5,338 | 26.4% |
| B | 2,321 | Mean | 1,551 | 60.3% | 5.7% | 55.5% | 38.4% | 2.7% | 3,168 | 30.1% |
| CCC or below | 142 | Mean | 1,270 | 87.3% | 11.8% | 71.2% | 38.1% | 5.3% | 3,970 | 30.8% |



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TABLE 3

Calibration Summary of the Article's Models

In Table 3, the mean variance in the model without the variance risk premium (VRP) is constant variance. The default boundary is endogenous. Columns 1 and 2 in Panel A describe the main calibrations in Sections III.A.1 and III.A.2. Column 3 describes the robustness-check calibration in Section C.1. Columns 4 and 5 describe the robustness-check calibrations in Section III.C.2. Appendix E has the calibration details and formulas. Detailed theoretical and empirical parameter definitions are available in Tables A1 and A2 of Appendix A, respectively. *Exposure estimates asset VRP as the correlation of asset variance with market variance times market VRP set to 4, following Ait-Sahalia and Kimmel (2007). **Only in the model with VRP.

*Panel A. Main Parameter Calibrations*

| Method | | Representative-Firm Calibration | | | Firm-Year Calibration | |
|---|---|---|---|---|---|---|
| Model Parameter | Definition | Without VRP Value 1 | With VRP Value 2 | With VRP Value 3 | Without VRP Value 4 | With VRP Value 5 |
| $\theta$ | Mean variance | Calibrated | Calibrated | Calibrated | Calibrated | Calibrated |
| $v$ | Unlevered asset value | Calibrated | Calibrated | Calibrated | Calibrated | Calibrated |
| $\lambda-\kappa$ | VRP | – | Calibrated | Exposure* | – | Calibrated |
| $V$ | Instant variance | – | $\theta$ | $\theta$ | – | Calibrated |
| $\kappa$ | Mean-reversion speed | – | 4 | 4 | – | 4 |
| $T$ | Period length | – | 2 | 2 | – | 1 |

*Panel B. Other Parameter Values*

| Parameter | Definition | Value |
|---|---|---|
| $r$ | Risk-free rate | 5% |
| TAX | Tax rate | 25% |
| $\rho$ | PBC rate | 45% |
| $M$ | Average debt maturity (1/m) | 10 |
| $\sigma**$ | Variance volatility | Standard deviation of unlevered equity variance in rating |
| $C$ | Coupon payment | Continuously compounded interest payments (log(1 + XINT/LT)) |
| $P$ | Debt's face value | Total liabilities |

matching these empirical values, 2-by-2 calibration without the VRP yields an asset value of $5,338 million and an asset variance of 6.9% for the BB representative firm. In the calibration, all the other parameters match their empirical values (e.g., the coupon rate is 3.6%). I use the calibrated parameters for representative firms without the VRP to calculate the model-implied leverage and yield spread. In the BB-rating example, all the earlier values in the model without the VRP imply 41% optimal leverage. Table 2's middle columns compare these model-implied values with their empirical twins.

When rating improves, calibrated asset size has an increasing trend, and historical asset risk has a declining trend, as shown in the last columns of Table 2. These trends together support the stylized observation that the historical probability of default for better ratings is smaller, as reported by the rating agencies. The trends are also similar to reports by earlier studies (Elkamhi et al. (2012), Schaefer and Strebulaev (2008), Eom, Helwege, and Huang (2004)). Figure 4 shows the counterintuitive positive correlation between asset volatility and leverage without the VRP from Table 4. Because IG firms' historical asset variance is lower than that of the other firms, and their size is larger, they have a lower historical default risk. Without the asset VRP, intuition is seemingly contradicted when they choose low leverage despite their low historical default risk.

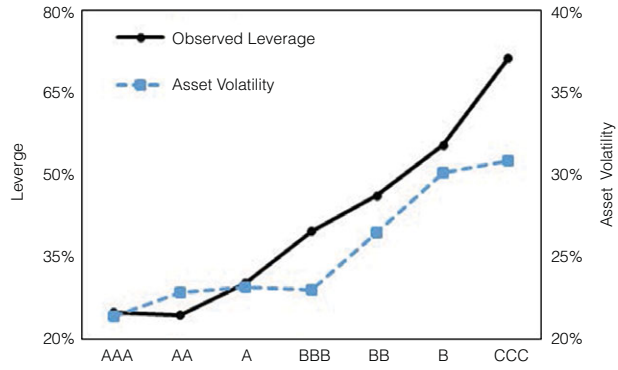
In Figure 5, I calculate the percentage difference between model-implied and empirical values. For speculative-grade (SG) firms, the model without the VRP

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FIGURE 4

Leverage and Calibrated Historical Asset Volatility across Risk Groups (1997–2015)

Figure 4 displays the trends in observed leverage and asset volatility across the ratings in terms of the risk group. "CCC" also includes ratings below CCC. This figure is based on Table 4. The *x*-axis shows the ratings. The *y*-axis shows observed leverage and calibrated asset volatility. Intuitively, low historical asset volatility implies higher leverage, but the trend seems puzzling because it is reversed: Firms with low business risk also have lower leverage.




reports lower optimal leverage than observed leverage. A similar trend appears later in the calibrations with the VRP for SG firms. This "overleverage" is normal and matches the intuition: Low-grade firms should have lower leverage, but they cannot afford to buy back debt because of facing financial distress and debt-buyback costs. In other words, they naturally are overlevered because their financial distress makes debt almost irreversible for them. As long as the costs to buy back debt are higher than those of default or issuing new debt, they remain overlevered. For example, one source of high refinancing cost for these firms is the stylized holdout problem. Empirical evidence supports this established observation. Simple transaction costs are one of the empirically identified reasons for overleverage (Gilson (1997)). Another empirical reason is financial distress for these firms. Very low "refinancing probability is consistent with market frictions preventing these highly overlevered firms from immediately unlevering, causing significant friction costs due to financial distress" (Korteweg (2010), p. 2167) reports that. Although optimal leverage for these firms is lower than their leverage, and they would benefit the most from unlevering, the overlevered firms are very unlikely to unlever. Thus, from now on, I mostly focus on IG firms.

Modeling a firm's decisions without the asset VRP leads to serious model misspecification, which is also not limited to leverage. For IG firms, the misspecification creates a gap between observed leverage and optimal leverage, with a one-on-one relationship with the mismatch in the credit spread. Of course, there is no expectation for the model to explain all the credit spread because of other priced factors, such as liquidity (Bongaerts, De Jong, and Driessen (2011)). However, the gap is the largest for IG firms that have liquid bonds because of their ratings. The calibrations ignore the asset VRP and, consequently, are misspecified. In the finance literature, the reports about the credit-premium puzzle

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TABLE 4

Sample Statistics and Calibration Results for Representative Firms across the Ratings in the Model with VRP (1997–2015)

Table 4 is comparable to Table 2. Table A2 in Appendix A has the details of the variable calculations. Columns 1–3 report the sample averages used as target values in calibration for the representative firm in each rating. Volatility is the square root of variance. Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap (equity value) is common shares times stock price. Empirical yield spreads are the Bank of America Merrill Lynch (BOA-ML) U.S. Corporate Option-Adjusted Spreads for an average rated firm. Columns 7–9 report the outcomes of 3-by-3 calibration in the model with the variance risk premium (VRP) to the sample averages in columns 1–3. Instant volatility is set equal to mean variance (details are in Appendix E). Compared with Table 2, the yield spread in the model with VRP matches its empirical value as part of calibration process. Size, $v_0$, is the unlevered value of the firm's assets. Mean volatility is the square root of the mean asset variance, $\sqrt{\theta}$. Asset VRP, $|\lambda - \kappa|$, is the price of variance risk. Columns 5 and 6 report the statistics for optimal leverage with and without VRP as implied by the calibrated parameters. Optimal leverage without VRP is borrowed from Table 2. Columns 5 and 6 are comparable with the actual leverage in column 4. Empirical leverage is book liabilities divided by book liabilities plus equity market cap.

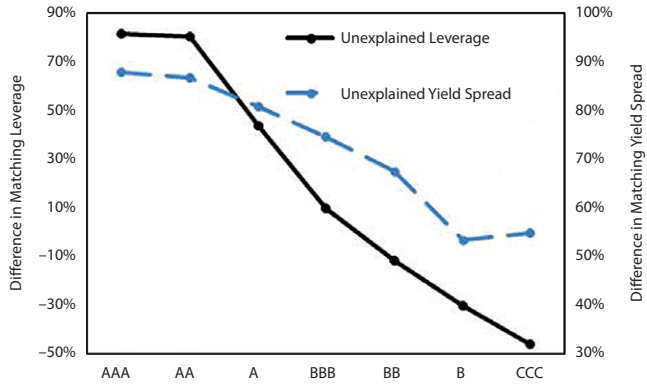
| | | | 1997–2015 Data Statistics | | | | Optimal Leverage | | Calibrated Values in the Model With VRP | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Equity Value | Equity Volatility | Yield Spread | Leverage | With VRP | Without VRP | Size | Mean Volatility | Asset VRP |
| Rating | No. of Obs. | Statistic | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| AAA | 113 | Mean | 183,896 | 26.5% | 0.83% | 24.8% | 36.5% | 45.0% | 228,050 | 21.3% | 2.96 |
| AA | 369 | Mean | 82,053 | 28.2% | 1.05% | 24.2% | 35.5% | 43.6% | 101,354 | 22.9% | 2.99 |
| A | 1,975 | Mean | 26,614 | 30.9% | 1.40% | 30.2% | 35.7% | 43.3% | 35,251 | 23.4% | 2.85 |
| BBB | 3,681 | Mean | 9,308 | 35.1% | 2.10% | 39.6% | 35.7% | 43.4% | 13,947 | 23.7% | 2.78 |
| Investment-grade (IG) firms | 6,138 | Mean | 22,464 | 33.1% | 1.61% | 35.3% | 36.1% | 43.3% | 31,784 | 23.5% | 2.66 |
| BB | 3,741 | Mean | 3,160 | 45.2% | 3.85% | 46.1% | 34.2% | 40.7% | 5,222 | 28.3% | 2.67 |
| B | 2,321 | Mean | 1,551 | 60.3% | 5.74% | 55.5% | 33.9% | 38.4% | 3,037 | 33.1% | 2.14 |
| CCC & below | 142 | Mean | 1,270 | 87.3% | 11.84% | 71.2% | 33.1% | 38.1% | 3,520 | 37.7% | 2.19 |

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FIGURE 5

Model Mismatch for Credit Spread and Leverage without the VRP (1997–2015)

Figure 5 shows the model error in predicting observed leverage and observed credit spreads across the risk groups after calibrating the model without the asset variance risk premium (VRP) to representative firms. This figure is based on Table 2. The *x*-axis shows the ratings. The *y*-axes show the relative difference in matching the observed leverage and observed yield spread in the data. The unexplained portion is (EMPIRICAL_MODEL_IMPLIED)/EMPIRICAL. "CCC" also includes ratings below CCC.




(in asset pricing) and the underleverage puzzle (in corporate finance) are isolated, and no notice is taken of any connection across ratings. This article connects these pieces in the literature through a deeper leverage analysis. Chen (2010) also links underleverage and the credit-premium puzzle by analyzing their relationship with common risk factors, but only in the time series at the macro level. However, this article determines the categories of the firms to which the puzzles are more relevant and shows the connection in the cross section of the firms. The unexplained leverage and credit spreads are larger only in the top ratings, which supports the idea that ignoring the VRP creates misspecification that is not limited to the credit premium. Moreover, I relax constant variance with priced variance risk. Next, I show that the asset VRP alleviates the mismatch in explaining the empirical trends.

## 2. Calibration with the VRP

Table 4 reports the calibration results. Compared with the model without the VRP, model calibration with the VRP keeps the same equations and parameters, but it has one extra equation and parameter: The asset VRP adjusts to match model-implied yield spread to empirical values (3-by-3 calibration: 3 equations, 3 unknowns).[11] For example, an average BB firm has an equity market cap of

[11]As a stylized finding, the structural models with constant variance do not produce large enough yield spreads (Eom et al. (2004), Huang and Zhou (2020)).“Once the structural models are calibrated … [model-implied] credit risk accounts for only a small fraction of observed yield spreads … (the credit spread puzzle)” (Huang and Huang (2012), p.155). Hence, there is no yield spread in the calibration for the model without the VRP because this model cannot match the observed yield spreads in the data. But the model with the VRP perfectly matches the yield spread. In Section III.C.1, when I drop yield spread from the calibrations and apply an alternative 2-by-2 calibration to both models with and without the VRP, the results do not change.

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$3,160 million, an equity volatility of 45%, and a yield spread of 3.9%. The 3-by-3 calibration with the VRP yields 2.7 VRP, $5,222 million asset value, and 8% mean variance to produce the earlier three empirical values in the model. In the calibration, all the other parameters match their empirical values (e.g., variance volatility is 12.8%, and the coupon rate is 3.6%). These calibrated values imply 34% optimal leverage. Appendix E has more calibration details. Similar to the IG firms in the calibrations without the VRP, IG firms have a relatively larger size and lower mean asset volatility in the model with the VRP. Thus, the model with the VRP also replicates the low historical risk and default probability of the top ratings.

Regarding the VRP, Ait-Sahalia and Kimmel (2007) report an RN mean-reversion speed of 5 on the VIX as the equity-market-variance index. In Table 4, the mean-reversion speed of asset variance is smaller than 5 in the RN measure for two reasons: idiosyncratic variance and no leverage. First, leverage inflates equity variance compared with asset variance (Merton (1974)). Ceteris paribus, high leverage increases the financial risk of a firm, which increases equity's variance almost linearly with leverage. Thus, the leverage of S&P 500 firms also inflates the mean-reversion speed of the equity-return variance in the VIX compared with the speed of asset variance. Second, individual firm-level speed is also deflated compared with the market's speed as a result of idiosyncratic variance, assuming that idiosyncratic variance does not time-vary as much as market variance (see equation (3)). Although the physical mean-reversion speed is constant for all the firms, the RN speed is smaller in IG firms because of the high VRP. Du et al. (2019) also report similar ranges for the VRP and mean-reversion speed.

First and foremost, Table 4 reports that optimal leverage is lower with the asset VRP (H1), especially for IG firms that tend to have the highest asset VRP. Contrary to the earlier low levels of historical variance and default probability for IG firms, the trend of the assets' VRP is reversed. A high VRP implies that IG firms have a relatively high RN variance and RN default probability, even with low historical variance and historical default probability. For IG firms, the comparison confirms that the model with the VRP significantly alleviates their underleverage compared with the model without the VRP. Hence, the following hypothesis is implied by the calibrations, which I verify statistically, in addition to H1, in the next section:

*Hypothesis 2 (H2).* The asset VRP reduces the IG firms' leverage more than the other firms' leverage, ceteris paribus.

It is interesting to notice how ignoring the VRP may mislead a researcher about leverage behavior: The historical asset variance and default risk for IG firms are lower than the average. Many theories of capital structure, as well as intuition, expect their low historical risk to create the potential for increasing debt in their capital structure. But this expected high optimal leverage conflicts with the low-leverage choice of IG firms. However, taking the VRP into account reconciles them: Some firms simply try to hedge their variance risk. Thus, model misspecification

(i.e., not considering the VRP) contributes to the seemingly paradoxical leverage choice of the firms, which appends to the other explanations.

## B. Regression Analysis

The regressions have the advantage of being model-free compared with the calibrations. In the linear regressions, the assets' VRP proxy has a negative effect on the leverage of the firms while there are controls for other factors, such as cash holdings and internal funds. In the same sample used for the calibrations, the ratio of equity's RN and historical volatility is the assets' VRP proxy. Equity RN volatility is the volatility implied by option prices. Because both RN and historical equity volatilities are inflated by leverage, the ratio cancels out the leverage effect. The regressions also have the classical factors used by earlier studies (e.g., Bae, Kang, and Wang (2011)): i) asset tangibility to control for bankruptcy costs, ii) profitability to control for pecking order, iii) cash holdings to control for financial flexibility, iv) natural logarithm of revenues to control for size, v) Tobin's Q to control for growth of the business, and vi) unlevered historical equity volatility to represent historical asset volatility (Appendix A gives more details about the calculation of the variables, such as the Compustat codes). Table 5 reports the descriptive statistics of these variables.

It is interesting to notice some anecdotal factors that imply that IG firms should have more debt (see Figure 6): IG firms have more tangible assets relative to their debt, which implies lower bankruptcy costs because tangible assets can be used as collateral for debt (Jensen and Meckling (1976)). They have less risk, have low historical asset variance[12] and asset beta, and they are larger in size; these factors reduce the chances of information asymmetry (Hennessy and Whited (2007)) or default (asset beta is unlevered equity beta). These trends in size and historical risk also appear in the calibrations. IG firms have a high payout rate, mostly to the shareholders, signaling less concern for financial flexibility, as suggested by DeAngelo, DeAngelo, and Whited (2011), and hold less cash for internal funding of their projects. Regardless of all these positive factors pushing them toward having higher leverage, they have the lowest leverage. In Figure 7, the only factor that sends a negative signal for low leverage is the asset VRP; the proxy for VRP and firm exposure to market variance risk for IG firms are the highest on average, which also anecdotally implies H2. Exposure to market variance risk is defined in Section III.C.1 with a more detailed analysis.

To verify these anecdotal findings, this article runs a regression that considers leverage adjustments by firms, following DeAngelo and Roll (2015), Banerjee, Dasgupta, and Kim (2008), and Fama and French (2002):

$$(13)\quad \mathrm{LEV}_{i,t} - \mathrm{LEV}_{i,t-j} = \Psi(\mathrm{TARGET}_{i,t} - \mathrm{LEV}_{i,t-1}),\quad \mathrm{TARGET}_{i,t} = \sum_k a_k X_{k,i,t},$$

[12]Only CCC and lower ratings have low unlevered volatility because of their extreme leverage: Their unlevered volatility is low because unlevered volatility is equity volatility $\times(1-\text{leverage})$, and this formula does not closely reflect their true asset volatility. Only calibrations address this issue (see their asset volatility in Table 2 and Figure 4).

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TABLE 5

## Descriptive Statistics for the Sample of Rated Firm-Years (1997–2015)

In Table 5, the sample is the same as in Table 4 from merged Compustat and Optionmetrics. Table A2 in Appendix A reports the details of the variable calculations. Tobin's Q is the market-to-book value of assets. Profitability is the operating income ratio. Cash is the ratio of cash holdings. Tangibility is the ratio of tangible assets and the proxy for bankruptcy costs. log(SALES) is the natural logarithm of the revenues. Asset payout is the total payout to debt and equity holders relative to size. Leverage is the total liabilities to the market cap of equity plus liabilities. Unlevered volatility is the historical volatility of stock returns unlevered with leverage to proxy for the assets' historical volatility. The variance risk premium (VRP) proxy represents the asset VRP and is the ratio of equity option-implied volatilities to historical volatilities.

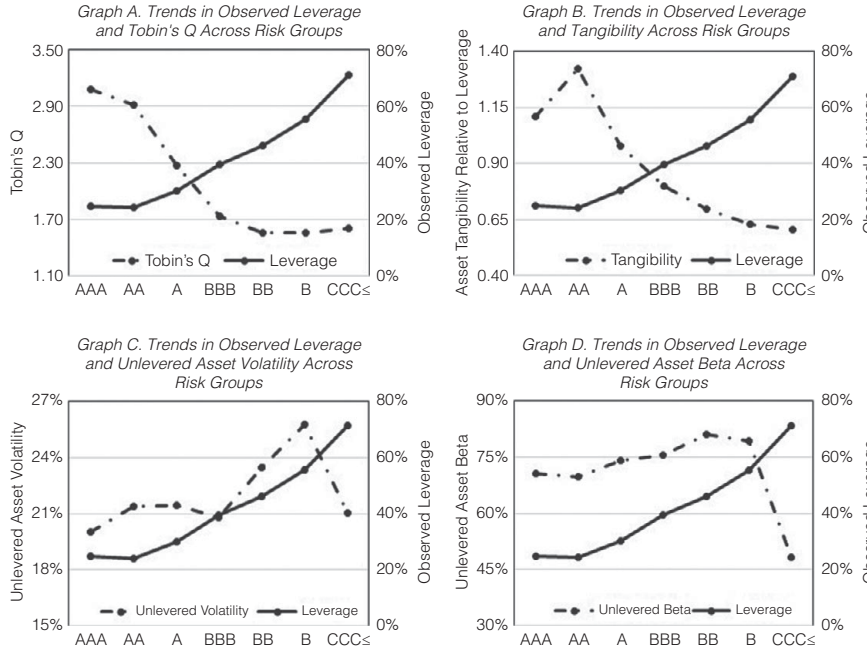
| Rating | Statistic | Leverage | Equity Value | Equity Volatility | Unlevered Volatility | VRP Proxy | Tobin's Q | Profitability | Cash | Tangibility | log(SALES) | Asset Payout |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AAA | Mean | 24.8% | 183,896 | 26.5% | 20.0% | 1.02 | 3.08 | 19.8% | 13.1% | 27.5% | 10.70 | 3.8% |
| 113 | Std. dev. | 16.5% | 127,046 | 10.1% | 9.2% | 0.19 | 1.86 | 8.3% | 11.8% | 21.2% | 1.23 | 1.8% |
| AA | Mean | 24.2% | 82,053 | 28.2% | 21.4% | 1.02 | 2.92 | 20.1% | 10.0% | 32.0% | 10.03 | 3.7% |
| 369 | Std. dev. | 12.7% | 85,452 | 10.5% | 8.7% | 0.20 | 1.72 | 6.6% | 10.6% | 16.7% | 1.20 | 2.0% |
| A | Mean | 30.2% | 26,614 | 30.9% | 21.5% | 1.02 | 2.27 | 17.9% | 9.7% | 29.5% | 9.09 | 3.8% |
| 1975 | Std. dev. | 14.2% | 35,312 | 12.4% | 10.1% | 0.17 | 1.23 | 7.1% | 10.9% | 19.2% | 1.17 | 2.4% |
| BBB | Mean | 39.6% | 9,308 | 35.1% | 20.8% | 1.02 | 1.73 | 14.8% | 8.6% | 31.5% | 8.49 | 3.9% |
| 3681 | Std. dev. | 16.3% | 14,043 | 14.5% | 10.2% | 0.19 | 1.01 | 7.1% | 9.7% | 23.3% | 1.12 | 3.5% |
| Investment-grade (IG) firms | Mean | 35.3% | 22,464 | 33.1% | 21.0% | 1.02 | 2.00 | 16.2% | 9.1% | 30.8% | 8.81 | 3.9% |
| 6138 | Std. dev. | 16.3% | 45,326 | 13.8% | 10.1% | 0.18 | 1.22 | 7.3% | 10.2% | 21.7% | 1.24 | 3.1% |
| BB | Mean | 46.1% | 3,160 | 45.2% | 23.5% | 1.01 | 1.56 | 13.1% | 9.5% | 32.0% | 7.56 | 3.5% |
| 3741 | Std. dev. | 18.4% | 5,436 | 20.4% | 12.8% | 0.21 | 0.94 | 8.9% | 10.9% | 25.1% | 1.05 | 3.1% |
| B | Mean | 55.5% | 1,551 | 60.3% | 25.8% | 1.03 | 1.55 | 8.9% | 12.9% | 34.9% | 6.88 | 3.8% |
| 2321 | Std. dev. | 21.8% | 3,094 | 27.9% | 18.1% | 0.31 | 1.20 | 11.0% | 15.7% | 25.5% | 1.30 | 3.5% |
| CCC and below | Mean | 71.2% | 1,270 | 87.3% | 21.1% | 0.92 | 1.61 | 0.7% | 12.5% | 43.1% | 6.74 | 4.6% |
| 142 | Std. dev. | 25.2% | 1,986 | 38.1% | 19.1% | 0.23 | 1.13 | 15.7% | 10.2% | 25.1% | 1.81 | 2.6% |
| Speculative-grade (SG) firms | Mean | 50.2% | 2,515 | 51.8% | 24.3% | 1.01 | 1.56 | 11.3% | 10.8% | 33.4% | 7.29 | 3.6% |
| 6204 | Std. dev. | 20.6% | 4,703 | 25.7% | 15.2% | 0.26 | 1.05 | 10.3% | 13.0% | 25.3% | 1.22 | 3.3% |
| All | Mean | 42.8% | 12,436 | 42.5% | 22.7% | 1.02 | 1.78 | 13.7% | 10.0% | 32.1% | 8.05 | 3.7% |
| 12342 | Std. dev. | 20.1% | 33,649 | 22.7% | 13.0% | 0.22 | 1.16 | 9.2% | 11.7% | 23.6% | 1.45 | 3.2% |

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FIGURE 6

Counterintuitive Trends of Classical Leverage Determinants and Leverage (1997–2015)

Figure 6 shows the trends in observed leverage and the leverage factors. Ratings are used because they categorize the observations based on historical risk. "CCC" also includes ratings below CCC. Variables are defined in Appendix A, and Table 5 reports the numbers. Graph C is comparable to Figure 4. The *x*-axis shows the ratings. The right *y*-axis shows observed leverage, and the left *y*-axis shows the factor. Intuitively, high Tobin's Q, relatively high asset tangibility, low historical asset volatility, and asset beta in these graphs imply expected higher leverage for investment-grade (IG) firms than for the others. But their reverse leverage trend implies that IG firms choose conservative leverage, which highlights them as the firms contributing to the underleverage puzzle.




where $\Psi$ is adjustment speed, TARGET is target leverage, LEV is leverage, $i$ is the firm index, $t$ is the time index, $X$ has the independent variables, and $k$ is the index for independent variables. With the assumption that $j$ is for 1 period and there is an intercept in the regression, the relation turns into the following:

$$\text{LEV}_{i,t} = (1-\Psi)\text{LEV}_{i,t-1} + a_0 + \sum_k \Psi a_k X_{k,i,t} + \epsilon_i + \epsilon_t + \epsilon, \quad (14)$$

where clustered-error controls are $\epsilon_i$, the firm, and $\epsilon_t$, time. There are several independent variables. The average leverage of the industry for each firm controls the industry effect, and there is no difference between having fixed industry dummies or the leverage average. Dummies for years and firms control the fixed effects. The other independent variables are the same as those listed in Table 5, without the asset payout rate. The regressions are on the whole sample and the subsamples of IG and SG firm-years. Factors are standardized to analyze the relative importance.

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FIGURE 7

The Trends of the Asset VRP Proxy and Exposure to Market Variance Risk across Ratings (1997–2015)

Figure 7 shows the trends in observed leverage and the asset variance risk premium (VRP). Ratings are used because they categorize the observations based on historical risk. "CCC" also includes ratings below CCC. Variables are defined in Appendix A. The *x*-axis shows the ratings. The right *y*-axis shows observed leverage, and the left *y*-axis shows the factor. A declining asset VRP proxy across ratings is the only factor (compared with Figure 6) that implies low leverage for investment-grade (IG) firms. This anecdotally supports the proposition that the asset VRP reduces leverage for IG firms (H2).

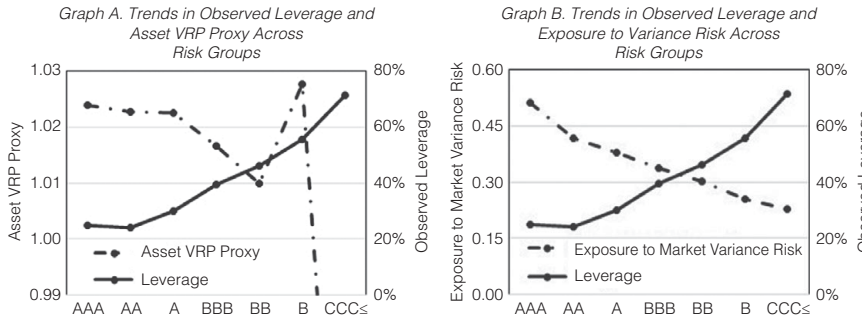



Table 6 reports the results.[13] As in Bae et al. (2011), $R^2$ is above 50% as a result of including fixed-effect dummies and lagged leverage. Assets' historical volatility has the largest effect on leverage. The overall strong effect of volatility is similar to the results of Chen, Wang, and Zhou (2015). One standard deviation of a firm-year from the average drops the target leverage by 6%, but historical volatility has a less strong effect in the IG subsample.

Conversely, 1 standard deviation of the assets' VRP from the average among IG firms has a stronger negative effect on leverage than in SG firms. The VRP coefficient in the IG sample is almost relatively twice that of the SG sample: The asset VRP coefficient is 50% of the historical volatility coefficient for the IG subsample, whereas it is 30% in the SG sample (3.1%/6.4% and 2.9%/8.6%). Similar results also appear in the subsample in the robustness check. Another noteworthy result is the relatively higher impact of the asset VRP than the profitability in the data. Titman and Wessels (1988) and Myers (1993) argue that profitability is a factor that is widely considered to have a first-order effect on leverage. If all the factors are ranked according to their standardized impact, the asset VRP stands second in line after asset volatility and above the other factors. In sum and similar to the calibrations, the asset VRP has a negative effect on leverage in the regressions (H1), and the negative effect is relatively stronger in IG firm-years (H2) than the average.

## C. Robustness Check

### 1. Calibration with Exposure to Market-Variance Risk

As an alternate measure of the asset VRP, I check the exposure of the firms' asset variances to market variance and, consequently, the market VRP. I show three

[13]None of the subsamples has zero-leverage observations, so the issues raised by Elsas and Florysiak (2015) and Lotfaliei (2018) do not apply to this article's analyses.

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TABLE 6

Regression Results (1997–2015)

Table 6 estimates $\text{LEV}_{i,t} = (1-\Psi)\text{LEV}_{i,t-1} + a_0 + \sum_k \Psi a_k X_{k,i,t}$, where $X$ has standardized independent variables and control dummies. Table A2 in Appendix A reports the details of the variable calculations. Descriptive statistics are in Table 5. The sample of investment-grade (IG) firms includes only the firm-quarters rated as IG by Standard & Poor's (S&P). The sample of speculative-grade (SG) firms has only the firm-quarters rated as non-IG or SG by S&P. Dummies for years and firms control for time and firm fixed effects. Average industry leverage controls for industry fixed effects. Standard errors (in parentheses) are corrected for clustered time and firm errors. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Coefficients for the standardized variables show the relative importance of each variable in determining target leverage. The asset variance risk premium (VRP) has a significantly negative effect on leverage (H1), especially for IG firms (H2).

| | Dependent Variable: Leverage | | | | | |
|---|---|---|---|---|---|---|
| | Parameter: Estimated Coefficients for Each Statistical Regression | | | | | |
| Sample | All | IG Firms | SG Firms | All | IG Firms | SG Firms |
| Model | 1 | 2 | 3 | 4 | 5 | 6 |
| Lag leverage | 0.438*** | 0.476*** | 0.346*** | 0.380*** | 0.380*** | 0.317*** |
| | (0.024) | (0.027) | (0.029) | (0.022) | (0.025) | (0.023) |
| Asset volatility | −0.0937*** | −0.0826*** | −0.102*** | −0.0790*** | −0.0645*** | −0.0861*** |
| | (0.006) | (0.009) | (0.006) | (0.005) | (0.007) | (0.005) |
| VRP proxy | −0.0358*** | −0.0370*** | −0.0336*** | −0.0310*** | −0.0306*** | −0.0289*** |
| | (0.005) | (0.006) | (0.006) | (0.005) | (0.005) | (0.006) |
| Tobin's Q | – | – | – | −0.0284*** | −0.0239*** | −0.0363*** |
| | – | – | – | (0.007) | (0.006) | (0.012) |
| Profitability | – | – | – | −0.0263*** | −0.0304*** | −0.0282*** |
| | – | – | – | (0.006) | (0.005) | (0.005) |
| Cash | – | – | – | −0.00821*** | −0.0118*** | −0.00648 |
| | – | – | – | (0.003) | (0.004) | (0.005) |
| Tangibility | – | – | – | 0.0105* | 0.00301 | 0.0175** |
| | – | – | – | (0.006) | (0.006) | (0.009) |
| log(SALES) | – | – | – | −0.00932 | 0.0137 | −0.00244 |
| | – | – | – | (0.007) | (0.010) | (0.008) |
| Intercept | 0.249*** | 0.248*** | 0.538*** | 0.221*** | 0.302*** | 0.489*** |
| | (0.012) | (0.013) | (0.022) | (0.014) | (0.019) | (0.033) |
| Industry average leverage | Yes | Yes | Yes | Yes | Yes | Yes |
| Time and firm fixed effects | Yes | Yes | Yes | Yes | Yes | Yes |
| Time and firm clustered errors | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 60% | 57% | 61% | 66% | 64% | 67% |
| BIC | −30,304.4 | −18,001.8 | −14,116.2 | −32,254.3 | −19,111.5 | −15,105.9 |
| No. of obs. | 12,342 | 6,138 | 6,204 | 12,342 | 6,138 | 6,204 |

outcomes. First, I confirm IG firms' high exposure to the market VRP, which explains their high asset VRP and its negative effect on their leverage. Second, I replicate the results of Du et al. (2019) and McQuade (2018) about improved credit spreads with the VRP to show that the use of this new method is reasonable. Finally, the calibration results are robust to a VRP estimation method that parsimoniously uses exposure to market-variance risk instead of the credit spread. The model with the VRP still shows a significantly closer leverage ratio to the observed leverage ratio.

For each firm-year between 1997 and 2015, I calculate the 90-day correlation between the squared Chicago Board Options Exchange (CBOE) VIX and the squared 30-day option-implied equity volatility. Because the VIX is based on 30-day options, I use the 30-day volatility for each firm rather than the 365-day volatility. The volatility is squared to represent variance. The average correlation across all the firm-years in each rating represents the exposure of an average firm in the rating to market variance and, subsequently, to the market VRP. Therefore, I use the correlation as a measure of exposure.

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FIGURE 8

Exposure of Variance to Market Variance Compared with Asset Beta (1997–2015)

In Figure 8, data are from 1997 to 2015 (see Table 5 for other sample statistics). The three columns represent the average correlation and unlevered beta for all investment-grade (IG) firm-years, all rated firm-years, and all speculative-grade (SG) firm-years. The exposure of variance to market variance is the 90-day average correlation between the squared Volatility Index (VIX) and the squared volatility of each firm-year. The Chicago Board Options Exchange (CBOE) reports and calculates the VIX from the 30-day option-implied Standard & Poor's (S&P) 500 volatility. For each firm-year, the correlation is for the past 90 days from the data date. The unlevered asset beta is the unlevered beta of equity for each firm-year. This figure shows relatively higher exposure of IG firms to market-variance shocks and relatively lower exposure to market-return shocks compared with SG firms.

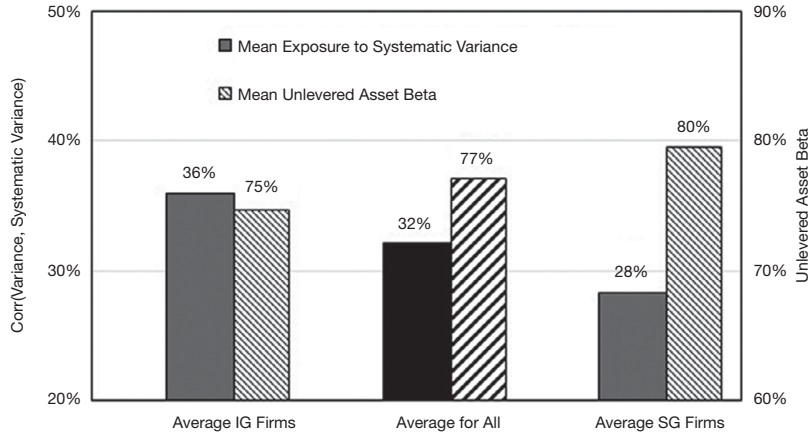



Figure 8 shows support for H2, where an average IG firm has a higher exposure to market variance than an average firm while having a lower asset beta. Compared with Figures 6 and 7, Figure 8 displays the averages across major categories rather than subcategories. The unlevered asset beta is the unlevered historical equity beta from the market returns in past years. The asset beta shows exposure to market return shocks, whereas correlation shows exposure to market-variance shocks. For example, across the extreme ratings, the IG subsample in Figure 8 has almost twice as much exposure than the SG subsample. Yet, SG firms have higher asset betas than IG firms. A similar trend in asset betas across ratings is also reported by Huang and Huang (2012), with a larger gap between IG and SG firms. Hence, the asset betas are more parsimonious with a lower gap in this article. If IG firms have higher exposure to market variance, then they are more likely to face a high asset VRP. This is different from the hedging asset beta, which is about shocks to market return. Indeed, IG firms have low asset betas, whereas their asset VRPs are high. These firms choose low leverage in this case to hedge the VRP and the risk of their asset variance being affected by a market-wide variance shock. Next, I verify this behavior through leverage calibration via exposure.

Table 7 reports the robust calibrations where the model with the VRP generates more realistic leverage ratios (Appendix E has the calibration details). Any error in measuring yield spread would only plague the calibrations in this model. Hence, I change the calibration method to check the robustness: For representative firms, both models with or without the VRP are only calibrated with 2-by-2 equations. In 2-by-2 equations, asset volatility and value are calibrated to equity volatility and

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TABLE 7

Calibration of Robustness-Check Results for Representative Firms in the Ratings (1997–2015)

Table 7 is comparable to Table 4. Compared with Table 4, the calibrations do not include the yield spread in the robustness check. This table only includes calibrations of asset volatility and asset value to equity volatility and equity value. Table 5 provides the statistics on firm-year data. Table A2 reports the details of the variable calculations. Columns 1–5 report the sample averages used as characteristics of the representative firm in each rating for calibration. Volatility is the square root of variance. The asset variance risk premium (VRP) exposure is estimated from the 90-day correlation between the squared Volatility Index (VIX) and the squared option-implied 30-day equity volatility. I choose 30-day volatility to match with the VIX horizon, which is also 30 days. The correlation is insensitive to leverage and does not require de-levering. Asset VRP, $|\lambda - \kappa|$, is the price of variance risk, which is calculated by multiplying exposure, $\alpha$, times market VRP. Market VRP is set to average 4, as reported by Ait-Sahalia and Kimmel (2007). Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap (equity value) is common shares times stock price. Columns 8–9 report the parameters in a 2-by-2 calibration with VRP, as in Appendix E, to the sample averages for equity market cap (equity value) and volatility without leverage. Size, $v_0$, is the unlevered value of the firm's assets. Mean volatility is the square root of the mean asset variance, $\sqrt{\theta}$. Instant volatility is equal to mean variance. Columns 6–7 report the statistics for the optimal leverages with and without VRP as implied by the calibrated parameters. Columns 6–7 are comparable with the actual leverage in column 5. Optimal leverage for the model without VRP is also implied by the parameters in a 2-by-2 calibration, as in Appendix E.

| | | | 1997–2015 Data Statistics | | | | | Optimal Leverage | | Calibrated Values in the Model with VRP | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Rating | No. of Obs. | Statistic | Asset VRP Exposure 1 | Asset VRP 2 | Equity Value 3 | Equity Volatility 4 | Leverage 5 | With VRP 6 | Without VRP 7 | Size 8 | Mean Volatility 9 |
| AAA | 113 | Mean | 51.02% | 2.04 | 183,896 | 26.5% | 24.8% | 39.5% | 45.0% | 227,885 | 21.3% |
| AA | 369 | Mean | 41.84% | 1.67 | 82,053 | 28.2% | 24.2% | 39.4% | 43.6% | 101,296 | 22.8% |
| A | 1,975 | Mean | 37.95% | 1.52 | 26,614 | 30.9% | 30.2% | 39.6% | 43.3% | 35,271 | 23.1% |
| BBB | 3,681 | Mean | 33.85% | 1.35 | 9,308 | 35.1% | 39.6% | 40.0% | 43.4% | 14,013 | 23.1% |
| Investment-grade (IG) firms | 6,138 | Mean | 35.96% | 1.44 | 22,464 | 33.1% | 35.3% | 39.7% | 43.3% | 31,836 | 23.2% |
| BB | 3,741 | Mean | 30.24% | 1.21 | 3,160 | 45.2% | 46.1% | 37.8% | 40.7% | 5,328 | 26.9% |
| B | 2,321 | Mean | 25.44% | 1.02 | 1,551 | 60.3% | 55.5% | 36.2% | 38.4% | 3,132 | 31.1% |
| CCC or below | 142 | Mean | 22.77% | 0.91 | 1,270 | 87.3% | 71.2% | 35.7% | 38.1% | 3,855 | 32.9% |



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value. The model with the VRP has two extra parameters: instant asset volatility and asset VRP. Instant volatility is set to mean asset volatility. The asset VRP is asset variance exposure, $\alpha$, times market VRP, as in equation (3). I multiply the exposure times 4, which is estimated by Ait-Sahalia and Kimmel (2007) as the market VRP. The asset VRP estimation is more parsimonious in this method and lower than the asset VRP in Table 4. Nevertheless, the results are similar to the earlier calibrations in Table 4. Not only do the results support the negative relationship between the asset VRP and leverage (H1), but also the model with the asset VRP still produces leverage closer to the observed leverage for IG firms relative to other firms (H2).

In order to validate this calibration method, I report the model-implied yield spreads in Table 8. If considering the asset VRP alleviates the underleverage puzzle (H1), and if there is a connection between credit and underleverage puzzles, as in Figure 5, then one expects also to see improvement in the credit puzzle using the asset VRP. There is no expectation to explain *all* the credit spreads as being due to other priced factors (e.g., liquidity). However, if the VRP intuition is valid, there is an expectation to improve the model-implied spreads, especially for IG firms. I use the parameters from Tables 7 and 2, such as the asset VRP, asset volatility, and value, to calculate model-implied yield spreads with and without the asset VRP. The calibration without the VRP shows the classical credit-premium puzzle, but the model with the VRP indeed produces yield spreads that are closer to the observed spreads. Moreover, the greatest improvement is in the IG category because of their high asset VRPs.

These results are similar to those of Du et al. (2019) and McQuade (2018). They report improved credit spreads with the asset VRP, but this article also connects the credit spreads to observed leverage. Regarding the exposure of firms' assets to the VRP, Dorion (2010) also reports that the ratio of systematic volatility for IG firms' assets is higher than for an average firm across ratings. Using a different method, he shows that the diversification gain in volatility is smaller by putting IG firms in a portfolio than with the others.

TABLE 8

Model-Implied Yield Spread Based on Calibration Results in Table 7 (1997–2015)

Table 8 reports the model-implied yield spread in comparison with the observed yield spread in the data based on the calibrated parameters in Table 7. Compared with Table 4 and for a robustness check, the calibrations do not use the yield spread. This table includes only calibrations of asset volatility and value to equity volatility and value. Appendix E gives more calibration details. Table A2 in Appendix A provides the details of the variable calculations. Column 1 reports the sample averages of the representative firm in each rating for comparison. Columns 2 and 3 report the model-implied yield spread with and without the asset variance risk premium (VRP) in the 2-by-2 calibration in Table 7. Column 2 replicates the classical credit-premium puzzle in which the model without VRP produces yield premiums much lower than their observed counterparts. Column 4 measures the improvement in explaining the yield spread by using the asset VRP. The greatest improvement is for investment-grade (IG) firms as a result of their high exposure to the market VRP (see also Figure 8).

| Rating | Observed Yield Spread | Yield Spread Without VRP | Yield Spread With VRP | VRP Contribution to Explaining Yield |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| AAA | 0.8% | 0.1% | 0.4% | 35% |
| AA | 1.1% | 0.1% | 0.4% | 24% |
| A | 1.4% | 0.3% | 0.6% | 23% |
| BBB | 2.1% | 0.5% | 1.0% | 20% |
| IG firms | 1.6% | 0.4% | 0.8% | 24% |
| BB | 3.9% | 1.3% | 1.9% | 17% |
| B | 5.7% | 2.7% | 3.7% | 17% |
| CCC or below | 11.8% | 5.3% | 7.0% | 14% |

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As explained in Section II.A, the differences in asset beta and the asset VRP are due to the impact of idiosyncratic volatility on the VRP. The magnitude of idiosyncratic volatility does not change asset beta, but it reduces the asset VRP. Idiosyncratic variance cushions the impact of the shocks of market variance from being transferred to the firms. When idiosyncratic variance is relatively low, the systematic portion of total variance is larger, and the effect of market-variance change is larger in IG firms. Because market variance has a premium, the premium results in a higher VRP on IG firms' assets. Hence, IG firms have higher VRPs than SG firms, although IG firms have lower asset betas. In sum, the firms, particularly IG firms, reduce leverage because, in addition to other factors, their assets have the VRP through the market VRP.

### 2. Calibration Using Firm-Year Data

In order to make sure that the results of the calibrations are not driven by the use of representative firms and Jensen's inequality, I run calibrations on firm-year data. It turns out that the earlier representative-firm calibrations are more parsimonious because underleverage in the model without the VRP is larger in firm-year calibrations. Calibrating to firm-year data has the advantages that statistical tests on the calibrations are possible, and it also delivers the time series of the estimated asset VRP. The disadvantage is a much smaller sample leaning toward IG firms. Nevertheless, the inferences for H1 and H2 are similar.

The data-collection process is similar to that in Section III.A, but I replace the data on representative-firm yield spreads with firm-years. The sample shrinks to the 2002–2015 period because of individual bond data availability from TRACE. Columns 1–4 of Table 9 report the statistics for all the firm-years in the sample. The sample leans more toward IG firms the availability of because bond data is better for these firms. As in Table 4, an average IG firm's equity is larger, and the equity volatility, yield spread, and financial leverage are lower.

The calibration results for all the firm-years are in Table 9, where the results are similar to those in Table 4. Calibration details are provided in Appendix E. For brevity, I report calibrated parameters only for the model with the VRP. The asset VRP is close to the numbers from the representative-firm calibration. The model with stochastic variance implies a flatter difference between historical instant and mean asset variance for IG firms. Therefore, if there exists a historical asset volatility term structure, it is flatter for IG firms than for the others. This means that the VRP proxy used in Graph A of Figure 7 for the regressions actually overestimates SG firms' asset VRP proxy and is more parsimonious. Because an SG firm's asset VRP is low and long-run historical volatility is more likely to be higher because of the steeper term structure, the ratio of long-run RN to historical volatility is more likely to be smaller than the same ratio based on 1-year values.

The model-implied leverages with and without the VRP are based on calibrated parameters. These results show that the calibrations to representative firms are more parsimonious because underleverage in the model without the VRP is more severe in the firm-year data. In other words, Jensen's inequality lowers

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TABLE 9

Sample Statistics and Calibration Results for All Rated Firm-Years (2002–2015)

Table 9 is comparable to Table 4. Columns 1–4 show the sample statistics for each rating. Volatility is the square root of variance. Yield spread is the difference between the corporate bond yield and the maturity-matching risk-free rate. Equity volatility is the standard deviation of stock returns for 365 days. Equity market cap is common shares times stock price. Leverage is the total book liabilities to the market cap of equity plus liabilities. Table A2 in Appendix A reports the details of the variable calculations. The optimal leverage for the model without the variance risk premium (VRP) is implied by the parameters in a 2-by-2 calibration, as in Appendix E. Columns 7–11 report the statistics for the calibrated parameters from 4-by-4 calibration with the VRP, as in Appendix E, and the error. Asset VRP, $|\lambda - \kappa|$, is the price of variance risk. Size, $v_0$, is the unlevered value of the firm's assets. Mean volatility is the square root of the mean asset variance, $\sqrt{\theta}$. Instant volatility is the square root of the instant asset variance, $\sqrt{V_0}$. MAPE is the mean average percentage error between the model-implied and observed values in the calibration. Columns 5 and 6 report the statistics for the optimal leverages with and without VRP as implied by the calibrated parameters. Columns 5 and 6 are comparable with the actual leverage in column 4.

| | | 2002–2015 Data Statistics | | | | Optimal Leverage | | Calibrated Values in the Model with VRP | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rating | Statistic | Equity Value 1 | Equity Volatility 2 | Yield Spread 3 | Leverage 4 | With VRP 5 | Without VRP 6 | Asset VRP 7 | Size 8 | Mean Volatility ($\sqrt{\theta}$) 9 | Instant Volatility ($\sqrt{V_0}$) 10 | MAPE 11 |
| AAA | Mean | 232,982 | 21.8% | 0.92% | 27.2% | 38.8% | 56.0% | 2.63 | 331,200 | 26.9% | 16.2% | 7% |
| 32 | Std. dev. | 78,960 | 10.8% | 0.50% | 19.3% | 7.3% | 11.6% | 0.49 | 236,242 | 7.1% | 8.4% | 3% |
| AA | Mean | 127,429 | 21.0% | 1.01% | 30.9% | 39.4% | 56.6% | 2.57 | 191,131 | 25.8% | 15.1% | 6% |
| 83 | Std. dev. | 85,303 | 8.4% | 0.64% | 14.6% | 6.6% | 9.7% | 0.44 | 172,741 | 8.1% | 6.4% | 2% |
| A | Mean | 37,169 | 26.0% | 1.51% | 33.0% | 37.7% | 52.1% | 2.55 | 48,209 | 27.4% | 18.2% | 6% |
| 365 | Std. dev. | 35,444 | 10.8% | 1.02% | 12.9% | 4.2% | 8.2% | 0.46 | 44,943 | 7.7% | 7.6% | 2% |
| BBB | Mean | 17,133 | 31.4% | 2.46% | 44.9% | 38.6% | 53.0% | 2.30 | 28,806 | 27.6% | 18.5% | 5% |
| 331 | Std. dev. | 23,291 | 13.3% | 1.59% | 14.7% | 5.9% | 9.3% | 0.67 | 41,950 | 7.9% | 7.9% | 2% |
| Investment-grade (IG) firms | Mean | 45,955 | 27.5% | 1.82% | 37.4% | 38.3% | 53.1% | 2.46 | 66,083 | 27.3% | 18.0% | 5% |
| 811 | Std. dev. | 64,785 | 12.2% | 1.37% | 15.5% | 5.4% | 9.1% | 0.57 | 108,960 | 7.8% | 7.7% | 2% |
| BB | Mean | 5,736 | 39.3% | 4.19% | 53.7% | 37.4% | 53.3% | 1.96 | 14,562 | 31.7% | 20.6% | 5% |
| 90 | Std. dev. | 7,361 | 15.3% | 2.07% | 17.6% | 4.5% | 11.8% | 0.79 | 33,129 | 8.5% | 10.4% | 2% |
| B | Mean | 4,943 | 63.0% | 9.50% | 71.9% | 38.5% | 57.0% | 1.17 | 23,331 | 34.0% | 22.6% | 4% |
| 51 | Std. dev. | 10,191 | 28.2% | 11.51% | 14.4% | 6.3% | 14.7% | 0.68 | 57,978 | 14.2% | 11.4% | 3% |
| CCC | Mean | 3,352 | 68.4% | 14.98% | 77.9% | 35.6% | 58.6% | 1.34 | 27,726 | 40.7% | 21.3% | 3% |
| 8 | Std. dev. | 1,793 | 21.5% | 10.65% | 11.2% | 2.1% | 11.8% | 0.44 | 52,060 | 12.8% | 8.8% | 2% |
| All | Mean | 39,651 | 30.8% | 2.56% | 41.1% | 38.2% | 53.4% | 2.33 | 58,662 | 28.2% | 18.5% | 5% |
| 960 | Std. dev. | 61,417 | 16.6% | 3.78% | 18.2% | 5.3% | 9.8% | 0.68 | 103,092 | 8.7% | 8.3% | 2% |

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model-implied leverage in the representative-firm calibration compared with calibrating to firm-years and then averaging. Because IG firms' historical asset variance is lower than that of the other firms and their size is larger, they have lower historical default risk. Without the asset VRP, it is counterintuitive when IG firms have relatively low historical default risk but also choose low leverage. However, calibrating with the VRP shows that these firms have a high VRP, which implies lower leverage.

Table 10 statistically compares the sample differences and thus controls for firm- and year-clustered errors using the method recommended by Petersen (2009). I estimate $\text{LEV}_{\text{DATA}} - \text{LEV}_{\text{MODEL}} = \alpha + \epsilon_i + \epsilon_t + \epsilon$, where $\alpha$ is the difference, and $\epsilon_i$ and $\epsilon_t$ control for clustered firm and time errors. Negative $\alpha$ implies underleverage where observed leverage is smaller than model-implied leverage, and vice versa. For SG firms, both the models with and without the VRP overestimate optimal leverage, which highlights these naturally overlevered firms, as discussed earlier. For IG firms, the model without the VRP significantly underestimates leverage, whereas the model with the VRP generates more reasonable results. Hence, ignoring the VRP exacerbates the seemingly paradoxical choice of these firms to have conservative leverage.

In addition to the cross-sectional analysis, this article reports the time-series behavior of an average firm's estimated asset VRP and compares it with leverage and the market's proxy for VRP. Figure 9 shows the time series. The asset VRP index is the size-weighted asset VRP from the calibrations. The leverage index is the size-weighted observed leverage of the firm-years in each year. The market VRP proxy is the ratio of the difference between markets' RN and historical volatility in relation to historical volatility. The market's RN volatility is the CBOE VIX, and historical volatility is for 30-day returns of S&P 500 firms. Both VIX and S&P 500 historical volatility are based on equity, which inflates them by the leverage of the S&P 500 firms. When the difference is divided by historical volatility, leverage cancels out, and the market VRP proxy is unlevered. The market VRP is based on the information from the options market. The market VRP looks more volatile than the asset VRP because the calculation of the asset VRP is based on a longer horizon than the market VRP proxy.

Even without including any information from the options market in this article's asset VRP index, the average asset VRP and market VRP proxy seem

TABLE 10

Differences between Model-Implied Optimal and Actual Leverage (2002–2015)

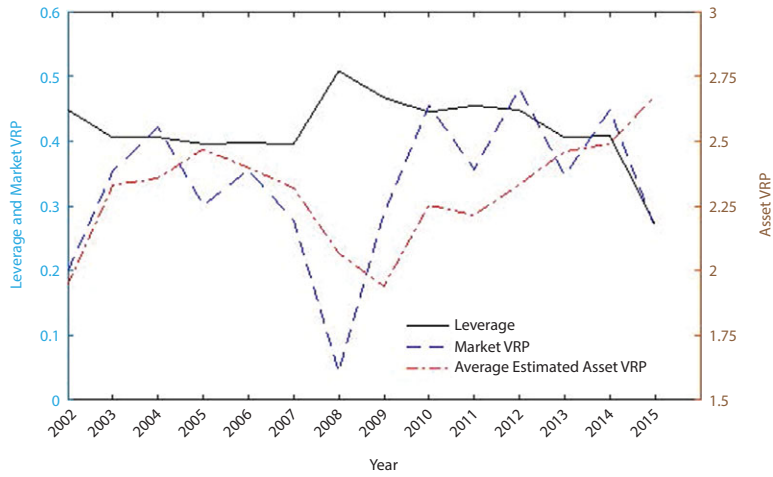
In Table 10, the differences are estimated with $\text{LEV}_{\text{DATA}} - \text{LEV}_{\text{MODEL}} = \alpha + \epsilon_i + \epsilon_t + \epsilon$, where $\alpha$ is the difference, and $\epsilon_i$ and $\epsilon_t$ control for clustered firm and time errors, respectively. Leverage statistics are available in Table 9. Actual leverage is from the data for each firm-year. Model-implied optimal leverage is the result of plugging the calibrated parameters into each model. The *p*-values test the null hypothesis that the coefficient is 0. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Negative $\alpha$ implies underleverage, and positive implies overleverage.

| Model | With VRP | Without VRP | With VRP | Without VRP | With VRP | Without VRP |
|---|---|---|---|---|---|---|
| Sample | IG Firms | IG Firms | All | All | SG Firms | SG Firms |
| $\text{LEV}_{\text{DATA}} - \text{LEV}_{\text{MODEL}}$ | −0.0091 | −0.16*** | 0.029 | −0.12*** | 0.24*** | 0.064** |
| *p*-value | (0.586) | (0.000) | (0.121) | (0.000) | (0.000) | (0.009) |
| Time and firm clustered errors | Yes | Yes | Yes | Yes | Yes | Yes |
| No. of obs. | 811 | 811 | 960 | 960 | 149 | 149 |

FIGURE 9

Time Series of the VRP and Leverage

In Figure 9, leverage and the asset variance risk premium (VRP) are, respectively, the asset-value-weighted averages of observed leverage and the calibrated asset VRP for all the firm-years in each year. The market VRP is the annual daily average for the difference between the Volatility Index (VIX) and the 30-day Standard & Poor's (S&P) 500 historical volatility divided by the historical S&P 500 volatility. This article's calibrated asset VRP index does not include any information content from the options market, whereas the VIX is based solely on option prices. This figure shows the positive correlation of the asset VRP index with the market VRP and the negative correlation of both with leverage in the time series.




to correlate. They also have a negative relationship with leverage. The negative relationship seems stronger during noncrisis times when the VRP indexes are high and imply high variance fears. The fear seems legitimate: During a crisis, the negative economy-wide shock drives historical variance up close to RN variance, which is evident in market VRP reduction. This increase validates the market-wide fear of increases in variance, which leads to conservative low leverage during noncrisis times.

## 3. Regression Analysis in Subsample

For the 2002–2015 period subsample, the empirical inferences from running model-free regressions also produce similar results in support of the model's implications: The VRP has a negative effect on the firms' leverage, especially for IG firms. For the 2002–2015 subsample, this article collects all the data following the same procedure as in the earlier section by merging Compustat and Optionmetrics, with similar filters.[14] Among these factors, only the asset VRP proxies and exposure to market-variance risk support IG firms' relatively low leverage.

[14]The descriptive statistics are provided in Appendix G of the Supplementary Material, and the classical factors, such as asset volatility and Tobin's Q, that determine leverage follow the same trends as in Figures 6 and 7. Although the sample seems small, Appendix H in the Supplementary Material reports similar regression results by estimation of a bias-corrected least-square dummy variable (LSDV) with bootstrapping to control for small sample size. Appendix H in the Supplementary Material also addresses Nickell's (1981) classical critique caused by including lagged leverage and fixed-effect dummies because it reports bias-corrected results using the method of Arellano and Bond (1991).

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Table 11 reports the results for the regressions. The asset VRP has a negative effect on leverage, as implied by the theory (H1), and it is the second factor in rank. The asset VRP is also relatively more important for IG firms (H2): The VRP coefficient in the IG sample is almost twice that of the SG sample. Another noteworthy result is the relatively higher impact of the asset VRP compared with profitability in the IG subsample. Profitability seems more important for SG firms, which are more likely to experience financial distress. In sum, both hypotheses are confirmed in model-free regressions of the subsample.

## IV. Conclusion

This article shows that the VRP at the asset level reduces leverage, especially for IG firms that seem to have greater exposure to market-variance risk. I present a theoretical model to support these hypotheses and verify them with empirical

TABLE 11

Regression Results (2002–2015)

Table 11 is comparable to Table 6 and has similar independent variables, but the sample is smaller than the sample in Table 6. It estimates $\text{LEV}_{i,t} = (1-\Psi)\text{LEV}_{i,t-1} + a_0 + \sum_k \Psi a_k X_{k,i,t}$, where $X$ has standardized independent variables and control dummies. Table A2 in Appendix A reports the details of the variable calculations. Independent-variable statistics are described in Table B.2 of Appendix G in the Supplementary Material. The sample of investment-grade (IG) firms includes only firm-quarters rated as IG by Standard & Poor's (S&P). The sample of speculative-grade (SG) firms includes only firm-quarters rated as non-IG or SG by S&P. Dummies for years and firms control for time and firm fixed effects. Average industry leverage controls for industry fixed effects. The coefficients of these control variables are omitted for brevity. Standard errors (in parentheses) are corrected for clustered time and firm errors. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The coefficients for the standardized variables show the relative importance of each variable in determining the target leverage. BIC is Bayesian information criterion.

| | Dependent Variable: Leverage | | | | | |
|---|---|---|---|---|---|---|
| | Parameter: Estimated Coefficients for Each Statistical Regression | | | | | |
| Sample | All | IG Firms | SG Firms | All | IG Firms | SG Firms |
| Model | 1 | 2 | 3 | 4 | 5 | 6 |
| Lag leverage | 0.468*** | 0.425*** | 0.436*** | 0.352*** | 0.253*** | 0.395*** |
| | (0.039) | (0.051) | (0.055) | (0.040) | (0.046) | (0.060) |
| Asset volatility | −0.0761*** | −0.0650*** | −0.0861*** | −0.0585*** | −0.0484*** | −0.0620*** |
| | (0.009) | (0.008) | (0.024) | (0.006) | (0.005) | (0.015) |
| Variance risk premium (VRP) proxy | −0.0405*** | −0.0375*** | −0.0342*** | −0.0321*** | −0.0297*** | −0.0181*** |
| | (0.006) | (0.006) | (0.013) | (0.005) | (0.005) | (0.005) |
| Tobin's Q | – | – | – | −0.0427*** | −0.0522*** | −0.108*** |
| | – | – | – | (0.009) | (0.009) | (0.030) |
| Profitability | – | – | – | −0.0300*** | −0.0216*** | −0.0246** |
| | – | – | – | (0.005) | (0.004) | (0.010) |
| Cash | – | – | – | −0.0066 | −0.00667 | −0.0119 |
| | – | – | – | (0.006) | (0.007) | (0.018) |
| Tangibility | – | – | – | 0.0340** | 0.0173 | 0.0746* |
| | – | – | – | (0.017) | (0.021) | (0.040) |
| log(SALES) | – | – | – | 0.0331 | 0.0108 | −0.0118 |
| | – | – | – | (0.025) | (0.016) | (0.041) |
| Intercept | 0.192*** | 0.351*** | 0.446*** | 0.229*** | 0.490*** | 0.441*** |
| | (0.027) | (0.042) | (0.051) | (0.032) | (0.053) | (0.116) |
| Industry average leverage | Yes | Yes | Yes | Yes | Yes | yes |
| Time and firm fixed effects | Yes | Yes | Yes | Yes | Yes | Yes |
| Time and firm clustered errors | Yes | Yes | Yes | Yes | Yes | Yes |
| $R^2$ | 64% | 61% | 71% | 74% | 74% | 84% |
| BIC | −2,997.9 | −2,783.3 | −366.3 | −3,294.5 | −3,106.9 | −455.1 |
| No. of obs. | 959 | 810 | 149 | 959 | 810 | 149 |

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tests. Risky time variations in asset variance produce the VRP that increases the wedge between historical and RN variance. RN asset variance raises the RN probability of default for the firm, which increases the costs of debt and reduces the tax savings. Hence, a firm with low historical variance and a high variance premium chooses conservative leverage. This behavior hedges future increases in asset variance induced by exposure to market-variance shocks. Empirically, the model calibrations with the VRP produce lower leverage that is closer to the actual leverage of the firms than the model that excludes the VRP. In the time series, the asset VRP is high after a crisis, which reflects the market's apprehension regarding a possible increase in variance during the crisis. This apprehension leads to lower leverage. In the cross section of firms ranked by their historical risk, the IG firms hold assets with high exposure to market-variance risk, which also seems to explain their conservative leverage. The regressions also validate a negative VRP–leverage relation, with a more negative correlation in the IG subsample.

The VRP effect cannot be detected in the earlier models with constant variance because they assume that historical and RN variances are equal. This is contrary to the stylized fact in other finance areas that variance is stochastic with priced risk. Focusing only on the historical variance of the firms' assets creates a model misspecification: Without the asset VRP, IG firms seem to have the potential to increase their leverage, considering their low historical risk of default, which remains underutilized. The misspecification not only is tractable in underleverage but also links underleverage to other seemingly odd observations, such as the unexpectedly high credit premium identified by Eom et al. (2004) and Huang and Huang (2012). Hence, models with the asset VRP not only explain long-run bond spreads (Du et al. (2019), but these models also improve the theory for the leverage choice of the firms.

A potential extension of this article for future research is to explain the low speed of leverage adjustments reported by Frank and Goyal (2007). Asset variance is an important factor in the speed of leverage adjustments modeled by Goldstein, Ju, and Leland (2001); high variance implies fewer readjustments. All these models assume constant variance. But the VRP may be a factor in the puzzling infrequent leverage adjustments and alleviate the debate on the structural models (Welch (2013), Strebulaev and Whited (2013)). Another potential subject for future research is to examine the underlying causes for high exposure to market variance risk in IG firms' assets; IG firms are large firms and may have high operating leverage or a well-diversified portfolio of assets with low idiosyncratic variance.

## Appendix A. Table of Variables

Table A1 presents the theoretical parameters, and Table A2 presents the definition of the empirical parameters.

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TABLE A1

Variables in the Model

| Symbol | Description | Symbol | Description |
|---|---|---|---|
| $v$ | Unlevered asset value | $\sqrt{V}$ | Volatility |
| $V$ | Variance of unlevered asset returns | $T$ | Period length for following optimal default policy |
| $\lambda$ | Risk-neutral (RN) speed of mean reversion | $\hat{V}$ | Future variance |
| $\kappa$ | Mean-reversion speed under physical measure | $I_{\tau<T}$ | Equal to 1 if default happens prior to time $T$ |
| $\lambda-\kappa$ | Variance risk premium (VRP) | $\delta$ | Asset-payout rate |
| $r$ | Risk-free rate | $\sigma$ | Variance volatility |
| $\theta$ | Mean variance (long-run or long-term variance) | $y$ | Debt yield |
| $\theta^*$ | RN mean variance | YSP | Credit spread, yield spread, or credit premium |
| TAX | Tax rate | $C$ | Debt's coupon (interest) payment |
| $\mu$ | Return drift under physical measure $\mathbb{P}$ | $P$ | Outstanding face value of the firm's debt |
| $D$ | Debt value | EQ | Equity value |
| $L^*$ | Optimal default boundary | $\rho$ | Proportional bankruptcy cost (PBC) rate |
| $d$ | New debt's value | $c$ | Newly issued debt's coupon |
| $p$ | Newly issued debt's face value | $M$ | Average debt maturity ( $=1/m$) |
| $\alpha$ | Asset variance exposure to market VRP | $\sqrt{b}$ | Asset beta |
| $W_i^P$ | Independent Brownian motions under physical measure $\mathbb{P}$, i={1,2} | $W_i$ | Independent Brownian motions under RN measure $\mathbb{Q}$, i={1,2} |

TABLE A2

List of Empirical Variables with Compustat Codes for the Calculation of Each Variable

| Variable | Description | | |
|---|---|---|---|
| Equity market cap | Total common shares times stock price (CSHO $\times$ PRCC_F) | | |
| Leverage | The total book value of liabilities to the market cap of equity plus the book liabilities (LEV = LT/(LT + CSHO $\times$ PRCC_F)) | | |
| Historical equity volatility | Historical standard deviation of equity returns for 365 days from Optionmetrics (RV) | | |
| Historical equity beta | Historical covariance of equity returns and Center for Research in Security Prices (CRSP) value-weighted index divided by its variance, with minimum 60 days and maximum 250 days of returns ( $\beta_{\text{EQUITY}}$) | | |
| Option-implied equity volatility | Daily average for 365-day option-implied equity standard deviation across all the strike prices in volatility surface from Optionmetrics (IV) | | |
| Asset variance risk premium (VRP) proxy | The ratio of the historical annual average of option-implied equity volatility to the historical equity volatility based on the same period ( $\text{MEAN}_{365\text{DAY}}(\text{IV})/\text{RV}$) | | |
| Market VRP proxy | The ratio of the difference between the Chicago Board Options Exchange (CBOE) Volatility Index (VIX) and the historical 30-day Standard & Poor's (S&P) volatility to the historical volatility ((VIX $-$ VOL)/VOL) | | |
| Exposure to market variance | 90-day correlation between squared option-implied 30-day volatility and squared VIX ( $\text{CORR}_{90\text{DAY}}(\text{IV}^2_{30\text{DAY}}, \text{VIX}^2)$ ) | | |
| Asset volatility proxy | Unlevered historical equity volatility $((1-\text{LEV})\times\text{RV})$ | | |
| Asset beta proxy | Unlevered historical equity beta $((1-\text{LEV})\times\beta_{\text{EQUITY}})$ | | |
| Corporate yield spread | Credit spread; yield of longest-maturity bond less the maturity-matching risk-free rate | | |
| Asset payout | Payout to shareholders (dividends and share repurchases) plus interest payments divided by assets ((DVPSP_F $\times$ CSHO + PRSTKC + XINT)/(LT + EQUITY)) | | |
| Variance volatility | The standard deviation of unlevered equity volatility for all the firm-years in the rating | | |
| Tobin's Q | Equity market cap; liabilities and preferred shares less taxes divided by book assets ((CSHO $\times$ PRCC_F + LT + PSTKL $-$ TXDITC)/AT) | | |
| Tangibility | Proxy for bankruptcy costs; the ratio of the property, plant, and equipment to assets (PPENT/AT) | | |
| Profitability | Operating income divided by book assets (OIBDP/AT) | Cash | The ratio of cash and equivalents to book assets (CHE/AT) |
| log(SALES) | The natural logarithm of the revenues (log(SALE)) | Risk-free rate | U.S. Treasury notes rates |
| Coupon rate, $C$ | Continuously compounded coupon rate ($\ln(1+\text{XINT/LT})$) | Outstanding debt, $P$ | Total liabilities (LT) |

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## Appendix B. Change of Variable

Let's define $\zeta = ln(v/L)$. By Ito's lemma, it follows that

$$d\zeta = (r-\delta-\frac{1}{2}V)dt+\sqrt{V}dW. \tag{15}$$

Integration over equation (15) results in the simple solution of the differential equation:

$$\zeta_T-\zeta_0 = (r-\delta)T-\frac{1}{2}\int_0^T Vdt+\int_0^T\sqrt{V}dW. \tag{16}$$

Defining $\hat{V} = (\int_0^T V_s ds)/T$ transforms equation (16) into

$$\zeta_T-\zeta_0 = (r-\delta-\frac{1}{2}\hat{V})T+\int_0^T\sqrt{V}dW. \tag{17}$$

The variable $\zeta_T$ follows a Brownian motion with mean $\zeta_0+(r-\delta-\frac{1}{2}\hat{V})T$. The variance for $\zeta_T$ is $\int_0^T Vdt$, which is also equal to $\hat{V}T$. Therefore, the differential of the process in equation (17) is

$$d\zeta_T = (r-\delta-\frac{1}{2}\hat{V})dt+\sqrt{\hat{V}}dW. \tag{18}$$

## Appendix C. Derivation of Security Formulas

### 1. Debt Formula

Because debt value will be conditioned by variance, I first analyze the debt value when the variance is constant in equation (4):

$$d(v_0) = E^{\mathbb{Q}}\left(\frac{c+mp}{m+r}+(1-I_{\tau<T})\left[e^{-rT}\left(D(v_T)-\frac{c+mp}{m+r}\right)\right]\right. \tag{19}$$
$$\left.+I_{\tau<T}\left[e^{-r\tau}\left((1-\rho)mL-\frac{c+mp}{m+r}\right)\right]\right).$$

With constant variance, debt value satisfies the following partial differential equation (PDE) and Dirichlet conditions; this is similar to the Black–Scholes PDE for contingent claim pricing:

$$\begin{cases}\frac{1}{2}Vv^2d_{vv}+(r-\delta)vd_v-(r+m)d+(C+mP) = 0,\\ d(L) = (1-\rho)mL, \quad d(v\to+\infty) = \frac{c+mp}{m+r}.\end{cases} \tag{20}$$

With constant variance, the default boundary is also constant, and the debt value has a closed-form solution:

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$$d = \frac{c+mp}{m+r} + e^{-H_b\zeta_0}\left((1-\rho)mL - \frac{c+mp}{m+r}\right) \tag{21}$$

$$h = \frac{r-\delta-\frac{1}{2}\hat{V}}{\sqrt{\hat{V}}} \qquad H_b = \frac{\sqrt{h^2+2(r+m)}+h}{\sqrt{\hat{V}}},$$

where future variance, $\hat{V}$, is equal to variance, $V$, because variance is constant.

I use conditioning on future variance. In equation (4), new state variables $\zeta$ and $\hat{V}$ replace $v$ and $V$:

$$d(\zeta_0,\hat{V}_0) = E^{\mathbb{Q}}\left[E\left(\frac{c+mp}{m+r} + (1-I_{\tau<T})\left[e^{-rT}\left(d(\zeta_T,\hat{V}_T) - \frac{c+mp}{m+r}\right)\right]\right.\right. \tag{22}$$
$$\left.\left. + I_{\tau<T}\left[e^{-r\tau}\left((1-\rho)mL - \frac{c+mp}{m+r}\right)\right] | \hat{V}\right)\right].$$

Note that $d(\zeta_0,\hat{V}_0)$ does not have the same form as $d(v_0,V_0)$ because it is not a linear transformation. Conditioning on $\hat{V}$ makes the expression within the parentheses similar to equation (19), which has a solution in the form of equation (21). Variance is constant from the conditioning, and the default boundary is decided and fixed at the start of the period:

$$d = \frac{c+mp}{m+r} + \left((1-\rho)\,mL - \frac{c+mp}{m+r}\right)E(e^{-H_b\zeta_0}). \tag{23}$$

The expectation term is presented in Appendix D. A similar method generates formulas for tax benefits, bankruptcy costs, and equity.

## 2. Tax Savings and Bankruptcy Cost Formulas

As with debt, the derivation uses conditioning on variance. First, I show the valuation when variance is constant. With constant variance, any claim, $F$, making continuous payments $C$ satisfies the following PDE, and the solution depends on Dirichlet conditions:

$$\frac{1}{2}Vv^2F_{vv} + (r-\delta)vF_v - rF + C = 0. \tag{24}$$

With constant variance, the default boundary is also constant. The tax savings formula is as follows:

$$\mathrm{TB}(L) = 0, \quad \mathrm{TB}(v\to+\infty) = \mathrm{TAX}\times C/r \tag{25}$$

$$\mathrm{TB}(\zeta_0,\hat{V}_0) = E\left(\mathrm{TAX}\times\frac{C}{r} + (1-I_{\tau<T})\left[e^{-rT}\left(\mathrm{TB}(\zeta_T,\hat{V}_T) - \mathrm{TAX}\times\frac{C}{r}\right)\right]\right.$$
$$\left. + I_{\tau<T}\left[e^{-r\tau}(0 - \mathrm{TAX}\times\frac{C}{r})\right]\right)$$
$$= \mathrm{TAX}\times\frac{C}{r} - \mathrm{TAX}\times\frac{C}{r}e^{-H\times\zeta_0}.$$

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For bankruptcy costs, the formula is as follows:

$$\text{(26)}\qquad \begin{aligned} &\mathrm{BC}(L)=\rho L,\ \ \mathrm{BC}(v\to+\infty)=0\\ &\mathrm{BC}(\zeta_0,\hat{V}_0)=E\big((1-I_{\tau<T})[e^{-rT}\mathrm{BC}(\zeta_T,\hat{V}_T)]+I_{\tau<T}[e^{-r\tau}\rho L]\big)\\ &\qquad\qquad=\rho L e^{-H\times\zeta_0}. \end{aligned}$$

A similar method of conditioning on future variance as in debt valuation (see equation (22)) generates formulas for tax benefits and bankruptcy costs:

$$\text{(27)}\qquad \mathrm{TB}(\zeta_0,\hat{V}_0)=\mathrm{TAX}\times\frac{C}{r}-\mathrm{TAX}\times\frac{C}{r}E\big(e^{-H\zeta_0}\big),$$

$$\text{(28)}\qquad \mathrm{BC}(\zeta_0,\hat{V}_0)=E\big(e^{-H\zeta_0}\big)\rho L.$$

The expectation term is derived in Appendix D.

## Appendix D. Discounted RN Default Probability and the Moment-Generating Function

### 1. Taylor Expansion

The following derivations apply to the expectation terms in equations (6), (8), and (9). Because the debt rolls over with rate $m$, the expectation term in equation (6) has $m$ as an extra parameter compared with the other two. Regardless of the form, Taylor expansion of any function within the expectation term up to the second degree is as follows:

$$\text{(29)}\qquad \begin{aligned} E\Big\{F(\zeta_0,\hat{V}_0)\Big\}\simeq\ & E\Big\{F\big(\zeta_0,E[\hat{V}_0]\big)+F'\big(\zeta_0,E[\hat{V}_0]\big)\big(\hat{V}_0-E[\hat{V}_0]\big)\\ &+\frac{1}{2}F''\big(\zeta_0,E[\hat{V}_0]\big)\big(\hat{V}_0-E[\hat{V}_0]\big)^2\Big\}\\ =\ & F\big(\zeta_0,E[\hat{V}_0]\big)+\frac{1}{2}F''\big(\zeta_0,E[\hat{V}_0]\big)\times E\Big[\big(\hat{V}_0-E[\hat{V}_0]\big)^2\Big], \end{aligned}$$

where $F(\zeta_0,\hat{V}_0)=\mathrm{EXP}(-\zeta_0 H_b)$ is in equation (6), and $F(\zeta_0,\hat{V}_0)=\mathrm{EXP}(-\zeta_0 H)$ is in equations (8) and (9). I show only the derivation for $F(\zeta_0,\hat{V}_0)=\mathrm{EXP}(-\zeta_0 H)$. The other derivations are similar by replacing $H$ with $H_b$. Because the function is exponential, deriving its higher-order derivatives is straightforward:

$$\text{(30)}\qquad \begin{aligned} &E\Big\{\mathrm{EXP}(-\zeta_0 H)\Big\}\simeq\mathrm{EXP}\big(-\zeta_0\hat{H}\big)\Big[1-\frac{1}{2}(A\zeta_0-B\zeta_0^2)\Big]\\ &A=\widehat{H''}E\Big[\big(\hat{V}_0-E[\hat{V}_0]\big)^2\Big],\quad B=\hat{H}'^2E\Big[\big(\hat{V}_0-E[\hat{V}_0]\big)^2\Big]\\ &\hat{H}=H|_{\hat{V}_0=E[\hat{V}_0]},\ \ \widehat{H'}=\frac{\partial H}{\partial\hat{V}_0}|_{\hat{V}_0=E[\hat{V}_0]},\ \ \widehat{H''}=\frac{\partial^2 H}{\partial\hat{V}_0^2}|_{\hat{V}_0=E[\hat{V}_0]}, \end{aligned}$$

where $E[\hat{V}_0]$ and $E[(\hat{V}_0-E[\hat{V}_0])^2]$ are derived next in equation (34).

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### 2. Future Variance Moments

To calculate the moments of future variance, this article uses the moment-generating function (MGF) derived from the Feynman–Kac formula and the method recommended by Tahani (2005). The MGF is as follows:

$$\text{MGF}(\hat{V},x) = E[\text{EXP}(x\hat{V})] = F(V_0,x). \tag{31}$$

The expected value follows the Feynman–Kac PDE as a function of variance, $V$, under $\mathbb{Q}$:

$$\frac{\partial F}{\partial t} + (\kappa\theta - \lambda V)\frac{\partial F}{\partial V} + \frac{1}{2}\sigma^2 V\frac{\partial F}{\partial V} + xVF = 0. \tag{32}$$

The solution is exponential linear in $V$. Hence, the MGF for $\hat{V}$ is log-linear in V:

$$\begin{aligned} &\text{MGF}(\hat{V},x) = \text{EXP}(V_0 M(T,x) + N(T,x)) \\ &M(T,x) = \frac{2x(1-e^{-\omega T})}{T(\lambda+\omega)(1+\phi e^{-\omega T})} \\ &N(T,x) = \frac{-2\kappa\theta}{\sigma^2}\left[log\left(\frac{1+\phi e^{-\omega T}}{1+\phi}\right) - \frac{(\omega-\lambda)T}{2}\right] \\ &\omega = \sqrt{\lambda^2 - \frac{2x\sigma^2}{T}} \;\; ; \;\; \phi = \frac{\omega-\lambda}{\omega+\lambda}. \end{aligned} \tag{33}$$

From the MGF, the first and second moments are calculated as $\text{MGF}'|_{x=0}$ and $\text{MGF}''|_{x=0}$:

$$\begin{aligned} &E(\hat{V}_0) = V_0 M_1 + N_1, \quad E\left([\hat{V}_0 - E(\hat{V}_0)]^2\right) = V_0 M_2 + N_2 \\ &M_1 = \frac{1-\text{EXP}(-T\lambda)}{T\lambda}, \quad M_2 = \frac{\sigma^2}{T^2\lambda^3}\left[1 - 2T\lambda\text{EXP}(-T\lambda) - \text{EXP}(-2T\lambda)\right] \\ &N_1 = \frac{\kappa\theta}{\lambda}\left[1 - M_1\right], \qquad N_2 = \frac{\kappa\theta\sigma^2}{T\lambda^3}\left[1 + \text{EXP}(-T\lambda) - 2M_1\right] - \frac{\kappa\theta}{2\lambda}M_2. \end{aligned} \tag{34}$$

## Appendix E. Calibration Procedure

Applying Ito's lemma to equation (11) generates the model-implied equity process:

$$\begin{aligned} &\frac{dEq}{\text{EQ}} = \mu_{\text{EQ}}dt + a_1 dW_1^p + b_1 dW_2^p \\ &a_1 = \frac{\partial \text{EQ}}{\partial v_0}\frac{v_0}{\text{EQ}}\sqrt{V_0}, \quad b_1 = \frac{\partial \text{EQ}}{\partial V_0}\frac{\sigma}{\text{EQ}}\sqrt{V_0}, \end{aligned} \tag{35}$$

where $\mu_{\text{EQ}}$ is the drift for the equity-return process. The variance of the equity return, $\sigma_r^{2\text{EQ}}$, is as follows:

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$$
\begin{aligned}
\sigma_r^{2\mathrm{EQ}} &= a_1^2 + b_1^2 \\
d\sigma_r^{2\mathrm{EQ}} &= \mu_r^{\mathrm{EQ}} dt + a_2 dW_1^p + b_2 dW_2^p \\
a_2 &= \frac{\partial \sigma_r^{2\mathrm{EQ}}}{\partial v_0} v_0 \sqrt{V_0}, \quad b_2 = \frac{\partial \sigma_r^{2\mathrm{EQ}}}{\partial V_0} \sigma \sqrt{V_0}.
\end{aligned} \tag{36}
$$

Empirical calibrations use equity value and equity variance.

## 1. Firm-Year Calibration

I collect corporate bond characteristics, prices, and yield from Mergent and TRACE. Bond data are for the longest-maturity senior unsecured corporate bonds without features such as being callable.[15] The yield spread is the bond yield minus the maturity-matching risk-free rate. Missing yield data for data dates are filled by linear interpolation of daily data.

For the model with the VRP, the calibration uses a 4-by-4 equation: The i) the instant variance, $V_0$; ii) long-run variance mean, $\theta$; iii) unlevered asset value, $v_0$; and iv) VRP, $|\lambda - \kappa|$, are set so that the model-implied values match the i) equity value, EQ; ii) historical volatility of equity return, $\sqrt{\sigma_r^{2\mathrm{EQ}}}$; iii) corporate yield spread, YSP; and iv) leverage of the firm, LEV, for each firm-year. In the model without the VRP, the stochastic variance parameters, VRP and $\theta$, do not exist. The process without the VRP follows the standard in the literature, and the calibration collapses into solving a 2-by-2 equation: Unlevered asset value, $v$, and variance, $V$, are calibrated to make the model-implied values match the leverage ratio and historical equity volatility:

$$
\begin{aligned}
&\text{With VRP} \begin{cases} \mathrm{EQ}(V_0, \theta, v_0, \lambda - \kappa) = \mathrm{EQ}_{\mathrm{DATA}} \\ \sqrt{\sigma_r^{2\mathrm{EQ}}(V_0, \theta, v_0, \lambda - \kappa)} = \sigma_{\mathrm{DATA}}^{\mathrm{EQ}} \\ \mathrm{YSP}(V_0, \theta, v_0, \lambda - \kappa) = \mathrm{YSP}_{\mathrm{DATA}} \\ \mathrm{LEV}(V_0, \theta, v_0, \lambda - \kappa) = \mathrm{LEV}_{\mathrm{DATA}} \end{cases}, \\
&\text{Without VRP} \begin{cases} \sqrt{\sigma_r^{2\mathrm{EQ}}(V_0, v_0)} = \sigma_{\mathrm{DATA}}^{\mathrm{EQ}} \\ \mathrm{LEV}(V_0, v_0) = \mathrm{LEV}_{\mathrm{DATA}} \end{cases},
\end{aligned} \tag{37}
$$

where $\mathrm{EQ}_{\mathrm{DATA}}$ is the equity market cap, $\sigma_{\mathrm{DATA}}^{\mathrm{EQ}}$ is the historical 365-day equity volatility, $\mathrm{YSP}_{\mathrm{DATA}}$ is the yield spread for the longest bond maturity, and $\mathrm{LEV}_{\mathrm{DATA}}$ is the leverage of the firm-year in the data.

With the VRP, the model-implied equity value and variance are as in equations (11) and (36). The model-implied market leverage is $D/(D+\mathrm{EQ})$,

[15] I use the longest bond maturity because the model does not have the term structure of yields spread and optimal debt maturity. Adding these features would make the model intractable, and it is already complex and requires approximation to solve.

where $D = d/m$, and $d$ is as in equation (7). The model-implied yield spread is calculated using the following equation:

$$\text{YSP} = \frac{C_s}{D(C_s)} - r, \quad C_s : P = D(C_s). \tag{38}$$

In sum, for each firm-year, asset value and volatility are calibrated in the model without the VRP so that the equity value and volatility match the empirical data (2-by-2 calibration). Calibration with the VRP is 4-by-4; it involves two extra equations (to match the empirical yield spread and empirical leverage) and two extra parameters (instant asset volatility and asset VRP).[16] Specifically, the model with the asset VRP implies yield spreads that fit their empirical twins as part of the calibration similar to that in Table 4.

To solve the equations, the calibration process minimizes the mean absolute percentage error (MAPE). Without the VRP, the model-implied value for equity is available by plugging equations (21), (25), and (26) from Appendix C into equation (11). From the equity value, equity volatility is easy to derive in a way similar to that in equation (36). Debt from equation (21) divided by equity plus debt also results in model-implied leverage. There is no yield spread in the model without the VRP because this model cannot match the observed yield spreads in the data as a stylized fact (Eom et al. (2004)).

Some other model parameters are set to match the data as well, and Table A2 in Appendix A gives the details of the calculations for each empirical value: Outstanding debt, $P$, is set equal to total liabilities.[17] The outstanding debt's coupon rate is the continuously compounded annual interest expenses relative to the liabilities. The asset-payout rate is 3% to match the average of the total asset payout. The variance volatility for each firm-year matches the volatility for all the firm-years' unlevered volatility in the rating. Other model parameters are the same as the values used in the comparative statics to match the empirical trends ($m = 0.1,\ r = 0.05,\ \rho = 0.45,\ \kappa = 4,\ T = 1$). The model without the VRP does not have variance volatility, the physical mean-reversion speed, and the decision period.

For example, AAR Corp. has a BB rating during 2006 with average 38% leverage, an equity market cap of $882 million, equity volatility of 35%, and a yield spread of 1.77%. The 4-by-4 calibration with the VRP yields 2.9 VRP, 23% instant volatility, $1,263 million asset value, and 5.7% mean variance. These calibrated values with the other parameters imply 36% optimal leverage. Without the VRP, the 2-by-2 calibration yields $1,184 million asset value and 4.7% asset variance, which implies 46% optimal leverage.

[16]Calibration to the leverage does not create an endogeneity problem. In both models, it is a comparison between "as is" versus "as expected" for the leverage. Nevertheless, this article also reports similar results in Appendix I of the Supplementary Material, where leverage is dropped from the calibrations in Table 9 for a robustness check.

[17]The debt and leverage calculations are similar to those of Elkamhi et al. (2012). The calculations include trade credits of the firm from its suppliers or customers because they are affected by the risk of the firm. There is also evidence that some firms substitute borrowing with trade credits (Petersen and Rajan (1997), Murfin and Njoroge (2015)).

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### 2. Representative-Firm Calibration

Because the calibrations are done to the representative firms instead of the firm-years, the process changes slightly compared with the firm-year calibrations: Instant variance is set equal to the long-run mean, and the decision period is 2 years. Hence, the last equation with leverage and instant variance from the 4-by-4 calibration in equation (37) is dropped, and the calibration becomes a 3-by-3 match. Equity value, volatility, leverage, outstanding debt, outstanding debt coupon rate, asset-payout rate, and variance volatility match the empirical averages in a way similar to the earlier calibration.[18] Other model parameters are the same as the values used in the comparative statics ( $m = 0.1,\ r = 0.05,\ \rho = 0.45,\ \kappa = 4$), which match the empirical reports. The model without the VRP lacks variance volatility, the physical mean-reversion speed, and the decision period, and its 2-by-2 calibration matches equity value and volatility by adjusting asset variance and size. Plugging the estimated parameters into each model yields the model-implied optimal leverage.

In Section III.C.1, the calibration is similar, with one slight change. The earlier calibrations for the model with the VRP include the yield spread. This extra parameter may imply an unfair calibration advantage for the model with the VRP. In addition, any error in measuring the yield spread would only affect the calibrations in the model with the VRP. Hence, the asset VRP and yield spread are dropped in the model calibration with the VRP. Both models are calibrated in a 2-by-2 manner; asset variance and size are adjusted so that model-implied equity value and volatility match their empirical values. In the model with the asset VRP, the asset VRP is simply the asset variance exposure times the market VRP. Assuming that leverage is almost constant during the 90-day period before the data date, the correlation is not inflated by the firm's leverage. Leverage inflates both the covariance of equity variance with the squared VIX and the standard deviation of equity variance. Leverage cancels out when the covariance is divided by the volatility of variance in the correlation calculation.

## Supplementary Material

To view supplementary material for this article, please visit http://dx.doi.org/10.1017/S0022109020000228.

## References

Ait-Sahalia, Y.; J. Fan; and Y. Li. "The Leverage Effect Puzzle: Disentangling Sources of Bias at High Frequency." *Journal of Financial Economics*, 109 (2013), 224–249.

Ait-Sahalia, Y., and R. Kimmel. "Maximum Likelihood Estimation of Stochastic Volatility Models." *Journal of Financial Economics*, 83 (2007), 413–452.

Arellano, M., and S. Bond. "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations." *Review of Economic Studies*, 58 (1991), 277–297.

[18]Appendix J in the Supplementary Material reports calibrations to median values without any change in the inferences. Both median and firm-year calibrations control for potential convexity in the data.

Bae, K.-H.; J.-K. Kang; and J. Wang. "Employee Treatment and Firm Leverage: A Test of the Stakeholder Theory of Capital Structure." *Journal of Financial Economics*, 100 (2011), 130–153.
Bakshi, G., and N. Kapadia. "Delta-Hedged Gains and the Negative Market Volatility Risk Premium." *Review of Financial Studies*, 16 (2003), 527–566.
Banerjee, S.; S. Dasgupta; and Y. Kim. "Buyer–Supplier Relationships and the Stakeholder Theory of Capital Structure." *Journal of Finance*, 63 (2008), 2507–2552.
Barras, L., and A. Malkhozov. "Does Variance Risk Have Two Prices? Evidence from the Equity and Option Markets." *Journal of Financial Economics*, 121 (2016), 79–92.
Bekaert, G., and G. Wu. "Asymmetric Volatility and Risk in Equity Markets." *Review of Financial Studies*, 13 (2000), 1–42.
Bongaerts, D.; F. De Jong; and J. Driessen. "Derivative Pricing with Liquidity Risk: Theory and Evidence from the Credit Default Swap Market." *Journal of Finance*, 66 (2011), 203–240.
Brennan, M. J., and E. S. Schwartz. "Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure." *Journal of Business*, 51 (1978), 103–114.
Bris, A.; I. Welch; and N. Zhu. "The Costs of Bankruptcy: Chapter 7 Liquidation versus Chapter 11 Reorganization." *Journal of Finance*, 61 (2006), 1253–1303.
Bruno, G. S. F. "Estimation and Inference in Dynamic Unbalanced Panel-Data Models with a Small Number of Individuals." *Stata Journal*, 5 (2005), 473–500.
Campbell, J. Y.; S. Giglio; C. Polk; and R. Turley. "An Intertemporal CAPM with Stochastic Volatility." *Journal of Financial Economics*, 158 (2018), 207–233.
Campbell, J. Y.; J. Hilscher; and J. Szilagyi. "In Search of Distress Risk." *Journal of Finance*, 63 (2008), 2899–2939.
Carr, P., and L. Wu. "Variance Risk Premiums." *Review of Financial Studies*, 22 (2009), 1311–1341.
Chen, H. "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure." *Journal of Finance*, 65 (2010), 2171–2212.
Chen, H.; H. Wang; and H. Zhou. "Stock Return Volatility and Capital Structure Decisions." *Paper presented at the American Finance Association Meeting, Boston, Massachusetts*, (2015).
Chen, H.; Y. Xu; and J. Yang. "Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads." Working Paper, available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2024329, (2013).
Chen, L.; P. Collin-Dufresne; and R. S. Goldstein. "On the Relation between the Credit Spread Puzzle and the Equity Premium Puzzle." *Review of Financial Studies*, 22 (2008), 3367–3409.
Choi, J., and M. Richardson. "The Volatility of a Firm's Assets and the Leverage Effect." *Journal of Financial Economics*, 121 (2016), 254–277.
Correia, R., and J. Población. "A Structural Model with Explicit Distress." *Journal of Banking and Finance*, 58 (2015), 112–130.
Cox, J. C.; J. E. Ingersoll; and S. A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica*, 53 (1985), 385–407.
Davydenko, S. A.; I. A. Strebulaev; and X. Zhao. "A Market-Based Study of the Cost of Default." *Review of Financial Studies*, 25 (2012), 2959–2999.
DeAngelo, H.; L. DeAngelo; and T. M. Whited. "Capital Structure Dynamics and Transitory Debt." *Journal of Financial Economics*, 99 (2011), 235–261.
DeAngelo, H., and R. Roll. "How Stable Are Corporate Capital Structures?" *Journal of Finance*, 70 (2015), 373–418.
Denis, D. J. "The Persistent Puzzle of Corporate Capital Structure: Current Challenges and New Directions." *Financial Review*, 47 (2012), 631–643.
Dixit, A. K., and R. S. Pindyck. *Investment Under Uncertainty*. Princeton, NJ: Princeton University Press (2012).
Dorion, C. *Pricing Financial Derivatives: The Impact of Business Conditions and Systematic Risk*. PhD thesis, McGill University, Montreal, Canada (2010).
Du, D.; R. Elkamhi; and J. Ericsson. "Time-Varying Asset Volatility and the Credit Spread Puzzle." *Journal of Finance*, 74 (2019), 1841–1885.
Elkamhi, R.; J. Ericsson; and C. A. Parsons. "The Cost and Timing of Financial Distress." *Journal of Financial Economics*, 105 (2012), 62–81.
Elsas, R., and D. Florysiak. "Dynamic Capital Structure Adjustment and the Impact of Fractional Dependent Variables." *Journal of Financial and Quantitative Analysis*, 50 (2015), 1105–1133.
Eom, Y. H.; J. Helwege; and J. Huang. "Structural Models of Corporate Bond Pricing: An Empirical Analysis." *Review of Financial Studies*, 17 (2004), 499.
Ericsson, J., and J. Reneby. "A Framework for Valuing Corporate Securities." *Applied Mathematical Finance*, 5 (1998), 143–163.
Fama, E. F., and K. R. French. "Testing Trade-Off and Pecking Order Predictions about Dividends and Debt." *Review of Financial Studies*, 15 (2002), 1–33.

https://doi.org/10.1017/S0022109020000228 Published online by Cambridge University Press

Fan, J. P. H.; S. Titman; and G. Twite. "An International Comparison of Capital Structure and Debt Maturity Choices." *Journal of Financial and Quantitative Analysis*, 47 (2012), 23–56.
Figlewski, S., and X. Wang. "Is the 'Leverage Effect' a Leverage Effect?" Working Paper, available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=256109, (2001).
Fouque, J.; R. Sircar; and K. Solna. "Stochastic Volatility Effects on Defaultable Bonds." *Applied Mathematical Finance*, 13 (2006), 215–244.
Frank, M. Z., and V. K. Goyal, "Trade-Off and Pecking Order Theories of Debt." B.E. Eckbo, ed., "Handbook of Empirical Corporate Finance," *Handbooks in Finance*, San Diego: Elsevier (2007), 135–202.
Frank, M. Z., and V. K. Goyal. "Capital Structure Decisions: Which Factors Are Reliably Important?" *Financial Management*, 38 (2009), 1–37.
Geske, R. "The Valuation of Corporate Liabilities as Compound Options." *Journal of Financial and Quantitative Analysis*, 12 (1977), 541–552.
Gilson, S. C. "Transactions Costs and Capital Structure Choice: Evidence from Financially Distressed Firms." *Journal of Finance*, 52 (1997), 161–196.
Glover, B. "The Expected Cost of Default." *Journal of Financial Economics*, 119 (2016), 284–299.
Goldstein, R.; N. Ju; and H. Leland. "An EBIT-Based Model of Dynamic Capital Structure." *Journal of Business*, 74 (2001), 483–512.
Graham, J. R. "How Big Are the Tax Benefits of Debt?" *Journal of Finance*, 55 (2000), 1901–1941.
Graham, J. R., and M. T. Leary. "A Review of Empirical Capital Structure Research and Directions for the Future." *Annual Review of Financial Economics*, 3 (2011).
Hackbarth, D.; C. A. Hennessy; and H. E. Leland. "Can the Trade-Off Theory Explain Debt Structure?" *Review of Financial Studies*, 20 (2007), 1389–1428.
Hennessy, C. A., and T. M. Whited. "How Costly Is External Financing? Evidence from a Structural Estimation." *Journal of Finance*, 62 (2007), 1705–1745.
Heston, S. L. "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options." *Review of Financial Studies*, 6 (1993), 327–343.
Hong, H., and D. Sraer. "Quiet Bubbles." *Journal of Financial Economics*, 110 (2013), 596–606.
Hsu, J.; J. Saá-Requejo; and P. Santa-Clara. "A Structural Model of Default Risk." *Journal of Fixed Income*, 19 (2010), 77–94.
Huang, J.-Z., and M. Huang. "How Much of the Corporate-Treasury Yield Spread Is Due to Credit Risk?" *Review of Asset Pricing Studies*, 2 (2012), 153–202.
Huang, J.-Z., and H. Zhou. "Specification Analysis of Structural Credit Risk Models." *Review of Finance*, 24 (2020), 45–98.
Hull, J., and A. White. "The Pricing of Options on Assets with Stochastic Volatilities." *Journal of Finance*, 42 (1987), 281–300.
Jensen, M. C., and W. H. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics*, 3 (1976), 305–360.
Jones, E. P.; S. P. Mason; and E. Rosenfeld. "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation." *Journal of Finance*, 39 (1984), 611–625.
Kisgen, D. J. "Credit Ratings and Capital Structure." *Journal of Finance*, 61 (2006), 1035–1072.
Kisgen, D. J. "Do Firms Target Credit Ratings or Leverage Levels?" *Journal of Financial and Quantitative Analysis*, 44 (2009), 1323–1344.
Korajczyk, R. A., and A. Levy. "Capital Structure Choice: Macroeconomic Conditions and Financial Constraints." *Journal of Financial Economics*, 68 (2003), 75–109.
Korteweg, A. "The Net Benefits to Leverage." *Journal of Finance*, 65 (2010), 2137–2170.
Leland, H. E. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure." *Journal of Finance*, 49 (1994), 1213–1252.
Leland, H. E. "Agency Costs, Risk Management, and Capital Structure." *Journal of Finance*, 53 (1998), 1213–1243.
Leland, H. E., and K. B. Toft. "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads." *Journal of Finance*, 51 (1996), 987–1019.
Lotfaliei, B. "Zero Leverage and the Value in Waiting to Issue Debt." *Journal of Banking and Finance*, 97 (2018), 335–349.
Luciano, E., and G. Nicodano. "Guarantees, Leverage, and Taxes." *Review of Financial Studies*, 27 (2014), 2736–2772.
McQuade, T. "Stochastic Volatility and Asset Pricing Puzzles." Working Paper, Stanford University (2018).
Merton, R. C. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance*, 29 (1974), 449–470.
Modigliani, F., and M. H. Miller. "The Cost of Capital, Corporation Finance and the Theory of Investment." *American Economic Review*, 48 (1958), 261–297.

https://doi.org/10.1017/S0022109020000228 Published online by Cambridge University Press

Murfin, J., and K. Njoroge. "The Implicit Costs of Trade Credit Borrowing by Large Firms." *Review of Financial Studies*, 28 (2015), 112–145.
Myers, S. C. "Still Searching for Optimal Capital Structure." *Journal of Applied Corporate Finance*, 6 (1993), 4–14.
Nickell, S. "Biases in Dynamic Models with Fixed Effects." *Econometrica*, 49 (1981), 1417–1426.
Öztekin, Ö. "Capital Structure Decisions Around the World: Which Factors Are Reliably Important?" *Journal of Financial and Quantitative Analysis*, 50 (2015), 301–323.
Petersen, M. A. "Estimating Standard Errors in Finance Panel Data Sets: Comparing Approaches." *Review of Financial Studies*, 22 (2009), 435–480.
Petersen, M. A., and R. G. Rajan. "Trade Credit: Theories and Evidence." *Review of Financial Studies*, 10 (1997), 661–691.
Rajan, R. G., and L. Zingales. "What Do We Know About Capital Structure? Some Evidence from International Data." *Journal of Finance*, 50 (1995), 1421–1460.
Romano, M., and N. Touzi. "Contingent Claims and Market Completeness in a Stochastic Volatility Model." *Mathematical Finance*, 7 (1997), 399–412.
Sabanis, S. "Stochastic Volatility and the Mean Reverting Process." *Journal of Futures Markets*, 23 (2003), 33–47.
Schaefer, S. M., and I. A. Strebulaev. "Structural Models of Credit Risk Are Useful: Evidence from Hedge Ratios on Corporate Bonds." *Journal of Financial Economics*, 90 (2008), 1–19.
Schwert, M., and I. A. Strebulaev. "Capital Structure and Systematic Risk." Working Paper #178, Stanford University (2014).
Stein, E. M., and J. C. Stein. "Stock Price Distributions with Stochastic Volatility: An Analytic Approach." *Review of Financial Studies*, 4 (1991), 727–752.
Strebulaev, I. A. "Do Tests of Capital Structure Theory Mean What They Say?" *Journal of Finance*, 62 (2007), 1747–1787.
Strebulaev, I. A., and T. M. Whited. "Dynamic Models and Structural Estimation in Corporate Finance." *Foundations and Trends in Finance*, 6 (2011).
Strebulaev, I. A., and T. M. Whited. "Dynamic Corporate Finance Is Useful: A Comment on Welch (2013)." *Critical Finance Review*, 2 (2013), 173–191.
Sundaresan, S. "A Review of Merton's Model of the Firm's Capital Structure with Its Wide Applications." *Annual Review of Financial Economics*, 5 (2013), 21–41.
Tahani, N. "Exotic Options Pricing under Stochastic Volatility." Working Paper 05-01, School of Administrative Studies, York University (2005).
Titman, S., and R. Wessels. "The Determinants of Capital Structure Choice." *Journal of Finance*, 43 (1988), 1–19.
Welch, I. "Capital Structure and Stock Returns." *Journal of Political Economy*, 112 (2004), 106–132.
Welch, I. "A Critique of Recent Quantitative and Deep-Structure Modeling in Capital Structure Research and Beyond." *Critical Finance Review*, 2 (2013), 131–172.
Wu, G. "The Determinants of Asymmetric Volatility." *Review of Financial Studies*, 14 (2001), 837–859.
Zhang, B. Y.; H. Zhou; and H. Zhu. "Explaining Credit Default Swap Spreads with the Equity Volatility and Jump Risks of Individual Firms." *Review of Financial Studies*, 22 (2009), 5099–5131.

https://doi.org/10.1017/S0022109020000228 Published online by Cambridge University Press