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Quantum robot: structure, algorithms and applications

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SUMMARY

A brand-new paradigm of robots–quantum robots–is proposed through the fusion of quantum theory with robot technology. A quantum robot is essentially a complex quantum system which generally consists of three fundamental components: multi-quantum computing units (MQCU), quantum controller/actuator, and information acquisition units. Corresponding to the system structure, several learning control algorithms, including quantum searching algorithms and quantum reinforcement learning algorithms, are presented for quantum robots. The theoretical results show that quantum robots using quantum searching algorithms can reduce the complexity of the search problem from $O(N^2)$ in classical robots to $O(N\sqrt{N})$. Simulation results demonstrate that quantum robots are also superior to classical robots in efficient learning under novel quantum reinforcement learning algorithms. Considering the advantages of quantum robots, some important potential applications are also analyzed and prospected.

KEYWORDS: Quantum robot; Quantum reinforcement learning; MQCU; Grover algorithm.

I. INTRODUCTION

The advances made in the field of robotics are among the most important achievements in science and technology in the 20th century.[1–3] With technological advancements,[4–8] robots are increasingly serving the community in a wide range of applications, such as industrial production, military affairs, national defense, medical treatment and sanitation, navigation and spaceflight, public security, and so on. Moreover, some new types of robots such as nanorobots, biorobots and medical robots have been developed through the fusion of nanotechnology, biological and medical engineering with robot technology. As viewed from the development trend of robots, increasing robotic intelligence and reducing the physical size of robots are two important research and development thrusts. The key to increasing robotic intelligence lies in improving the performance of sensors and increasing the speed of learning and decision making. At the same time, the reduction of physical size will cause quantum effects to become more prominent. Hence many new challenges must be addressed from new angles.

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Quantum theory is one of the crowning achievements in the 20th century. In particular, many results in quantum information technology have shown that quantum computers can effectively increase the efficiency for solving some important classical problems and can even solve some hard problems that classical computers cannot solve efficiently. Furthermore, quantum systems can realize higher degrees of security in telecommunication systems and can increase the capacity of communication channels.[9] As a result, quantum systems and their applications have become important research topics for many scientists.

In this paper, we consider the fusion of quantum theory and robotics, and use quantum systems to construct a new paradigm of robots — quantum robots. The concept of quantum robots has been presented from a physics perspective by Benioff in 1998.[10,11] Benioff emphasized the applications of quantum computers in quantum robots; however, the quantum robot he described is not aware of its environment, does not make decisions, and does not carry out experiments or make measurements. We propose a brand-new quantum robot from an engineering perspective and consider its information exchanges and learning control. Since quantum robots apply quantum effects, it solves difficulties resulting from the reduction of physical size. Moreover, the performance of sensors can be improved through equipping quantum robots with quantum sensors,[12–16] and the speed of robot learning and behavior decision-making can be increased using parallel computing, fast searching and efficient learning from quantum algorithms.

The organization of this paper is as follows: Section II presents a system structure for quantum robots and describes the functions of three fundamental components, including multi-quantum computing units (MQCU), quantum controller/actuator, and information acquisition units. In Section III we apply the Grover algorithm to a quantum robot search problem. In addition, we propose a novel machine learning algorithm–quantum reinforcement learning (QRL) for quantum robots. The theoretical results show that quantum robots can reduce the complexity of the search problem from $O(N^2)$ in classical robots to $O(N\sqrt{N})$ using the Grover algorithm. Simulation results also demonstrate that quantum robots are superior to classical robots in efficient learning under the QRL algorithm. Section IV compares quantum robots with classical robots and suggests some possible applications for quantum robots given their advantages. Section V presents conclusions and remarks.

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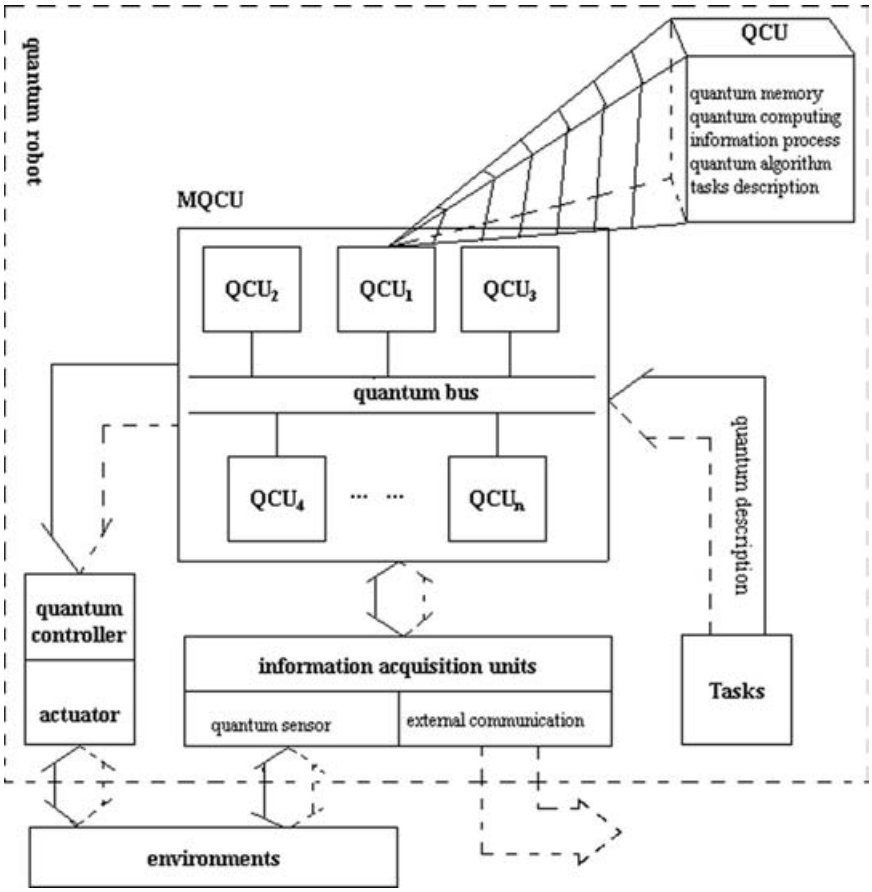


Fig. 1. System structure of a quantum robot. A quantum robot includes three fundamental components: MQCU, quantum controller and actuator, and information acquisition units.

II. SYSTEM STRUCTURE OF QUANTUM ROBOTS

In 1998, Benioff first proposed the concept of quantum robot, where a quantum robot is described as a mobile quantum system that includes an on-board quantum computer and needed ancillary systems.[10] He emphasized the importance of quantum computer in quantum robots, but the robot described there has no awareness of its environment and does not make decisions or measurements. In this paper, we provide an alternative definition for quantum robots, which considers interaction with the external environment via sensing and information processing. *A quantum robot is a mobile physical apparatus designed for using quantum effects of quantum systems, which can sense the environment and its own state, process quantum information and accomplish meaningful tasks*. In particular, a quantum robot system includes three interacting components: multi-quantum computing units (MQCU), quantum controller and actuator, and information acquisition units. A detailed description of the system and each component is shown in Fig. 1.

II.1. MQCU

MQCU is the information processing center and acts as the cerebrum of a quantum robot. It receives tasks described in quantum languages and exchanges information with its environment via quantum sensors or external communications. By storing, analyzing, computing and processing a variety of information including task information, environment information and sensing information, the cerebrum can conduct appropriate quantum control algorithms and generate indication signals for the quantum controller to direct the actuator to perform certain operations. Usually, a MQCU is made up of many quantum computing units (QCUs) and each QCU accomplishes some specific tasks and exchanges information with each other through a quantum bus. The quantum bus may be a refreshable entanglement resource[17] or some other quantum circuits. Besides general functions of a classical bus, such as CAN bus, PCI bus and PC104 bus, the quantum bus should also be able to exchange or preprocess quantum information. According to quantum information theory, the QCUs can exchange information with each other more rapidly and secretly than MCUs of classical robots. Each QCU is a quantum information processor and it can perform some concrete tasks such as quantum computing, quantum memory and task description. In the actual applications, a quantum computer can be used as the main part of a QCU.

A quantum computer is a physical apparatus that can process quantum information and perform parallel computation by manipulating quantum states in a controlled fashion. In the quantum computer, an information unit (called quantum bit, or qubit) can lie in the coherent superposition state of logical states 0 and 1; that is to say, it can simultaneously store 0 and 1, so it can effectively speed

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up the computation of some classical problems, and can even solve some hard problems that classical computer cannot solve efficiently. The essential characteristics of quantum computation are quantum superposition and quantum coherence. With the rapid development of quantum computation technology, some quantum computer models can be constructed using nuclear magnetic resonance (NMR), ion traps, and photons. In the present robot system, quantum computers can act as QCUs and accomplish some specific tasks such as storing, analyzing, computing, and processing a variety of information to help the MQCU conduct appropriate quantum control algorithms.

II.2. Quantum controller and actuator

These are the execution and control apparatus of a quantum robot. A quantum controller receives and processes indication signals from the MQCU and informs the actuator to carry out corresponding actions. It acts as the connection between the cerebrum (MQCU) and the arm (actuator) of a quantum robot. Usually, a quantum controller is a quantum system, such as a quantum CNOT gate. Moreover, one may design useful quantum controllers under the direction of the rapidly developing quantum control theory.[18–23]

The actuator executes some actions determined by the indication signal from the quantum controller. An actuator may be a pure quantum system or a semiclassical apparatus capable of processing quantum information. Generally, the actuator can exchange quantum information as well as classical information with the quantum controller. In some specific circumstances, the actuator and quantum controller can be considered as a single apparatus, which can directly receive information from the MQCU and carry out some actions on its environments. In addition to quantum sensors, the actuator is another interaction channel between a quantum robot and its environment.

II.3. Information acquisition units

As with a classical robot, a quantum robot also needs to perceive its environment and acquire information through information acquisition units. In a quantum robot, quantum sensors perceive the information of its environment; the robot may also receive some other information from distant mainframes or other quantum robots through external communication units. Usually, the information to be acquired includes quantum information and classical information. However, according to quantum theory, the acquisition of quantum information is difficult since quantum measurement destroys the quantum state of a system. Hence quantum nondemolition (QND) measurement is an important task in quantum robots. A quantum sensor is a kind of microstructure sensor, which is designed by applying quantum effect. To detect faint classical signals, two kinds of quantum sensors, superconduction quantum interference device (SQUID) sensor[13,14] and quantum well Hall sensor,[15,16] can be used in a quantum robot.

SQUID sensors are extremely sensitive magnetic sensors that are constructed based on the principles of superconductivity, Meissner effect, flux quantization, and Josephson effect. Under Josephson effect, SQUID sensors can convert minute changes in the current or magnetic field to a measurable voltage and can detect magnetic fields as small as 10^{-10} Tesla. Quantum well Hall sensors are a kind of high-performance micro Hall sensors and use two-dimensional electron gases to obtain a compromise between high mobility and high carrier concentration while maintaining a reasonably high sheet resistance. For example, we can construct a quantum well Hall sensor through sandwiching thin InAs layers between AlGaSb layers. The sensor has high magnetic sensitivity and excellent temperature stability as the result of good confinement of two dimensional electron gases in a quantum well structure. Hence quantum well Hall sensors can be used to detect faint electromagnetic fields under different circumstances. Due to the high sensitivity and good temperature stability of quantum sensors, we can equip the present quantum robot with these sensors to measure extremely weak electromagnetic fields. Simultaneously, scientists are studying other new types of quantum sensors which can acquire quantum information. Once they are realized, we may also equip quantum robots with these devices to sense a variety of quantum signals and feed them back into the MQCU.

Moreover, quantum robots have communication interfaces to exchange information with distant mainframes or other quantum robots, which can constitute a multi quantum robot system as shown in Fig. 2. With external communication, quantum information can be exchanged and the advantages of quantum communication such as high channel capability, perfect security and quantum teleportation can be fully utilized.

Based on the above structure, it is obvious that quantum robots also belong to an important class of robotic systems that has the ability to accomplish certain tasks through perceiving environments via sensors and acting upon those environments via actuators. The characteristics rest with the physical implementation and their particular information processing capabilities. Suppose the task of the quantum robot is to assist doctors with medical service in biomedicine, the information of task decomposition described by quantum languages is sent to the MQCU. Moreover, the MQCU also receives sensing information from the external environments, and every QCU is in charge of some specific works such as tracking, navigation, estimation, diagnosis, etc. According to the processing results of the MQCU, the quantum controller obtains indication signals from the MQCU and informs the actuator to carry out corresponding actions on external environments. Repeatedly, information acquisition units

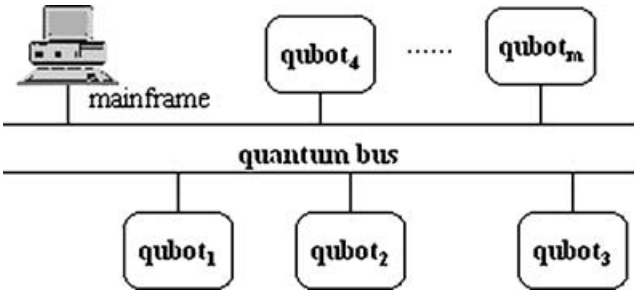


Fig. 2. Multi quantum robot system (where qubot, i.e. quantum robot).

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perceive environments and send sensing information to the MQCU. The MQCU processes the information and generates new signals or learning control algorithms to the quantum controller and actuator until the task is accomplished. In this process, the design of learning control algorithm is a key aspect. Considering the characteristics of the quantum robot, we present two algorithms: Grover algorithm[24] for robot searching and quantum reinforcement learning (QRL) algorithm for robot learning.

III. LEARNING CONTROL ALGORITHMS FOR QUANTUM ROBOTS

A key aspect of quantum robot design is the development of high-efficiency learning control algorithms. As for classical robots, it is difficult to perform algorithms with high complexity while satisfying the requirement for real-time processing because the CPU cannot compute fast enough and the integration of chips is limited. However, a quantum robot is essentially made up of many quantum systems and the MQCU lies in the central position of the entire quantum robot system. Hence one can use the ability of quantum parallel processing to design corresponding learning control algorithms, which can effectively reduce the complexity of solving problems and speed up information processing. After introducing fundamental concepts of quantum parallel computation in subsection III.1, we continue to describe Grover algorithm for quantum robot search problems and propose a novel QRL algorithm for quantum robot learning.

III.1. Parallel processing in quantum robots

A quantum robot is essentially a complex quantum system and its state is represented with a quantum state. To conveniently process all kinds of information, we considered encoding all information according to qubits. In quantum information theory, the state of an arbitrary qubit can be written by a superposition state as follows:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

where α and β are complex coefficients that satisfy $|\alpha|^2 + |\beta|^2 = 1$. $|0\rangle$ and $|1\rangle$ are two orthogonal states (also called eigenstates of quantum state $|\psi\rangle$), and they correspond to logic states 0 and 1 respectively. $|\alpha|^2$ represents the occurrence probability of $|0\rangle$ when the qubit is measured, and $|\beta|^2$ is the probability of obtaining the result $|1\rangle$. The value of a classical bit is a Boolean value either 0 or 1, but a qubit can be prepared in the coherent superposition state of 0 and 1, i.e. a qubit can simultaneously store 0 and 1, which is one of the main differences between classical robots and quantum robots.

According to quantum information theory, the quantum computing process can be looked upon as a unitary linear transformation U from an input qubit to an output qubit. Since a quantum robot is essentially a quantum system and the MQCU lies in the central position of the quantum robot, the processing of the quantum robot can also be looked upon as corresponding transformation. If one applies a transformation U to a superposition state, the transformation will act on all eigenstates of this superposition state and the output will be a new superposition state by superposing the results of eigenstates. When the quantum robot processes a function $f(x)$ by this method, the transformation U can simultaneously work out many different results for a certain input x. This is analogous to classical parallel processing, so we call it parallel processing of a quantum robot. The ability of strong parallel processing is an important advantage of quantum robots over classical robots.

Consider an n-qubit cluster which lies in the following superposition state:

$$|\psi\rangle = \sum_{x=00\cdots0}^{11\cdots1} C_x|x\rangle, \qquad \sum_{x=00\cdots0}^{11\cdots1} |C_x|^2 = 1 \tag{2}$$

where the length of x is n, C_x is complex coefficients and $|C_x|^2$ represents the occurrence probability of $|x\rangle$ measuring state $|\psi\rangle$. $|x\rangle$ has 2^n values, so the superposition state can be looked upon as the superposition state of all integers from 0 to $2^n - 1$. Since U is linear, processing function $f(x)$ produces:

$$U\sum_{x=00\cdots0}^{11\cdots1} C_x|x,0\rangle = \sum_{x=00\cdots0}^{11\cdots1} C_xU|x,0\rangle = \sum_{x=00\cdots0}^{11\cdots1} C_x|x,f(x)\rangle \tag{3}$$

where $|x,0\rangle$ represents the input joint state and $|x,f(x)\rangle$ represents the output joint state. Based on the above analysis, it is easy to see that an n-qubit cluster can simultaneously process 2^n states. However, this is different from classical parallel processing as quantum parallel processing does not necessarily make a tradeoff between processing time and required physical space. In fact, it provides an exponential-scale processing space in the n-qubit linear physical space. So a quantum robot can effectively speed up information processing for some problems such as navigation and decision.

III.2. Searching algorithm for quantum robots

Just as classical robots, most planning and control problems of a quantum robot can be posed as search problems. Thus we will put forth an abstract robot planning problem, and apply Grover algorithm to it as an example.

First consider a robot planning system whose state evolves according to certain transition probabilities that depend on a control u. If the robot is in state i and chooses control u according to a policy π, it will move to state j with probability $p_{ij}(u)$ and a cost $g(i,u,j)$. Now suppose that the desirability of a state is defined as V, which means the value of a state. The task of a robot planning system is to find out the optimal sequence of $V(state)$, which satisfies some forms of Bellman's equation

$$V^*(i) = \min_u E[g(i,u,j) + V^*(j)\,|\,i,u] \quad (\text{for all } i,j) \tag{4}$$

where $E[\cdot|i,u]$ is the expected value. At a certain state i, the robot planning problem is simply an unstructured search problem to find the action a_i (or the next state j) which is optimal.

For unstructured search problems of search space N, the computational complexity in classical computation is $O(N)$. In robotics, it is an important task to search for suitable

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actions from the action space based on the current state of the robot. If the complexity of the state (or action) space is O(N), the problem complexity for classical robots is O(N^2), whereas a quantum robot can reduce the complexity to O($N\sqrt{N}$) by using the Grover algorithm.

If there are N alternative actions ($2^n \geq N \geq 2^{n-1}$), we can express them with n qubits:

$$|\psi_0\rangle = \sum_{l=00\cdots0}^{11\cdots1} a_l|l\rangle \tag{5}$$

where the length of l is n. For convenience, (5) can also be expressed as

$$|\psi_0\rangle = \sum_{i=1}^{2^n} a_i|i\rangle \tag{6}$$

Assuming the action to be found corresponds to $|k\rangle$, now we use Grover algorithm to complete the search task.

First, we prepare a quantum state

$$|s\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=1}^{2^n} |i\rangle \tag{7}$$

which is an equal weight superposition state. This can be accomplished by applying the Hadamard transformation to each qubit of the n-qubit state $|00\cdots00\rangle$.[9,25] Then we construct a reflection transform

$$U_s = 2|s\rangle\langle s| - I \tag{8}$$

Geometrically, when U_s acts on an arbitrary vector, it preserves the component along $|s\rangle$ and flips the component in the hyperplane orthogonal to $|s\rangle$. Thus if we apply U_s to $|\psi_0\rangle$, we get

$$\begin{aligned} U_s|\psi_0\rangle &= 2|s\rangle\langle s\,|\psi_0\rangle - |\psi_0\rangle = 2|s\rangle\sqrt{2^n}\langle a\rangle - |\psi_0\rangle \\ &= 2\frac{1}{\sqrt{2^n}} \sum_{i=1}^{2^n} |i\rangle\sqrt{2^n}\,\langle a\rangle - \sum_{i=1}^{2^n} a_i\,|i\rangle \\ &= \sum_{i=1}^{2^n} (2\,\langle a\rangle - a_i)\,|i\rangle \end{aligned} \tag{9}$$

where

$$\langle a\rangle = \frac{1}{2^n} \sum_{i=1}^{2^n} a_i \tag{10}$$

Now we give another reflection transform

$$U_k = -2|k\rangle\langle k| + I \tag{11}$$

where $|k\rangle$ is the k-th eigenstate and by applying it to state $|\psi_0\rangle$, we obtain

$$\begin{aligned} U_k|\psi_0\rangle &= -2|k\rangle\langle k\,|\psi_0\rangle + |\psi_0\rangle = -2|k\rangle a_k + |\psi_0\rangle \\ &= \sum_{i=1,i\neq k}^{2^n} a_i|i\rangle - a_k|k\rangle \end{aligned} \tag{12}$$

It is easy to see that U_k only changes the amplitude's sign of $|k\rangle$ in the superposition state. Thus we can form a unitary transformation (Grover iteration)[9,24]

$$U_G = U_s U_k \tag{13}$$

By repeatedly applying the transformation U_G on $|\psi_0\rangle$, we can enhance the probability amplitude of $|k\rangle$ while suppressing the amplitude of any other states $|i \neq k\rangle$.[24] After enough number of iterations of the transformation, we can perform a measurement on the system to make the state $|\psi_0\rangle$ collapse into $|k\rangle$ with a probability of almost 1.

Define angle θ satisfying $\sin^2\theta = 1/2^n$. After applying the $U_G\,j$ times to $|\psi_0\rangle$, the amplitude of $|k\rangle$ becomes

$$a_k^j = \sin((2j+1)\theta) \tag{14}$$

If $j = (\pi - 2\theta)/4\theta$, then $(2j+1)\theta = \pi/2$ and $a_k^j = 1$. However, we must perform an integer number of iterations. Boyer has shown in reference [26] that the probability of failure is no more than $1/2^N$ if we perform the Grover iteration int($\pi/4\theta$) times (here the function int(x) return the integer part of x). According to Grover algorithm, when N is large, quantum robots can find the action corresponding to $|k\rangle$ with a high probability of $[1 - \mathrm{O}(1/N)]$. Since Grover algorithm can find desired result with a probability of almost 1 in $\sqrt{N}$ steps, the quantum robot reduces the complexity and can find a suitable action from the action space based on the current state of the robot. If the number of states is 10^4, considering different number of actions, the problem complexities in classical robots and quantum robots can be compared as shown in Table I. From the Table, we can conclude that the advantage of quantum robots is more and more prominent with the increase in the number of actions.

III.3. Learning algorithm for quantum robots

The essence of robot learning is to deal with state-action pair {State(t), Action(t)}.[27] Learning methods are generally classified into supervised, unsupervised and reinforcement learning (RL). Supervised learning requires explicit feedback provided by input-output pairs and gives a map from inputs to outputs. Unsupervised learning only processes on the input data. However, RL uses a scalar value named reward to evaluate the input-output pairs and learns by interaction with environments through trial-and-error. Since 1980s, RL has become an important approach to machine intelligence,[28–31] and is widely used in artificial intelligence, especially in robots,[32–34] due to its good performance of on-line adaptation and powerful learning abilities of complex

Table I. Comparisons of problem complexities in classical robots and quantum robots.

Number of actions	10^2	10^3	10^4	10^5	10^6	10^7	10^8
Complexity in classical robots	10^6	10^7	10^8	10^9	10^{10}	10^{11}	10^{12}
Complexity in quantum robots	10^5	3.2×10^5	10^6	3.2×10^6	10^7	3.2×10^7	10^8

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nonlinear systems.[28,29,35] To adapt learning algorithms to a quantum robot, we propose a novel learning method–quantum reinforcement learning (QRL), inspired by quantum superposition and quantum parallelism.

Let N_s and N_a be the number of states and actions, respectively. Choose numbers m and n satisfying the following inequations:

$$N_s \le 2^m \le 2N_s, N_a \le 2^n \le 2N_a \tag{15}$$

Use m and n qubits to represent state set $S=\{s\}$ and action set $A=\{a\}$ respectively:

$$s : \left[\begin{array}{c|c|c|c} a_1 & a_2 & \cdots & a_m \\ b_1 & b_2 & & b_m \end{array}\right], \quad \text{where } |a_i|^2+|b_i|^2=1, \; i=1,2,\ldots m$$

$$a : \left[\begin{array}{c|c|c|c} \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \beta_1 & \beta_2 & & \beta_n \end{array}\right], \quad \text{where } |\alpha_i|^2+|\beta_i|^2=1, \; i=1,2,\ldots n$$

Thus these may be in superposition state:

$$\left|s^{(m)}\right\rangle = \sum_{s=00\cdots0}^{11\cdots1} C_s|s\rangle, \quad \left|a^{(n)}\right\rangle = \sum_{a=00\cdots0}^{11\cdots1} C_a|a\rangle \tag{16}$$

where $C_s = x_s + iy_s$ and $C_a = u_a + iv_a$ are complex numbers. The mapping from states to actions is $f(s)=\pi : S \to A$ and we will get:

$$f(s) = \left|a_s^{(n)}\right\rangle = \sum_{a=00\cdots0}^{11\cdots1} C_a|a\rangle \tag{17}$$

$|C_a|^2$ denotes the probability of $|a\rangle$ when $|a_s^{(n)}\rangle$ is measured. Based on the above expressions, the procedural form of QRL can be described as follows.

Procedure QRL. Initialize $|s^{(m)}\rangle = \sum_{s=00\cdots0}^{11\cdots1} C_s|s\rangle$, $f(s) = |a_s^{(n)}\rangle = \sum_{a=00\cdots0}^{11\cdots1} C_a|a\rangle$ and $V(s)$ arbitrarily

Repeat (for each episode)

For all states $|s^{(m)}\rangle = \sum_{s=00\cdots0}^{11\cdots1} C_s|s\rangle$:

(1) Observe $f(s)$ and get $|a\rangle$;

(2) Take action $|a\rangle$, observe next state $|s'^{(m)}\rangle$, reward r

Then update: $V(s) \leftarrow V(s) + \alpha(r + \gamma V(s') - V(s))$

$$C_a \leftarrow e^{\lambda(r+V(s'))}C_a$$

Until for all states $|\Delta V(s)| \le \varepsilon$.

QRL is inspired by the state superposition principle of quantum states and quantum parallel computation. The state value can be represented with a quantum state and be obtained by randomly observing the simulated quantum state, which will lead to state collapse according to the quantum measurement postulate. And the occurrence probability of eigenvalue is denoted by probability amplitude, which is updated according to rewards. This approach represents the whole state-action space with the superposition of quantum state and makes a good tradeoff between exploration and exploitation using probability. Moreover, the representation method is consistent with quantum parallel computation and can speed up learning dramatically.

In reference [31], Bertsekas and Tsitsiklis have verified that stochastic iterative algorithms, under certain exploration policies, converge at the optimal state value function $V(s)^*$ with probability 1 when the following conditions hold (where α_k is the stepsize):

$$\lim_{T\to\infty}\sum_{k=1}^{T}\alpha_k = \infty, \quad \lim_{T\to\infty}\sum_{k=1}^{T}\alpha_k^2 < \infty \tag{18}$$

QRL is similar to RL, with the following differences: (1) exploration policy is based on the collapse theory of quantum measurement while being observed; (2) parallel computation. Hence the modification of RL does not affect the characteristic of convergence and QRL algorithms converge when (18) holds.

To evaluate the QRL algorithm in practice, consider an example of a grid environment with four rooms and surrounding corridors as shown in Fig. 3. Each cell of the grid corresponds to an individual state of the environment. From any state, the robot (or agent) can perform one of four primary actions: up, down, left and right; and actions that would lead into a blocked cell are not executed. The task of the algorithm is to find an optimal policy that will let the robot move from S to G with minimized cost (or maximized rewards). The simulation environment is based on Windows 2000 and *Visual C++*. The results are processed with Matlab 6.5.

Experimental set-up. In QRL, the action selecting policy is obviously different from classical RL algorithms, which is inspired by the collapse theory of quantum measurement. The probability amplitude $|C_a|^2$ is used to denote the probability of an action defined as $f(s) = |a_s^{(n)}\rangle = \sum_{a=00\cdots0}^{11\cdots1} C_a|a\rangle$. For

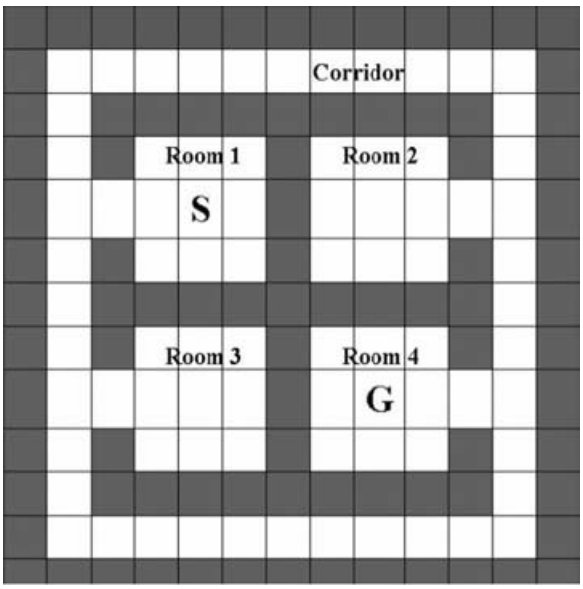


Fig. 3. The example of rooms with corridors in a gridworld environment with cell-to-cell actions (up, down, left and right). The labels S and G indicate the initial state and the goal in the simulation experiment described in the text.

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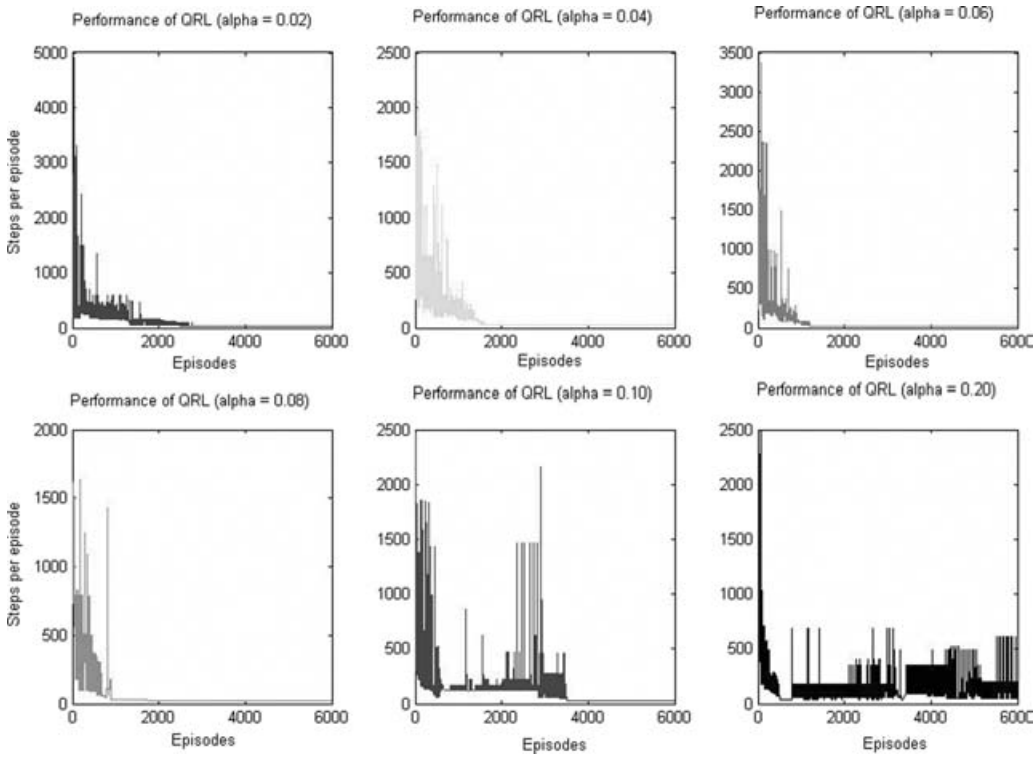


Fig. 4. Performance of QRL for the example shown in Fig. 3.

the four cell-to-cell actions $|C_a|^2$ is initialized uniformly. In this $13 \times 13(0 \sim 12)$ grid world, the initial state and the goal are cell(4,4) and cell(8,8), respectively.

Results and analysis. The learning performance for QRL is plotted in Fig. 4. We observe that given a proper stepsize (alpha < 0.10) this algorithm learns extraordinarily fast at the beginning phase, and then steadily converges to the optimal policy that costs 25 steps to the goal G. As the stepsize increases from 0.02 to 0.10, this algorithm learns faster but more unsteadily. When the stepsize is 0.20, it cannot converge to the optimal policy. These results show that this QRL algorithms are superior to other RL algorithms in two main aspects: (1) Action selecting policy makes a good tradeoff between exploration and exploitation using probability, which speeds up the learning and guarantees the search over the whole state-action space as well. (2) Updating is carried out in parallel, which will be much more prominent in the future when practical quantum apparatus comes into use instead of being simulated on classical computers.

IV. APPLICATIONS OF QUANTUM ROBOTS

Before presenting the potential applications of quantum robots, we first compare quantum robots with classical robots in detail in Table II.

Thus it can be seen that the quantum robot, as a brand-new robot, can solve difficulties resulting from the reduction of physical size of robots. It makes use of quantum sensors to acquire quantum information as well as classical information

Table II. Comparisons between quantum robots and classical robots.

Compared items	Quantum robot	Classical robot
System property	Quantum system	Mechanical system
On-board sensors	Quantum sensors such as SQUID sensors and quantum well Hall sensors	Infrared sensors, ultrasonic sensors, CCD camera and etc.
Physical Law	Quantum mechanical law	Classical mechanical law
Information processing centre	MQCU (Multi quantum computing units)	Classical processor such as classical computer
Information	Quantum information and classical information	Classical information
Sensitivity	High	Relatively lower
Physical size	Microcosmic	Mostly macro
Technical difficulty	More difficult	Easier
Communication manner	Quantum communication and classical communication	Classical communication
Parallel processing ability	Powerful	Weak

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from its environments and has extremely high delicacy, which effectively overcomes the limitation of existing sensor performances. At the same time, it can directly use the advantages of quantum information technology to speed up the control and decision making. So it will have wide application prospect in many fields.

Here, we outline some potential applications of quantum robots. A quantum robot can be used as a patrol warrior in military areas and national defense. If the magnetic fields near important ports and military bases have been measured in advance, the high sensitivity of quantum sensors allows quantum robots to perceive the faint changes in magnetic fields resulting from the closing of nukes and scouts to effectively forewarn decision-makers. In aviation and spaceflight, quantum robots can be used as Mars and lunar exploration vehicles with its advantage of high sensitivity in perceiving environments, its powerful ability to process information and its inherence nature that allows for more secure communication. In biomedicine, quantum robots can be used for patients with a contagious disease for examining or tracking state of an illness, and can act as SARS nurses and contagion doctors. Once a quantum robot is successfully constructed, it may be possible to establish molecular-scale medical robots. Thus quantum robots can be injected into the body and move along with blood circulation to detect potential pathological changes in body. At the same time, it may provide a novel way to study life. In scientific research, it is possible to use quantum robots to solve most complex problems with less physical resources and provide a test-bed for experimental research in physics, chemistry and information technologies so that some experimental realizations of quantum communication, quantum computation and quantum control may be possible. In safety engineering, one may take advantage of the high sensitivity and small size of quantum robots to apply them to many tasks such as anti-terror forewarning, fire forecasting, surveillance and traffic control.

V. CONCLUSION

With technological advancements, robots have been widely applied to many fields. At the same time, new types of robots, such as nanorobots, biorobots and medical robots have also been under development. This paper proposes another new type of robots—quantum robot, by the fusion of quantum theory with robotic technology. A quantum robot is a mobile physical apparatus designed using quantum effect of quantum systems, which can sense the environment and its own state, process quantum information and accomplish meaningful tasks. It consists of three fundamental components: MQCU, quantum controller and actuator, and information acquisition units.

To adapt algorithms to the system structure of quantum robots, a quantum searching algorithm and a quantum reinforcement learning algorithm are proposed for quantum robots. The theoretical results show that quantum robots can reduce the complexity of the search problem from of $O(N^2)$ in classical robots to $O(N\sqrt{N})$ using the Grover algorithm. Simulation experiments demonstrate that quantum robots are also superior to classical robots in efficient learning since quantum reinforcement learning makes a good tradeoff between exploration and exploitation using probability. Hence quantum robots have many potentially important applications in military affairs, national defense, aviation and spaceflight, biomedicine, scientific research, safety engineering and other daily life tasks.

To implement a real quantum robot is no doubt a very challenging mission, which consists of three kinds of tasks: (1) synthesis architecture for quantum robot systems; (2) physical implementation including computing units, sensors, actuators and communication hardware; and (3) software level researches such as related theories and programming issues. Though our work is only the first step towards practical quantum robots, it has a promising future given the rapid development and gradual maturation of quantum information technology and quantum control theory.

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