Proof: We suppose ABC acute with circumdiameter 1 as in Figure 3. (The proof is similar in the obtuse case for any obtuse vertex.) One has $AH = \cos A$, $AE = \cos A \sin C$, $\angle CAD = 90° - \angle C$ and $\angle AEF = \angle B$. If S is the intersection of the altitude from A with EF, the sine law for triangle ASE and the formula for the cosine of a difference give thus

$$AS = \frac{\sin B}{\sin(90° + \angle C - \angle B)} \cos A \sin C = \frac{\sin B \sin C}{\cos B \cos C + \sin B \sin C} \cos A.$$

Hence $\dfrac{AH}{AS} = 1 + \cot B \cot C$. By Theorem 1, the homothety h about A with this ratio maps D to X. As the circumorthic triangle is twice as large as the orthic triangle, the image of the orthic triangle under h is the image of the circumorthic triangle under a homothety of half ratio centred at X.

References

1. G. Leversha, *The Geometry of the triangle*, United Kingdom Mathematics Trust (2013).
2. J. Kocik and J. Solecki, Disentangling a triangle, *Amer. Math. Monthly* **116** (2009) pp. 228-237.
3. G. Nicollier, Area of the orthic quadrilaterals of a convex cyclic orthodiagonal quadrilateral, *Forum Geom.* **16** (2016) pp. 233-239. Available at: http://forumgeom.fau.edu/FG2016volume16/FG201628index.html

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101.22 The hyperbolic version of the Steiner-Lehmus theorem

The Steiner-Lehmus theorem states that if ABC is a (Euclidean) triangle in which the internal angle bisectors of $\angle B$ and $\angle C$ are equal, then $AB = AC$. This theorem was also proved for spherical triangles by Steiner, according to [1].

In hyperbolic geometry, conflicting statements have appeared recently. Thus, it is proved in [2, p. 152, Theorem 4.36] that

> if ABC is a hyperbolic triangle in which the internal angle bisectors of $\angle B$ and $\angle C$ are equal, then AB and AC *must* be equal. (1)

However, a contradictory statement is established in [3]. Namely,

> if ABC is a hyperbolic triangle in which the internal angle bisectors of $\angle B$ and $\angle C$ are equal, then AB and AC *cannot* be equal. (2)

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Statement (1) is quoted in [4] and seconded in [5], and statement (2) is seconded in [6].

When I was asked to write a review of [6] for Zentralblatt Math. earlier this year, I was confronted by the dilemma and, not being sufficiently versed in hyperbolic geometry, I decided to write to the authors of the above mentioned references. I received two replies: one confirmed that (1) is true, and the other confirmed that (2) is true. Since these replies came form experts, I decided that

there cannot exist any hyperbolic triangle in which two internal angle bisectors are equal, (3)

and I explicitly expressed this in the review [7] that I wrote. Of course, the conclusion (3) was based on common sense and, for logic fans, on the logical wisdom that

$$[(p \Rightarrow q) \wedge (p \Rightarrow \sim q)] \Rightarrow \sim p.$$

Ever since I wrote that review, I have been looking forward to receiving a note from someone who is well-versed in hyperbolic geometry confirming the validity of (some of) the statements (1), (2), and (3). I hope this note will popularise this hope of mine, and even fulfil it!

References

1. A. Hendersen, The Lehmus-Steiner-Terquem problem in a global survey, *Scripta Mathematica* **21** (1955), pp. 223-232, pp. 309-312.
2. A. A. Ungar, *A gyrovector space approach to hyperbolic geometry*, Synthesis Lectures on Mathematics and Statistics 4, san Rafael, CA: Morgan and Claypool Publishers (2009).
3. N. Sönmez, Trigonometric proof of Steiner-Lehmus theorem in hyperbolic geometry, *KoG* **12** (2008) pp. 35-36.
4. O. Demirel and E. Soytürk Seyrantepe, The theorems of Urquhart and Steiner-Lehmus in the Poincaré ball model of hyperbolic geometry, *Mat. Vesn.* **63** (2011) pp. 263-274.
5. K. Kiyota, A trigonometric proof of the Steiner-Lehmus theorem in hyperbolic geometry, arXiv:1508.03248v [math.MG] 13 Aug. 2015.
6. C. Barbu, Trigonometric proof of Steiner-Lehmus theorem in hyperbolic geometry, *Acta Univ. Apulensis, Math. Inform.* **23** (2010) pp. 63-67.
7. M. Hajja, Review of [6], Zentralblatt Math., Zbl 1265.30184.

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