Contents

Preface *page* ix
Acknowledgments xiv
Organization of the Book xv

PART I BASICS: FOUNDATIONAL MATERIAL, ELEMENTARY ASPECTS, AND EXAMPLES

1 Introduction 3
1.1 Linear Partial Differential Equations, the Classical Laplace Operator, and Eigenvalue Problems 3
1.2 Laplace-Type Operators on Graphs 9
1.3 Other Sources and Directions of the Theory 12

2 The Abstract Setting 14
2.1 A Category Theoretical Perspective 14
2.1.1 The Static Picture 14
2.1.2 The Dynamical Picture 15
2.1.3 Probing a (Dynamical) Structure 17
2.2 Relations and Norms 19
2.3 Complexes and Orientations 21
2.4 (Oriented) Hypergraphs and their Higher Order Analogs 23
2.4.1 Generalizations 23
2.4.2 Hypergraphs and Hypersimplical Complexes 25
2.4.3 Oriented Hypergraphs and Oriented Hypersimplical Complexes 26
2.5 Scalar Products and Laplace Operators 28
2.5.1 Simplicial Complexes 28
2.5.2 Relative Complexes 32

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 15 Nov 2026 at 20:10:38, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781009641821

2.5.3 Laplace Operators as Averaging Operators 34
2.5.4 Oriented Hypergraphs 36
2.6 Rayleigh Quotients and their Generalizations 37
2.6.1 Rayleigh Quotients 37
2.6.2 Critical Points and p-Laplacians 40
2.7 Symmetries 44
2.8 Homomorphisms 47
2.9 General Setting 48

3 The Classical Case: Eigenvalues on Euclidean Domains and Riemannian Manifolds 50
3.1 The Laplacian on Euclidean Domains 51
3.2 The Laplace–Beltrami Operator and the Hodge Laplacian on Riemannian Manifolds 59
3.3 The p-Laplacian 68

4 First Properties of the Spectrum 76
4.1 The Laplacian and Its Spectrum on Graphs 76
4.1.1 General Aspects 76
4.1.2 The Algebraic Graph Laplacian 80
4.1.3 The Normalized Graph Laplacian 85
4.2 Some Elementary Estimates 109
4.3 Cutting a Graph Optimally, or the Pólya–Cheeger Constant and Eigenvalue Estimates 113
4.4 Dividing a Graph Optimally into Opposite Classes, or the Dual Cheeger Constant and Estimates for the Largest Eigenvalue 123
4.5 Neighborhood Graphs 128
4.6 Signed Graphs 138
4.7 The General Case: Directed Graphs with Arbitrary Weights 150
4.8 Simplicial Complexes 155
4.9 Oriented Hypergraphs 161

5 Eigenvalues and Eigenfunctions of the Laplace Operator via Floer Theory 172
5.1 Rayleigh Quotients, Eigenfunctions, and Eigenvalues 172
5.2 Gradient Flow 175

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 15 Nov 2026 at 20:10:38, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781009641821

PART II EIGENVALUES AND EIGENFUNCTIONS ON SIMPLICIAL COMPLEXES AND HYPERGRAPHS

6 Lovász Extensions 181
6.1 Extensions and Eigenvalue Problems 181
6.2 Definitions and Basic Properties of Lovász Extensions 183
6.3 Eigenvalue Problem for a Pair of Lovász Extensions 188
6.4 Lovász Extension and Cheeger Inequalities 198

7 Discrete p-Laplacians 205
7.1 Graph p-Laplacians ($p > 1$) 205
7.2 The Graph 1-Laplacian 213
7.3 The Graph ∞-Laplacian 224
7.4 p-Laplacians on Oriented Hypergraphs 226

8 Cheeger Inequalities 232
8.1 Cheeger Inequalities on Graphs 232
8.2 Cheeger Inequalities on Signed Graphs 233
8.3 Cheeger Inequalities on Simplicial Complexes 237
8.4 Cheeger Inequalities on Hypergraphs 255

9 Nodal Domains 269
9.1 Nodal Domain Theorems on Graphs and Matrices 270
9.2 Nonlinear Nodal Domain Theorems on Graphs 276
9.3 Nodal Domain Theorems on Hypergraphs 287

PART III ADDITIONAL TOPICS: INTERLACING, TENSORS, NONBACKTRACKING LAPLACIANS, AND APPLICATIONS

10 Interlacing and Spectral Classes 297
10.1 Interlacing 297
10.1.1 Deletion of Subcomplexes 299
10.1.2 Simplicial Maps 305
10.1.3 Coverings 311
10.1.4 Contraction 314
10.2 Spectral Classes 319
10.2.1 Asymptotic Classes of Graphs 319
10.2.2 Spectral Asymptotics 322

11 Spectral Theory of Weighted Hypergraphs via Tensors 327
11.1 Tensors 327
11.2 Hypergraph Tensors 331

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 15 Nov 2026 at 20:10:38, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781009641821

11.3 First Spectral Properties 333
11.4 Θ-Duplicate Vertices 341
11.5 The Hyperflower 343
11.6 Spectral Symmetries 348

12 The Nonbacktracking Laplacian 351
12.1 Basic Definitions 352
12.2 The Nonbacktracking Graph 356
12.3 First Spectral Properties of the Nonbacktracking Laplacian 357
12.4 Spectral Gap from 1 360
12.5 Cycles 363
12.6 Isospectrality 370

13 Applications 373
13.1 Social Sciences 373
13.1.1 Eigenvector Centrality 374
13.1.2 Cohesion of Networks 375
13.2 Biological Networks 376
13.3 Computer Science 378
13.4 Nonlinear Dynamics 384

Remarks on Notation 388
References 389
Index 403

Downloaded from https://www.cambridge.org/core. UNIVERSIDAD AUTONOMA METROPOLITANA, on 15 Nov 2026 at 20:10:38, subject to the Cambridge Core terms of use, available at https://www.cambridge.org/core/terms. https://doi.org/10.1017/9781009641821