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Robust adaptive control and observer for a robot with pneumatic actuators

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SUMMARY
In this paper, the robust adaptive control and observer are considered for a pneumatic robot leg. This approach does not require a precise knowledge of the robot model physical parameters, only the adaptive upper bounds on the norm of the matrices model are used, which with the sliding mode guarantee the robustness of system. Convergence conditions and simulation results for the adaptive control and observer are presented.

KEYWORDS: Adaptive control and observer; Sliding modes; Nonlinear system; Pneumatic robot leg.

INTRODUCTION

Due to their high power output and their low cost, pneumatic actuators appear to be of great interest in robotic applications. In particular, legged robots may use their natural compliance to deaden any shock when dealing with an unknown environment. Thus a pneumatic actuated legged robot[1] has been built to study control problems when dealing with fast dynamic gaits. The dynamic model of one leg, as considered here, is highly nonlinear and time variant. This is due mainly to the mechanical coupling effects and the compressibility of air.

The control law considered previously of such systems,[2] assume that joint displacements, velocities and accelerations are available. In pratice, this is not the case. High precision measurements of the joint displacement are available. By contrast, the speed and acceleration measurements are contamined by noise. To overcome this problem an observer system may be used to generate estimates of the unavailable states from available measurements. In the literature, several observer structures are possible,[3–4] these schemes need the knowledge of the robot model's physical parameters.

This paper deals with the problem of controlling a pneumatic robot leg, based solely on the knowledge of the upper bounds on the norm of the matrices model. Non linear sliding mode observers are used here to estimate the state vector. However, sometimes these bounds may not be easily obtained because of the uncertainties, and varying payload. As an alternative, we propose simple adaptation laws for these bounds which, with the sliding mode, guarantee the stability and robustness of system. The results of simulation are presented to show the effectiveness of the proposed control.

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2. SYSTEM DESCRIPTION AND MODELLING

The systems model is composed of two stages: a dynamic one for the mechanical part, and a thermodynamic stage for the pneumatic part.

The dynamic equation of the system can be written by using the Lagrangian's method.[5–7]

Let us recall that q, $\dot{q}$, $\ddot{q}$ and τ denote, respectively, the (2×1) vectors of joint position, speed, acceleration and torque.

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau \tag{1}$$

$M(q)$ is the (2×2) generalized inertia matrix which is Symmetric Positive Definite (SPD), $G(q)$ is the (2×1) vector of gravitational forces and $C(q,\dot{q})\dot{q}$ is the vector of centripetal and Coriolis forces. We note that the matrix $(\dot{M}-2C)$ is skew symmetric.

The dynamics of the pneumatic actuator can be represented by the nominal model described by the following equation:

$$\dot{\tau}=Ji-B\tau-E\dot{q} \tag{2}$$

where J, B, E are (2×2) diagonal matrices called thermodynamics parameters that depend on the temperature gas characteristics and initial conditions and i is the vector of the applied currents. For more details on the way this equation is obtained, we can refer to thermodynamical studies in reference [7].

The equations (1) and (2) can be rewritten in one stage by differentiating (1) and substituting $\dot{\tau}$, in (2). Thus, we obtain the following equation

$$\dddot{q}=-M^{-1}[(\dot{M}+C+BM)\ddot{q}+(\dot{C}+BC+E)\dot{q}+(BG+\dot{G})-Ji] \tag{3}$$

(For ease of notation, we omit the arguments in the used matrices)

Introducing the state vector $x_1=q$, $x_2=\dot{q}$, $x_3=\ddot{q}$, the model (3) can be rewritten in the following state-space representation:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=x_3\\ \dot{x}_3=f_1(x,i)\end{cases} \tag{4}$$

with,

$$f_1(x,i)=-M^{-1}[(\dot{M}+C+BM)x_3+(\dot{C}+BC+E)x_2 +(BG+\dot{G})-Ji] \tag{5}$$

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which defines a locally observable representation in the sense of the Lie-algebra, as defined by Hermann and Krener.[8]

3. ROBUST SLIDING MODE OBSERVER

The problem is to design a state estimate function only of the measurement link positions x_1. The state estimation is performed by the following observer:

$$\begin{aligned}\dot{\hat{x}}_1&=\hat{x}_2-\Gamma_1\tilde{x}_1-\Lambda_1\, sgn(\tilde{x}_1)\\ \dot{\hat{x}}_2&=\hat{x}_3-\Gamma_2\tilde{x}_1-\Lambda_2\, sgn(\tilde{x}_1)\\ \dot{\hat{x}}_3&=\hat{f}-i-\Gamma_3\tilde{x}_1-\Lambda_3\, sgn(\tilde{x}_1)\end{aligned} \tag{6}$$

where the estimated state is denoted, $\hat{x}=(\hat{x}_1^T, \hat{x}_2^T, \hat{x}_3^T)^T$ and the observation error is $\tilde{x}=\hat{x}-x$.

The term $\hat{f}$ is introduced to cope with the influence of f_1 and to account for the differences between tracking errors and observation errors. Γ_1, Γ_2, Γ_3 are positive diagonal matrices; they are chosen so that the linear part of the system is asymptotically stable. The matrices Λ_1, Λ_2 and Λ_3 are chosen positive diagonal matrices, $\Lambda_1=\lambda_1 I$, $\Lambda_2=\lambda_2 I$ and $\Lambda_3=\lambda_3 I$.

The observation error system is obtained from equations (4) and (6)

$$\begin{aligned}\dot{\tilde{x}}_1&=\tilde{x}_2-\Gamma_1\tilde{x}_1-\Lambda_1\, sgn(\tilde{x}_1)\\ \dot{\tilde{x}}_2&=\tilde{x}_3-\Gamma_2\tilde{x}_1-\Lambda_2\, sgn(\tilde{x}_1)\\ \dot{\tilde{x}}_3&=\hat{f}-i-\Gamma_3\tilde{x}_1-\Lambda_3\, sgn(\tilde{x}_1)-f_1(x,i)\end{aligned} \tag{7}$$

In order to obtain the system's dynamics behavior inside the attractive region called sliding path,[4] we will proceed step by step.

First, let us prove that the hypersurface $\tilde{x}_1=0$ is invariant.

Consider a first Lyapounov function candidate:

$$V_1=\frac{1}{2}\tilde{x}_1^T\tilde{x}_1.$$

The condition $\dot{V}_1<0$ is satisfied if condition $|\tilde{x}_2^i|<\lambda_1$; $i\in\{1,2\}$ holds. A generalized notion of the solution of a differential equation with discontinuities on the right hand side is as in Fillipov;[9] the dynamics on the switching surface is an average of the dynamics on each side of the discontinuity surface. The region within which the switching surface is invariant is termed 'sliding patch' (for more details, see Utkin[10]).

On this patch, $\dot{\tilde{x}}_1=0$ and then $\overline{sgn}(\tilde{x}_1)=\Lambda_1^{-1}\tilde{x}_2$.

($\overline{sgn}$ denotes the function equivalent, in the mean, to the *sgn* effect).

The dynamics of $\tilde{x}_2$ on the patch is then given by: $\dot{\tilde{x}}_2=\tilde{x}_3-\Lambda_2\Lambda_1^{-1}\tilde{x}_2$

In a second step, to show that the hypersurface $\tilde{x}_2=0$ is invariant, we define the following Lyapounov function:

$$V_2=\frac{1}{2}\tilde{x}_2^T\tilde{x}_2.$$

Then,

$$\begin{aligned}\dot{V}_2&=\tilde{x}_2^T\dot{\tilde{x}}_2\\ &=\tilde{x}_2^T(\tilde{x}_3-\Lambda_2\Lambda_1^{-1}\tilde{x}_2)\\ &=\tilde{x}_2^T\tilde{x}_3-\tilde{x}_2^T\Lambda_2\Lambda_1^{-1}\tilde{x}_2\end{aligned}$$

So, $\dot{V}_2<0$ under the conditions

$$\|\tilde{x}_3\|<(\lambda_2/\lambda_1)\|\tilde{x}_2\| \tag{8}$$

The intersection of the regions (8), and the hypersurfaces $\tilde{x}_1=0$, $\tilde{x}_2=0$ is reduced to a segment which is the new sliding patch. On this patch, $\dot{\tilde{x}}_2=0$; that is $\overline{sgn}(\tilde{x}_1)=\Lambda_2^{-1}\tilde{x}_3$.

Then, the reduced dynamics of the observation error can be written as:

$$\dot{\tilde{x}}_3=\hat{f}-i-f_1(x,i)-\Lambda_3\Lambda_2^{-1}\tilde{x}_3 \tag{9}$$

We have finally obtained the reduced dynamics of the observation error. The convergence condition on $\tilde{x}_3$ will be studied in a closed-loop, with the equation of a tracking error.

4. TRACKING ERROR EQUATION

The control objective is to track the desired position, velocity, and acceleration $\{x_d, \dot{x}_d, \ddot{x}_d\}$, time-varying trajectories.

Let us consider for the input current, the following control expression:

$$i=\hat{f}-v \tag{10}$$

$$v=\ddot{x}_d-K_p e_1-K_v e_2-K_a e_3 \tag{11}$$

with, $e_1=x_1-x_d$, $e_2=\hat{x}_2-\dot{x}_d$ and $e_3=\hat{x}_3-\ddot{x}_d$.

K_p, K_v and K_a are the feedback gains on the position error e_1, velocity error e_2 and acceleration error e_3, respectively.

Applying this control law to the observer error (9) yields:

$$\dot{\tilde{x}}_3=v-f_1(x,i)-\Lambda_3\Lambda_2^{-1}\tilde{x}_3 \tag{12}$$

by substituting (5) and (11) in (12), one obtains

$$\dot{\tilde{x}}_3=M^{-1}J[f-\hat{f}]-M^{-1}C\tilde{x}_3-\Lambda\tilde{x}_3 \tag{13}$$

with, $\Lambda=\Lambda_3\Lambda_2^{-1}$ and

$$\begin{aligned}f=J^{-1}[&(\dot{M}+C+BM)\hat{x}_3-(\dot{M}+BM)\tilde{x}_3+(\dot{C}+BC+E)x_2\\ &+BG+\dot{G}+(M+J)v]\end{aligned}$$

a function f can be upper bounded as follows

$$\begin{aligned}\|f\|\le\|J^{-1}\|[&\|\dot{M}+C+BM\|\|\hat{x}_3\|+\|\dot{M}+BM\|\|\tilde{x}_3\|\\ &+\|\dot{C}+BC+E\|\|x_2\|+\|BG+\dot{G}\|+\|M+J\|\|v\|]\end{aligned}$$

Furthermore, using the properties of the system[11] and the Bernestein lemma,[12] we have

- $\|M(x_1)\|\le\alpha_1$
- $\|\dot{M}(x_1)\|\le\alpha_1 w_m$
- $\|C(x_1,x_2)\|\le\alpha_2\|x_2\|$
- $\|\dot{C}(x_1,x_2)\|\le\alpha_2 w_m\|x_2\|$

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- $\|G(x_1)\| \le g_0$
- $\|\dot{G}(x_1)\| \le g_0 w_m$
- $\|J^{-1}\| \le J''_m$
- $\|B\| \le b_m$
- $\|E\| \le e_m$

where, α_1, α_2, g_0, J''_m, b_m, e_m and w_m are the positive constants.

Therefore, on the sliding patch f becomes upper bounded by

$$\begin{aligned}\|f\| \le{}& J''_m \alpha_1 w_m \|\hat{x}_3\| + J''_m \alpha_2 \|x_2\| \|\hat{x}_3\| + J''_m b_m \alpha_1 \|\hat{x}_3\| \\ &+ J''_m \alpha_1 w_m \lambda_2 + J''_m b_m \alpha_1 \lambda_2 + J''_m \alpha_2 w_m \|x_2\|^2 \\ &+ J''_m b_m \alpha_2 \|x_2\|^2 + J''_m e_m \|x_2\| + J''_m g_0 w_m \\ &+ J''_m b_m g_0 + (J''_m \alpha_1 + 1)\|v\|\end{aligned}$$

Let

$$\begin{aligned}b(x_2, \hat{x}_3, v) ={}& (J''_m \alpha_1 w_m + J''_m b_m \alpha_1)\|\hat{x}_3\| + J''_m \alpha_2 \|x_2''\| \|\hat{x}_3\| \\ &+ (J''_m \alpha_2 w_m + J''_m b_m \alpha_2)\|x_2\|^2 + J''_m e_m \|x_2\| \\ &+ (J''_m \alpha_1 w_m + J''_m b_m \alpha_1)\lambda_2 + J''_m g_0 w_m \\ &+ J''_m b_m g_0 + (J''_m \alpha_1 + 1)\|v\|\end{aligned} \tag{14}$$

To achieve our objective, the expression of $\hat{f}$ is chosen as follows

$$\hat{f} = \lambda_2 b(\hat{x}_2, \hat{x}_3, v)\, sgn(\tilde{x}_1) \tag{15}$$

The dynamics within the sliding patch ($\tilde{x}_1 = 0$, $\dot{\tilde{x}}_1 = 0$) of the augmented state vector $\hat{x} = (\hat{x}_1^T, \hat{x}_2^T, \hat{x}_3^T, \tilde{x}_3^T)^T$ is then given by the equations (4), (6), (10) and (13).

$$\begin{cases}\dot{x}_1 = \hat{x}_2 \\ \dot{\hat{x}}_2 = \hat{x}_3 - \tilde{x}_3 \\ \dot{\hat{x}}_3 = v - \Lambda \tilde{x}_3 \\ \dot{\tilde{x}}_3 = M^{-1} J[f - \hat{f}] - M^{-1} C \tilde{x}_3 - \Lambda \tilde{x}_3\end{cases}$$

Hence, the dynamics of the tracking error vector $e^T = (e_1^T, e_2^T, e_3^T)^T$ is given by the error equation

$$\dot{e} = Fe + G\tilde{x}_3 \tag{16}$$

where

$$F = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ -K_p & -K_v & -K_a \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ I \\ -\Lambda \end{bmatrix}$$

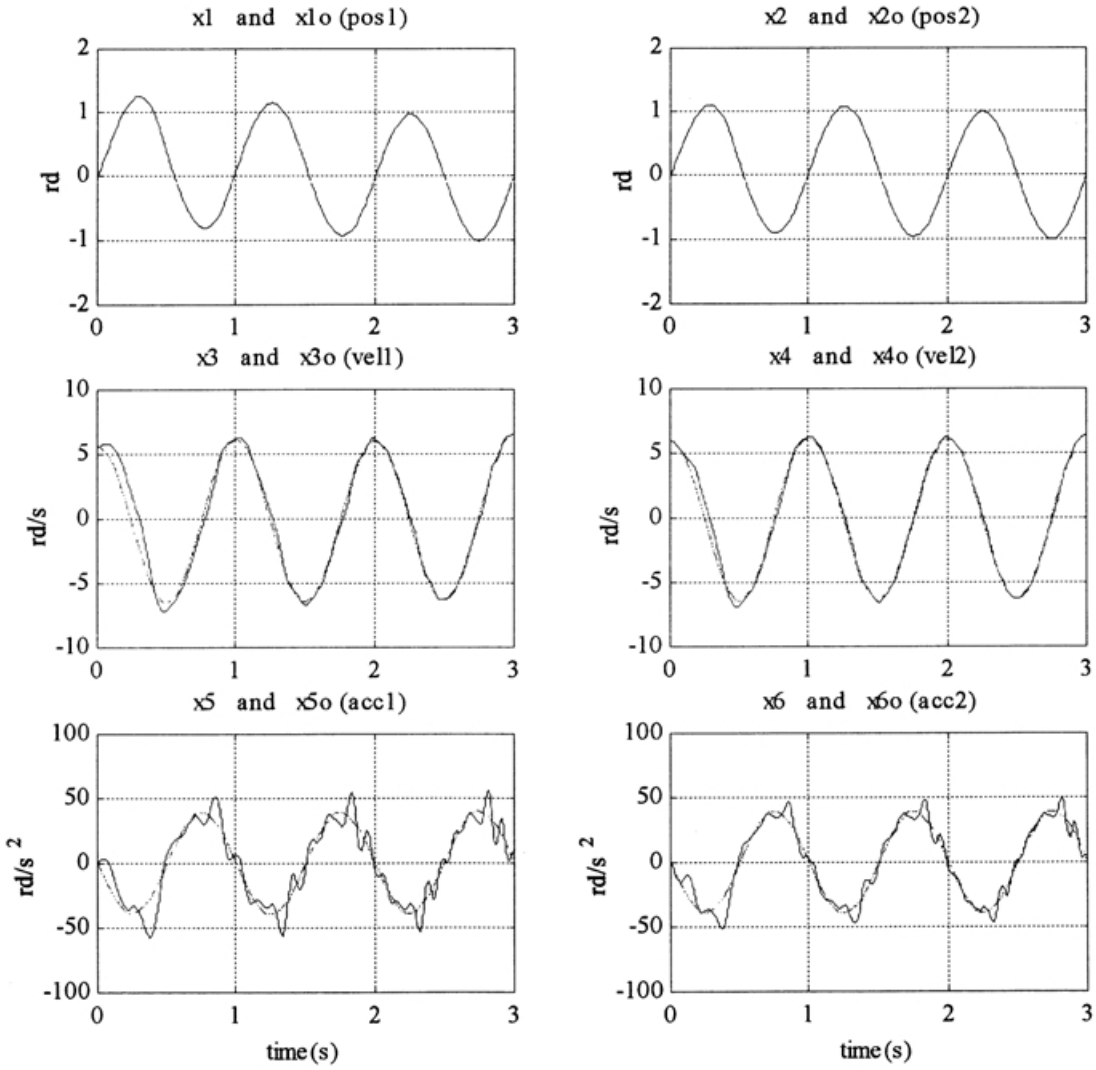


Fig. 1. Actual and observed state variables.

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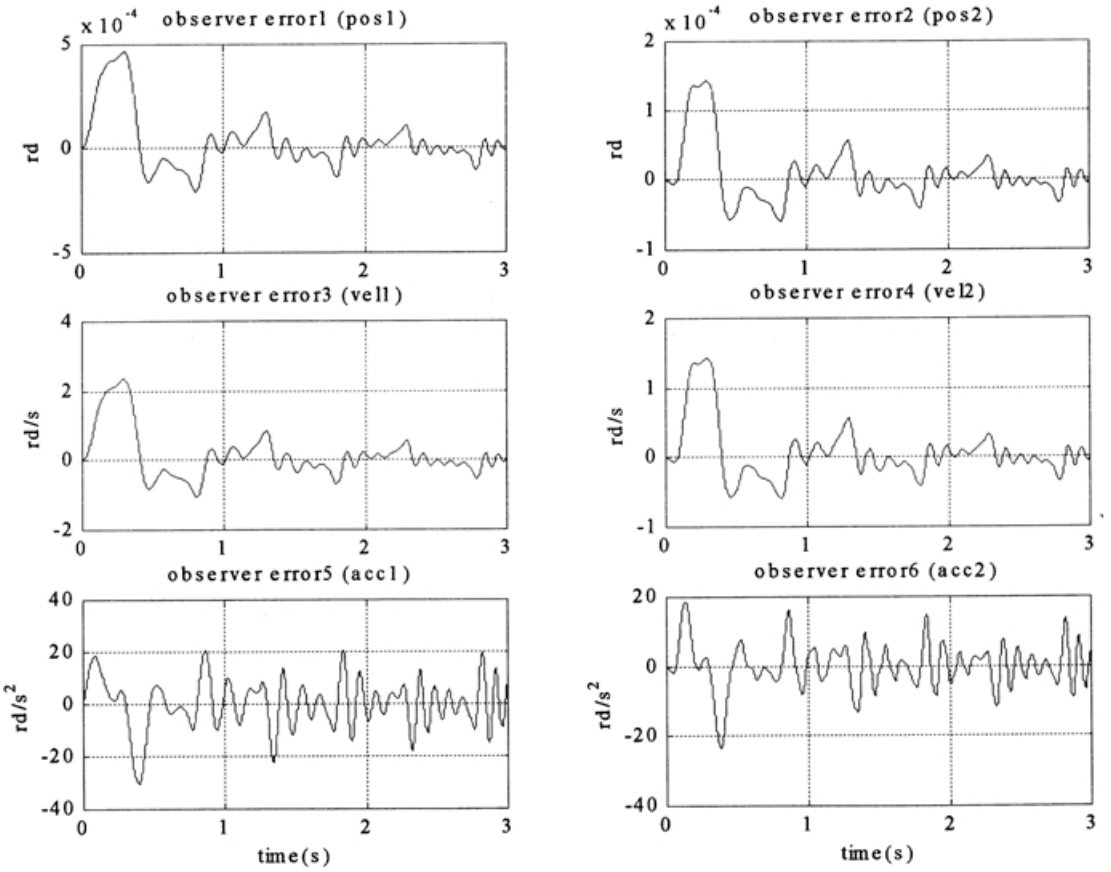


Fig. 2. Observer error.

5. STABILITY OF OBSERVER BASED CONTROL

At first, let's assume that the parameters α_1, α_2, g_0, J''_m, b_m, e_m and w_m are known.

In the analysis to follow, we need the following lemma

Lemma 1. *The following design matrix is chosen*:

$K_p=k_pI$, $K_v=k_vI$, $K_a=k_aI$; *with* k_p, k_v, $k_a>0$ *and* $k_vk_a>k_p$ *renders the matrix* F *asymptotically stable*.

Proof: See the Appendix.

The stability properties of the proposed scheme are summarized in the following theorem.

Theorem 2. *Given the system described by (1), the observer governed by equation (6) and the control law (10), (11) with* K_p, K_v *and* K_a *chosen so that* F *is a stable matrix* (*see lemma 1) and* $\hat{f}$ *given by (15), then the observer error (13) and the tracking error (16) are globally asymptotically stable.*

Proof: First, we show that the observer error $\tilde{x}_3(t)$ tend asymptotically to zero.

The following Lyapunov function is considered,

$$V_3=\frac{1}{2}\tilde{x}_3^TJ^{-1}M\tilde{x}_3$$

The time derivative of V_3 is given by:

$$\dot{V}_3=\tilde{x}_3^TJ^{-1}M\dot{\tilde{x}}_3+\frac{1}{2}\tilde{x}_3^TJ^{-1}\dot{M}\tilde{x}_3$$

$$=-\tilde{x}_3^TJ^{-1}M\Lambda\tilde{x}_3+\tilde{x}_3^T(f-\hat{f})-\tilde{x}_3^TJ^{-1}C\tilde{x}_3+\frac{1}{2}\tilde{x}_3^TJ^{-1}\dot{M}\tilde{x}_3$$

Let $Q=J^{-1}M\Lambda$ which is SPD. Then, on the sliding patch $\dot{V}$ can be upper bounded as

$$\dot{V}_3\le-\lambda_{\min}\{Q\}\|\tilde{x}_3\|^2+\tilde{x}_3^T(f-\hat{f})$$

$$\le-\lambda_{\min}\{Q\}\|\tilde{x}_3\|^2+\tilde{x}_3^T(f-\lambda_2b(x_2,\hat{x}_3,v)\,sgn(\tilde{x}_1))$$

$$\le-\lambda_{\min}\{Q\}\|\tilde{x}_3\|^2+\|\tilde{x}_3\|(b(x_2,\hat{x}_3,v)-b(\hat{x}_2,\hat{x}_3,v))$$

$$\le-\lambda_{\min}\{Q\}\|\tilde{x}_3\|^2$$

$$\le0$$

therefore, $\tilde{x}_3$ tend asymptotically to zero.

Now, to prove that the tracking error (16) is globally asymptotically stable we use the following lemma.[13]

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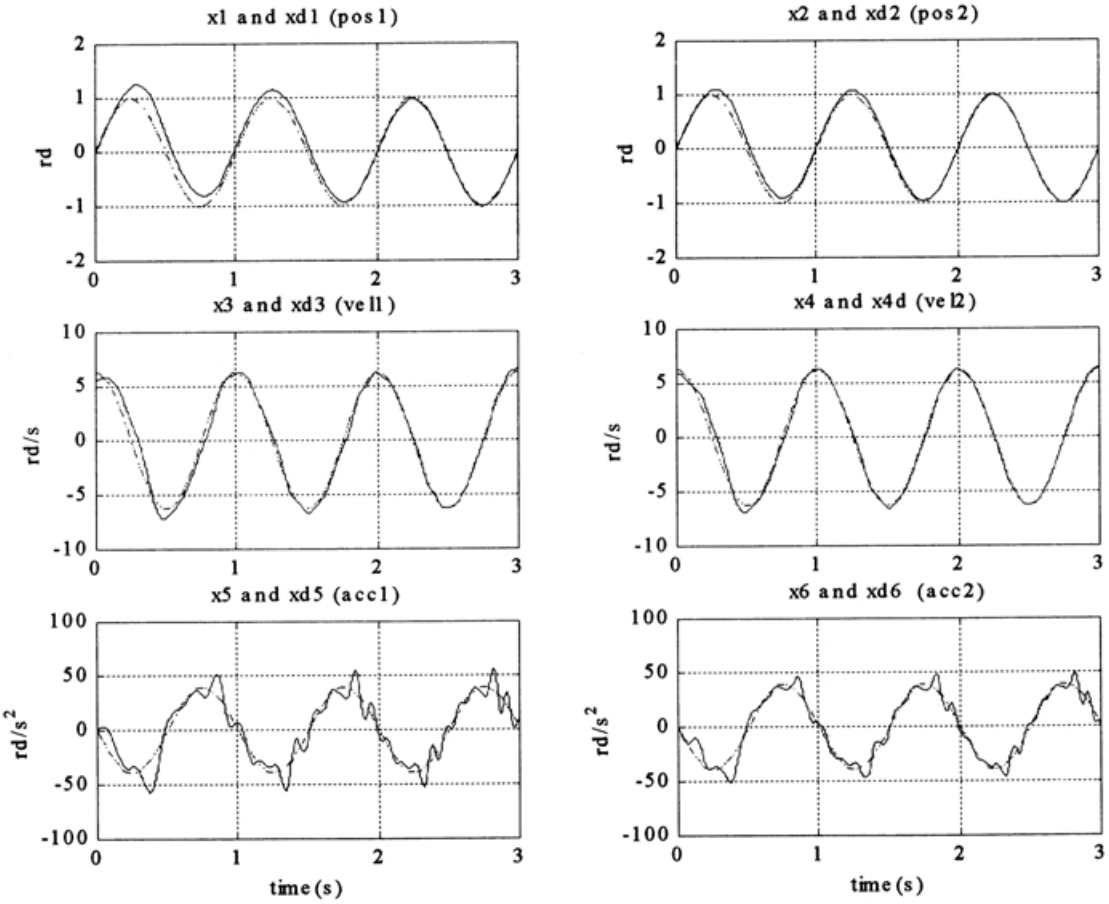


Fig. 3. Actual and desired state variables.

Lemma 3. *Let* $y=H(s)x$, *with* $H(s)$ *an* $(n\times m)$ *transfer matrix which is strictly proper and exponentially stable. Then, if* $x\in L_2^m$, *we have*

(*i*) $y\in L_2^n\cap L_\infty^n$, *and* $\dot{y}\in L_2^n$
(*ii*) y *is continuous, and* $y\to 0$ *as* $t\to\infty$.
(*iii*) *If* $x\to 0$ *as* $t\to$, *then* $\dot{y}\to 0$.

With K_p, K_v and K_a chosen such that the matrix F is asymptotically stable (lemma 1), the transfer matrix of system described by (16) is strictly proper and exponentially stable. Then, we can apply lemma 2 to conclude that the tracking error e tends asymptotically to zero.

6. ROBUST ADAPTIVE CONTROL AND OBSERVER

Sometimes the upper bounds on the norm of the matrices model may not be easily obtained because of the uncertainties and varying payload. Here, in order to establish $\hat{f}$ we propose the simple adaptation laws for these bounds.
The function $b(x_2, \hat{x}_3, v)$ defined in (14) can be written as

$$b(x_2, \hat{x}_3, v)=\Psi^T(x_2, \hat{x}_3, v)\theta$$

where

$$\Psi^T(x_2, \hat{x}_3, v)=(\|\hat{x}_3\|, \|\hat{x}_2\|, \|\hat{x}_3\|, \|\hat{x}_2\|^2, \|\hat{x}_2\|, 1, \|v\|)$$

and

$$\theta^T=(J_m''\alpha_1(w_m+b_m), J_m''\alpha_2, J_m''\alpha_2(w_m+b_m), J_m''e_m$$
$$J_m''g_0(w_m+b_m)\lambda_2+J_m''\alpha_1(w_m+b_m), J_m''\alpha_1+1)$$

The parameter vector θ is adjusted according to the following update law:

$$\dot{\hat{\theta}}=\lambda_2 L\Psi^T(x_2,\hat{x}_3,v)\|sgn(\tilde{x}_1)\| \tag{17}$$

with the gain L as a positive-definite symmetric matrix.
The adaptive version of $\hat{f}$ is then

$$\hat{f}=\lambda_2\Psi^T(x_2, \hat{x}_3, v)\hat{\theta}\, sgn(\tilde{x}_1) \tag{18}$$

Theorem 4. *Given the system described by (1), the observer governed by equation (6), the control law (10), (11) and the adaptive law (17), then with* K_p,K_v *and* K_a *chosen so that F is a stable matrix (see lemma 1) and* $\hat{f}$ *defined by (18), then the observer error (13) and the tracking error (16) are globally asymptotically stable.*

Proof: to show the convergence of observer error, we consider the following lyapunov function:

$$V_4=\frac{1}{2}\tilde{x}_3^T J^{-1}M\tilde{x}_3+\frac{1}{2}\tilde{\theta}^T L^{-1}\tilde{\theta}$$

with, $\tilde{\theta}=\hat{\theta}-\theta$ the estimation error.

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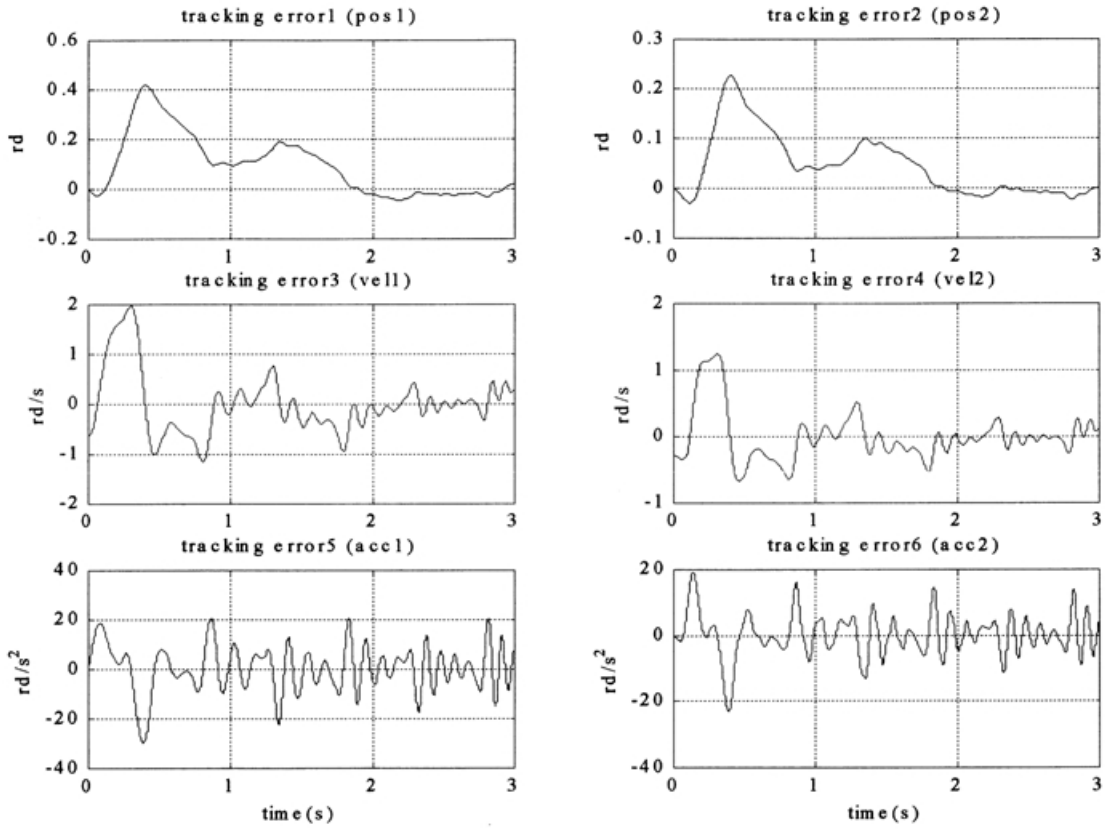


Fig. 4. Tracking error.

The time derivative of V_4 can be upper bounded as

$$\begin{aligned}\dot{V}_4 &\leq -\lambda_{\min}\{Q\}\|\tilde{x}_3\|^2+\tilde{x}_3^T(f-\hat{f})+\tilde{\theta}^T L^{-1}\dot{\hat{\theta}}\\ &\leq -\lambda_{\min}\{Q\}\|\tilde{x}_3\|^2+\|\tilde{x}_3\|\Psi^T(x_2,\hat{x}_3,v)\theta\\ &\quad -\tilde{x}_3^T\lambda_2\Psi^T(x_2,\hat{x}_3,v)\hat{\theta}\,sgn(\tilde{x}_1)+\tilde{\theta}^T\lambda_2\Psi^T(x_2,\hat{x}_3,v)\|sgn(\tilde{x}_1)\|\\ &\leq -\lambda_{\min}\{Q\}\|\tilde{x}_3\|^2\\ &\leq 0\end{aligned}$$

thus, the observer error $\tilde{x}_3$ tend asymptotically to zero and the estimation error $\tilde{\theta}$ is bounded.

The proof of the tracking error convergence is the same as the non adaptive case.

SIMULATION RESULTS

Simulations have been done to verify the proposed robust adaptive control and observer law. The feedback gains were chosen to be $k_p=10$, $k_v=5$ and $k_a=2{,}5$.

The obtained results confirm that the complete system, controller and observer, remains stable and shows a good estimation of the velocities and accelerations (Figure 1 and Figure 2). Furthermore, (Figure 3 and Figure 4) prove that the robust adaptive control and observer achieve good tracking. Indeed, they show the convergence of the real state towards the desired one. Note that we have also taken into account the limitations on the current of $\pm 20\ mA$.

CONCLUSION

In this paper a robust adaptive controller and observer based on the full nonlinear robot dynamics are applied for control and estimation of velocities and accelerations for a pneumatic leg of a robot. This approach does not require a precise knowledge of the robot model physical parameters, only the adaptive upper bounds on the norm of the matrices model are used. We have proved the convergence of the observer in a closed loop with the controller. Some simulation results have been reported which show that a reasonable performance is obtained.

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APPENDIX

Proof of Lemma 1: Let $C \in \mathcal{R}^{6\times 6}$ be composed of block matrices $C_1 \in \mathcal{R}^{4\times 4}$, $C_2 \in \mathcal{R}^{4\times 2}$, $C_3 \in \mathcal{R}^{2\times 4}$ and $C_4 \in \mathcal{R}^{2\times 2}$, and define $C = sI - F$ as

$$C = (sI - F) = \begin{pmatrix} sI & -I & 0 \\ 0 & sI & -I \\ k_pI & k_vI & (s+k_a)I \end{pmatrix}$$

$$\det(C) = -\det\begin{pmatrix} sI & -I & 0 \\ k_pI & k_vI & (s+k_a)I \\ 0 & sI & -I \end{pmatrix} = -\det\begin{pmatrix} C_1 & C_2 \\ C_3 & C_4 \end{pmatrix}$$

From the block determinant properties we have

$$\det(C) = -\det(C_4)\ .\ \det(C_1 - C_2C_4^{-1}C_3)$$

Identifying the C_i as

$$C_1 = \begin{pmatrix} sI & -I \\ k_pI & k_vI \end{pmatrix},\ C_2 = \begin{pmatrix} 0 \\ (s+k_a)I \end{pmatrix}$$

$$C_3 = (0 \quad sI),\ C_4 = -I$$

then,

$$\det(C) = \det(C_1 + C_2C_3)$$

$$= \det\begin{pmatrix} sI & -I \\ k_pI & (k_v + s(s+k_a))I \end{pmatrix}$$

and from the block determinant property, we have

$$\det(C) = (k_v + s(s+k_a))^2\ .\ \det\left(sI + \frac{k_pI}{k_v + s(s+k_a)}\right)$$

$$= (s^3 + k_as^2 + k_vs + k_p)^2$$

so that positive value for k_p, k_v and k_a are such that $k_v\ .\ k_a > k_p$ yield strictly stable eigenvalues on F.

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