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Material selection in engineering design based on nearest neighbor search under uncertainty: a spatial approach by harmonizing the Euclidean and Taxicab geometry

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Abstract

Material selection is a fundamental step in mechanical design that has to meet all the functional requirements of the component. Multiple-attributed decision-making (MADM) processes are already well established to choose the preeminent alternative from the finite set of alternatives, but there is some lack of geometrical evidence if the alternatives are considered as multi-dimensional points. In this paper, a fresh spatial approach is proposed based on nearest neighbor search (NNS) in which the nearness parameter is considered as a Manhattan norm (Taxicab geometry) in turn which is a function of the Euclidean norm and cosine similarity to raise a preeminent alternative under the MADM framework. Cryogenic storage tank and flywheel are considered as two case studies to check the validity of the proposed spatial approach based on NNS in material selection. The result shows the right choice for the cryogenic storage tank is the austenitic steel (SS 301 FH), and for the flywheel, it is a composite material (Kevler 49-epoxy FRP) those are consistent with the real-world practice and the results are compared with other MADM methods of previous works.

Introduction

It was the Bronze Age; the revolutionary invention was the wheel that begot a chariot. Through the passage of time and advancement of technology, the chariot entered into the Industrial Age. Now it has many varieties in terms of brand, the source of energy, body style, and features like safety, comfortability, and entertainment. As a whole, a wide range of multi-attributed alternatives are generated according to the customer requirements. In the engineering point of view, it is termed as product design and development (Otto and Wood, 2001; Zha *et al*., 2004). A successful product has to fulfill all the functional requirements and to be economical where the right choice of constituent material of a product from the wide range of materials takes the vital role (Pfeifer, 2009). The design is a sequential decision-making process that requires data where some degree of uncertainty always exists. The existence of a designer depends on the satisfaction of a customer. A designer and customer both are in the same situation of "which alternative" is to choose under uncertainty.

The aim of this paper is to investigate the "which" under uncertainty by implementing the Manhattan distance (Taxicab geometry) (Krause, 1986) as a choice parameter in multi-attributed decision-making (MADM) framework to select the preeminent material (Fig. 1).

Material selection

Material selection in machine design perspective can be delineated as the outcomes of the alternatives generation followed by the alternatives evaluation.

Alternatives generation

There is no one-way method to generate the alternatives. Significantly, from the Suh's design axioms (Suh, 1990), alternatives can be generated by mapping the functional requirements with the design parameters (Dieter, 1983). Ashby (1999) formalized the Suh's design axioms in a noetic way and proposed a chart known as Ashby's Chart to generate the potential materials under a specific condition. It can be regarded as a *satisficing* (Simon, 1988) model in the field of material selection in engineering design where he showed the performance of the machine elements is the function of the material properties. The chart is a log-log plot whose abscissa and ordinate are the material properties, but we are left in this chart with many alternatives or a set of potential alternatives. Due to the lack of information, uncertainty

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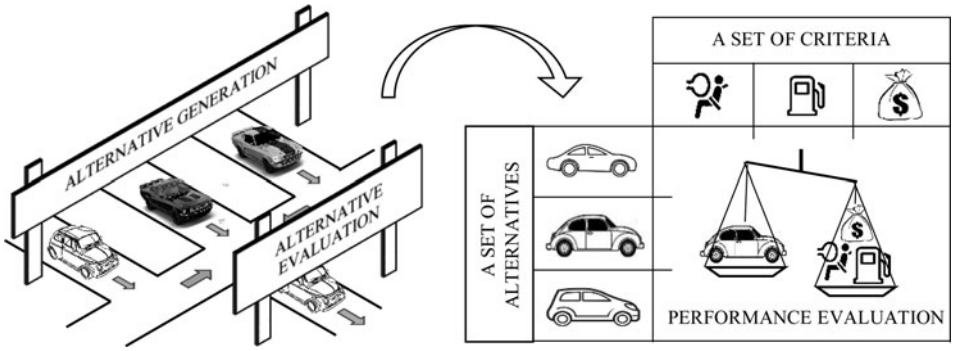


Fig. 1. Graphical multi-attributed decision-making framework.

takes place in the design formulation and thereby alternative generation in material selection. Hazelrigg (2003) proposed an axiomatic framework based on Neuman–Morgenstern utility axioms to evaluate the alternatives under uncertainty. It is known as decision-based design framework where the alternatives are evaluated by their expected utility values to ensure a rational choice (Wassenaar and Chen, 2003; Das *et al.*, 2016). However, a structured method is required, which is germane to the material selection under uncertainty to evaluate the alternatives and thereby to raise the preeminent one.

Alternatives evaluation

Alternatives are generally evaluated in the domain of multiple-criteria decision analysis (MCDA) where the alternatives are described in terms of evaluative criteria. MCDA methods are further categorized as:

- MADM where the decision space is explicitly known and discrete.
- Multi-objective decision-making (MODM) where the decision space is continuous.

The sole attention of this paper is about the MADM framework. MADM approaches are already well demonstrated that addresses the uncertainty by considering a set of alternatives with uncertain values for the attributes or criteria of the alternatives (Wallenius *et al.*, 2008; Shahinur *et al.*, 2017). The basic steps in all MADM approaches are generally considered as:

- *Decision matrix* is the discrete decision space where a finite set of alternatives is expressed by its performance ratings in multiple attribute or criteria.
- *Weightage* or priority is assigned to each criterion according to the design requirements to satisfy the customer's demands and desires. Analytical hierarchy process (AHP) (Mujgan *et al.*, 2004; Hu *et al.*, 2014), fuzzy AHP (Chan *et al.*, 2008), digital logic (DL), modified DL (Dehghan-Manshadi *et al.*, 2007), and entropy are some popular methods for priority distribution (Jahan *et al.*, 2012b).
- *Normalization* is the process by which the performance ratings measured in the different unit are converted into a common unit. Vector normalization based on Euclidean distance, standardization based on standard deviation, and feature scaling based on the difference between the best and worst values are widely used (Hatush and Skitmore, 1998; Opricovic and Gwo-Hshiung, 2004; Rao and Patel, 2010; Girubha and Vinodh, 2012).
- *Overall performance analysis* that assigns the overall performance rating to each alternative and the alternatives are ranked accordingly. According to the nature of the mathematical models used in the analysis, the MADA methods are named (Shanian *et al.*, 2008; Peng and Xiao, 2013; Mousavi-Nasab and Sotoudeh-Anvari, 2017).

Compiling the above-mentioned four steps, various MADM models are formed. There is an ample literature where varieties of MADM methods are discussed, modified, and applied in the field of material selection. Shahinur *et al.* (2017) judiciously introduced the fuzzy logic to select the material for vehicle body and the aluminum alloys are chosen as the best materials. Cryogenic storage tank materials are analyzed by WPM (Dehghan-Manshadi *et al.*, 2007), TOPSIS (Jahan *et al.*, 2012a), MOORA (Karande and Chakraborty, 2012), Fuzzy logic (Khabbaz *et al.*, 2009), and the preeminent material is considered as Austenitic steel (SS 301-FH). Flywheel materials are evaluated by TOPSIS (Jee and Kang, 2000), VIKOR, ELECTRE (Chatterjee *et al.*, 2009), and TOPSIS (Jahan *et al.*, 2012a) and the analyses show the best material is Kevler 49-epoxy FRP. Gear materials are divisioned by Extended PROMETHEE II (Chatterjee and Chakraborty, 2012), TOPSIS (Milani *et al.*, 2005), and MULTIMOORA (Hafezalkotob *et al.*, 2016) and raise the best material as Carburised steel. Metallic bipolar plate materials are interpreted by VIKOR (Rao, 2008) and TOPSIS (Shanian and Savadogo, 2006) and the optimal choice shows the Austenitic stainless steel 316. Meanwhile, with time and the advancement of technology, new materials are being used besides the traditional materials. For example, shape memory alloy (smart material) that can memorize the shape at various operating conditions and behave accordingly. Unlike traditional MADM approaches, Huang (2002) introduced a series of performance index charts under various operating conditions to select the preeminent material for actuators and suggested the NiTi popularly known as Nitinol as a good choice.

From the above discussion, it can be concluded that the TOPSIS and VIKOR are widely used. In most of the cases, different types of normalization process are suggested and implemented (Fayazbakhsh *et al.*, 2009), but the basic structure of overall performance analysis of the alternatives in all processes is same. Cables *et al.* (2016) proposed a method, reference ideal method where the only "ideal point" is considered as a reference point (in case of TOPSIS, there is an "ideal point" and a "negative ideal point") and the outcomes are compared with TOPSIS and VIKOR methods. Lourenzutti and Krohling (2014) introduced the Hellinger distance in TOPSIS (H-TOPSIS), whereas the original TOPSIS is based on Euclidean distance.

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Gap analysis and aim

From the above discussion, some of the methods (Lourenzutti and Krohling, 2014; Cables *et al.*, 2016) can be regarded to certain extent as a nearest neighbor search (NNS) approach (Barrena *et al.*, 2010). NNS is the finding of a point (a) in a given set which is nearer to a reference point or query point (q) in a multi-dimensional Euclidean space. In the above-mentioned TOPSIS-based methods, the closeness or nearness between two points is decided by a ratio that lies between 0 and 1 where the higher value dictates the close proximity. All the methods are structured with strong mathematical background, but if we consider the alternative as a point in the metric space, there is some lack of spatial relationship evidence from "closeness" point of view. From a mathematical point of view, the closeness between two points can be described as a distance parameter, $d(a, q) \to 0$ as $a \to q$ (Arkhangel'skii and Fedorchuk, 1990).

At this point, very specifically, the aim of this paper is to select the preeminent material in design based on NNS on the basis of $d(a, q) \to 0$ in the Cartesian plane under MADM framework through the eyes of TOPSIS. If the points (alternatives) are mapped in the two-dimensional plane, then we can easily visualize and compare the points with the query point. The designers rather enjoy the spatial relationship which gives a confidence and courage under uncertainty.

Proposed NNS-based material selection in the Cartesian plane

The NNS is generally pronounced in the field of machine learning, pattern recognition, geometrical information systems, and many others (Papadopoulos and Manolopoulos, 2005; Moghtadaiee and Dempster, 2015). It deals with spatial data in an effective and efficient way. The data can be characterized as points or lines or higher entities oriented in two-dimensional or multi-dimensional space. The scenario is the same in nature in the sense of material selection where the material is considered as an n-dimensional point in the above-mentioned MADM methods. There is a wide range of NNS methods.

The proposed approach does not consider the traditional NNS methods, but the basic definition of the NNS, that is, searching the nearness of a set of points ($a_{i=1,2,\ldots,m}$) to a query point (q) by means of a distance parameter in n-dimensional Euclidean space ($a_i, q \in \mathbb{R}^n$) shown in Figure 2. From the material selection point of view, the absolute nearness or similarity between a_i (alternatives) and q (reference alternative) depends upon the length of the position vectors ($|\overrightarrow{oa_i}|, |\overrightarrow{oq}|$) and the angle between the vectors (θ_i). Cosine similarity is a popular approach (Kou and Lin, 2014; Xia *et al.*, 2015) to find the θ_i that investigates the similarity among alternatives and given by,

$$\cos\theta_i = \frac{\overrightarrow{oa_i} \cdot \overrightarrow{oq}}{|\overrightarrow{oa_i}| \cdot |\overrightarrow{oq}|} \qquad (i = 1, 2, \ldots, m). \tag{1}$$

Therefore, the comparison between a_i and q is the function of the length and orientation of the alternatives and should be mapped in the Cartesian plane ($(|\overrightarrow{oa_i}|, \theta_i, |\overrightarrow{oq}|) : \mathrm{R}^n \to \mathrm{R}^2$) in respect of the query vector to investigate the exact nearness between a_i and q by means of the distance parameter (Fig. 2). Actually, these distance parameters are the dissimilarity functions of the alternatives and the less value of the dissimilarity function dictates the choice which is very nearer to the query point. Now the question is that which distance parameter should be considered, Euclidean distance (l_2 norm) or Manhattan distance (l_1 norm). In Cartesian plane, the Manhattan distance is the sum of the projected length of a Euclidean distance along the axes. The proximity of any two points depends upon the adjustment of the coordinates along the axes which preserve the absolute similarity information. Therefore, our decision favors the Manhattan distance. From the Cartesian plane in Figure 2, the coordinates of a_i and q are $a_i = (|\overrightarrow{oa_i}| \cos(\theta_i + \varphi), |\overrightarrow{oa_i}| \sin(\theta_i + \varphi))$ and $q = (|\overrightarrow{oq}| \cos\varphi, |\overrightarrow{oq}| \sin\varphi)$. The Manhattan distance between a_i and q is given by,

$$d_{\mathrm{M}}(a_i, q) = \overline{a_i b_i} + \overline{b_i q} \qquad i = 1, 2, \ldots, m, \tag{2}$$

where

$$\overline{a_i b_i} = ||\overrightarrow{oq}| \cos\varphi - |\overrightarrow{oa_i}| \cos(\theta_i + \varphi)|, \tag{3}$$

$$\overline{b_i q} = ||\overrightarrow{oq}| \sin\varphi - |\overrightarrow{oa_i}| \sin(\theta_i + \varphi)|. \tag{4}$$

The most favorable orientation of $|\overrightarrow{oq}|$ in Cartesian plane is the equi-inclination with the axes, that is, $\varphi = 45^\circ$ that gives the optimal position of q and the maximum equivalent Manhattan distance of the length $|\overrightarrow{oq}|$ ($d_{\mathrm{M}}(o, q) = \overline{oh} + \overline{hq}$). In the Cartesian plane (Fig. 2), the line $\overline{oa_i}$ can be either any side of $\overline{oq}$ and θ_i may or may not be greater or less than 45°, but the ultimate Manhattan distance ($d_{\mathrm{M}}(a_i, q)$) will be the same in all cases and the generalized Manhattan distance between a_i and q following the expressions (2) to (4) is given by,

$$\begin{aligned} d_{\mathrm{M}}(a_i, q) = \frac{1}{\sqrt{2}}\{&||\overrightarrow{oq}| - |\overrightarrow{oa_i}|(\cos\theta_i - \sin\theta_i)| \\ &+ ||\overrightarrow{oq}| - |\overrightarrow{oa_i}|(\cos\theta_i + \sin\theta_i)|\}. \end{aligned} \tag{5}$$

In the Cartesian plane (Fig. 2), the line $\overline{oa_i}$ can be either any side of $\overline{oq}$ and θ_i can be $>45^\circ$, but the ultimate Manhattan distance ($d_{\mathrm{M}}(a_i, q)$) will be the same. The minimum value of the expression (5) addresses the preeminent alternative.

Implementation of the proposed model in the MADM framework

A set of alternatives (A) with performance ratings (D) is decided according to the design requirements. Performance is the measure of effectiveness in the form of attributes (E). Attributes are set according to the demands and desires of the design requirements and can be split up in benefit and cost attributes. The steps are followed to capture the preeminent alternative:

Step 1: A set of alternatives with performance ratings in the form of attributes

$$A = \{a_i | i = 1, 2, \ldots, m\}, \tag{6}$$

$$E = \left\{ e_j | \underbrace{j = 1, 2, \ldots, n - r}_{\text{benefit attribute}}; \underbrace{j = n - r + 1, \ldots, n}_{\text{cost attribute}} \right\}, \tag{7}$$

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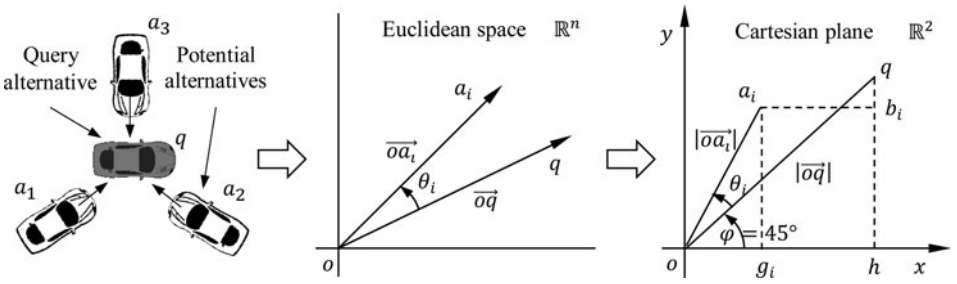


Fig. 2. Spatial representation of alternatives and choice parameter.

$$D = \{d_{ij} | i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n\}. \tag{8}$$

Step 2: Weightage to the attributes

$$W = \{w_j | j = 1, 2, \ldots, n\}. \tag{9}$$

Step 3: Normalization of the performance rating matrix

$$r_{ij} = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}} \qquad j = 1, 2, \ldots, n. \tag{10}$$

Step 4: Weighted normalization of the performance rating matrix

$$x_{ij} = r_{ij} \cdot w_j = \frac{d_{ij}}{\sqrt{\sum_{i=1}^{m} d_{ij}^2}} \cdot w_j \qquad j = 1, 2, \ldots, n. \tag{11}$$

Step 5: Query alternative with performance ratings

$$\begin{aligned} q = \{&y_j(\max x_{ij}) | j = 1, 2, \ldots, n - r;\\ &y_j(\min x_{ij}) | j = n - r + 1, \ldots, n\}. \end{aligned} \tag{12}$$

Step 6: Mapping of the alternatives and query alternative in Euclidean space

$$\overrightarrow{oa_i} = \sum_{j=1}^{n} x_{ij} \cdot \hat{e}_j \qquad i = 1, 2, \ldots, n, \tag{13}$$

$$|\overrightarrow{oa_i}| = \sqrt{\sum_{j=1}^{n} x_{ij}^2} \qquad i = 1, 2, \ldots, n, \tag{14}$$

$$\overrightarrow{oq} = \sum_{j=1}^{n} y_j \cdot \hat{e}_j \qquad i = 1, 2, \ldots, n, \tag{15}$$

$$|\overrightarrow{oq}| = \sqrt{\sum_{j=1}^{n} y_j^2}. \tag{16}$$

Step 7: Cosine similarity following the expressions (1), (14), and (16)

$$\cos\theta_i = \frac{\sum_{j=1}^{n} x_{ij} \cdot y_j}{\sqrt{\sum_{j=1}^{n} x_{ij}^2} \cdot \sqrt{\sum_{j=1}^{n} y_j^2}} \qquad i = 1, 2, \ldots, m, \tag{17}$$

$$\sin\theta_i = \sqrt{1 - \cos^2\theta_i} \qquad i = 1, 2, \ldots, m\,. \tag{18}$$

Step 8: Calculation of Manhattan distance following the expressions (2)–(4), (14), (16)–(18) or directly from the expressions (5), (14), (16)–(18)

$$\begin{aligned} d_M(a_i, q) = \frac{1}{\sqrt{2}}\{&||\overrightarrow{oq}| - |\overrightarrow{oa_i}|(\cos\theta_i - \sin\theta_i)|\\ &+||\overrightarrow{oq}| - |\overrightarrow{oa_i}|(\cos\theta_i + \sin\theta_i)|\}\,. \end{aligned} \tag{19}$$

The alternatives are ranked according to the minimum value of the Manhattan distance ($d_M(a_{i=1,2,\ldots m}, q)$). The steps followed and the respective input/output of the proposed method are shown in Figure 3.

Case study 1: cryogenic storage tank

The cryogenic storage tank is also referred to cryogenic storage dewar (named after James Dewar, who first introduced liquid hydrogen and oxygen storage) is basically a double-walled vacuum/super-insulators vessel shown in Figure 4. It carries the liquid nitrogen, oxygen, hydrogen, argon, and helium gas with <110 K/−163 °C temperature. Liquid hydrogen has already been accepted as an excellent energy source. It is non-toxic and extremely environmentally benign as water is only the exhaust product when it is converted into the energy. The choice of material in cryogenic tank design is generally dictated by safety and economy. In case of cryogenic tank, the main issue is the safety factors and the design requirements under the consideration of low-temperature embrittlement can be stated as (Flynn, 2005; Farag, 2014):

- *Fracture toughness*: The boiling temperature of liquid nitrogen gas is about −196°C or liquid hydrogen gas is about −253°C. At this temperature, the materials lose their ductile behavior and become brittle in nature. Therefore, materials should be sufficiently tough to resist the brittle fracture. The metals having face-centered cube (fcc) lattice are suitable because of their insensitiveness at low temperature. Copper, nickel, all of the copper–nickel alloys, aluminum and its alloys, and the austenitic stainless steels that contain more than approximately 7% nickel are suitable for designing any cryogenic storage tank (Flynn, 2005).
- *Heat transfer*: The flow of heat through the wall of the cryogenic tank is generally conduction type. Materials having low thermal conductivity are preferable.
- *Thermal stress*: Due to the low temperature, the inner wall is subjected to a contraction that causes thermal stresses. Therefore, materials having a low coefficient of thermal expansion are suitable.

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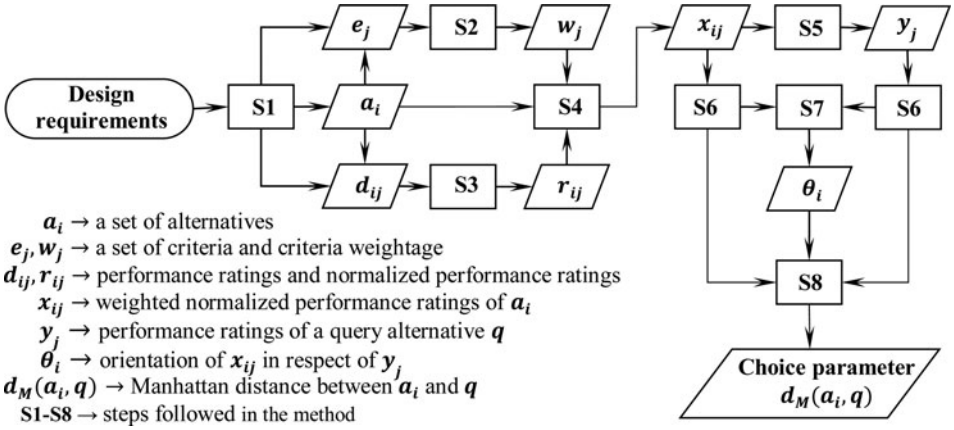


Fig. 3. Nearest neighbor search-based material selection flow diagram.

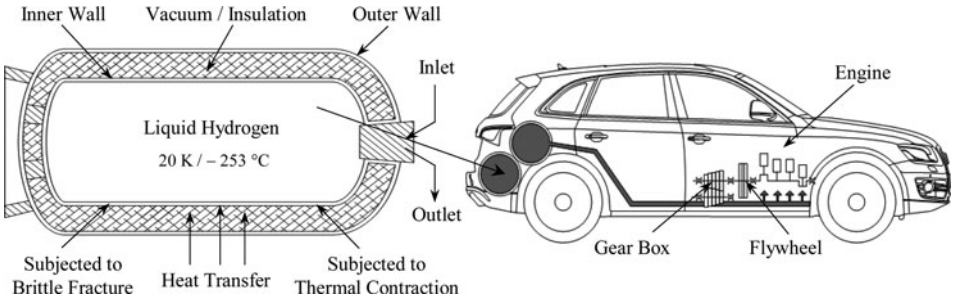


Fig. 4. Cryogenic storage tank and application.

- *Thermal diffusivity*: Perfect thermal insulation is not possible practically. Materials should be chosen that they can release the heat as quickly as possible. Diffusivity measures the rate of transfer of heat, which is inversely proportional to the specific heat of the materials.
- *Transportation*: Materials having lower specific gravity is suitable for transportation.

From the above-mentioned discussion and MADM point of view, all attributes can be categorized as:

- *Benefit attribute*: Toughness index (e_1); yield stress (e_2); Young's modulus (e_3);
- *Cost attribute*: Specific gravity (e_4); coefficient of thermal expansion (e_5); thermal conductivity (e_6); specific heat (e_7).

Following the expressions (6)–(8), some of the candidate materials under uncertainty having fcc lattice are tabulated in Table 1 with their performance ratings. The relative weights to the attributes in expression (9) are assigned following the previous works (Dehghan-Manshadi *et al.*, 2007; Jahan *et al.*, 2012a, b; Karande and Chakraborty, 2012) and given by,

$$w_{j=1,2,\ldots,n} = [0.28,\ 0.14,\ 0.05,\ 0.24,\ 0.19,\ 0.05,\ 0.05]. \quad (20)$$

The weighted normalized performance ratings are tabulated in Table 2 following the expressions (10)–(11) and (20). From Table 2 and following the expression (12), the query alternative with performance ratings is given by,

$$q = [0.2401,\ 0.0813,\ 0.0269,\ 0.0369,\ 0.0394,\ 0.0012,\ 0.0106]. \quad (21)$$

The overall performances of the alternatives are evaluated through the expressions (13)–(19) and (21) and tabulated in Table 3.

Case study 2: flywheel

The flywheel is an energy storage device used to maintain the minimum fluctuation of speed of a machine. Energy density, that is, ratio of the energy (e) and mass (m) is an important parameter used to qualify the energy storage device. The energy density is proportional to the ratio between maximum stress (σ_u) the material can withstand and its density (ρ) (Genta, 1985; Ashby, 1999) given by,

$$\frac{e}{m} = K \cdot \frac{\sigma_u}{\rho}, \quad (22)$$

where K is known as the "shape factor" that depends on the geometrical configuration of the flywheel, and the choice of flywheel material depends on the specific strength (σ_u/ρ). Composite materials are widely used due to its excellent specific strength. As the flywheel is subjected to the cyclic loading, the fatigue strength (σ_{limit}) is considered instead of ultimate strength (σ_u) (Jee and Kang, 2000; Chatterjee *et al.*, 2009).

Another important phenomenon is about the burst containment which is associated with the rotor. All materials have a fatigue life when they are subjected to cyclic loading or may have inconsistencies in the structure and the fracture may take place even below the strength limit. In some literature (Jee and Kang, 2000), this fracture phenomenon is addressed by the ratio (K_{1c}/ρ) of fracture toughness (K_{1c}) and density (ρ), but there is no clear explanation of choosing density in fracture toughness. According to Griffith crack theory, the critical strain energy release rate (G_{1c}) that propagates the crack is given by (Broek, 1984),

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Table 1. Performance ratings matrix (d_{ij}) of case study 1

Candidate material ($a_{i=1,2,\ldots,7}$)		Toughness index e_1 ($w_1 = 0.28$)	Yield stress (MPa) e_2 ($w_2 = 0.14$)	Young's modulus (GPa) e_3 ($w_3 = 0.05$)	Specific gravity e_4 ($w_4 = 0.24$)	Coeff. of thermal exp. (10^{-6}/°C) e_5 ($w_5 = 0.19$)	Thermal conductivity (cal/cm^2/cm/°C/s) e_6 ($w_6 = 0.05$)	Specific heat (cal/g/°C) e_7 ($w_7 = 0.05$)
a_1	Al 2014 – T6	75.50	420.00	74.20	2.80	21.40	0.37	0.16
a_2	Al 5052 – O	95.00	91.00	70.00	2.68	22.10	0.33	0.16
a_3	SS 301 – FH	770.00	1365.00	189.00	7.90	16.90	0.04	0.08
a_4	SS 310–3/4H	187.00	1120.00	210.00	7.90	14.40	0.03	0.08
a_5	Ti–6Al – 4V	179.00	875.00	112.00	4.43	9.40	0.02	0.09
a_6	Inconel 718	239.00	1190.00	217.00	8.51	11.50	0.31	0.07
a_7	70Cu – 30Zn	273.00	200.00	112.00	8.53	19.90	0.29	0.06

Table 2. Weighted normalized performance ratings matrix (x_{ij}) of case study 1

Candidate material		e_1 (max)	e_2 (max)	e_3 (max)	e_4 (min)	e_5 (min)	e_6 (min)	e_7 (min)
a_1	Al 2014 – T6	0.0235	0.0250	0.0092	0.0385	0.0897	0.0283	0.0282
a_2	Al 5052 – O	0.0296	0.0054	0.0087	0.0369	0.0926	0.0252	0.0282
a_3	SS 301 – FH	0.2401	0.0813	0.0234	0.1086	0.0708	0.0031	0.0141
a_4	SS 310–3/4H	0.0583	0.0667	0.0261	0.1086	0.0603	0.0023	0.0141
a_5	Ti–6Al – 4V	0.0558	0.0521	0.0139	0.0609	0.0394	0.0012	0.0159
a_6	Inconel 718	0.0745	0.0709	0.0269	0.1170	0.0482	0.0237	0.0123
a_7	70Cu – 30Zn	0.0851	0.0119	0.0139	0.1173	0.0834	0.0221	0.0106

Table 3. Overall performance evaluation of case study 1

Candidate material		$\lvert\overrightarrow{oa_i}\rvert$	$\overrightarrow{oa_i} \cdot \overrightarrow{oq}$	$\lvert\overrightarrow{oq}\rvert$	$\cos\theta_i$	$\sin\theta_i$	$\overline{a_i b_i}$	$\overline{b_i q}$	$d_M(a_i, q)$	Rank
a_1	Al 2014 – T6	0.1113	0.0132	0.2608	0.4547	0.8906	0.2187	0.0785	0.2972	6
a_2	Al 5052 – O	0.1111	0.0131	0.2608	0.4521	0.8920	0.2190	0.0788	0.2978	7
a_3	SS 301 – FH	0.2861	0.0718	0.2608	0.9623	0.2720	0.0448	0.0653	0.1101	1
a_4	SS 310–3/4H	0.1555	0.0267	0.2608	0.6584	0.7527	0.1948	0.0293	0.2241	4
a_5	Ti–6Al – 4V	0.1074	0.0220	0.2608	0.7854	0.6190	0.1718	0.0778	0.2496	5
a_6	Inconel 718	0.1674	0.0308	0.2608	0.7055	0.7087	0.1848	0.0170	0.2018	2
a_7	70Cu – 30Zn	0.1700	0.0295	0.2608	0.6654	0.7465	0.1942	0.0147	0.2089	3

$$\left.\begin{aligned} G_{1c} &= \frac{\pi\sigma_c^2 a}{E} \\ \text{with } K_{1c} &= \sigma_c \cdot \sqrt{\pi a} \end{aligned}\right\} \overset{\text{yields}}{\rightarrow} G_{1c} = \frac{K_{1c}^2}{E}, \tag{23}$$

where a is the half crack length ($2a$), σ_c is the critical stress at which the fracture takes place and K_{1c} is known as the critical stress intensity factor or popularly fracture toughness. We would like to consider K_{1c} as a material selection attribute instead of K_{1c}/ρ. If the fracture happens, the wheel breaks up into the number of fragments and the kinetic energy of the wheel is transferred to the surrounding through these fragments that cause the damage to the system. In case of monolithic metal, the number of fragments is three or four, whereas for composite materials, there are many tiny fragments. On that ground, the composite materials are the better choice. One of the popular materials for the flywheel is Kevlar but it is costly. However, following the previous literature (Jee and Kang, 2000), the specific strength (σ_{limit}/ρ), fracture toughness to density ratio (K_{1c}), fragmentability, and cost are considered as material attributes for the flywheel. All attributes can be summarized as:

- *Benefit attribute*: specific strength (e_1); fracture toughness (e_2); fragmentability (e_3);
- *Cost attribute*: cost (e_4).

Some of the potential materials under uncertainty with their performance ratings are tabulated in Table 4 where basically Al alloys, Ti alloys, and Composite materials have been considered. Some material attributes are rather ordinal like fragmentability, creep resistance, or corrosion resistance and can be converted

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Table 4. Performance ratings matrix (d_{ij}) of case study 2

Candidate material ($a_{i=1,2,\ldots,7}$)		Specific strength, σ_{limit}/ρ (MJ/kg) e_1 ($w_1 = 0.4$)	Fracture toughness, K_{1c} (MPa $m^{1/2}$) e_2 ($w_2 = 0.3$)	Fragmentability e_3 ($w_3 = 0.1$)	Cost (US$/ton) e_4 ($w_4 = 0.2$)
a_1	300M	100.00	68.90	Poor (2)	4200
a_2	2024 – T3	49.65	38.00	Poor (2)	2100
a_3	7050 – T73651	78.00	35.40	Poor (2)	2100
a_4	Ti–6Al – 4V	108.88	123.00	Poor (2)	10,500
a_5	E glass – epoxy FRP	70.00	20.00	Excellent (5)	2735
a_6	S glass – epoxy FRP	165.00	50.00	Excellent (5)	4095
a_7	Carbon – epoxy FRP	440.25	35.00	Fairly good (4)	35,470
a_8	Kevlar 29 – epoxy FRP	242.86	40.00	Fairly good (4)	11,000
a_9	Kevlar 49 – epoxy FRP	616.44	50.00	Fairly good (4)	25,000
a_{10}	Boron – epoxy FRP	500.00	46.00	Good (3)	315,000

Table 5. Weighted normalized performance ratings matrix (x_{ij}) of case study 2

Candidate material		e_1 (max)	e_2 (max)	e_3 (max)	e_4 (min)
a_1	300M	0.0411	0.1139	0.0180	0.0026
a_2	2024 – T3	0.0204	0.0628	0.0180	0.0013
a_3	7050 – T73651	0.0321	0.0585	0.0180	0.0013
a_4	Ti–6Al – 4V	0.0448	0.2034	0.0180	0.0066
a_5	E glass – epoxy FRP	0.0288	0.0331	0.0451	0.0017
a_6	S glass – epoxy FRP	0.0679	0.0827	0.0451	0.0026
a_7	Carbon – epoxy FRP	0.1811	0.0579	0.0361	0.0223
a_8	Kevlar 29 – epoxy FRP	0.0999	0.0661	0.0361	0.0069
a_9	Kevlar 49 – epoxy FRP	0.2536	0.0827	0.0361	0.0157
a_{10}	Boron – epoxy FRP	0.2057	0.0761	0.0271	0.1979

into cardinal by following the five-point *Likert scale* (Likert, 1932) as: very weak – 1; weak – 2; average – 3; good – 4; excellent – 5. The priority to the attributes is assigned following the previous works (Jee and Kang, 2000; Chatterjee *et al.*, 2009) and given by,

$$w_{j=1,2,\ldots,n} = [0.4,\ 0.3,\ 0.1,\ 0.2]. \tag{24}$$

Following the same steps as case study1, the weighted normalized performance ratings are tabulated in Table 5 and the query alternative with performance ratings is given by,

$$q = [0.2536,\ 0.2034,\ 0.0451,\ 0.0013]. \tag{25}$$

The overall performances of the alternatives are systematized in Table 6 according to steps 6–7 formalized in the proposed framework.

Results and discussion

In this analysis, the material properties are considered as discrete and average, but these are rather variable in nature. The true value cannot be known precisely (aleatory uncertainty). As the material selection is initiated at the conceptual stage of the design process, there is a further scope to analyze the suitability of the chosen material at the embodiment stage.

Cryogenic storage tank: For low temperature, <−163°C, the fcc materials are widely used. The austenitic steel SS 301 – FH is ranked 1 material in Table 3, which is a consistent result with the previous works and real-world practice. Austenitic steel still now is very popular and widely used liquid nitrogen or hydrogen storage tank (Godula-Jopek *et al.*, 2012). Now it is the time to check the consistency of the ranking of the materials in Table 3 with previous works as:

Proposed method with vector normalization: 6-7-1-4-5-2-3;
MOORA with linear normalization (Karande and Chakraborty, 2012): 7-6-1-4-5-3-2;
WPM with non-linear normalization (Dehghan-Manshadi *et al.*, 2007): 5-7-1-4-2-3-6;
TOPSIS with target-based normalization (Jahan *et al.*, 2012a, b): 4-5-1-6-2-3-7.

Sometimes the second option is very important in the ranking. In Dehghan-Manshadi *et al.* (2007); Jahan *et al.* (2012a, b), the titanium alloy (Ti-6Al-4V) got second rank. Ti-6Al-4V is excellent in the aerospace industry, but for low-temperature embrittlement cases, titanium alloys are very weak (Flynn, 2005), whereas Inconel is a good alternative than titanium alloy as a cryogenic storage tank material.

Flywheel: The result in Table 6 gives some interesting points and a clear delineation of past and present. As the technology is changing rapidly, new challenging alternatives are taking place in the market, despite being costly a designer or decision maker bends toward the composite materials due to its excellent specific strength. The ranking in Table 6 is compared with the previous works as:

Proposed method: 7-9-8-3-10-6-2-4-1-5;
TOPSIS (Jee and Kang, 2000): 5-9-7-6-8-3-4-2-1-10;
ELECTRE-II (Chatterjee *et al.*, 2009): 10-9-8-6-7-3-2-4-1-5;
VIKOR (Chatterjee *et al.*, 2009): 9-10-8-6-7-5-2-4-1-3.

There is no confusion about ranked 1 material, Kevlar 49 due to its excellent specific strength and consistent with the real-world practice (Genta, 1985). Carbon-epoxy FRP also holds the second

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Table 6. Overall performance evaluation of case study 2

Candidate material		$\lvert\overrightarrow{oa_i}\rvert$	$\overrightarrow{oa_i}\cdot\overrightarrow{oq}$	$\lvert\overrightarrow{oq}\rvert$	$\cos\theta_i$	$\sin\theta_i$	$\overline{a_ib_i}$	$\overline{b_iq}$	$d_M(a_i, q)$	Rank
a_1	300M	0.1224	0.0344	0.3282	0.8563	0.5165	0.2027	0.1132	0.3159	7
a_2	2024 – T3	0.0685	0.0188	0.3282	0.8362	0.5484	0.2181	0.1650	0.3831	9
a_3	7050 – T73651	0.0691	0.0209	0.3282	0.9216	0.3881	0.2060	0.1681	0.3741	8
a_4	Ti–6Al – 4V	0.2092	0.0536	0.3282	0.7807	0.6249	0.2090	0.0241	0.2331	3
a_5	E glass – epoxy FRP	0.0629	0.0161	0.3282	0.7799	0.6259	0.2252	0.1695	0.3947	10
a_6	S glass – epoxy FRP	0.1161	0.0361	0.3282	0.9474	0.3200	0.1805	0.1280	0.3085	6
a_7	Carbon – epoxy FRP	0.1948	0.0594	0.3282	0.9291	0.3698	0.1550	0.0531	0.2081	2
a_8	Kevlar 29 – epoxy FRP	0.1253	0.0404	0.3282	0.9824	0.1868	0.1616	0.1285	0.2901	4
a_9	Kevlar 49 – epoxy FRP	0.2696	0.0828	0.3282	0.9358	0.3525	0.1209	0.0135	0.1344	1
a_{10}	Boron – epoxy FRP	0.2967	0.0691	0.3282	0.7096	0.7046	0.2310	0.0646	0.2956	5

position in all methods. Composite materials reinforced with glass fiber, aramid, or carbon are similar in specific strength point of view and the choice among them differs by their cost mainly.

It is the versatility of the MADM approaches blended with mathematics and cognitions that different decision-making methods yield different rankings considering the same input data. Material selection takes place at the earliest stage of the design process when there is a lack of information and variability in the available information in the decision space. Under these circumstances, the proposed method gives a confidence in visualizing (geometrical) point of view and courage in ranking consistency point of view as compared with other rankings.

Conclusions

Engineering design is the judicious trade-off among shape, materials, and manufacturing that requires a wide range of decisions. Decision-making in engineering design allocates all the resources optimally while fulfilling the design objectives within economic constraints, quality constraints, safety constraints, environmental constraints, etc., under uncertainty.

To ensure an optimal choice, the problem should be precisely structured according to the decision requirements. In this paper, a simplified spatial approach is proposed to select the preeminent material by Manhattan distance along with the cosine similarity. In this approach, we can visualize the impact of the selection process. One may conclude, why not directly the Manhattan distance without considering the cosine similarity as given by,

$$d_M(a_i, q) = \sum_{j=1}^{n} |x_{ij} - y_j| \qquad i = 1, 2, \ldots m. \tag{26}$$

In this case, for example, a cryogenic storage tank, the ranking will be changed as 6-7-1-4-2-3-5, but the rank 1 (austenitic steel SS 301 – FH) will be unchanged and the titanium alloy (Ti-6Al-4V) holds the rank 2. Whatever the methods, if the methods have definite logic, the fittest materials will be always survived, but there should have some consistency among the rankings inside a particular method. Inconel is much better than titanium alloy in terms of all low-temperature embrittlement attributes. Therefore, in the choice analysis, the cosine similarity parameter takes the vital role to address the similarity with the reference or query alternative.

Decision-making is the process to choose an appropriate alternative based on the belief of the decision maker and the quality of a decision depends on the justifications for that belief. In the proposed method, a spatial relationship is introduced to make the belief as the justified belief that gives us a confidence and courage to select a preeminent alternative under uncertainty.

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