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THE OUTPUT EULER EQUATION AND REAL INTEREST RATE REGIMES

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Output Euler equations (OEE) for the US deliver slope estimates that are not significantly different from zero. This finding is counterintuitive as it implies a zero elasticity of intertemporal substitution (EIS) and aggregate demand movements that are nonresponsive to the short-term real interest rate. This paper shows that failure to account for regime changes in the dynamics of the real interest rate is responsible for this result. Based on a joint specification for the OEE and the real interest rate in an unobserved components model framework with Markov-switching parameters, the means, variances, and degrees of persistence of the real interest rate are different for the periods 1966–1980, 1980–1985, and 1985–2015. Once these regime changes are taken into account, the EIS estimate is 0.1 and no longer statistically insignificant. This finding is robust to alternative measures of the output gap as well as different specifications for the natural real interest rate.

Keywords: Output Euler Equation, Markov-Switching, Real Interest Rate, Elasticity of Intertemporal Substitution

1. INTRODUCTION

During recent decades, New Keynesian macroeconomic models have become a popular framework for monetary analysis. In its simplest form, the New Keynesian model comprises of the output Euler equation (OEE), which generalizes the consumption Euler equation to output, and an inflation equation, which resembles the expectation-augmented Phillips curve. While both equations are equally important for macroeconomic analyses, the dynamics of aggregate demand as captured by the OEE is severely understudied, especially when compared to its aggregate supply counterpart.

This paper aims to fill this gap in the literature by focusing on an important empirical puzzle facing the OEE. According to theory, the slope of the OEE relates aggregate demand to the real interest rate, which captures the key channel in which monetary policy is able to affect the real economy through the mechanism of intertemporal substitution in consumption. However, a robust empirical finding is that the OEE slope is zero, which challenges the standard view on the effectiveness of the short-term interest rate as a monetary policy instrument. Furthermore, with

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theory implying that the OEE slope is the negative of the elasticity of intertemporal substitution (EIS), the empirical challenge facing the OEE becomes all the more perplexing since it infers that agents do not substitute consumption over time.[1]

To investigate the zero EIS puzzle, this paper proposes an empirical model for the OEE based on a Markov-switching unobserved components (UC) model. The UC framework developed in this paper contains at least two attractive features that distinguishes it from the earlier literature. First, despite widespread evidence of regime-switching in the means, variances, and degrees of persistence in the US real interest rate [Garcia and Perron (1996), Rapach and Wohar (2005), Manopimoke (2009)], these forms of instabilities are often overlooked when estimating the OEE. However, given that the OEE in its closed-form specification implies that current output depends on the expected path of future real interest rates, changes in the dynamic properties of the real interest rate such as shifts in its degree of persistence or its long-run mean should have important implications for the OEE. To ensure that these interactions are properly accounted for, the UC framework as proposed in this paper is a joint specification of output and the real interest rate, where real interest rate dynamics are allowed to undergo regime-switching at unknown dates.

Second, the UC model estimates key variables of the OEE, which are the output and real interest rate gaps, as deviations of actual values from their respective natural rates, which are treated as latent state variables in the empirical model. Dealing with gap constructs in this way can be seen as more desirable than the usual approach of resorting to statistical detrending methods to obtain estimates of the unobserved natural rates, as these approaches often impose strong priors on the statistical properties of the trend. In contrast, latent state variables estimated within the UC framework depend on principle economic relationships as specified by the OEE, which should help minimize measurement error and provide gap estimates with more economic content.[2]

Bilbiie and Straub (2012) offer the closest study to this paper. Based on a split sample analysis, these authors argue that the EIS puzzle stems from failure to account for a structural break in the EIS parameter. More specifically, they show that the EIS in the OEE is a convolution of two nonzero coefficients with opposite signs across the samples 1965–1979 and 1982–2003, where the incorrect sign in the earlier period can be explained by institutional evidence of limited asset market participation in the United States. While this paper also associates the EIS puzzle with instability in the OEE relation, it differs from Bilbiie and Straub (2012) by attributing the puzzle to regime-switching behavior in the real interest rate process, rather than a one-time shift in the EIS parameter. This is because in previous work, there is limited evidence of time variation in the EIS parameter [Atkeson and Ogaki (1996), Attanasio and Browning (1995), Chiappori and Paiella (2011)]. The EIS is also a deep structural parameter of the economy, making it unlikely that it would have undergone a discrete structural break.

This paper is also a part of a larger literature that examines the EIS puzzle with consumption Euler equations. Dating back to Mankiw (1981), Hall (1988),

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and Campbell and Mankiw (1989), these authors report that there is virtually no evidence of intertemporal substitution in US consumption data. Subsequent work by Campbell (2003) and Yogo (2004) estimate the EIS for several countries while carefully accounting for statistical problems stemming from weak instruments, yet arrive at similar conclusions.[3] This paper takes on an alternative approach by being one of the few papers that examine the EIS puzzle within the context of the OEE. As pointed out by Bilbiie and Straub (2012), studying the EIS puzzle with the OEE can be advantageous since estimating the consumption Euler equation can become complicated by issues related to the nonseparability of consumption and leisure in the utility function. At the same time, focusing on the OEE can also advance our knowledge about the proper modeling of the monetary transmission mechanism based on the New Keynesian output relation.[4]

As a preview of the empirical results, this paper finds that the US real interest rate underwent two distinct regime changes around 1980 and 1985. These structural shifts separate real interest rate behavior into three distinct regimes, with means and variances that can be characterized by medium, high, and low levels. Furthermore, the natural real rate became highly persistent after the Great Moderation in 1985, while the persistence and variability of shocks to the real interest rate gap declined dramatically. A key finding from the joint estimation of the OEE is that once regime-switching behavior in the real interest rate is properly accounted for, the EIS parameter becomes statistically significant and is estimated with the correct sign throughout the postwar period. In particular, the estimated magnitude of the EIS parameter in the OEE relation is 0.1 across the three regimes and is surprisingly robust against alternative measures of the output gap as well as different specifications for the natural real rate.

This paper is structured as follows. Section 2 provides some background on the OEE and outlines the empirical model based on its closed-form specification. Section 3 introduces the UC model that is a joint specification of the OEE and the real interest rate process with Markov-switching parameters. Section 4 presents the estimation results, and Section 5 provides some further discussion to gain added insight on the empirical findings. Section 6 concludes.

2. THE OUTPUT EULER EQUATION

The simplest form of the intertemporal OEE, also known as the New Keynesian IS curve, is a microfounded, optimization-based relation that takes on the following form:

$$x_t = E_t x_{t+1} - \sigma \tilde{r}_t + \eta_t, \tag{1}$$

where x_t is the output gap, $E_t x_{t+1}$ is the expectation formed at time t of the one-period-ahead output gap, $\tilde{r}_t$ is the real interest rate gap, and η_t is the aggregate demand shock. The real interest rate gap is defined as the difference between the real interest rate and its natural level, or $\tilde{r}_t = r_t - r_t^*$, where r_t is the real interest rate and r_t^* is the natural real rate of interest. According to the "Wicksellian"

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concept for the natural real interest rate, r_t^* is the unobservable real rate that is consistent with real output at potential and stable inflation [Woodford (2011), Kiley (2015)]. In practice, r_t^* may vary over time with structural factors such as the rate of output trend growth rate and time preferences. The real interest rate gap, on the other hand, reflects the existence of nominal rigidities in the economy and moves with short-run fluctuations in the business cycle.

In a closed economy without capital, investment in durable goods and a government, consumption equals output and the OEE slope reflects the aggregate EIS parameter.[5] Theory predicts that the slope of the OEE is negative, because following an increase in the short-term interest rate, the representative agent that practices intertemporal substitution will substitute consumption (output) today for consumption (output) tomorrow, ultimately lowering aggregate demand. Note that this mechanism of intertemporal substitution is an important channel in which monetary policy can affect the economy through its influence on short-term interest rates.

While theoretically sound, the OEE in (1) provides a remarkably poor description of the key dynamic features of the aggregate data due to two main empirical challenges. First, the forward-looking element, $E_t x_{t+1}$, fails to completely capture the persistent responses of output to monetary policy shocks. To this end, "hybrid" specifications that augment the OEE with lags of the output gap have become widespread, as these "backward-looking" elements help the model match the high degree of inertia in the data [Svensson (1999), Fuhrer (2000), Rudebusch (2002)]. The presence of output lags in the OEE, while seemingly ad hoc, is typically justified by theoretical models related to habit formation, in which consumers' utility depends partly on current output relative to past output [Fuhrer (2000)]. However, although these backward-looking elements afford the OEE to be better reconciled with the data, many economists are still puzzled as to why future output expectations do not play a more important role in determining current output. Fuhrer and Rudebusch (2004) and Fuhrer and Olivei (2004) investigate this issue, and explain that due to the problem of weak identification which arises from the use of conventional Generalized Method of Moments (GMM) techniques for OEE estimation, obtaining accurate estimates of OEE parameters are in general difficult.

The second empirical challenge that plagues the OEE is that the model typically fails to deliver slope estimates that are significantly different from zero. This finding is counterintuitive because it implies a zero EIS, which suggests that agents do not substitute consumption over time. While a number of papers acknowledge this empirical irregularity, most studies merely treat zero slope estimates as an anomaly and offer no further explanations for their results. Such studies include Fuhrer and Rudebusch (2004), who report zero EIS estimates for the OEE based on a variety of output detrending methods, real interest rate definitions, and instrument sets while estimating the OEE with GMM and maximum likelihood (ML) methods. Fuhrer and Olivei (2004) reach similar conclusions despite employing an improved GMM approach that better deals with the problem of weak identification. In a recent study, Stracca (2010) estimates the OEE and several possible extensions

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using panel data from 22 OECD countries to increase the statistical power of the empirical exercise, but still fails to establish a significant link between the output gap and the real interest rate.

To focus on the second empirical puzzle pertaining to zero slope estimates for the OEE, this paper considers an approach that sidesteps the difficulty of dealing with the forward-looking element, $E_t x_{t+1}$. This can be achieved by forward iteration of (1), arriving at the following closed-form specification:

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} - \sigma \sum_{j=0}^{\infty} E_t \tilde{r}_{t+j} + \eta_t. \quad (2)$$

As is standard in the literature, two lags of the output gap have been added to the closed-form OEE to help the model capture the high degrees of inertia in the data.

According to (2), the current output gap now depends on the sum of current and future forecasts of the real interest rate gap, which approximates a long-run real interest rate. This makes it apparent that the central bank's primary impact on the economy comes about not only through the level at which it sets current overnight rates, but also from the way it affects private-sector expectations about the likely future path of short-term overnight rates. Therefore, the future behavior of real rates becomes an important determining factor of current business cycle conditions, which has led many policy makers to stress the importance of credibility and commitment in their conduct of monetary policy.

3. THE UNOBSERVED COMPONENTS FRAMEWORK WITH MARKOV-SWITCHING PARAMETERS

Given that the output gap depends on the future path of expected real interest rate gaps, a key challenge for estimating the closed-form OEE in (2) is obtaining an expression for the infinite sum term. Multistep forecasts of future real interest rate gaps can be computed in a number of ways depending on how the dynamics of the real interest rate are specified. One way to define the dynamics of the real interest rate jointly with the OEE in (2) is according to the UC model as shown below, henceforth referred to as the baseline UC model.

Baseline UC model

Output gap equation:

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} - \sigma \sum_{j=0}^{\infty} E_{t-1} \tilde{r}_{t+j} + v_t, \; v_t \sim \text{i.i.d.} N(0, \sigma^2_{v,D_t}), \quad (3)$$

Real interest rate equations:

$$r_t = r^*_t + \tilde{r}_t, \quad (4)$$

$$r^*_t = (1 - \rho_{S_t}) r^{\text{LR}}_{S_t} + \rho_{S_t} r^*_{t-1} + e_t, \; e_t \sim \text{i.i.d.} N(0, \sigma^2_{e,S_t}), \quad (5)$$

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$$\tilde{r}_t = \phi_{S_t}\tilde{r}_{t-1} + \epsilon_t, \epsilon_t \sim \text{i.i.d.}N(0, \sigma^2_{\epsilon,S_t}). \quad (6)$$

The following assumptions are made in the above specification. First, (3) follows the closed-form OEE in (2), but with one slight modification. For feasible estimation of the model, the expectational element $E_t(\cdot)$ in the infinite sum term is replaced with $E_{t-1}(\cdot)$; otherwise current forecasts of the real interest rate gap may be correlated with the error term. Accordingly, the error term in the output gap equation is rewritten as $v_t = \eta_t - \sigma \sum_{j=0}^{\infty}(E_t\tilde{r}_{t+j} - E_{t-1}\tilde{r}_{t+j})$ where the second element in v_t is related to economic agents' revision on expectations of current and future real interest rate gaps. Since this term may be correlated with innovations to $\tilde{r}_t$, the correlation coefficient between v_t and ϵ_t is allowed to be nonzero during estimation.

To complete the specification for the output gap equation, the variance of shocks to the output gap are defined as follows:

$$\sigma^2_{v,D_t} = (1 - D_t)\sigma^2_{v,1} + D_t\sigma^2_{v,2}, \quad (7)$$

$$D_t = \begin{cases} 0 & \text{if } t \leq 1984\text{Q3} \\ 1 & \text{otherwise,} \end{cases} \quad (8)$$

where the dummy variable D_t helps capture any decline in the variance of shocks to real output during the onset of the Great Moderation in 1984Q3 [see Kim and Nelson (1999), McConnell and Perez-Quiros (2000)].

Next, (4) decomposes the real interest rate into a natural real interest rate r^*_t and a real interest rate gap $\tilde{r}_t$. In (5), r^*_t is described as a stationary time-varying process with unconditional long-run mean r^{LR} and persistence parameter ρ. According to this description, r^*_t can be interpreted as the neutral monetary policy stance consistent with the short-term goals of the Federal Reserve Bank, to be set period by period under the situation where the economy has not necessarily settled at its natural long-run level r^{LR}. According to this interpretation, the natural real rate is treated as an exogenous process in the UC framework; thus, shocks to the natural real rate e_t are assumed to be uncorrelated with all other shocks in the system.

It is worth mentioning that the chosen specification for r^*_t is purely statistical. This is similar in spirit to the empirical specifications of Larsen and McKeown (2004), Basdevant et al. (2004), and Cour-Thimann et al. (2006). Another approach that is more in line with theory is to let r^*_t vary with real fundamentals, such as the determinants of trend GDP growth [Giammarioli and Valla (2003), Laubach and Williams (2003), Neiss and Nelson (2003)]. This definition for r^*_t is explored later for robustness checks, but for the time being, the baseline UC model is based on the statistical definition of r^*_t to remain rather agnostic about how r^*_t is related to other economic variables. This is seen as preferable given that there is typically a high degree of specification uncertainty surrounding natural real interest rate estimates.

Last, an expression for the real interest rate gap in (6) defines $\tilde{r}_t$ as an autoregressive process (AR) of order 1 with persistence parameter ϕ. Based on this

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specification, the sum of multistep forecasts of the real interest gap that drives the infinite sum term in the closed-form equation for the OEE can now be computed as follows:

$$\begin{aligned}\sigma \sum_{j=0}^{\infty} E_{t-1}\tilde{r}_{t+j} &= \sigma E_{t-1}(\tilde{r}_t + \tilde{r}_{t+1} + \tilde{r}_{t+2} + \cdots) \\ &= \sigma(\phi_{S_t} + \phi_{S_t}^2 + \phi_{S_t}^3 + \cdots)\tilde{r}_{t-1} \\ &= \frac{\sigma\phi_{S_t}}{(1-\phi_{S_t})}\tilde{r}_{t-1}.\end{aligned}$$

Substituting this expression into (3) yields

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} - \frac{\sigma\phi_{S_t}}{(1-\phi_{S_t})}\tilde{r}_{t-1} + v_t,\ v_t \sim \text{i.i.d.}N(0, \sigma^2_{v,D_t}). \qquad (\mathbf{3}')$$

As shown, the slope of the OEE now depends on both the EIS parameter and the persistence parameter of the real interest rate gap. This specification makes it apparent that changes in the times-series properties of the short-term real interest rate, in particular its degree of persistence, can have an important impact on the link between the output and real interest rate gaps.

Based on the literature on real interest rate dynamics, there is widespread evidence of regime-switching behavior in the mean, variance, and persistence of the US real interest rate [Huizinga and Mishkin (1986), Bai and Perron (2003), Neely and Rapach (2008), Manopimoke (2009), Kim and Kim (2015)].[6] Therefore, to account for the possibility that the real interest rate in the OEE relation may have undergone discrete changes, as well as to examine how this behavior may have affected estimates of the OEE, all parameters that describe the real interest rate series in the baseline UC model, r^{LR}, ρ, ϕ, σ_e, and σ_ϵ, are allowed to undergo regime changes at unknown dates. More specifically, their dynamics as shown in (5)–(6) are governed by the first-order Markov-Switching process S_t, which represents the state in which the economy is in at time t.

The number of regimes in the UC model is assumed to be unknown and is determined during estimation via a series of diagnostic tests.[7] The dynamics of S_t are then specified accordingly. For example, if diagnostic test results point towards three recurring regimes in the real interest rate, S_t will follow a three-state Markov process with transition probability matrix:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}, \qquad (\mathbf{9})$$

where $P_{ij} = Pr[S_t = j | S_{t-1} = i]$ and $\sum_{j=1}^{3} P_{ij} = 1$ for $i = 1, 2$, and 3. Note that in the empirical model, changes in the behavior of the real interest rate need not be recurring, as the model can also accommodate for one-time discrete shifts

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or structural breaks. For the case of a two structural break model (three distinct states), the transition probabilities P_{21}, P_{31}, and P_{32} will equal zero and P_{33} will be an absorbing state with probability equal to one.

3.1. An Alternative Measure for the Output Gap

One shortcoming of the baseline UC model is that it treats the output gap, x_t, as an observable variable. Accordingly, x_t will need to be proxied by estimates of the output gap, such as those obtained by applying statistical detrending methods to real GDP. Although this approach is standard in the literature, it is well known that statistical constructs of x_t can be subject to considerable measurement error. Therefore, this paper also considers estimating the output gap as a latent state variable in the UC framework. As a byproduct, this approach produces estimates for the output gap that are consistent with the OEE, which thereby contains more economic content when compared to output gap measures that are simply obtained from statistical methods.

Treating the output gap as an unobserved variable requires augmenting the baseline UC specification in (3)–(6) with a UC model for output. To help identify the latent output gap, an equation for inflation is also jointly estimated resulting in the following trivariate UC model for the real interest rate, output and inflation:

Trivariate UC model

Output equations:

$$y_t = \tau_t + x_t, \tag{10}$$

$$\tau_t = \delta_t + \tau_{t-1} + w_t, w_t \sim \text{i.i.d.} N(0, \sigma^2_{w,D_t}) \tag{11}$$

$$\delta_t = \delta_{t-1} + \eta_t, \eta_t \sim \text{i.i.d.} N(0, \sigma^2_{\eta}), \tag{12}$$

Inflation equation:

$$\pi_t = c_1\pi_{t-1} + c_2\pi_{t-2} + c_3\pi_{t-3} + kx_{t-1} + z_t, z_t \sim \text{i.i.d.} N(0, \sigma^2_{z,\tilde{D}_t}). \tag{13}$$

Each equation can be interpreted as follows. In (10), the equilibrium real output y_t is decomposed into a stochastic trend component τ_t, and a cyclical component x_t, which follows the OEE as outlined in (3). As for the trend component of output, it follows a random walk with time-varying drift, as specified by (11). In (12), the drift term δ_t represents a time-varying output trend growth rate, which helps capture any changes in productivity that may have occurred, such as the slowdown in productivity in 1974. Finally, to ensure that τ_t is estimated as the country's productive capacity that is consistent with stable inflation, a reduced form Phillips curve as specified by (13) is jointly specified alongside the OEE and real interest rate equations.

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As before, the variance of shocks in the output equations undergoes a one-time change in 1984Q3 to account for the Great Moderation:

$$\sigma^2_{v,D_t} = (1 - D_t)\sigma^2_{v,1} + D_t\sigma^2_{v,2}, \tag{14}$$

$$\sigma^2_{w,D_t} = (1 - D_t)\sigma^2_{w,1} + D_t\sigma^2_{w,2}, \tag{15}$$

where D_t follows (8). Heteroskedasticity in the inflation process can also be captured in a similar fashion. During the Great Inflation of the 1970s, US inflation was significantly more volatile compared to other periods. The variance of shocks to the inflation process thus follows:

$$\sigma^2_{z,\tilde{D}_t} = \tilde{D}_{1t}\sigma^2_{z,2} + \tilde{D}_{2t}\sigma^2_{z,2} + \tilde{D}_{3t}\sigma^2_{z,3}, \tag{16}$$

where

$$\tilde{D}_{1t} = \begin{cases} 1 & \text{if } t_0 \le t \le 1971\text{Q1} \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

$$\tilde{D}_{2t} = \begin{cases} 1 & \text{if } 1971\text{Q2} \le t \le 1981\text{Q1} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

$$\tilde{D}_{3t} = \begin{cases} 1 & \text{if } 1981\text{Q2} \le t \le T \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

Finally, to estimate the empirical models described in this section, the UC specifications are written in state-space form and estimated with maximum-likelihood methods based on Kim's (1994) approximate Kalman filter. Since the trivariate UC model is a generalization of the baseline UC model, only the state-space representation of the trivariate UC model is outlined in Appendix A.

4. EMPIRICAL RESULTS

4.1. Data

All data is taken from the Federal Reserve Economic Database (FRED), and the study is based on quarterly data. For the baseline UC model, four measures of the output gap are used as proxies for x_t, based on the following definitions of potential output: (1) the Congressional Budget Office's (CBO) official estimate of potential output; (2) the Hodrick–Prescott (HP) filter of log real GDP; (3) a deterministic quadratic trend for log real GDP; and (4) a segmented deterministic linear trend for log real GDP with a breakpoint in the first quarter of 1974 to capture the productivity slowdown that may have occurred at that time. In the trivariate UC model, x_t is extracted directly from y_t, which is the log of real GDP. The inflation measure in the Phillips curve equation is calculated as $\pi_t = 400 \times (\ln P_t - \ln P_{t-1})$, where P_t is the GDP chain-weighted price index.

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The relevant real interest rate measure is the ex-ante real interest rate (EARR), defined as the difference between the nominal short-term interest rate and current period inflation expectations. However, inflation expectations are unobserved; thus, the ex-post real interest rate (EPRR), which is equivalent to the EARR under the assumption of rational expectations, is employed for the empirical analysis instead. To construct the EPRR, the current period inflation rate is subtracted from the nominal interest rate. For consistency with the Phillips curve equation, the relevant inflation measure for the EPRR is computed from the GDP chain-weighted price index, and the nominal interest rate is the quarterly average of the overnight federal funds rate.

Finally, the sample period of study spans 1966Q1–2015Q3. The start of the sample marks the beginning of the period in which the federal funds rate is recognized as a primary instrument of monetary policy. Note that in the period prior to 1966, the federal funds rate did not exceed the discount rate, rendering it as an invalid instrument of monetary policy.

4.2. Empirical Results

The empirical investigation of the EIS puzzle is carried out as follows. First, both the baseline and trivariate UC models are estimated with no regime-switching parameters to verify the EIS puzzle, as well as to obtain benchmark results in which further analyses can be compared against. Next, to determine the appropriate number of states, the modified Ljung–Box Portmanteau test statistic is calculated for each UC model to determine whether there is remaining serial correlation in the models' residuals and their squares.[8] This process is repeated for UC models with incremental number of regimes until the model with no remaining serial correlation is identified. Due to space considerations, all diagnostic test results and plots of the empirical findings are only reported for the trivariate UC model, as results belonging to the baseline UC models are quantitatively similar.

Table 1 reports the empirical findings for the UC models with no regime switching. The first four columns describe the results for the baseline UC model with different proxies for the output gap, while the last column reports the results for the trivariate UC model. Parameter estimates describing the output, real interest rate, and inflation equations are analyzed in turn. For the output equation, the estimated sum $\alpha_1 + \alpha_2$ is close to 1 for all output gap measures, suggesting that the transitory component of output is highly persistent. There is also strong evidence in support of the Great Moderation, as observed by the significant decline in the variance of shocks to US output after 1984Q3.

For the real interest rate equation, both the natural real interest rate and the real interest rate gap are highly persistent based on estimates of ρ and ϕ, respectively. While the estimate of the long-run steady-state level r_{LR} is associated with a large standard error, this is not surprising given that natural real rate estimates are often attached to a large measure of uncertainty. Turning to examine the estimated variance of shocks, both components of the real interest rate are, in general,

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TABLE 1. Estimation results for the UC models with no regime change

Parameters	Output gap measures				
	CBO	HP-filter	Quadratic	Segmented	Trivariate UC
	Output equation				
σ	0.077	0.000	0.126	0.139	0.067
	(0.047)	(0.000)	(0.129)	(0.171)	(0.047)
α_1	1.233***	1.091***	0.994***	1.016***	1.759***
	(0.083)	(0.067)	(0.186)	(0.213)	(0.067)
α_2	−0.267***	−0.253***	−0.075	−0.054	−0.811
	(0.081)	(0.067)	(0.148)	(0.173)	(0.069)
$\sigma_{v,1}$	0.990***	0.966***	0.900***	0.908***	0.180**
	(0.091)	(0.083)	(0.142)	(0.162)	(0.089)
$\sigma_{v,2}$	0.508***	0.487***	0.372***	0.384**	0.000
	(0.037)	(0.031)	(0.113)	(0.185)	(0.037)
$\rho_{v,\epsilon}$	−0.007	0.418***	0.216	0.178	−0.999***
	(0.093)	(0.128)	(0.513)	(0.688)	(0.000)
$\sigma_{w,1}$	–	–	–	–	0.885***
	–	–	–	–	(0.087)
$\sigma_{w,2}$	–	–	–	–	0.490***
	–	–	–	–	(0.038)
σ_η	–	–	–	–	0.052
	–	–	–	–	(0.040)
	Real interest rate equation				
r^{LR}	1.559	1.742	1.824**	1.777*	2.076
	(1.492)	(1.720)	(0.895)	(1.012)	(1.511)
ϕ	0.562***	0.968***	0.814***	0.798***	0.397***
	(0.140)	(0.023)	(0.151)	(0.179)	(0.124)
ρ	0.963***	0.364***	0.898***	0.909***	0.964***
	(0.025)	(0.137)	(0.041)	(0.051)	(0.023)
σ_e	0.726***	0.955***	1.240***	1.235***	0.696***
	(0.167)	(0.106)	(0.139)	(0.226)	(0.119)
σ_ϵ	1.000***	0.723***	0.419	0.436	0.971***
	(0.132)	(0.116)	(0.366)	(0.600)	(0.104)
Likelihood	−332.546	−313.826	−334.628	−337.516	−411.807

Note: Standard errors are in parentheses. *,**,*** denotes statistical significance at the 10%, 5% and 1% levels, respectively.

quite volatile. This finding contrasts with Neiss and Nelson (2003), who report a more smooth natural real rate. Last, the parameter estimates for the inflation equation in the trivariate UC model are reported separately in Table 2 due to space considerations. As shown, US inflation is a highly persistent process as suggested

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TABLE 2. Estimation results for the inflation equation in the trivariate UC model

	One regime	Two regimes	Three regimes
k	0.099	0.187**	0.139*
	(0.065)	(0.085)	(0.081)
c_1	0.556***	0.544***	0.538***
	(0.074)	(0.073)	(0.078)
c_2	0.166**	0.157*	0.164*
	(0.079)	(0.080)	(0.093)
c_3	0.233***	0.256***	0.258***
	(0.072)	(0.073)	(0.075)
$\sigma_{z,1}$	1.074***	1.076***	1.100***
	(0.233)	(0.237)	(0.245)
$\sigma_{z,2}$	1.539***	1.522***	1.561***
	(0.182)	(0.047)	(0.183)
$\sigma_{z,3}$	0.773***	0.766***	0.769***
	(0.048)	(0.047)	(0.048)

Note: Standard errors are in parentheses. *,**,*** denotes statistical significance at the 10%, 5% and 1% levels, respectively.

by the sum of $c_1 + c_2 + c_3$, and as expected, the variability of shocks to inflation was higher during the Great Inflation when compared to other periods.[9]

Thus far, most parameter estimates in Table 1 are of the correct sign and display reasonable magnitudes. However, the key parameter of interest, σ, is not statistically significant, making a strong case for the EIS puzzle. Furthermore, parameter estimates describing the EPRR process also differ to a considerable degree across the various output gap measures, suggesting some form of misspecification in the UC model with no-regime-switching behavior. This is confirmed by residual diagnostic test results that are reported in the first block of Table 3. As shown, the reported p-values associated with the Ljung–Box test statistics suggest that the null of no serial correlation in the model's squared residuals is firmly rejected at the 1% level.

With misspecification in the UC model with no regime-switching, the UC models are estimated again, but this time the parameters in the real interest rate equations follow a two-state Markov-switching process. The filtered probabilities associated with the latent state variable S_t are plotted in Figure 1, which displays convincing evidence that the dynamics of the US real interest rate underwent a significant change during the Great Moderation in the mid-1980s. As shown in Table 4, once this regime shift has been taken into account, the EIS parameter σ now becomes statistically significant with the correct sign. The magnitude of σ is more or less equal to 0.1 across all output gap measures, which is an interesting result insofar as the various output gap measures all differ to some considerable degree. Compared to existing studies, an estimate of 0.1 for σ is a reasonable.

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TABLE 3. Residual diagnostic tests for the trivariate UC model

Lag	Standardized residuals	Squared standardized residuals
	No regime changes	
1	0.308	0.000
4	0.162	0.000
8	0.374	0.000
12	0.379	0.000
16	0.589	0.000
20	0.538	0.000
	Two Markov-switching regimes	
1	0.670	0.238
4	0.329	0.716
8	0.614	0.378
12	0.847	0.628
16	0.924	0.822
20	0.958	0.853
	Three Markov-switching regimes	
1	0.567	0.100
4	0.283	0.437
8	0.654	0.258
12	0.870	0.545
16	0.844	0.590
20	0.880	0.690

Note: Reported are the *p*-values associated with the modified Ljung–Box Portmanteau test statistic under the null of no serial correlation.

Based on aggregate consumption data, Hall (1988), Mankiw (1981), and Yogo (2004), estimate the EIS to be around 0.2. By extending the New Keynesian output equation to include asset prices and monetary aggregates, Goodhart and Hofmann (2005) report a statistically significant EIS estimate of 0.04.[10]

Examining the results in Table 4 further, estimates for the output equation are similar to the no regime-switching case, except that now, a nonzero σ_η provides evidence of a time-varying output trend growth rate. Based on the estimated real interest rate parameters across the two regimes, it can be inferred that the regime shift in the mid-1980s has been in a large part driven by the significant decline in the level of r_{LR}, as well as the persistence and the variability of shocks to the real interest rate gap. On the other hand, the persistence of the natural real rate ρ and the variability of its shocks σ_e remain relatively constant over the two regimes.[11] Finally, compared to the no regime-switching case, estimates of the persistence parameters in the real interest rate equation as well as the variability

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TABLE 4. Estimation results for the two-state Markov-switching UC models

	Output gaps				
Parameters	CBO	HP-filter	Quadratic	Segmented	Trivariate UC
	Output equation				
σ	0.100**	0.082**	0.099*	0.096**	0.105***
	(0.046)	(0.040)	(0.054)	(0.047)	(0.033)
α_1	1.270***	1.119***	1.293***	1.173***	1.135***
	(0.072)	(0.069)	(0.070)	(0.101)	(0.086)
α_2	−0.302***	−0.261***	−0.316***	−0.187**	−0.217***
	(0.071)	(0.069)	(0.070)	(0.092)	(0.081)
$\sigma_{v,1}$	0.975***	0.934***	0.980***	0.967***	0.946***
	(0.089)	(0.085)	(0.090)	(0.092)	(0.089)
$\sigma_{v,2}$	0.523***	0.478***	0.530***	0.481***	0.481***
	(0.034)	(0.030)	(0.034)	(0.042)	(0.035)
$\rho_{v,\epsilon}$	−0.029	−0.028	−0.029	0.026	−0.005
	(0.113)	(0.109)	(0.105)	(0.125)	(0.002)
$\sigma_{w,1}$	–	–	–	–	0.000
					(0.023)
$\sigma_{w,2}$	–	–	–	–	0.000
					(0.026)
σ_{η}	–	–	–	–	0.094**
					(0.035)
	Real interest rate equation				
r_1^{LR}	4.292	4.603	4.392	3.819	6.278
	(5.690)	(4.089)	(5.096)	(4.030)	(6.400)
r_2^{LR}	0.636	0.491	0.547	0.746	0.468
	(1.867)	(1.650)	(1.854)	(1.719)	(0.368)
ϕ_1	0.521***	0.509***	0.517***	0.533***	0.518***
	(0.134)	(0.080)	(0.176)	(0.132)	(0.029)
ϕ_2	0.120	0.080	0.090	0.850***	0.143
	(0.461)	(0.341)	(0.405)	(0.057)	(0.266)
ρ_1	0.981***	0.981***	0.978***	0.974***	0.975***
	(0.034)	(0.033)	(0.035)	(0.036)	(0.033)
ρ_2	0.956***	0.956***	0.956***	0.951***	0.950***
	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)
σ_e	0.661***	0.703***	0.665***	0.748***	0.703***
	(0.092)	(0.096)	(0.091)	(0.056)	(0.093)
$\sigma_{\epsilon,1}$	1.552***	1.553***	1.556***	1.535***	1.566***
	(0.183)	(0.182)	(0.187)	(0.181)	(0.180)
$\sigma_{\epsilon,2}$	0.295**	0.254*	0.290**	0.144	0.270*
	(0.137)	(0.144)	(0.132)	(0.095)	(0.151)
Likelihood	−307.200	−292.589	−308.773	−306.337	−390.592

Note: Standard errors are in parentheses. *,**,*** denotes statistical significance at the 10%, 5%, and 1% levels, respectively.

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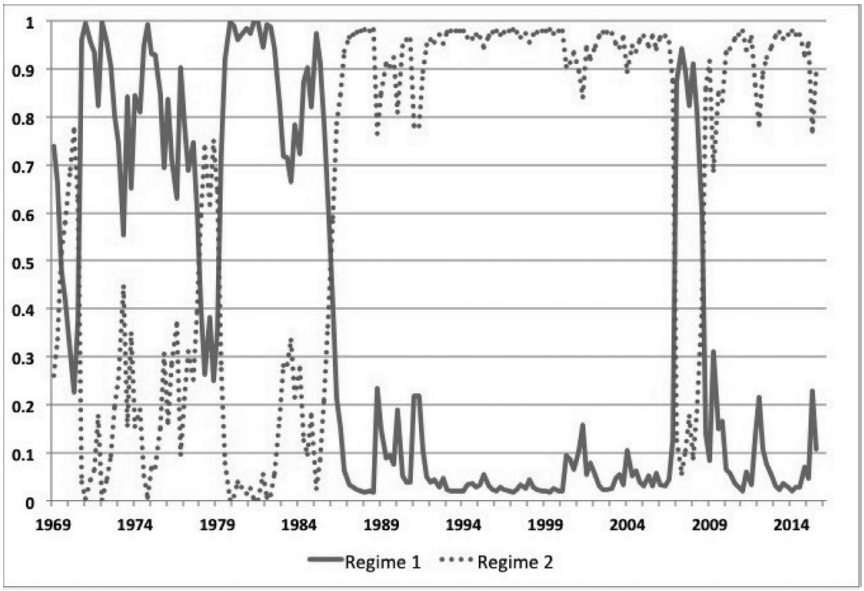


FIGURE 1. Filtered probabilities from the two-state Markov-switching trivariate UC model.

of the shock innovations now appear to be less sensitive to the different output gap specifications.

Although the two-state UC model appears to describe the joint dynamics of output and the real interest rate well, it is still necessary to evaluate its empirical fit. According to the residual diagnostic test results in the second block of Table 3, the two-state model appears well specified. However, judging from the filtered probabilities in Figure 1, allowing for a third state in the UC model may be necessary. This is because while the filtered probabilities show that a regime change occurred around the mid-1980s, prior to the Great Moderation, the distinction between states 1 and 2 are not necessarily clear-cut. In particular, although there exists higher probability that the model is in state 1, the probability that the model is in state 2 is also nontrivial, being as high as 0.7 in the early 1980s.

For the above reason, a three-state Markov-switching UC model is estimated, with filtered probabilities plotted in Figure 2. Compared to the two-state case, the transition between the three regimes are now sharp and more distinct. As expected, the model identifies an additional regime change in the early 1980s, separating the dynamics of the EPRR into three regimes: 1966–1980, 1980–1985, and 1985–2015.[12] The empirical models of Huizinga and Mishkin (1986), Garcia and Perron (1996), Rapach and Wohar (2005), and Manopimoke (2009) also find similar dates of change for the EPRR process, and interpret them as a monetary phenomenon.[13] This interpretation seems appropriate given that the first shift in the early 1980s closely coincides with the beginning of Paul Volcker's chairmanship at the Federal Reserve Bank, which is associated with a monetary policy stance

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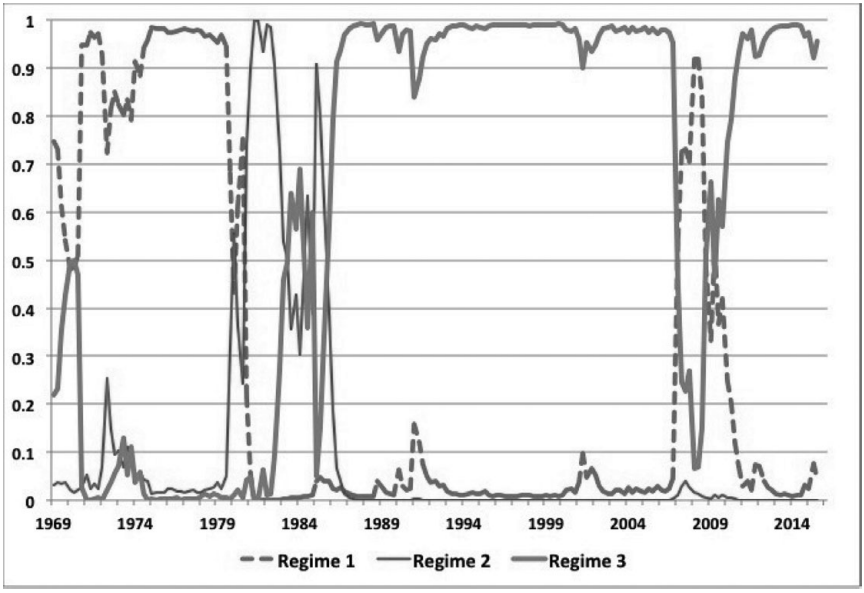


FIGURE 2. Filtered probabilities from the three-state Markov-switching trivariate UC model.

that is more focused on achieving stability in inflation and economic activity. The timing of the second regime shift in 1985 occurred during the onset of the Great Moderation. While the driving force behind the Great Moderation is still a subject of debate, a number of influential studies argue that improved monetary policy was an important contributing factor [Clarida et al. (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006), Boivin et al. (2010)].[14]

Table 5 reports the parameter estimates associated with the three-state Markov-switching UC models. Similar to the two-state model results, the key parameter of interest, σ, is statistically significant and is estimated with a magnitude that is approximately 0.1 across all output gap measures. This finding again highlights the importance of accounting for regime-switching behavior in the real interest rate series when estimating the OEE.

The estimated output dynamics for the three-state case does not differ much from the two-state model. However, once three regimes are accounted for, the amount of uncertainty attached to the estimates of the long-run real interest rate, r_{LR}, is significantly reduced, especially during the period leading up to the Great Moderation. Based on the levels of r_{LR} across the three states, the three regimes can be described by medium, high, and low steady-state levels. In particular, the high mean state corresponds to a high variance state for the real interest rate gap and vice versa. Accordingly, the findings in this paper in large part echo the results of Garcia and Perron (1996), who also find a strong positive relationship between the level of the long-run mean and the variability of shocks to the EPRR process.

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TABLE 5. Estimation results for the three-state Markov-switching UC models

	Output gaps				
Parameters	CBO	HP-filter	Quadratic	Segmented	Trivariate UC
			Output equation		
σ	0.111**	0.083**	0.110***	0.103**	0.100**
	(0.047)	(0.040)	(0.038)	(0.041)	(0.044)
α_1	1.277***	1.119***	1.295***	1.328****	1.156***
	(0.068)	(0.068)	(0.068)	(0.068)	(0.089)
α_2	−0.306***	−0.259***	−0.319***	−0.331***	−0.242***
	(0.069)	(0.069)	(0.068)	(0.069)	(0.086)
$\sigma_{v,1}$	0.973***	0.936***	0.976***	0.978***	0.955***
	(0.089)	(0.084)	(0.090)	(0.090)	(0.093)
$\sigma_{v,2}$	0.529***	0.480***	0.533***	0.546***	0.488***
	(0.033)	(0.030)	(0.034)	(0.035)	(0.037)
$\rho_{v,\epsilon}$	0.000	0.071	0.000	0.010	0.039
	(0.080)	(0.116)	(0.085)	(0.014)	(0.073)
$\sigma_{w,1}$	–	–			0.000
					(0.006)
$\sigma_{w,2}$	–	–	–	–	0.000
					(0.036)
σ_{η}	–	–	–	–	0.084**
					(0.003)
			Real interest rate equation		
r_1^{LR}	0.747*	0.835*	0.781*	0.772**	1.152
	(0.442)	(0.481)	(0.452)	(0.381)	(1.391)
r_2^{LR}	6.370***	6.182***	6.304***	6.527***	6.525***
	(0.810)	(0.775)	(0.845)	(0.827)	(1.407)
r_3^{LR}	0.289	0.338	0.305	0.290	0.300
	(2.033)	(2.783)	(1.990)	(1.925)	(3.633)
ϕ_1	0.513***	0.509***	0.505***	0.586***	0.455**
	(0.114)	(0.156)	(0.054)	(0.170)	(0.229)
ϕ_2	0.483**	0.500***	0.503***	0.502***	0.520***
	(0.238)	(0.039)	(0.028)	(0.079)	(0.088)
ϕ_3	0.033	0.000	0.000	0.000	0.048
	(0.300)	(0.004)	(0.000)	(0.000)	(0.274)
ρ_1	0.661**	0.679**	0.675**	0.338	0.847***
	(0.316)	(0.345)	(0.281)	(0.892)	(0.206)
ρ_2	0.696***	0.656**	0.725***	0.716***	0.855***
	(0.236)	(0.306)	(0.210)	(0.209)	(0.124)
ρ_3	0.960***	0.959***	0.960***	0.959***	0.960***
	(0.027)	(0.032)	(0.027)	(0.027)	(0.037)
σ_e	0.664***	0.677***	0.667***	0.668***	0.664***
	(0.088)	(0.088)	(0.084)	(0.084)	(0.096)

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TABLE 5. Continued

	Output gaps				
Parameters	CBO	HP-filter	Quadratic	Segmented	Trivariate UC
$\sigma_{\epsilon,1}$	1.274***	1.264***	1.273***	1.269***	1.255***
	(0.169)	(0.172)	(0.170)	(0.185)	(0.210)
$\sigma_{\epsilon,2}$	1.855***	1.849***	1.870***	1.850***	1.909***
	(0.330)	(0.327)	(0.337)	(0.321)	(0.344)
$\sigma_{\epsilon,3}$	0.386***	0.370***	0.379***	0.376***	0.390***
	(0.115)	(0.095)	(0.090)	(0.090)	(0.110)
Likelihood	−303.085	−287.766	−304.420	−307.334	−388.450

Note: Standard errors are in parentheses. *,**,*** denotes statistical significance at the 10%, 5% , and 1% levels, respectively.

Turning to analyze the persistence of the real rate, deviations of r_t^* from its long-run mean has become more pronounced since the Great Moderation, as ρ increases in magnitude from approximately 0.7 to 0.9. On the other hand, the persistence of the real interest rate gap, while relatively large in the pre-1985 period, fell dramatically to zero after the Great Moderation. This finding stands in contrast with Garcia and Perron (1996), who show that deviations of the EPRR process from its regime-switching long-run mean is essentially a random white noise process throughout the entire postwar period. This difference in results most likely stems from how the long-run mean is handled. While Garcia and Perron only allow for structural shifts in the long-run mean, this paper also incorporates time-variation around that mean, thus providing a more generalized framework for US real interest rate dynamics.

Figure 3 contains a plot of the estimated natural real interest rate and the actual EPRR. It can be observed that while these two series more or less moved in the same general direction prior to the mid-1980s, the actual real rate only began to track the natural real interest rate closely after the Great Moderation. Interestingly, the implied behavior of the natural real interest rate in this post-1985 period is surprisingly similar to the one advocated by Barksy et al. (2014). Based on an extension of the well-known Smets and Wouters (2003) DSGE model with price and wage rigidities, these authors along with Justiniano et al. (2013) explain that a policy path in which the actual EPRR tracks the natural real rate can significantly stabilize the output gap and decrease the variability of wage and price inflation. Based on this line of reasoning, the behavior of the actual and natural real rate processes as implied by the UC model in Figure 3 may be a signal of improved monetary policy that took place in the post Great Moderation period.

Last, the empirical fit of the three-state UC models are evaluated with residual diagnostic tests. According to the final block in Table 3, the three-state model is well specified. However, given that the two-state model also passed the residual diagnostic tests, information criterions are employed to formally select between

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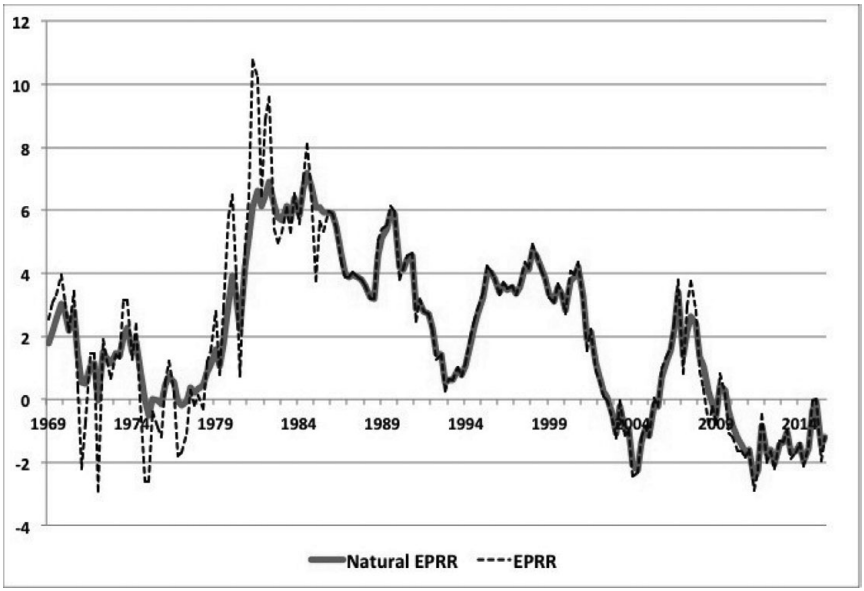


FIGURE 3. Estimated natural real rate from the three-state Markov-switching trivariate UC model and the actual ex-post real interest rate.

the two and three-state models. The first test is based on the Akaike information criterion (AIC), which is calculated as $\ln L - k$, where $\ln L$ denotes the log-likelihood and k corresponds to the number of parameters in the model. For the two and three-state models, the AIC is computed as -417.59 and -423.45, respectively. A second Bayesian information criterion (BIC), defined as $-2\ln L + k \cdot \ln(n)$ where n denotes the sample size, is also computed to be -591.63 and -638.26 for the two and three-state models, respectively. Since the model with the lowest AIC and BIC is preferred, the best-fitting specification for the joint dynamics of output and the real interest rate process appears to be the three-state Markov-switching UC model.

Based on the empirical analyses in this section, the bottom line result is that OEEs in previous studies might be misspecified. More specifically, the results from the Markov-switching UC models suggest that the EIS puzzle stems from failure to account for regime-switching in the dynamics of the real interest rate. This conclusion is confirmed by a number of additional robustness tests, which are available upon request. For example, the UC model is also estimated with alternative specifications for the natural real rate, such as one that ties the dynamics of the natural real interest rate to the time-varying growth rate of output (see Appendix B). This specification is more or less an extension of the Laubach and Williams (2003) model with regime-switching parameters in the real interest rate process. Similar to the main findings in this paper, while this variant of the UC model with no regime-switching replicates the EIS puzzle, estimates of σ are

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statistically significant with magnitudes of around 0.08 for all output gap measures once regime-switching dynamics in the real rate process are properly taken into account.

5. FURTHER DISCUSSION OF RESULTS

Notice that the closed-form OEE in (3′) is statistically indistinguishable from the following reduced form IS relation:

$$x_t = c + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} - \beta r_{t-1} + w_t, \tag{20}$$

with $\beta = \frac{\sigma\phi}{1-\phi}$. Therefore, one way to check whether the estimates of σ from the UC models are reasonable, implied estimates of β from the three-state Markov-switching UC model can be compared against actual estimates of β from the reduced form IS relation in (20). However, to allow for the possibility that β may vary gradually over time rather than undergo discrete regime shifts, a rolling estimation over the sample 1966Q1–2015Q3 is carried out when estimating the reduced form model. The rolling estimation is based on a 15-year fixed window with x_t proxied by the CBO output gap. As before, r_t is the EPRR and is calculated as the difference between the federal funds rate and GDP chain-weighted inflation.

Figure 4 plots the rolling estimates of β, where the value on the x-axis marks the beginning of the estimation window. Overall, the magnitude of β appears reasonable, given that reduced form IS models typically yield estimates of the slope that are between 0.05 and 0.10 [Rudebusch and Svensson (1999), Rudebusch (2002), Fuhrer and Estrella (2003)]. The rolling estimates also suggest that β underwent a distinct decline during the early to mid 1980s. This finding implies that in the post-1980s period, the output gap has become less sensitive to movements in the real interest rate process, which is a finding that is consistent with the implications drawn from many earlier VAR analyses. For example, the split sample results of Boivin and Giannoni (2002, 2006) show that exogenous monetary policy shocks as obtained by a recursive identification scheme had a smaller effect on aggregate demand in the post-Volcker period. Based on factor augmented VARs and a DSGE model, Boivin et al. (2010) show that the effect of monetary policy shocks on real GDP has become more muted since the early 1980s.

To verify the validity of the estimated EIS parameters from the preceding section, implied values of β are calculated from the trivariate UC model results in Table 5. They are 0.083, 0.108, and 0.005 in the three regimes. These values are roughly in line with the reduced form estimates of β that are shown in Figure 4, providing support for the empirical relevance of the EIS parameter as estimated by the three-state Markov-switching UC model.

Finally, an important implication that we can draw from the analysis in this section is that a zero β estimate from a reduced form IS model may not necessarily translate to the lack of intertemporal substitution behavior. With β being a convolution of both the EIS parameter and a persistence parameter of the

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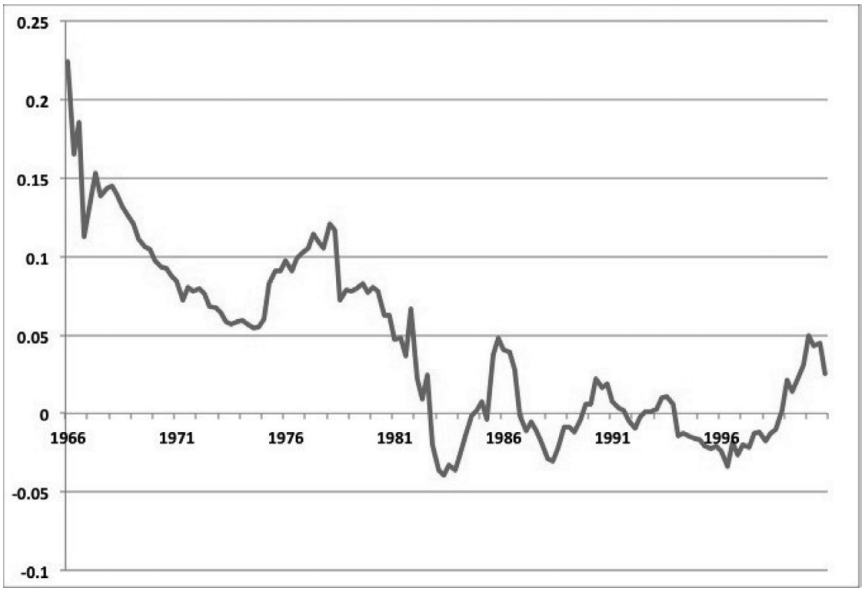


FIGURE 4. Rolling estimates of β from the reduced form IS curve.

real interest rate gap, the slope of the IS relation may appear negligible simply because of exceptionally low degrees of persistence in the real interest rate gap. According to the regime-switching analysis for the joint behavior of the US real interest rate and the output gap as presented in this paper, this appears to be the case for the post 1980s period.

6. CONCLUSION

While the mainstay of much macroeconomic research relies on the OEE, a disconcerting result is that the model fails to capture many key dynamic properties of the data. An important concern is that the OEE fails to establish a significant link between output and the real interest rate, leading to the counterintuitive result of a zero EIS. Also known as the EIS puzzle, this finding is a key challenge to longstanding economic theory and any standard macroeconomic model that relies on the mechanism of intertemporal substitution for monetary analysis.

This paper shows that the EIS puzzle is due to the failure of past studies to take into account the changing times-series properties of the real interest rate when estimating the OEE. In doing so, this paper develops a joint specification for output and the real interest rate, where the long-run means, variances, and degrees of real rate persistence are allowed to undergo regime changes at unknown dates. Time variation in potential output and the natural real rate of interest are features that are also carefully accounted for when estimating the empirical model by utilizing an UC framework.

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The best-fitting model for the joint behavior of output and the real interest rate process describes postwar real rate dynamics by three distinct regimes: 1966–1980, 1980–1985, and 1985–2015. While the UC model with no regime-switching fails to establish any link between output and the real interest rate, allowing the real interest rate to undergo three recurring regimes deliver estimates of the EIS that are statistically significant. Furthermore, estimates of the EIS parameter are consistently 0.1 regardless of the output gap measure used. These findings caution future work against assuming constant dynamics for macroeconomic variables that are subject to regime change; otherwise, it could lead to macroeconomic inferences and predictions that are highly misleading.

The findings in this paper also contribute to the longstanding literature that deals with measurement issues of the real interest rate. Of particular interest are the findings that since the Great Moderation in 1985, the variance and persistence of the real interest rate gap declined significantly, and the natural real rate of interest started to track the actual real interest rate closely. With the real interest rate being a key measure of the monetary stance, these results contribute to our understanding of historical monetary policy conduct in the United States. However, to gain added intuition on what forces are actually driving real interest rate behavior, future research along the lines of Neely and Rapach (2008) that examines how real interest rate persistence may be tied to a particular monetary policy regime in place would be highly beneficial.

NOTES

1. The empirical relevance of the EIS parameter is related to a number of other macroeconomic issues as well. For example, Summers (1984), King and Rebelo (1990), and Trostel (1993) show that the size of the EIS parameter is important towards determining the macroeconomic effects of capital income taxation. Jones et al. (2000) argue that the degree of intertemporal substitution is a key determinant of whether uncertainty boosts or slows down long-run economic growth.

2. Some studies do not detrend the real interest rate prior to estimating the OEE due to the natural real rate being notoriously difficult to estimate. However, failure to account for the long-run properties of the natural real rate in the OEE relation may result in misleading inferences. With the real interest rate being a highly persistent process whereas the output gap a stationary one, failure to detrend the real rate could result in a persistent mismatch problem, which could potentially explain why existing OEEs struggle to establish significant slope estimates. Furthermore, in the New Keynesian Phillips curve (NKPC) literature, Kozicki and Tinsley (2002), Cogley and Sbordone (2008) and Kim et al. (2014), among others, show that the empirical implications of the NKPC can change markedly once variations in the long-run properties of the inflation process are properly accounted for.

3. While these findings have been challenged by a few studies, particularly those employing microeconomic data to show that the zero EIS result is due to limited asset market participation in bond and stock markets by the majority of households in the economy [Attanasio and Weber (1993), Atkeson and Ogaki (1996), Vissing-Jørgensen (2002)], the general consensus in the macroeconomic literature seems to be that the average EIS for the US economy as a whole is zero.

4. Apart from Bilbiie and Straub (2012), there appears to be only one other study that examines the EIS puzzle with the OEE. Goodhart and Hofmann (2005) estimate the New Keynesian output equation for G7 countries and find a significant link between output and interest rates once the model is augmented to include additional determinants of aggregate demand, such as asset prices and monetary aggregates.

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5. Under the assumption of CRRA utility, the EIS parameter is the inverse of the coefficient of relative risk aversion.

6. The real interest rate is a highly persistent process and a number of authors find evidence of a unit root in the US real interest rate series [Mishkin (1981), Rose (1988), King et al. (1991), Gali (1992), Koustas and Serletis (1999)]. However, many authors including Perron (1990) argue that one may mistaken the high levels of persistence in a times-series process for a unit root if regime-switching in the mean of the process is unaccounted for. Following an influential study by Garcia and Perron (1996), a number of studies show that this is the case. In the paper by Garcia and Perron, once regime-switching in the long-run mean of the ex-post real interest rate has been accounted for, the US real interest rate series between 1961–1986 essentially becomes a random white noise process.

7. By determining the number of regimes in this way, this paper does not impose any restrictions on the number of states that can occur in the empirical model. Bilbiie and Straub (2012) on the other hand assume that there is only one structural break in the OEE and rely on the Wald test for GMM estimators to identify the timing of change. Then, they proceed to estimate the model parameters via a split sample estimation, whereas in this paper, dates of regime changes are determined endogenously with all other model parameters.

8. The presence of serial correlation in the residuals can signal model misspecification while serial correlation in their squares can signify remaining ARCH effects. For Markov-switching models, diagnostic tests based on the Ljung–Box Portmanteau test statistic are often carried out because formal hypothesis tests such as the likelihood ratio, Wald and Lagrange multiplier are invalid in the presence of regime changes. In particular, these tests do not have standard asymptotic distributions due to the problem of identically zero scores under the null and the presence of nuisance parameters under the alternative (Hansen, 1992).

9. Estimation results for the inflation equation in subsequent analyses are also reported in Table 2. Since the findings are similar to the no regime-switching case, they are not discussed further in the remainder of this section.

10. Admittedly, an EIS estimate of 0.1 is much smaller than what is required by theory. Calibrated macroeconomic models designed to match growth and business cycle facts typically require that the EIS be close to 1 [Kydland and Prescott (1982), Weil (1989)]. While small empirical estimates for the EIS still remains a puzzle, many studies explain that, in aggregate, the measures of the EIS are low because of limited participation in the stock and bond markets by the majority of households in the economy [Vissing-Jørgensen (2002), Guvenen (2006)].

11. In the three-state UC model that will be discussed next, σ_e also does not vary significantly across regimes and is thus treated as a constant to reduce the number of parameters to be estimated. However, ρ is left unconstrained as it shows time-variation across the three regimes.

12. Note that there is a brief period during the global financial crisis in which the model switches back to regime 1, but this regime change is largely ignored because it does not show up in the corresponding smoothed probability estimates.

13. Note that these authors also report an additional regime shift in 1973. This is not surprising since their measure of the EPRR is based on CPI inflation that is influenced more heavily by sharp rises in oil prices that occurred during the Great Inflation period.

14. The other prominent view is the "good-luck" hypothesis, which suggests that the substantial decline in macroeconomic volatility during the Great Moderation was largely the result of smaller shocks affecting the economy [Canova and Gambetti (2005), Sims and Zha (2006), Justiniano and Primiceri (2008)].

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APPENDIX A

The corresponding state-space representation of the trivariate UC model can be written as follows:

Measurement equation:

$$\begin{bmatrix} y_t \\ r_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & k & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tau_t \\ x_t \\ x_{t-1} \\ \delta_t \\ r_t^* \\ \tilde{r}_t \\ z_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ c_1\pi_{t-1} + c_2\pi_{t-2} + c_3\pi_{t-3} \end{bmatrix},$$

Transition equation:

$$\begin{bmatrix} \tau_t \\ x_t \\ x_{t-1} \\ \delta_t \\ r_t^* \\ \tilde{r}_t \\ z_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ (1-\rho_{S_t})r_{S_t}^{\mathrm{LR}} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & 0 & 0 & \frac{-\sigma\phi_{S_t}}{1-\phi_{S_t}} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{S_t} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_{S_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ x_{t-1} \\ x_{t-2} \\ \delta_{t-1} \\ r_{t-1}^* \\ \tilde{r}_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} w_t \\ v_t \\ 0 \\ \eta_t \\ e_t \\ \epsilon_t \\ z_t \end{bmatrix}$$

$$\begin{bmatrix} w_t \\ v_t \\ \eta_t \\ e_t \\ \epsilon_t \\ z_t \end{bmatrix} \sim \text{i.i.d.}N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{w,D_t}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{v,D_t}^2 & 0 & 0 & \rho\sigma_{v,D_t}\sigma_\epsilon & 0 \\ 0 & 0 & \sigma_\eta^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_e^2 & 0 & 0 \\ 0 & \rho\sigma_{v,D_t}\sigma_\epsilon & 0 & 0 & \sigma_\epsilon^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{z,\tilde{D}_t}^2 \end{bmatrix} \right),$$

where the dynamics of D_t and $\tilde{D}_t$ are as defined in (8) and (17)–(19), respectively.

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APPENDIX B

Building upon the approach of Laubach and Williams (2003), an alternative specification for the UC model with the natural real interest rate expressed as a function of the time-varying trend output growth rate can be written as follows:

Output equations:

$$y_t = \tau_t + x_t$$

$$\tau_t = \delta_t + \tau_{t-1} + w_t, w_t \sim \text{i.i.d.} N(0, \sigma^2_{w,D_t})$$

$$\delta_t = \delta_{t-1} + \eta_t, \eta_t \sim \text{i.i.d.} N(0, \sigma^2_\eta),$$

Real interest rate equations:

$$r_t = r^*_t + \tilde{r}_t,$$

$$r^*_t = c_0\delta_t + d_t$$

$$d_t = \psi_{S_t} d_{t-1} + e_t, e_t \sim \text{i.i.d.} N(0, \sigma^2_e)$$

$$\tilde{r}_t = \phi_{S_t}\tilde{r}_{t-1} + \epsilon_t, \epsilon_t \sim \text{i.i.d.} N(0, \sigma^2_{\epsilon_{S,t}}),$$

Inflation equation:

$$\pi_t = c_1\pi_{t-1} + c_2\pi_{t-2} + c_3\pi_{t-3} + kx_{t-1} + z_t, z_t \sim \text{i.i.d.} N(0, \sigma^2_{z,\tilde{D}_t}).$$

The above specification is similar to the trivariate UC model with the main difference being that the natural real interest rate is now a function of the trend output growth rate and d_t. The dynamics of d_t is assumed to follow an AR(1) process, and it captures other determinants of the natural real interest rate aside from the trend output growth rate, such as households' rate of time preference. Note that alternative specifications for d_t such as a random walk process are also considered, but the results are not discussed here as they yield findings that are qualitatively similar.

The shocks in the above specification are defined in the same way as the trivariate UC model. As before, S_t follows a first-order Markov-switching process, and the number of regimes in the UC model is determined via a series of residual diagnostic tests. As described in Laubach and Williams (2003), a pile-up problem can occur during estimation causing the innovations to the trend output growth rate, σ_η, to be biased toward zero. The median unbiased estimator of Stock and Watson (1998) is thus used to obtain an estimate of the ratio $\lambda_g = \sigma_\eta/\sigma_w$, which is then imposed during estimation. In general, the estimation method follows the sequential steps as described by Laubach and Williams (2003) which is not replicated again here. The estimation results are available from the author upon request.

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