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Damage Identification of Truss Structures Based on Force Method

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Abstract. An computationally efficient damage identification technique for the planar and space truss structures is presented based on the force method and the micro genetic algorithm. For this purpose, the general equilibrium equations and the kinematic relations in which the reaction forces and the displacements at nodes are take into account, respectively, are formulated. The compatibility equations in terms of forces are explicitly presented using the singular value decomposition (SVD) technique. Then governing equations with unknown reaction forces and initial elongations are derived. Next, the micro genetic algorithm (MGA) is used to properly identify the site and extent of multiple damage cases in truss structures. In order to verify the accuracy and the superiority of the proposed damage detection technique, the numerical solutions are presented for the planar and space truss models. The numerical results indicate that the combination of the force method and the MGA can provide a reliable tool to accurately and efficiently identify the multiple damages of the truss structures.

AMS subject classifications: 65F99

Key words: Damage identification, truss, force method, micro genetic algorithm.

1 Introduction

The matrix structural analysis is usually carried out through two counterpart approaches known as displacement method and force method. Due to its generality and simplicity in the computer implementation, the displacement method can be considered as dominated structural analysis method and a lot of commercial software have been developed based on this method. Although, this method is an efficient approach for the stress-displacement type analysis, but it presents some disadvantages in the optimization problems when the number of stress constraints are larger than the displacement constraints [1]. In comparison with the displacement method, the force method of the

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structural analysis is appealing to engineers since the properties of members of the structure most often depend on the member forces rather than joint displacements and the formulation for optimization problems including stress constraints is relatively simple [2–5].

In general, for the force method, equilibrium equations are not adequate enough in solving the structural analysis problems and need to be augmented by the compatibility conditions. This force method can be classified into four different approaches [6]: (i) topological force method; (ii) algebraic force method; (iii) mixed algebraic-combination force method; (iv) integrated force method. Topological method was developed by several researchers [7–10] and algebraic force method was performed by many investigators [11–13]. Also the mixed algebraic-combinational force method has been used by Coleman and Pothen [14, 15], Pothen [16], Gilbert and Heath [17]. Patnaik [18, 19] developed the integrated force method, in which member forces were used as variables, the equilibrium equations and the compatibility conditions were satisfied simultaneously in terms of these variables.

Turning now to the problems of the detecting the site and extent of damage in the structural systems, considerable research efforts have been made. Among the variety of damage detection methods, the genetic algorithms (GAs) are one of the most successful search methods, and they have been applied to the problems of the multiple structural damage identification. Villalba and Laier [20] proposed the use of a self-adaptive multi-chromosome GA to locate and quantify damage in structures. A combined GA and eigensensitivity algorithm has been used by Friswell et al. [21] to identify the location and magnitude of damage from measured vibration data. He and Hwang [22] combined a real-parameter GA with simulated annealing algorithm and adaptive mechanism for finding the actual damage. Koh and Dyke [23] investigated the use of correlation-based damage detection methods for long-span cable-stayed bridges. The locations of damage were determined by iteratively searching for the combination of structural responses that maximizes the correlation coefficient through the application of GA. Gomes and Silva [24] used a genetic real coded algorithm as an optimization tool to match the damaged natural frequencies with those obtained by a parametric finite element code through a modified multiple damage location assurance criterion index. Moslem and Nafaspour [25] used a technique based on a residual force vector to define the probably damaged elements and Ananda Rao et al. [26] proposed a simple GA with binary representation, and the optimization function was formulated in terms of diagonal terms in residual force matrix. Mares and Surace [27] presented the concept of residual force vectors to specify the objective function for the optimization procedure which was then implemented by using GA.

Even though a significant amount of research has been conducted on development of the damage detection methods in structural systems, to the authors' knowledge, there was no study reported on the damage detection techniques using the force method in the literature.

In this study, the efficient numerical method to determine the site and extent of the multiple structural damages in planar and space trusses is proposed through the force

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method. The general equilibrium equations and kinematic relations are formulated and the compatibility matrix is obtained directly using the displacement-deformation relations and the singular value decomposition (SVD) technique. Both member and reaction forces are simultaneously and directly obtained. Next to identify the multiple structural damages in truss structures, the micro genetic algorithm (MGA) is utilized. In order to show the superiority of the proposed methodology, the numerical solutions are presented for the planar and space truss models.

2 Formulation

2.1 Equilibrium equations

For the structural analysis of a d-dimensional truss with b members, n free nodes and n_c constrained nodes, its topology can be expressed by a connectivity matrix $\mathbf{C}_s$ $(\in\mathbb{R}^{b\times(n+n_c)})$ [28,29]. If the free nodes are numbered first, then to the constrained nodes, $\mathbf{C}_s$ is divided into two parts as follows:

$$\mathbf{C}_s=[\mathbf{C},\mathbf{C}_c], \tag{2.1}$$

where $\mathbf{C}$ $(\in\mathbb{R}^{b\times n})$ and $\mathbf{C}_c$ $(\in\mathbb{R}^{b\times n_c})$ denote the connectivities of members to the free and constrained nodes, respectively.

The coordinate difference vectors $\mathbf{l}_x$ $(=l_x^k)$, $\mathbf{l}_y$ $(=l_y^k)$ and $\mathbf{l}_z$ $(=l_z^k)\in\mathbb{R}^b$ $(k=1,2,\cdots,b)$ of members in x-, y- and z-directions, respectively, are expressed as follows:

$$\mathbf{l}_x=\mathbf{C}\mathbf{x}+\mathbf{C}_c\mathbf{x}_c, \tag{2.2a}$$

$$\mathbf{l}_y=\mathbf{C}\mathbf{y}+\mathbf{C}_c\mathbf{y}_c, \tag{2.2b}$$

$$\mathbf{l}_z=\mathbf{C}\mathbf{z}+\mathbf{C}_c\mathbf{z}_c, \tag{2.2c}$$

where $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ $(\in\mathbb{R}^n)$ and $\mathbf{x}_c$, $\mathbf{y}_c$, $\mathbf{z}_c$ $(\in\mathbb{R}^{n_c})$ are the nodal coordinate vectors of the free and constrained nodes, respectively, in x-, y- and z-directions; $\mathbf{l}$ $(l^k=\sqrt{(l_x^k)^2+(l_y^k)^2+(l_z^k)^2})\in\mathbb{R}^b$ is the length vector; $\mathbf{L}_x=diag(\mathbf{l}_x)$, $\mathbf{L}_y=diag(\mathbf{l}_y)$, $\mathbf{L}_z=diag(\mathbf{l}_z)$ and $\mathbf{L}=diag(\mathbf{l})\in\mathbb{R}^{b\times b}$ are the diagonal square matrices of the coordinate difference vectors and the member length vector, respectively. The equilibrium equations of the free nodes in each direction of a truss structure can be stated as

$$\mathbf{A}\mathbf{f}=\mathbf{p}, \tag{2.3}$$

where $\mathbf{A}$ $(\in\mathbb{R}^{dn\times b})$ is the equilibrium matrix that transforms the vector of member forces $\mathbf{f}$ of the system to the vector of external loads $\mathbf{p}$ of the free nodes, defined by

$$\mathbf{A}=\begin{pmatrix}\mathbf{C}^T\mathbf{L}_x\\ \mathbf{C}^T\mathbf{L}_y\\ \mathbf{C}^T\mathbf{L}_z\end{pmatrix}\mathbf{L}^{-1}. \tag{2.4}$$

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It should be noted that the equilibrium equations given in Eq. (2.3) do not include any boundary constraints. There are three types of statically indeterminate trusses which are (i) externally, (ii) internally, and (iii) mixed (i.e., both externally and internally) statically indeterminate trusses. Let r_e, r_i and r denote the number of external, internal and total indeterminacies, respectively. The external indeterminacy is related to the boundary constraints while the internal indeterminacy is associated with the number of bars in the truss. They are defined, respectively, as follows:

$$r_e = c - r_b \geq 0, \tag{2.5a}$$

$$r_i = b - dn_s + r_b, \tag{2.5b}$$

where c and r_b $(=d(d+1)/2)$ are the numbers of boundary constraints and independent rigid-body motions, respectively; n_s $(=n+n_c)$ is the total number of nodes of the truss. The number of total statically indeterminacy is given by

$$r = r_e + r_i \tag{2.6}$$

or

$$r = b_s - dn_s, \tag{2.7}$$

where b_s $(=b+c)$ is the sum of the number of members and boundary constraints.

Let $\mathbf{r}_c$ $(\in \mathbb{R}^c)$ denote the unknown vector of reaction forces employed to remove all boundary constraints. Accordingly, the constrained nodes can now be treated as the free nodes. In this context, the general equilibrium equations for all nodes (including the constrained nodes) of the discrete truss can be written as follows:

$$\bar{\mathbf{A}}\bar{\mathbf{f}} = \bar{\mathbf{p}}, \tag{2.8}$$

where $\bar{\mathbf{f}}$ $(=\{\mathbf{f}^T, \mathbf{r}_c^T\}^T) \in \mathbb{R}^{b_s}$ is the unknown member and reaction force vector and $\bar{\mathbf{p}}$ $(\in \mathbb{R}^{dn_s})$ is the external load vector of all nodes. $\bar{\mathbf{A}}$ $(=[\mathbf{A}|\mathbf{A}_c]) \in \mathbb{R}^{dn_s \times b_s}$ is called the general equilibrium matrix of all nodes in this study, which transforms the vector of member and reaction forces $\bar{\mathbf{f}}$ to the vector of external loads $\bar{\mathbf{p}}$; $\mathbf{A}_c$ $(\in \mathbb{R}^{dn_s \times c})$ is the coefficient matrix of c unknown reaction forces in which its c columns correspond to the last c unknown components of $\bar{\mathbf{f}}$. The c columns of the matrix $\mathbf{A}_c$ are vectors with all of their entries being zero except the entries in the rows corresponding to the constrained degrees of freedom having the negative unit value.

2.2 Kinematic relations

Let $\mathbf{e}$ $(\in \mathbb{R}^{b_s})$ and $\mathbf{d}$ $(\in \mathbb{R}^{dn_s})$ denote the member deformation vector and the displacement vector, respectively, due to the external loads $\bar{\mathbf{p}}$. The elongations of all members are related to the displacements of all nodes by the following kinematic relation:

$$\mathbf{e} = \mathbf{B}\mathbf{d}, \tag{2.9}$$

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where $\mathbf{B}$ ($\in \mathbb{R}^{b_s \times dn_s}$) is the kinematic matrix. From the conservation of energy, the work done by the external loads should be equal to the increase of strain energy stored in the structure in the field of small displacements for the elastic conservative systems

$$\frac{1}{2}\bar{\mathbf{p}}^T\mathbf{d} = \frac{1}{2}\bar{\mathbf{f}}^T\mathbf{e}. \tag{2.10}$$

By substituting Eqs. (2.8) and (2.9) into Eq. (2.10), the following relation is obtained

$$\bar{\mathbf{f}}^T\bar{\mathbf{A}}^T\mathbf{d} = \bar{\mathbf{f}}^T\mathbf{B}\mathbf{d}. \tag{2.11}$$

Based on the principle of virtual work [30], the above equation can be ensured to be true only if the following relation holds:

$$\mathbf{B} = \bar{\mathbf{A}}^T. \tag{2.12}$$

Hence, the kinematic relation in Eq. (2.9) can be rewritten as

$$\mathbf{e} = \bar{\mathbf{A}}^T\mathbf{d}. \tag{2.13}$$

The expression given by Eq. (2.13) is the general form of the deformation-displacement relationship of the discrete system. In Eq. (2.13), there are r ($= b_s - dn_s$) compatibility equations which can be achieved through the elimination of the dn_s nodal displacements from the b_s elongations. The b_s components of member and reaction force vector $\bar{\mathbf{f}}$ must satisfy the dn_s equilibrium equations in Eq. (2.8) along with r compatibility equations obtained from Eq. (2.13).

2.3 Compatibility conditions

There are many numerical algorithms for the generation of compatibility equations based on various matrix algorithms, including the Gauss-Jordan elimination, QR, LU, LQ, turn-back method [12, 13], and singular value decomposition (SVD) [31]. In this study, the SVD is adopted as an effective and numerically stable algorithm. In order to extract the r compatibility conditions from Eq. (2.13), the SVD is performed on the general equilibrium matrix $\bar{\mathbf{A}}$ instead of the kinematic matrix $\mathbf{B}$ (i.e., the transpose of the general equilibrium matrix $\bar{\mathbf{A}}$) due to the static-kinematic duality.

$$\bar{\mathbf{A}} = \mathbf{U}\mathbf{V}\mathbf{W}^T, \tag{2.14}$$

where $\mathbf{U}$ ($\in \mathbb{R}^{dn_s \times dn_s}$) $= [\mathbf{u}_1\mathbf{u}_2\cdots\mathbf{u}_{dn_s}]$ and $\mathbf{W}$ ($\in \mathbb{R}^{b_s \times b_s}$) $= [\mathbf{w}_1\mathbf{w}_2\cdots\mathbf{w}_{b_s}]$ are the orthogonal matrices. $\mathbf{V}$ ($\in \mathbb{R}^{dn_s \times b_s}$) is a diagonal matrix with non-negative singular values of $\bar{\mathbf{A}}$ in decreasing order as

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{b_s} \geq 0. \tag{2.15}$$

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The vector space bases of compatibility equations of any discrete truss structures are calculated from the transpose of the null space of the general equilibrium matrix $\bar{\mathbf{A}}$. In this case, the matrix $\mathbf{W}$ in Eq. (2.14) can be expressed as follow:

$$\mathbf{W}=[\mathbf{w}_1\mathbf{w}_2\cdots\mathbf{w}_{dn_s}|\mathbf{s}_1\mathbf{s}_2\cdots\mathbf{s}_r]. \tag{2.16}$$

From Eqs. (2.13) and (2.14), for any displacement vector $\mathbf{d}$, the elongation vector $\mathbf{e}$ must be in the subspace spanned by the column vectors of $[\mathbf{w}_1\mathbf{w}_2\cdots\mathbf{w}_{dn_s}]$. Hence, it has to be orthogonal to the columns of $[\mathbf{s}_1\mathbf{s}_2\cdots\mathbf{s}_r]^T$; i.e., the transpose of the last r vectors $\mathbf{s}_i$ $(\in\mathbb{R}^{b_s})$ in $\mathbf{W}$ corresponding to the last r zero singular values σ_i $(i=1,2,\cdots,r)$ in $\mathbf{V}$ forms a set of r compatibility equations as follows:

$$\mathbf{Se}=\mathbf{0}, \tag{2.17}$$

where $\mathbf{S}$ $(\in\mathbb{R}^{r\times b_s})$ is the compatibility matrix in terms of elongations. These compatibility equations assure that the elongation vector $\mathbf{e}$ is constrained such that the deformed elements fit together to form the structure free of voids and discontinuities.

2.4 Determination of member forces and nodal displacements

Assuming linear elastic material, the elongation-force relation is given as

$$\mathbf{e}=\mathbf{e}_0+\mathbf{G}\bar{\mathbf{f}}, \tag{2.18}$$

where $\mathbf{e}_0$ $(\in\mathbb{R}^{b_s})$ is the initial elongation vector due to initial stresses, imperfections and temperature effects; $\mathbf{G}$ $(\in\mathbb{R}^{b_s\times b_s})$ is the diagonal flexibility matrix. Substituting Eq. (2.18) into (2.17) yields the following expression.

$$\mathbf{D}\bar{\mathbf{f}}=\mathbf{p}_{e_0}, \tag{2.19}$$

where

$$\mathbf{D}=\mathbf{SG},\quad \mathbf{p}_{e_0}=-\mathbf{Se}_0. \tag{2.20}$$

In which $\mathbf{D}$ $(\in\mathbb{R}^{r\times b_s})$ is the compatibility matrix in terms of forces of all members and $\mathbf{p}_{e_0}$ $(\in\mathbb{R}^r)$ is a virtual force vector in the indeterminate truss structures caused by member initial elongations $\mathbf{e}_0$. Combining Eq. (2.8) and Eq. (2.19) yields

$$\left[\begin{array}{c|c}\mathbf{A} & \mathbf{A}_c\\ \hline \mathbf{D} & \mathbf{0}\end{array}\right]\left\{\begin{array}{c}\mathbf{f}\\ \hline \mathbf{r}_c\end{array}\right\}=\left\{\begin{array}{c}\bar{\mathbf{p}}\\ \hline \mathbf{p}_{e_0}\end{array}\right\} \tag{2.21}$$

or

$$\tilde{\bar{\mathbf{A}}}\bar{\mathbf{f}}=\tilde{\bar{\mathbf{p}}}, \tag{2.22}$$

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where $\tilde{\bar{\mathbf{A}}}$ $(\in \mathbb{R}^{b_s \times b_s})$ is a square full rank matrix coupling the general equilibrium matrix $\bar{\mathbf{A}}$ and the compatibility matrix in terms of forces $\mathbf{D}$. $\tilde{\bar{\mathbf{p}}}$ $(=\{\bar{\mathbf{p}}^T, \mathbf{p}_{e_0}^T\}^T) \in \mathbb{R}^{b_s}$ is the external actions coupling the external loads and the virtual forces due to member initial elongations.

It is obvious that the member elongations are determined using the nodal displacements from Eq. (2.13). Regarding indeterminate truss structures, however, it is not sufficient to conversely obtain the nodal displacements using the member elongations since the general equilibrium matrix $\bar{\mathbf{A}}$ is rectangular and cannot be inverted. This implies that the compatibility equations should be melted into the general equilibrium equations. For this reason, the nodal displacements $\mathbf{d}$ must be calculated using $\tilde{\bar{\mathbf{A}}}$ in Eq. (2.22) instead of $\bar{\mathbf{A}}$ as

$$\mathbf{d} = \mathbf{M}\mathbf{e}, \tag{2.23}$$

where $\mathbf{M}$ $(\in \mathbb{R}^{dn_s \times dn_s})$ is given by

$$\mathbf{M} = \begin{cases} \bar{\mathbf{A}}^{-T}, & \text{if } r = 0, \\ dn_s \text{ rows of } \tilde{\bar{\mathbf{A}}}^{-T}, & \text{if } r > 0. \end{cases} \tag{2.24}$$

The member and reaction forces $\bar{\mathbf{f}}$ and all nodal displacements $\mathbf{d}$ for the truss structure can be defined by the proposed method through the following force method analysis procedure

Step 1 Convert the truss with boundary constraints into the equivalent free-standing model.

Step 2 Define $\mathbf{C}$, $\mathbf{A}$ and $\mathbf{A}_c$ by Eqs. (2.2), (2.4) and (2.8), respectively. Then determine $\bar{\mathbf{A}}$ by combining $\mathbf{A}$ and $\mathbf{A}_c$.

Step 3 Calculate r by Eq. (2.6) or Eq. (2.7). If $r = 0$, go to Step 5. Otherwise, generate the compatibility matrix $\mathbf{S}$ by Eqs. (2.14) and (2.17).

Step 4 Define $\mathbf{D}$ and $\mathbf{p}_{e_0}$ by Eq. (2.20).

Step 5 If $r = 0$, define $\bar{\mathbf{f}}$ by Eq. (2.8). Otherwise, define $\bar{\mathbf{f}}$ by Eq. (2.22).

Step 6 Calculate $\mathbf{M}$ by Eq. (2.24). Then define $\mathbf{d}$ by Eq. (2.23).

3 Damage detection by micro genetic algorithm

The genetic algorithms (GAs) are optimization techniques based on the Darwin's principle of survival of the fittest [32]. The mechanics of natural genetics is based on operations that result in structured yet randomized exchange of genetic information between the chromosomal strings of the reproducing parents and consists of reproduction, crossover, occasional mutation, and inversion of the chromosomal strings [32]. GAs take the population of solutions to find the optimum point, while the heuristic search methods such as simulated annealing or taboo search exploit one solution on their process to find it. It

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is known that it is difficult to find the optimum point with mathematical optimization methods which include a gradient vector and Hessian if there are a lot of local optima or a steep gradient around the optimum point [33]. On the other hand, GAs do not use a gradient vector and Hessian, but use the object function value during their search. This GA has been shown to be capable of solving various optimization problems via some basic concepts and operators. However, it has been revealed that it is not easy for the simple GA to solve the damage quantification problems properly from some research works [23, 24]. To acquire better identification result and high efficiency, the various optimization algorithms were improved. Krishnakumar [34] developed the micro genetic algorithm (MGA) based on the simple GA. Carroll [35] then improved MGA with another operator.

In this study, the micro genetic algorithm (MGA) is employed to detect the site and extent of damage in truss structures. This MGA utilizes a relatively smaller population size than the simple GA resulting in less computation time. The objective function to be minimized in the optimization process is demonstrated as follows:

$$obj(x) = \|\varepsilon_i^d - \varepsilon_i(x)\|, \tag{3.1}$$

where ε_i^d is the strain of the ith member for the damaged truss and $\varepsilon_i(x)$ is one can be predicted from an analytic model. Some basic concepts and operations of the current algorithm are as follows:

- *Coding*: The coding of variables that describe the problem is an essential characteristic of the genetic algorithm. A binary coding method is used in this study. This method is to transform variables to a binary string of specific length.
- *Selection operator*: The selection procedure used in current study is based on the fitness of each individual. There are number of reproduction schemes commonly used in the genetic algorithm. These include tournament selection, ranking selection, steady-state selection, greedy over selection, and proportionate reproduction. In this study, the tournament selection is used.
- *Crossover*: Crossover is the operator that produces new individuals by exchanging some bits of a couple of randomly selected individuals. The two-point crossover is taken since the two-point crossover performs much better among the multi-point crossover techniques [36]. In addition, the crossover rate is set as 1.0 therefore all populations must perform a crossover operation at every generation.
- *Mutation and restart operation*: There is no use of mutation operation because 1.0 crossover rate provides adequate variability. In addition, we do not use the restart operation in this study.

The flowchart of the present MGA is illustrated in Fig. 1.

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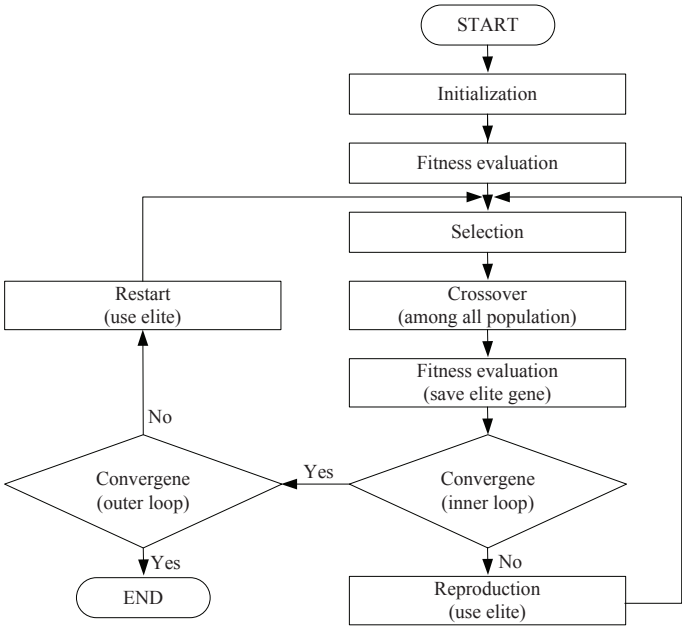


Figure 1: Flowchart of the micro genetic algorithm.

4 Numerical examples

In order to show the superiority of the damage detection method developed by this study based on the force method (FM) and the MGA, two types of indeterminate truss structures are considered. For analysis, the initial elongation vector $\mathbf{e}_0$ is assumed to be zero and all members are assumed to have the same axial stiffness of 1kN. In many studies related to the damage identification problem, the damage has been simulated by decreasing one of the stiffness parameters of the element such as the elasticity modulus, moment of inertia, cross-sectional area, and so on. In this study, the damage of the truss structure is simulated via the reduction of the elasticity modulus in each element.

4.1 21-bar planar truss

Consider the simply supported 21-bar planar truss subjected to a vertical force acting at node 5 as shown in Fig. 2. The truss is modelled using 21 consistent finite element without internal nodes giving 17 DOFs. The number of external indeterminacy in Eq. (2.5a) is $r_e=3-3=0$, while the number of internal indeterminacy in Eq. (2.5b) $r_i=21-2\times10+3=4$. Thus, the number of redundancy in Eq. (2.6) is $r=0+4=4$. The structure is said to be four degrees of internal indeterminacy, which implies only 4 redundant forces obtained from the compatibility condition is required to determine the unknown member and reaction force vector $\bar{\mathbf{f}}$.

In the first damage scenario, the damage is introduced in element 8 by reducing its

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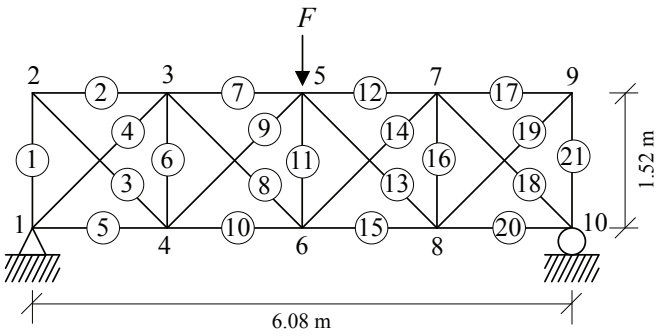


Figure 2: Planar truss having 21 members.

stiffness to 40% of the initial value. The measurement noise is simulated and entered into the analysis. The identified damage expressed in the ratio of elasticity modulus reduction evaluated by this study is depicted in Fig. 3. The convergence history of the objective function value with generations is also shown in Fig. 4. For comparison, the result from the finite element model based on the displacement method (DM) is presented together. It can be observed from Figs. 3 and 4 that the two algorithms based on the force and displacement methods combined with MGA are properly achieved to detect the site and extent of the hypothetical damage. In Table 1, the CPU time for analyses based on both the force and displacement methods are presented. It is seen that the required computational time for the FM is significantly lower than that for the DM, illustrating the efficiency of the FM over the DM for the analysis of planar truss structure.

In the second scenario, the damage is induced in members 10 and 16. Damaged members are assumed to have 40 and 30% of their initial stiffness, respectively. The identified damage is plotted in Fig. 5 and the convergence history is in Fig. 6. It is observed that the MGA can identify the locations as well as the extent of the damaged in both cases,

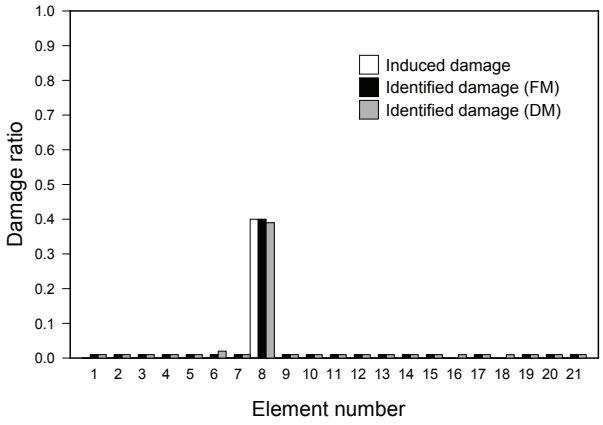


Figure 3: Identified damages of the 21-bar planar truss for scenario 1.

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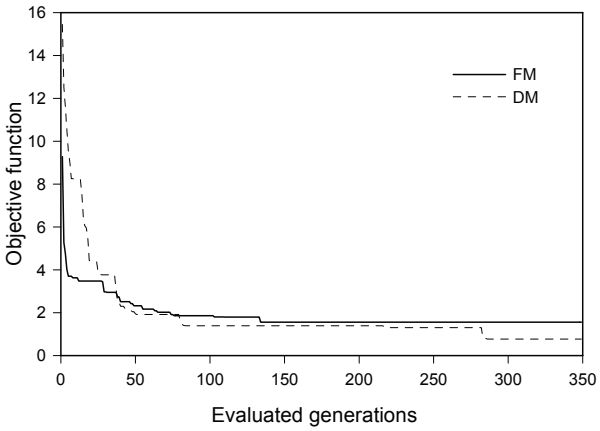


Figure 4: Convergence history the 21-bar planar truss for scenario 1.

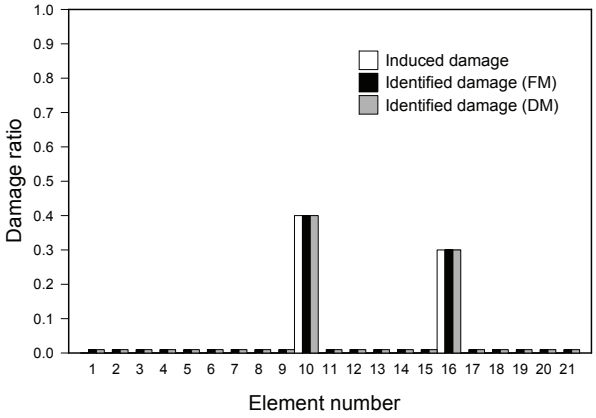


Figure 5: Identified damages of the 21-bar planar truss for scenario 2.

whereas the FM shows the superior efficiency on the CPU time required when compared with the DM as can be seen in Table 1.

In the third scenario of the 21-bar truss, members 7, 10, and 16 are damaged by reduction of their stiffness's to 60, 50, 30% of the initial values, respectively. Figs. 7 and 8 show the identified damages and the convergence history, respectively. It is seen that the present study detects the location and the extent of the damage with good accuracy. It

Table 1: Computational time for the 21-bar planar truss (s).

	Force method (FM)	Displacement method (DM)
Scenario 1	138	815
Scenario 2	104	479
Scenario 3	88	313

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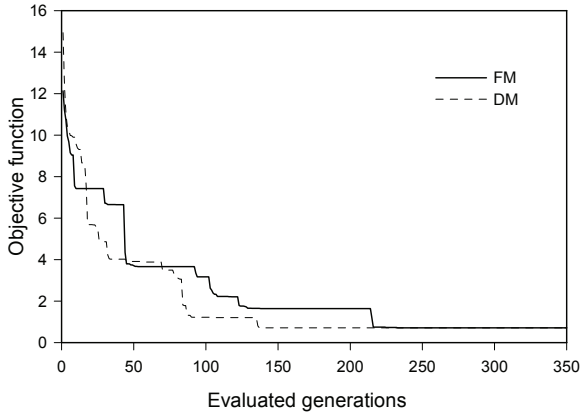


Figure 6: Convergence history the 21-bar planar truss for scenario 2.

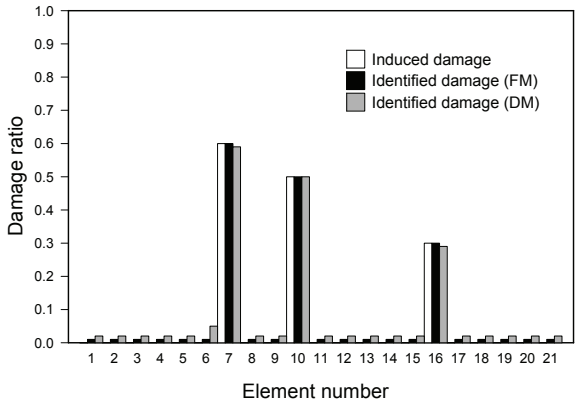


Figure 7: Identified damages of the 21-bar planar truss for scenario 3.

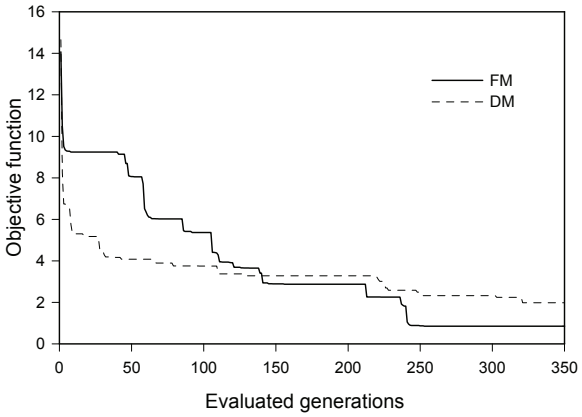


Figure 8: Convergence history the 21-bar planar truss for scenario 3.

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can also be found from the result shown in Table 1 that the computational time for the FM is significantly lower than that required by the DM, again pointing out the efficiency of the FM.

4.2 25-bar space truss

Fig. 9 shows the 25-bar space truss with 10 nodes leading to 18 DOFs subjected to two forces acting at node 1 in x and $-z$ direction, respectively. The truss has mixed indeterminacy of seven in which six are external indeterminacy, and the other is internally generated. Hence, seven redundant forces defined from seven equations of compatibility conditions are needed to determine the unknown member and reaction forces.

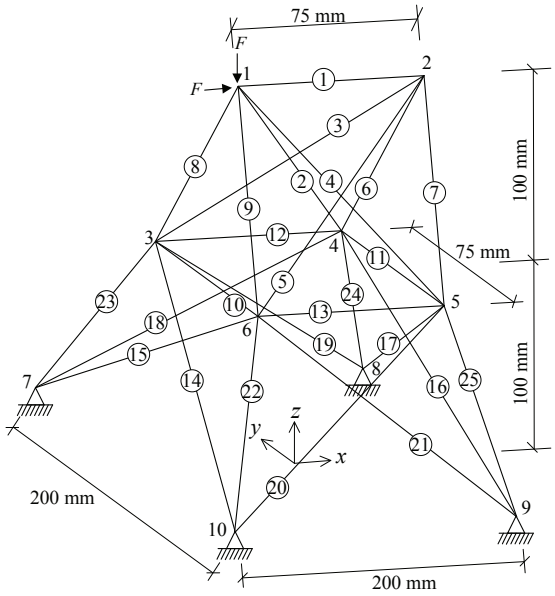


Figure 9: Space truss having 25 members.

In the damage scenario, the stiffness of members 20 and 21 are reduced to 50 and 40% of their initial values, respectively. The damage identification results are plotted in Fig. 10. It is seen that the current study clearly detects the site and extent of the damage in the members and the results using the FM and the DM are found to be almost identical, while the computational time for the FM is approximately one third of that required by the DM as shown in Table 2.

Table 2: Computational time for the 25-bar space truss (s).

Force method (FM)	Displacement method (DM)
205	658

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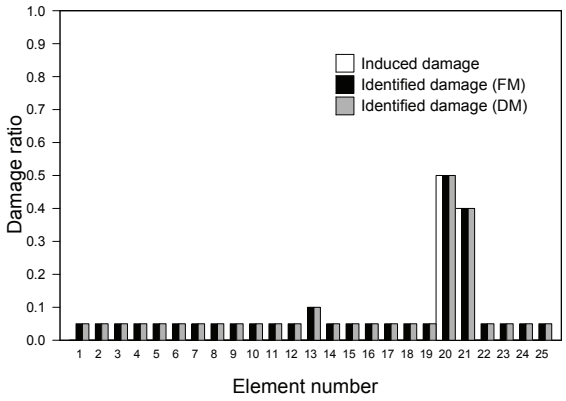


Figure 10: Identified damages of the 25-bar space truss.

5 Conclusions

An efficient damage identification method has been developed to properly identify the sites and extents of multiple damages in truss structures using the force method and the micro genetic algorithm. The general equilibrium equations and the kinematic relations have been formulated, and the compatibility equations in terms of forces using the SVD technique were presented. The optimization problem have been solved using the micro genetic algorithm to identify the actual damage. In order to show the effectiveness and superiority of the proposed methodology, the planar and space truss structures are considered through the numerical examples. The main objective of this study is to investigate the relative performance of the force and displacement methods in the damage detection analysis of the truss structures. It is found that the damage detection technique that is based on the force method is computationally far more efficient than the displacement one.

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